Structural Equation Modeling: A Multidisciplinary Journal

ISSN: 1070-5511 (Print) 1532-8007 (Online) Journal homepage: https://www.tandfonline.com/loi/hsem20

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Structural Equation Modeling: A Multidisciplinary Journal, 27: 396–413, 2020
ISSN: 1070-5511 print / 1532-8007 online
DOI: https://doi.org/10.1080/10705511.2019.1652613

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In science, the gold standard for evidence is an empirical result which is consistent across multiple studies. Meta-analysis techniques allow researchers to combine the results of different studies. Due to the increasing availability of longitudinal data, studying lagged effects is increasingly popular also in meta-analytic studies. However, in current practice, little attention is paid to the unique challenges of meta-analyzing these lagged effects. Namely, it is well known that lagged effects estimates change depending on the time that elapses between measurement waves. This means that studies that use different uniform time intervals between observations (e.g., 1 hour vs 3 hours or 1 month vs 2 months) can come to very different parameter estimates, and seemingly contradictory conclusions, about the same underlying process. In this article, we introduce, describe, and illustrate a new meta-analysis method (CTmeta) which assumes an underlying continuous-time process, and compare it with current practice.

Keywords: Meta-analysis, first-order vector autoregressive (VAR(1)) model, cross-lagged panel model (CLPM), continuous-time SEM, differential equation model

INTRODUCTION

In science, the gold standard for evidence is an empirical result which is consistent across multiple studies. Meta-analysis techniques allow researchers to combine the results of different studies, usually in the form of effect-size estimates, to render a more accurate estimate of this effect size in the population (Borenstein, Hedges, Higgins, & Rothstein, 2009; Wolf, 1986). Recent work has broadened the scope of traditional meta-analysis to include structural equation modeling (SEM) based techniques, using the correlation matrices reported by empirical studies to build (weighted) path models of interest (cf. Cheung, 2015; Viswesvaran & Ones, 2007).

One popular class of longitudinal SEM models is based on the estimation of (first-order) lagged effects, such as the vector-autoregressive (VAR(1); Hamilton, 1994) and cross-lagged panel models (CLPMS; Bollen & Curran, 2006; Mayer, 1986). Typically, researchers use these models to assess the Granger-causal relationships between pairs of variables, through the estimation of cross-lagged regression parameters (Granger, 1969; Rogosa, 1979, 1980). These lagged effects models are increasingly targeted for meta-analytic studies (for instance, Jacobson & Newman, 2017; Maricuţoiu, Sulea, & Iancu, 2017; Masselink et al., 2018; Nohe, Meier, Sonntag, & Michel, 2015). In current practice, however, little attention is paid to the unique challenges of meta-analyzing models estimated from repeated measurement data. Specifically, it is well known that lagged regression models suffer from the problem of *time-interval dependency*, that is, the fact that the parameter estimates change depending on the time that elapses between measurement waves (Gollob & Reichardt, 1987; Kuiper & Ryan, 2018; Pelz & Lew, 1970; Voelkle & Oud, 2013). This means that studies using different time intervals between observations (e.g., 1 hour vs 3 hours or 1 month vs 2 months) can come to very different parameter

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estimates, and *seemingly* contradictory conclusions, about the same underlying process.

In the dynamic modeling literature, researchers have proposed the use of *continuous-time (CT) models* to overcome the time-interval problem (Boker, 2002; Boker, Neale, & Rausch, 2004; Chow et al., 2005; Oravecz, Tuerlinckx, & Vandekerckhove, 2009; Oud, 2002; Oud & Delsing, 2010; Voelkle, Oud, Davidov, & Schmidt, 2012). This approach explicitly models lagged parameters between observed measurement waves as a non-linear function of i) dynamic relationships operating on a moment-to-moment basis and ii) the time interval that elapses between waves. This means that CT models offer the possibility to map results from studies using different measurement regimes onto a single underlying set of dynamic relationships. While this feature makes CT modeling an attractive possibility for the meta-analysis of lagged relationships, so far CT models have only been applied to individual studies.

In this paper, we introduce, describe, and illustrate a new meta-analysis method for lagged-regression models which assumes an underlying continuous-time process. We implement this new methodology in an easy-to-use R package *CTmeta* and associated Shiny applications (Chang, Cheng, Allaire, Xie, & McPherson, 2019; R Core Team, 2019). We begin by briefly reviewing meta-analysis in the context of discrete-time (DT) lagged regression models, and the problems which arise in this practice due to time-interval dependency. Second, we describe the CT approach to lagged models, and propose a meta-analysis technique based on this approach. Third, we demonstrate the application of our new CT meta-analysis method and compare its performance to current best practice methods.

BACKGROUND

We first briefly introduce the core concepts of meta-analysis and, second, the most commonly model used for lagged regression analysis, the so-called first-order vector autoregressive (VAR(1)) model or the cross-lagged panel model (CLPM). Third, we describe the challenge of meta-analyzing these models due to the time-interval dependency problem. The focus of the remainder of this article is on creating and illustrating a solution for this challenge.

Meta-Analysis

Meta-analysis aims to synthesize evidence from several different studies, usually in the form of a statistical parameter or set of parameters (e.g., an effect size measure or standardized regression parameter(s)) estimated from different samples, to come to an overall estimate of some population parameter(s) (cf. Becker & Wu, 2007; Borenstein et al., 2009). In principle, meta-analysis combines evidence from different studies by taking a *weighted* average of their parameter estimates, with the contribution of each study being weighted by the amount of information or certainty in a given estimate, as reported by or inferred from properties (e.g., the sample size) of that study. This weighting procedure can be applied univariately (via WLS) or multivariately (via GLS) to take into account dependencies between the parameters which are a target of the meta-analysis. For a more in-depth treatment of these weighting procedures, the reader is referred to Becker and Wu (2007) and Demidenko, Sargent, and Onega (2012).

As well as the method of weighting parameter estimates, meta-analytic techniques primarily differ depending on the underlying model that is assumed for the/a population parameter of interest: Researchers can assume a single underlying parameter, in a *fixed-effects* (FE) analysis, or a distribution of population parameters, in a *random-effects* (RE) model. The former can be seen as a special case of the latter, in which all variance in parameter estimates is assumed to come from sampling variance alone. For a further overview of meta-analytic techniques, see for instance Borenstein et al. (2009) and Becker and Wu (2007). In this paper, we will introduce a meta-analytic technique that can facilitate both FE and RE models, with both uni- and multi-variate weighting.

The Discrete-time VAR(1) Model

When researchers have multiple repeated measurements of some set of variables, a popular choice of model with which to analyze these data is based on lagged regression parameters. These *lagged effects* describe the relationship between current observations and past observations: when current observations are regressed on those directly preceding them, these models can be described as *first-order*. In the context of panel data (i.e., data for a large number of participants but with relatively few observations spaced far apart in time), this is referred to as the cross-lagged panel model (CLPM); and, in the context of time-series data (i.e., single-subjects data with many observations at a higher frequency), this is referred to as the (discrete-time) first-order vector autoregressive (DT-VAR(1)) model. As both models are conceptually very similar, we use only the DT-VAR(1) terminology throughout. Figure 1 depicts a bivariate DT-VAR(1) model as a path model.

Let $\boldsymbol{y}_{i,m}$ be the vector with q observed variables for individual i ($i = 1, \ldots, N$) at measurement occasion m. In the DT-VAR(1) model, this vector is regressed on the preceding observation through

$$\boldsymbol{y}_{i,m} = \boldsymbol{c}_i + \boldsymbol{\Phi}\boldsymbol{y}_{i,m-1} + \boldsymbol{\varepsilon}_{i,m} \tag{1}$$

where $\boldsymbol{c}_i$ is a q-vector of intercepts which is related to the mean of $\boldsymbol{y}_{i,m}$ by $\boldsymbol{\mu}_i = (\boldsymbol{I} - \boldsymbol{\Phi})^{-1}\boldsymbol{c}_i$; $\boldsymbol{\varepsilon}_{i,m}$ represents a q-vector of errors for measurement m that are independent and identically distributed: $\boldsymbol{\varepsilon}_{i,m} \sim (\boldsymbol{0}, \boldsymbol{\Sigma}_\varepsilon)$; and $\boldsymbol{\Phi}$ is the $q \times q$ matrix of lagged regression parameters, that is, autoregressive (ϕ_{jj}) and cross-lagged ($\phi_{jk}, j \neq k$) effects.[1] For time-series data of

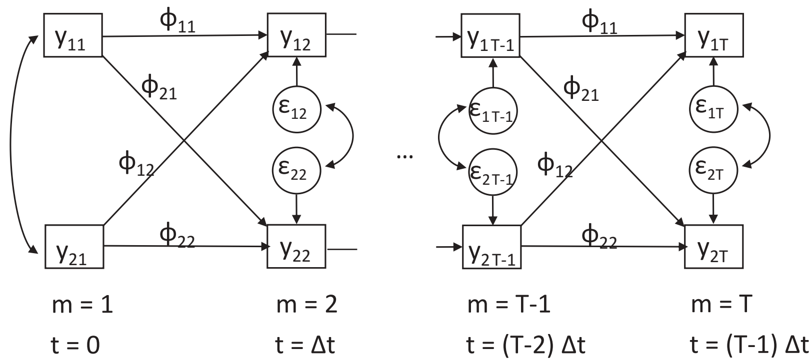


FIGURE 1 A graphical representation of a bivariate single-subject VAR(1) model.

a single individual ($N = 1$), Equation 1 is only identified when $m \geq 2$ and $\mathbf{\Phi}$ is assumed to remain invariant over time. In the panel data setting, we further assume throughout that the lagged effects matrix $\mathbf{\Phi}$ and the residual variance matrix $\mathbf{\Sigma}_\varepsilon$ are fixed across time and equal across all individuals. Notably, we make no restrictions on whether the intercepts vary across individuals, as specified in the popular random-intercept CLPM (Hamaker, Kuiper, & Grasman, 2015), or whether they are equal across individuals ($\boldsymbol{c}_i = \boldsymbol{c}$). In the former case, this model is only identified for $m \geq 3$.

The parameters of lagged effects models have become increasingly the target of meta-analytic studies (e.g. Maricuţoiu et al., 2017; Masselink et al., 2018; Nohe et al., 2015). Typically, when researchers estimate a lagged regression model, their primary aim is to compare the size and sign of the estimated cross-lagged parameters ($\mathbf{\Phi}$); where the relative strength is often referred to as "causal dominance" (Bentler & Speckart, 1981; Finkel, 1995; Hamaker et al., 2015; Rogosa, 1980). For example, Moberly and Watkins (2008) investigate which of momentary ruminative self-focus (RSF) and negative affect (NA) can be considered the 'driving force' of the pair, by comparing the size of the cross-lagged effects of RSF and NA on each other at the next measurement occasion. The meta-analysis of Nohe et al. (2015) aims to synthesize evidence from multiple different studies on the reciprocal lagged relationships between work-family conflict and strain. The main motivation for this meta-analysis is to come to a more conclusive conclusion regarding 'causal dominance' relations between this pair of variables in the population.

[1] Depending on the setting, researchers may specify the initial value y_0 as either fixed or drawn from the stationary distribution of y, a function of $\mathbf{\Phi}$ and $\mathbf{\Sigma}_\varepsilon$ (cf. Hamerle et al., 1991) As the focus of the methods described in this paper are on the meta-analysis of $\mathbf{\Phi}$, the methods described here are applicable regardless of choice of initial value, as well as fixed or random intercepts, as long as the same approach is used across all studies in the meta-analysis. This ensures that the lagged regression parameters are in principle comparable, that is, they are estimates of the same population quantity.

As a necessary first condition for this comparison to be sensible, one should use *standardized lagged parameters* to obtain parameters that are on the same scale and thus comparable (Bentler & Speckart, 1981). As such, we will throughout treat the population standardized lagged parameters as the target of any meta-analytic technique. Details on how to calculate the standardized parameters from correlation matrices is given in Appendix A.

Time-interval Dependency: A Headache for Meta-analysts

A major obstacle for any meta-analytic study of lagged effects is the well-known time-interval dependency problem. This refers to the phenomenon that studying the same underlying dynamic process can result in autoregressive and cross-lagged parameter estimates of different signs, size, and relative ordering due only to the use of different time intervals between measurements (Chatfield, 2004; Dormann & Griffin, 2015; Gollob & Reichardt, 1987; Hamilton, 1994; Kuiper & Ryan, 2018; Oud, 2002). Stated otherwise, these lagged parameters are a *function of the time interval* and are therefore denoted by $\mathbf{\Phi}(\Delta t)$ in the remainder. We can understand this problem with reference to the path model in Figure 1. If we were to fit a model on every second measurement wave, as if we would have measured every, say, 2 hours instead of 1 hour, we would come to $\mathbf{\Phi}(\Delta t = 2) = \mathbf{\Phi}(\Delta t = 1)^2$, a well-known result from both the time-series literature Hamilton (1994) and path-tracing rules (Bollen, 1989). Because of the matrix multiplication, elements in $\mathbf{\Phi}(\Delta t = 2)$ are a sum of multiple different products of parameters in the original matrix $\mathbf{\Phi}(\Delta t = 1)$: For instance, leaving the subscript i out for ease of notation, the cross-lagged parameter relating y_1 to y_2 at the longer time interval, that is, $\phi_{21}(2)$, can be found by path-tracing rules to equal $\phi_{11}(1)\phi_{21}(1) + \phi_{21}(1)\phi_{22}(1)$. The lower our sampling rate, the more paths must be traced through to connect y_1 to y_2 and so the more complex this function becomes. As such, the parameter estimates yielded using different time intervals are not directly comparable, as they describe fundamentally different structural relationships between different variables (e.g., y_2 an hour later or y_2 2 hours later).

The implications of the time-interval problem for meta-analysis is immediately apparent: Taking a weighted average of studies which use different time intervals may result in a set of parameters which do not accurately reflect the true underlying process for *any time interval*. As a corollary, even if researchers were to analyze only studies that use the same time interval, the relative ordering of cross-lagged parameter estimates, in terms of their absolute values, and thus the substantive conclusions drawn about the underlying process, may not generalize to any other time interval: The cross-lagged effect of y_1 on y_3 may be larger than the reciprocal effect for some intervals ($\phi_{13}(\Delta t) > \phi_{31}(\Delta t), \forall \Delta t < s$)) and smaller for others ($\phi_{13}(\Delta t) < \phi_{31}(\Delta t), \forall \Delta t > s$)).

Currently, there is no established best practice for dealing with different time intervals in the meta-analysis of lagged regression models. Broadly, there appear to be four different approaches used in the literature:

1. **Ignore** Ignoring the time interval (e.g., Masselink et al., 2018); sometimes this is done as a first step (e.g., Jacobson & Newman, 2017; Maricuţoiu et al., 2017; Nohe et al., 2015).
2. **Linear** Including the time interval as linear predictor (i.e., moderator) of the effect (cf. Card, 2019).
3. **Quadratic** Including the time interval as linear and quadratic moderator of the effect (cf. Card, 2019).
4. **Dummy** Doing separate analyses for each of the unique time intervals (groups) used by the studies in the meta-analysis (e.g., Maricuţoiu et al., 2017; Nohe et al., 2015) or, equivalently, using dummy moderator variables for each time-interval group.[2]

However, each of these approaches are problematic in principle because 1) when ignoring the time interval, you weight incomparable parameter estimates; 2) the linear approach cannot capture the non-linear relationships between the lagged effects and the time interval, that is, the non-linear relationship implied by exponentiating a matrix (as will become clear in the next section); 3) the quadratic approach also fails to capture the exponential relationship; and 4) doing separate analyses reduces your certainty/power because you split your data, and does not produce a model which describes how these parameter estimates relate to one another.

What is missing from the literature is a technique which takes account of the well-known time-interval problem in a principled way, allowing for the non-linear relationships between lagged parameters at different time intervals. In the remainder of the paper, we propose and illustrate a meta-analysis method based on a continuous-time VAR(1) model for this purpose.

CONTINUOUS-TIME META-ANALYSIS

In this section, we review continuous-time models, which follow a generalization of the logic described in the above subsection regarding measuring less frequently. Then, we briefly describe our proposed meta-analytic technique, which assumes an underlying data-generating CT model.

Continuous-time Models: A Primer

Continuous-time (CT) modeling overcomes the time-interval dependency problem by explicitly taking the time-interval dependent nature of the relationships between variables at different measurement occasions into account (Boker & Laurenceau, 2006; Oravecz et al., 2009; Oravecz, Tuerlinckx, & Vandekerckhove, 2011; Oud & Delsing, 2010; Voelkle & Oud, 2013). Conceptually, the CT model assumes that the processes of interest take on values and, moreover, influence each other at every moment in time, not only on the occasions at which the researcher measures them. As such, we can say that infinitely many latent values of these processes are present between each measurement occasion. Multiple authors have argued that psychological processes in particular may be more appropriately modeled as CT rather than DT processes (Boker & Laurenceau, 2006; van Montfort, Oud, & Voelkle, 2018).

We can model these processes using first-order differential equations, where the change over a very small time interval (i.e., the derivative) is a function of the value of the processes at that moment in time. This first-order differential equation can be thought of as a DT-VAR(1) model over an infinitesimally small time interval between measurement occasions: the corresponding effects matrix describing the dynamics is denoted by A and is called the drift matrix (see Oravecz et al., 2011; Oud & Delsing, 2010; Voelkle & Oud, 2013, for more details).

The first-order stochastic differential equation can also be represented as a (single-subject) CT-VAR(1) model also referred to as the multivariate Ornstein-Uhlenbeck process:

$$\frac{d\boldsymbol{y}(t)}{dt} = \boldsymbol{\alpha} + \boldsymbol{A}y(t) + \boldsymbol{\Omega}\frac{d\boldsymbol{w}(t)}{dt}. \quad (2)$$

where $\boldsymbol{\alpha}$ is a q-vector with intercepts of the differential equation model, and the $q \times q$ drift matrix $\boldsymbol{A}$ contains the CT-VAR (1) lagged parameters which relate the values of the variables in y at a particular time t to the derivative (i.e., rate of change) in $\boldsymbol{y}(t)$ with respect to time. In case of a stable and therefore stationary process, the eigenvalues of $\boldsymbol{A}$ are negative. The last term on the right-hand side of Equation 2 is the continuous-time version of a random error process that are independent and identically distributed, but maybe contemporaneously

[2] In an FE model, doing separate analyses for each time-interval group is the same as using dummy variables to account for the time interval. However, in case of a RE model, including dummy variables uses one random effect and thus results in one variance of that random effect ($\hat{\tau}^2$), while doing separate analyses per time-interval group uses a random effect per analysis and thus renders a $\hat{\tau}^2$ per time-interval group.

correlated. For a detailed discussion and for details on how this model can specified as a multi-subject panel data model and estimated in a SEM-framework, see, among others, Boker (2002) and Oud and Delsing (2010).

While the DT and CT models represent distinct ways of modeling longitudinal processes, the DT and CT lagged effects (i.e., $\mathbf{\Phi}(\Delta t)$ and $\boldsymbol{A}$, respectively) are related in the following way:

$$\mathbf{\Phi}(\Delta t) = e^{\boldsymbol{A}\Delta t}, \quad (3)$$

where $\mathbf{\Phi}(\Delta t)$ and Δt are as defined above; e is the matrix exponential (if $q>1$; Moler & Van Loan, 2013); and $\boldsymbol{A}$ the $q \times q$ *drift matrix*. The crucial aspect is that the lagged regression parameters are a non-linear function of the drift matrix. This means that the lagged regression parameters at any time interval in the DT model are a result of taking the appropriate power (using the exponential function and the time interval) of the lagged relationships at a very short time interval.

Under the condition that the eigenvalues of $\mathbf{\Phi}(\Delta t)$ are real and lie between zero and one, the mapping from DT-VAR(1) effects matrix to the underlying CT drift matrix in Equation 3 is unique (Hamerle, Nagl, & Singer, 1991; Kuiper & Ryan, 2018). This means that, using the lagged parameters which are obtained at a particular time interval Δt can be said to directly imply a set of lagged parameters at any different time interval Δt^*:

$$\begin{aligned}\mathbf{\Phi}(\Delta t^*) &= \left(e^{\boldsymbol{A}\Delta t}\right)^{\frac{\Delta t^*}{\Delta t}} \\ &= \left(\mathbf{\Phi}(\Delta t)\right)^{\frac{\Delta t^*}{\Delta t}}. \end{aligned} \quad (4)$$

Stated otherwise, one can use these equations to transform the parameter estimates for a given interval to a new set of parameters corresponding to a different time interval. This is the core of our proposed method and we will refer to the new set of parameters as transformed lagged parameters. Notably, if the eigenvalues of $\mathbf{\Phi}(\Delta t)$ are complex or when at least one of them is negative, then this transformation is only unique and possible, respectively, when Δt^* is a whole-numbered multiple of Δt. A more detailed discussion on the conditions under which this is (uniquely) possible is given by Kuiper and Ryan (2018).

Continuous-time Meta-analysis: CTmeta

If we are willing to assume an underlying CT process, the results from studies using different time intervals (e.g., $\mathbf{\Phi}(\Delta t = 1)$, $\mathbf{\Phi}(\Delta t = 2)$, and $\mathbf{\Phi}(\Delta t = 3.5)$) can all be mapped back to one underlying effects matrix $\boldsymbol{A}$. This means that, using the CT-VAR(1) model, results from studies using different time intervals can be considered informative about one another in a principled way: We can use the (transformed) standardized lagged effects from all studies in a single meta-analysis.

As we believe that researchers are primarily interested in lagged effects matrices and how they change depending on the time interval, our method involves approximating the CT function $\mathbf{\Phi}(\Delta t)$ over different values of Δt. We do this by making use of Equation 3 and mapping each $\mathbf{\Phi}$ matrix, and their covariance matrices, back onto a target time interval or set of time intervals, generally taken to be the time intervals of substantive interest or which are present in the meta-analyzed studies. Per time interval, the corresponding standardized lagged effects are then weighted in the normal way (using the aforementioned covariance matrices) to obtain the overall estimates (and their covariance matrix). This procedure can be repeated for each target time interval of interest, each time using all of the input study parameters in the calculation of the weighted overall estimates for that time interval. In the discussion, we address some advantages of this approach over the direct meta-analysis of the drift matrix.

This method for continuous-time meta-analysis of lagged regression parameters is implemented in the R-package *CTmeta*, available for download from https://github.com/rebeccakuiper/CTmeta, the github page of the first author. To apply the CTmeta method, researchers need sets of parameters' point-estimates from different studies as well as the covariance matrix of those sets of parameters. Appendix B describes how these can be retrieved from either the same-time-moment (contemporaneous) and lagged correlations between all variables or, alternatively, the (un)standardized lagged parameter estimates with either the residual covariance matrix or the contemporaneous covariance matrix. These functions are also implemented in the *CTmeta* R-package, as well as in Shiny applications described in more detail in Appendix C.

ILLUSTRATION AND COMPARISON

In contrast to the four commonly used meta-analysis approaches, the CTmeta method proposed above would seem advantageous because it 1) explicitly accounts for the various different time intervals used by studies in the meta-analysis, 2) accounts for the non-linear relationship between the time interval and the lagged variables, and 3) employs all sets of reported parameters to estimate a single underlying matrix of dynamics. Here, we illustrate the application of CTmeta, and compare its performance to the current best practice in this field. Our illustration consists of simulated data which mimics the set-up of a published empirical meta-analysis. The code to reproduce all of the analyses shown below is available on https://github.com/rebeccakuiper/CTmeta-analysis_Example_and_Simulation, the github page of the first author.

Example Data Set

Suppose we are interested in meta-analyzing a set of studies with $q = 2$ variables (e.g., work engagement and burnout). These two variables are assumed to have a contemporaneous pairwise correlation of $cor(y_{1m}, y_{2m}) = 0.3$, and the dynamic relationships between these two variables can be described by a CT-VAR(1) model with drift matrix

$$\boldsymbol{A} = \begin{bmatrix} -0.79 & 0.36 \\ 0.60 & -1.03 \end{bmatrix}.$$

This corresponds to the following population $\boldsymbol{\Phi}(\Delta t)$ for $\Delta t = 1$ and 2:

$$\boldsymbol{\Phi}(1) = \begin{bmatrix} 0.50 & 0.15 \\ 0.25 & 0.40 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Phi}(2) = \begin{bmatrix} 0.29 & 0.14 \\ 0.23 & 0.20 \end{bmatrix}.$$

Using Equation 3, we can derive the discrete-time matrix $\boldsymbol{\Phi}(\Delta t)$ for any time interval Δt. This is plotted (using the Shiny web application of Kuiper (2018b)) in Figure 2.

The primary aim of any meta-analysis applied to this process is to recover this underlying function, at time intervals for which the data is available, as closely and reliably as possible. The secondary aim is to capture the form of this underlying function beyond those time intervals where data was available (where model assumptions/extrapolation are more heavily relied on). To evaluate the CTmeta method, we mimic a typical setting for meta-analysis on longitudinal studies where studies of different sample sizes use different time intervals of measurement and thus obtain different sets of lagged regression parameter estimates. This allows us to see with one analysis how different meta-analysis methods perform when different amounts of information of the underlying process is available (i.e., more or larger studies for some time intervals than others).

We base our design on Maricuţoiu et al. (2017), who performed a meta-analysis on the estimated cross-lagged relationships between work engagement and burnout in 25 panel data studies, with a wide range of study-specific time intervals between measurement occasions, $\Delta t_s \in \left[\frac{1}{365}, 3\right]$ years, and a range of study-specific different sample sizes, $T_s \in [67, 2897]$, where the subscript s is used as study indicator. Based on the characteristics of these studies, shown in full in Table 1, we generate data for the studies which will be the target of our meta-analysis. For each target study, we generate T_s measurements from a VAR(1) model with lagged parameters $\boldsymbol{\Phi}(\Delta t_s)$ (based on the drift matrix given above) using the R-package *tsDyn* (Fabio Di Narzo, Aznarte, & Stigler, 2009).[3] We then fit a DT-VAR(1) model to each data set using the *vars* package (Pfaff, 2008). The parameter estimates from each simulated data set serve as input to the meta-analysis methods assessed in the following.

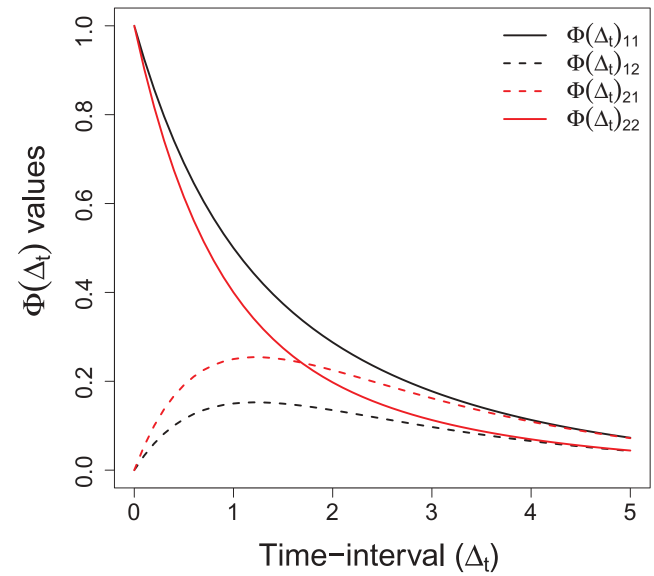


FIGURE 2 The lagged regression parameters $\boldsymbol{\Phi}(\Delta t)$ of the bivariate example as a function of the time interval Δt.

TABLE 1
The Study-specific Sample Sizes T_s and Study-specific Time Intervals Δt_s (in Years) Used in the Working Example

Study (s)	T_s	*Reported Time Interval*	Δt_s
1	643	12 months	1
2	651	12 months	1
3	473	12 months	1
4	387	4 months, 2.5 months and 6.5 months	$\frac{1}{3}$
5	18	9 months	$\frac{3}{4}$
6	209	12 months	1
7	2897	12 months	1
8	160	2 months	$\frac{1}{6}$
9	1964	36 months, 48 months	3
10	848	12 months	1
11	926	12 months	1
12	274	8 months	$\frac{2}{3}$
13	433	24 months	2
14	256	1 day	$\frac{1}{365}$
15	409	24 months	2
16	926	12 months	1
17	162	2 months	$\frac{1}{6}$
18	262	14 months	$\frac{14}{12}$
19	247	12 months	1
20	102	10 months	$\frac{10}{12}$
21	171	48 months	4
22	201	12 months	1
23	309	12 months	1
24	77	1 month	$\frac{1}{12}$
25	67	8 months	$\frac{2}{3}$

[3] Note that Maricuţoiu et al. (2017) meta-analyze contemporaneous and lagged correlations. They meta-analyze correlations for multiple studies, where in each study these correlations are based on multiple persons (for two waves, rendering six correlations; and sometimes averaged over multiple waves to again obtain six correlations). For the sake of simplicity, we will use their study-specific number of persons as our study-specific number of measurements (denoted by T_s).

With this input, we can compare the performances of the four current best-practice meta-analysis approaches (i.e., ignore, linear, quadratic, and dummy method) to the CTmeta method, in terms of recovering the true underlying lagged relationships across different intervals. For each of the five methods, we use a GLS approach in weighting the parameter estimates to account for dependencies within each set of lagged regression parameters. The covariance matrices of the (transformed) standardized lagged effect parameters, necessary for the GLS approach, are calculated based on the derivations shown in Appendix B. The GLS meta-analysis method, using the *rma.mv* function of the *metafor* package in R (Viechtbauer, 2010), renders the full covariance matrix of the parameter estimates and thus allows us to calculate elliptical 95% confidence intervals, which take into account the covariances of the parameter estimates. If one would disregard these covariances and combine the univariate 95% confidence intervals per parameter estimate based on only the variances, one obtains the simultaneous confidence interval, which covers more than 95%. As an example, in case of two parameter estimates, the elliptical 95% confidence interval is an ellipses and the simultaneous confidence interval is a rectangle covering this ellipses and thus covers parameter estimate combinations that are not part of the true, multivariate 95% confidence interval. Therefore, we will use the elliptical 95% confidence intervals (from now referred to as 95CI), which can be used to properly quantify the uncertainty in parameter estimates for each method.

Comparison of Methods

Figures 3 and 4 depict the results of the five different meta-analyses, where subfigures (a) through (d) show the overall estimates of ϕ_{11}, ϕ_{12}, ϕ_{21} and ϕ_{22}, respectively. The overall lagged effect parameters for each of the 12 unique time intervals are denoted by dots and the triangles depict the upper and lower bound of the corresponding 95CI. Each meta-analysis method is depicted by a different color, and for clarity, the ignore, linear, and quadratic meta-analysis results are shown in Figure 3, while the dummy method and CTmeta results are shown in Figure 4. For each method, we compare how these capture the true, population lagged effects at different intervals, denoted by a green line in each figure.

Ignoring Time Interval

We start with aggregating the 25 sets of four parameter estimates ignoring the time interval. Since we neglect the time interval, there is one overall estimate per parameter, indicated by red dots in Figure 3, which lie consequently on a horizontal line. Similarly, the 95CI (per parameter estimate) is the same for all time intervals. When inspecting the overall parameter estimates for time intervals present in our study, only for $\phi_{12}(\frac{1}{3})$ the 95CI of the overall estimate contains the true value, while for all other parameters and/or time intervals it does not.

The overall estimate is a mix of the true parameter values for each of the used time intervals. Evidently, a mix of these true parameter values is not particularly informative about the (underlying) process. This method leads to a biased overall effect estimate matrix and may lead to incorrect conclusions regarding dominance (i.e., predictive strength) and sign (cf. Kuiper & Ryan, 2018). Only when (approximately) the same time intervals are used in each study, this method is valid.

Time Interval as a Linear (and Quadratic) Moderator

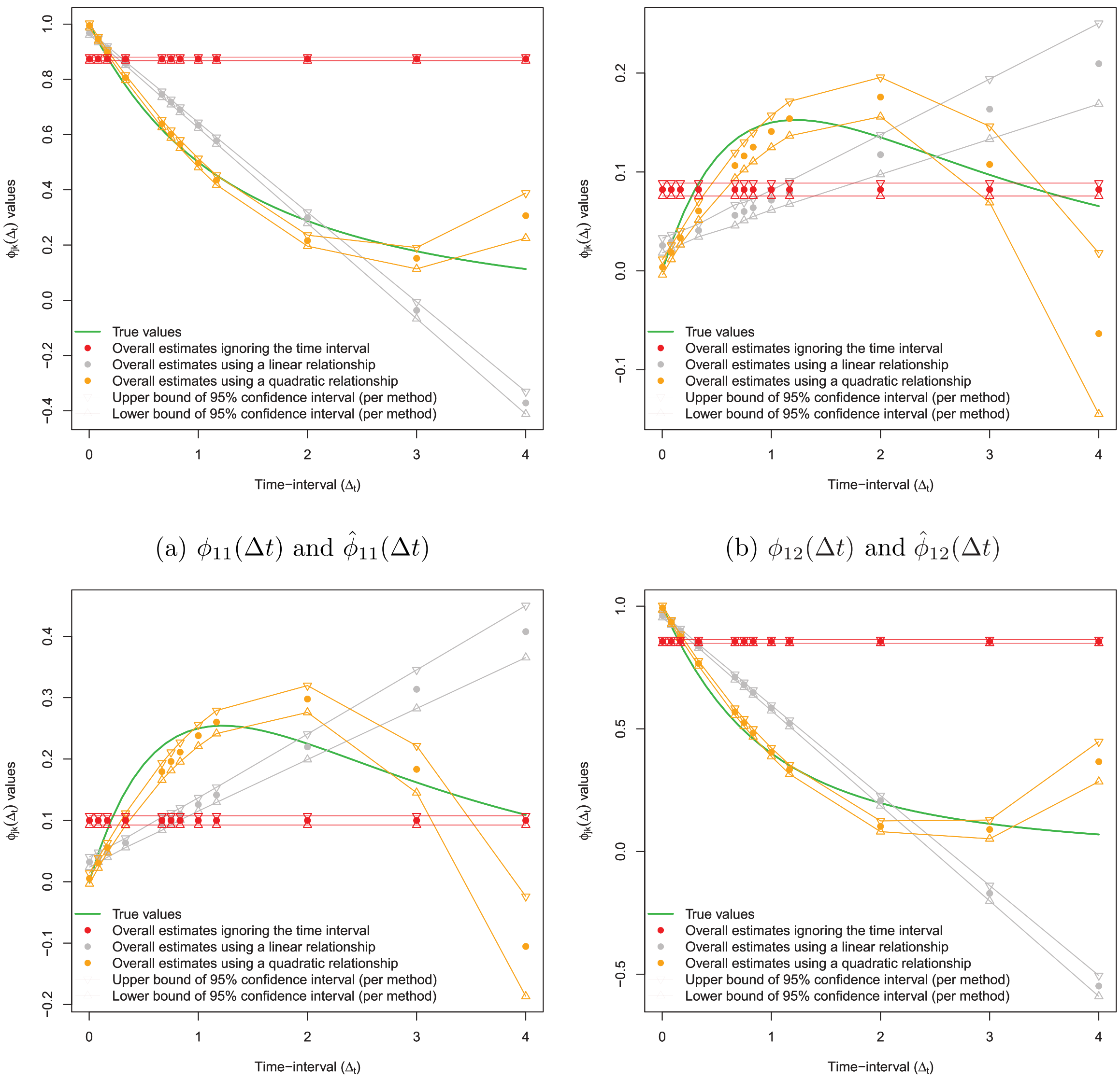
When adding the time interval as a linear predictor, we obtain per parameter an intercept and slope estimate from which we can derive the overall estimates per time interval. The results of this method are depicted in gray in Figure 3. Note that this, by construction, renders a straight line for each standardized lagged effect. Thus, using the study-specific time interval as a linear moderator results in a linear prediction of a non-linear relationship (cf. green line and Equation 3). Especially, if the time intervals are far apart, the linear relationship is a bad prediction. If the time intervals are close together, one could benefit from the linear model since there is only one time interval parameter to be estimated.

From the subfigures, we can see that for each standardized lagged effect there are many time intervals for which the 95CI does not contain the true population value, that is, for which the green line is not between the gray lines. When adding the time interval also as a quadratic predictor, one obtains (evidently) a non-linear relationship, see the orange parts in Figure 3. Nevertheless, for only 4 out of 12 time interval groups, the 95CI contains the true population value.

Time Interval Groups as Dummies

In the working example, we have 12 unique time intervals, that is, $G = 12$ time interval groups, and thus the dummy method leads to 12 overall estimates.[4] Figure 4 displays the results in black. From the subfigures, we can see that some of the overall estimates are not very accurate but the coverage is good: in most instances, the 95CIs does contain the true population value (green). The reason for this is that the overall estimate is based on relatively few observations, resulting in a high(er) standard error of that overall estimate. Only for the overall estimate of $\phi_{11}\left(1\frac{1}{6}\right)$, the true parameter value lies outside of the 95CI. From the subfigures, we can see that the certainty of the overall

[4] Notably, for interpretational and calculational ease, one should use G dummies and leave the intercept out (instead of including $G-1$ dummies). This leads to G overall estimates, namely one per time interval group, with their covariance matrix (instead of group differences in those estimates and the covariance matrix of these differences).



(a) $\phi_{11}(\Delta t)$ and $\hat{\phi}_{11}(\Delta t)$

(b) $\phi_{12}(\Delta t)$ and $\hat{\phi}_{12}(\Delta t)$

(c) $\phi_{21}(\Delta t)$ and $\hat{\phi}_{21}(\Delta t)$

(d) $\phi_{22}(\Delta t)$ and $\hat{\phi}_{22}(\Delta t)$

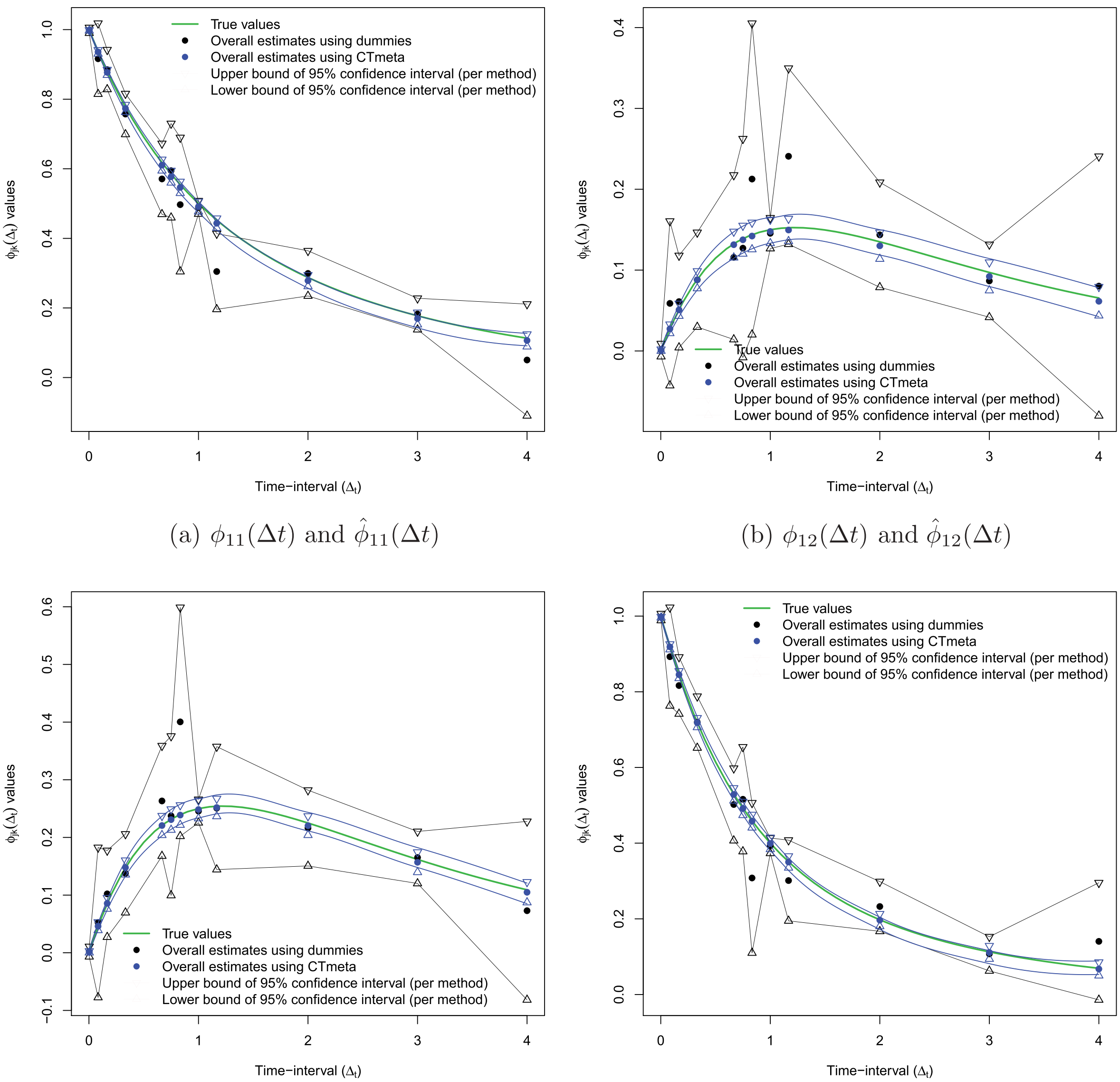
FIGURE 3 Subfigures (a–d) display one of the four $\phi_{jk}(\Delta t)$s (green line) and its estimated overall standardized lagged effect $\hat{\phi}_{jk}(\Delta t)$ (dots) together with their elliptical 95% confidence intervals (triangles) based on the following three methods: ignoring time interval (red), using a linear relationship (gray), and using a linear and quadratic relationship (orange).

estimate increases with total sample size used for that time interval group: for $\Delta t = 1$, the total number of observations is the highest and the width of the 95CIs is the smallest.

In estimating each interval-specific overall estimate, the dummy method only uses information from studies with that exact time interval, akin to splitting the sample and performing separate analyses. Therefore, the 95CIs is wider than when all observations could have been used. Because of this, 0 is sometimes included in the 95CIs while the true parameter differs from zero. Another consequence is that this approach needs more than one study per time interval group to obtain additional information, otherwise your overall estimate for that time interval equals the estimate of that single study.

Continuous-Time Meta-Analysis (CTmeta)

Since the estimates from all studies are all informative about the underlying process, we use information from all 25 longitudinal studies in the aggregation by meta-analyzing the transformed lagged parameters. Much like the dummy analysis method, we perform the meta-analysis technique 12 times, one for each time interval used. However, the input for each time interval analysis includes all observations, by using



(a) $\phi_{11}(\Delta t)$ and $\hat{\phi}_{11}(\Delta t)$ (b) $\phi_{12}(\Delta t)$ and $\hat{\phi}_{12}(\Delta t)$

(c) $\phi_{21}(\Delta t)$ and $\hat{\phi}_{21}(\Delta t)$ (d) $\phi_{22}(\Delta t)$ and $\hat{\phi}_{22}(\Delta t)$

FIGURE 4 Subfigures (a–d) display one of the four $\phi_{jk}(\Delta t)$s (green line) and its estimated overall standardized lagged effect $\hat{\phi}_{jk}(\Delta t)$ (dots) together with their elliptical 95% confidence intervals (triangles) based on the following three methods: dummy (black) and CTmeta (blue).

all transformed lagged parameters at that time interval. The results are plotted in blue in Figure 4. From the subfigures, we can see that the overall standardized lagged effects are almost exactly equal to the true values and that all the 95CIs contain the true population value. Additionally, the 95CIs do not contain 0 except for one case: the overall estimate of $\phi_{12}\left(\frac{1}{365}\right)$.

When comparing this method (blue) to the dummy approach (black), we see that the 95CIs are much narrower than the ones of the dummy approach and thus also contain the value 0 less often. This is of course due to using all available data from all studies for each of the four lagged effects.

Comparison of Dummy Method and CTmeta: Simulation

From the example above, it is clear that only the dummy method and CTmeta capture the non-linear relationship between the lagged effects and the time interval. To examine which of these two methods performs best, we also conducted a simulation study based on the example above. We repeated the exact GLS meta-analysis (for the four lagged effects), as described above, 10,000 times.[5]

Table 2 shows results of this simulation: the coverage, bias, and root-mean-square error (RMSE) of both methods for each of the four standardized lagged effects and 12 unique time intervals The table shows that both methods seem to be unbiased and that their coverage rates are comparable. However, the variance of the dummy method estimates are much larger, as we can see from the RMSE in Table 2 (which ranges, depending on the time interval, from equivalent to 19 times larger than that of CTmeta).

As we would expect, given that the CTmeta method uses all parameter estimates for every time interval, CTmeta is more efficient than the dummy method: The 95CI of the dummy method is wider for all lagged effects and all time intervals (depending on the time interval, between 1.26 and 15.85 times wider). As a consequence, we also see that the CTmeta method has more power to detect non-zero relationships than the dummy method: For the dummy analysis, zero is more often included in the 95CI for non-zero parameters than for the CTmeta method (for the dummy method the average proportions over the 12 time intervals are 0.07, 0.28, 0.44, and 0.08 for the four respective parameters; while for CTmeta these are 0, 0, 0.06, and 0, respectively, where 0 is only contained in the 95CI for the overall estimate of $\phi_{21}(1/365)$). While both the CTmeta and the dummy method are unbiased, with acceptable coverage rates, the CTmeta method outperforms the dummy method in these other key metrics.

DISCUSSION

In this article, we proposed a new method for the meta-analysis of lagged regression models which overcomes the well-known problem of time-interval dependency. Based on the assumption of an underlying continuous-time model, we showed that this method outperforms current best-available methods in a representative example and simulation study. While many researchers have explicitly promoted the potential of CT models in aiding researchers to compare cross-lagged effects in studies with different measurement regimes (e.g., Oud, 2007), this paper provides the first implementation of a method by which researchers can use CT models directly for this purpose. This new method, implemented in the R package *CTmeta* and accompanying Shiny application (Kuiper, 2018a), allows researchers to combine evidence from such studies in a principled way, and to come to an understanding of how lagged relationships vary and evolve as a function of the time interval.

While the current implementation of the CTmeta method allows for some degree of flexibility regarding the type of input (correlation matrices, lagged regression parameters), weighting method (WLS or GLS) and meta-analysis model (FE or RE), further work is needed to address common issues which arise in meta-analytic studies. For example, accounting for studies with more than one set of parameters estimates (i.e., multiple samples; as Nohe et al. (2015) have in their selected studies and account for by using a multilevel meta-analysis).

In the current paper, we focus on the meta-analysis of cross-lagged and auto-regressive parameters resulting from fixed effects DT-VAR(1) (CLPM) models. The standardized lagged effect parameters from the random-intercept CLPM (RI-CLPM; Hamaker et al., 2015) are also increasingly the target of meta-analyses (e.g. Masselink et al., 2018). In these models, random intercepts are used to separate the between from the within-persons variance components by decomposing observations into a stable mean, specific to an individual, and a deviation from that mean at a given time point, and the DT-VAR(1) model is applied to these latter deviations. The CTmeta method can be readily applied to lagged parameters estimated from RI-CLPM models without adjustment. However, the meta-analysis of the random intercept parameters themselves is not yet implemented, that is, we did inspect the meta-analysis of random-intercept parameters (their means and variances). Another, promising extension would for future studies would be the meta-analysis of random auto-regressive and cross-lagged effects, such as those yielded by multilevel DT-VAR(1) models, increasingly popular in experience sampling studies (e.g., Bringmann et al., 2013). The CTmeta method can currently only meta-analyze the fixed auto-regressive and cross-lagged effects of such models.

At its core, this method leverages assumptions about the form of the underlying process to achieve gains in performance in terms of efficiency and power, in comparison to the next-best performing dummy variable method. The key assumption is that the underlying process is continuous-time in nature. Boker (2002) and van Montfort et al. (2018), among others, have argued that psychological processes are more appropriately conceptualized as CT processes, reflecting the notion that psychological processes evolve and vary in a smooth, continuous manner over time. In the current paper, the simplest possible continuous-time model, the first-order differential equation with integral solution the CT-VAR(1), is assumed as the underlying data-generating mechanism. This is an attractive choice as the mapping from this CT model to the DT-VAR(1) (and vice versa) is well-known and unique in many instances. However, in light of that assumption, two limitations of

[5] When simulating data, we discarded the samples where the DT-VAR (tr1) lagged parameter matrix had at least one negative eigenvalue (since in that case there does not exist a CT equivalent) and where the covariance matrices were not positive definite (comparable to negative variance; since in that case, we cannot perform a GLS meta-analysis).

TABLE 2
Coverage, Bias, and Root-Mean-Square Error (RMSE) of the Dummy Method and CTmeta Method, for Each of the 4 Standardized Lagged Effects and 12 Unique Time Intervals Considered in the Meta-analysis

	Coverage								*Bias*			
	dummy method				*CTmeta*				*dummy method*			
Δt^*	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
1/365	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	−0.002	0.000	0.000	−0.002
1/12	0.997	0.995	0.995	0.996	1.000	1.000	1.000	1.000	−0.006	0.002	0.003	−0.007
2/12	0.992	0.991	0.986	0.991	0.991	0.997	0.991	0.996	−0.016	0.000	0.002	−0.016
4/12	0.977	0.978	0.968	0.970	0.969	0.982	0.970	0.973	−0.005	0.002	0.004	−0.006
8/12	0.963	0.959	0.961	0.958	0.969	0.978	0.977	0.968	−0.001	−0.001	−0.001	−0.000
9/12	0.963	0.965	0.962	0.963	0.976	0.984	0.983	0.974	−0.011	−0.000	0.003	−0.012
10/12	0.962	0.962	0.954	0.963	0.966	0.980	0.977	0.971	−0.005	−0.002	−0.004	−0.002
12/12	0.944	0.965	0.954	0.960	0.933	0.964	0.952	0.956	−0.000	0.000	0.000	−0.000
14/12	0.963	0.965	0.958	0.969	0.914	0.961	0.937	0.954	−0.011	−0.002	−0.002	−0.009
24/12	0.971	0.981	0.984	0.992	0.959	0.974	0.982	0.991	−0.028	−0.002	−0.001	−0.025
36/12	0.993	0.997	0.999	1.000	0.989	0.992	0.999	1.000	−0.007	−0.005	−0.008	−0.004
48/12	0.999	1.000	1.000	1.000	0.999	1.000	1.000	1.000	−0.011	0.004	0.007	−0.013

	Bias				RMSE							
	CTmeta				dummy method				CTmeta			
Δt^*	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
1/365	−0.003	0.000	0.000	−0.003	0.000	0.001	0.001	0.001	0.000	0.001	0.001	0.000
1/12	−0.001	0.000	0.001	−0.002	0.029	0.040	0.031	0.036	0.002	0.019	0.019	0.002
2/12	−0.003	0.000	0.001	−0.003	0.020	0.044	0.038	0.025	0.003	0.035	0.034	0.003
4/12	−0.001	0.000	0.001	−0.001	0.029	0.070	0.064	0.035	0.005	0.060	0.059	0.006
8/12	−0.003	−0.001	−0.003	−0.001	0.045	0.103	0.099	0.049	0.007	0.090	0.088	0.008
9/12	−0.003	0.001	0.001	−0.003	0.064	0.114	0.111	0.068	0.008	0.094	0.092	0.008
10/12	−0.003	−0.001	−0.003	−0.001	0.090	0.130	0.132	0.093	0.008	0.097	0.096	0.008
12/12	−0.000	−0.000	0.000	−0.000	0.010	0.101	0.100	0.010	0.008	0.101	0.100	0.008
14/12	−0.003	−0.000	−0.000	−0.002	0.056	0.114	0.117	0.055	0.008	0.102	0.102	0.008
24/12	−0.003	0.000	0.001	−0.003	0.030	0.090	0.096	0.025	0.008	0.088	0.092	0.006
36/12	−0.002	−0.001	−0.002	−0.001	0.016	0.066	0.067	0.013	0.007	0.062	0.066	0.005
48/12	−0.000	0.000	0.000	−0.000	0.043	0.054	0.059	0.031	0.005	0.041	0.045	0.004

Note: A value of (-)0.000 means a value smaller than 0.0005 in absolute sense.

the current methodology are of note here. First, following the Nyquist-Shannon theorem (Shannon, 1984) we must assume throughout that the sampling frequency in the target studies is sufficiently high to capture the dynamics of interest. While for oscillating processes, it is straightforward to derive the necessary sampling frequency (less than or equal to half the wavelength), an exploration of whether this condition is met in typical psychological settings is beyond the scope of the current paper. Second, when oscillating behavior is present in the system of interest, as indicated by complex eigenvalues of the lagged effects matrix, the meta-analytic strategy described in the current paper breaks down (cf. Hamerle et al., 1991). In that case, there is no unique mapping from the discrete-time to the continuous-time effects matrices (that is, the inverse of Equation 3 does not lead to a unique $\boldsymbol{A}$). A potential topic for future research would be to investigate how better to utilize observations at many unequal time intervals to better identify oscillating systems (as suggested by Voelkle & Oud, 2013, Section 1.2). In general, it remains to be seen how the CTmeta method performs under conditions of model misspecification, be that a non-continuous-time process or a more complex or higher-order dynamic process. Future research could potentially generalize the CTmeta method to any continuous-time model for which the integral solution is known.

The particular implementation of continuous-time meta-analysis described in the current paper reflects our beliefs that researchers are primarily interested in how lagged effects vary and evolve over time. An alternative approach would be to perform a meta-analysis directly on the parameters of the CT model, that is, the drift matrix. As well as entailing potential difficulties in interpretation, there are a number of practical considerations which led us to the CTmeta method described here. Primarily, while the transformation from DT-VAR(1) lagged effects to the CT-VAR(1) parameters is known, the transformation of their covariance matrices is less straightforward. This implies that proper weighting of evidence from different studies is difficult to implement and, moreover, it complicates determining the elliptical/multivariate 95% confidence intervals of the overall DT-VAR(1) lagged effects. For a further discussion of this issue, see Appendix D.

Finally, while the CTmeta method allows us to overcome the issue of time-interval dependency, there remain a number of unsolved conceptual issues with comparing cross-lagged effects using meta-analytic techniques. For example, simple comparison of the size of overall cross-lagged estimates is a useful first step, but not a principled hypothesis test of relative magnitudes. Furthermore, while the interest of researchers applying these methods is typically in making some inference about (Granger-)causal relationships (cf. Usami, Murayama, & Hamaker, 2019), the order of magnitude of cross-lagged effects does not necessarily ensure the same ordering of 'causal dominance' relations: further assumptions are necessary for these lagged parameters to reflect causal relationships. Nonetheless, the availability of a method to meta-analyze cross-lagged parameters from studies with different measurement designs is a necessary first-step on the road to establishing any reliable conclusions regarding the nature of these relationships in principle.

FUNDING

This work was supported by the Netherlands Organisation for Scientific Research (NWO) under VENI Grant 451-16-019 (Kuiper) and Talent Grant 406-15-128 (Ryan).

REFERENCES

Becker, B. J., & Wu, M.-J. (2007). The synthesis of regression slopes in meta-analysis. *Statistical Science*, *22*, 414–429. doi:10.1214/07-STS243

Bentler, P. M., & Speckart, G. (1981). Attitudes" cause" behaviors: A structural equation analysis. *Journal of Personality and Social Psychology*, *40*, 226. doi:10.1037/0022-3514.40.2.226

Boker, S. M. (2002). Consequences of continuity: The hunt for intrinsic properties within parameters of dynamics in psychological processes. *Multivariate Behavioral Research*, *37*, 405–422. doi:10.1207/S15327906MBR3703_5

Boker, S. M., & Laurenceau, J.-P. (2006). Dynamic system modeling: An application to the regulation of intimicy and disclosure in marriage. In T. A. Walls & J. L. Schafer (Eds.), *Models for intensive longitudinal data* (pp. 195–218). New York, NY: Oxford University Press.

Boker, S. M., Neale, M., & Rausch, J. (2004). Latent differential equation modeling with multivariate multi-occasion indicators. In J. Oud, K. van Montfort, & A. Satorra (Eds.), *Recent developments on structural equation models* (pp. 151–174). Dordrecht, The Netherlands: Kluwer Academic.

Bollen, K. A. (1989). *Structural equations with latent variables*. Oxford, UK: Wiley.

Bollen, K. A., & Curran, P. (2006). *Latent curve models: A structural equation approach*. Hoboken, NJ: Wiley and Sons.

Borenstein, M., Hedges, L. V., Higgins, J. P. T., & Rothstein, H. R. (2009). *Introduction to meta-analysis*. New York, NY: Wiley.

Bringmann, L. F., Vissers, N., Wichers, M., Geschwind, N., Kuppens, P., Peeters, F., & Tuerlinckx, F. (2013). A network approach to psychopathology: New insights into clinical longitudinal data. *PloS One*, *8*, e60188, 1–13. doi:10.1371/journal.pone.0060188

Card, N. A. (2019). Lag as moderator meta-analysis: A methodological approach for synthesizing longitudinal data. *International Journal of Behavioral Development*, *43*, 80–89. doi:10.1177/0165025418773461

Chang, W., Cheng, J., Allaire, J., Xie, Y., & McPherson, J. (2019). Shiny: Web application framework for r (R package version 1.3.2) [Computer software manual]. Retrieved from https://CRAN.R-project.org/package=shiny

Chatfield, C. (2004). *The analysis of time series: An introduction* (6th ed.). London, UK: Chapman and Hall.

Cheung, M. W.-L. (2015). *Meta-analysis: A structural equation modeling approach*. Chichester, UK: Wiley.

Chow, S., Ram, N., Boker, S., Fujita, F., Clore, G., & Nesselroade, J. (2005). Capturing weekly fluctuation in emotion using a latent di□erential structural approach. *Emotion*, *5*, 208–225. doi:10.1037/1528-3542.5.2.208

Demidenko, E., Sargent, J., & Onega, T. (2012). Random effects coe□cient of determination for mixed and metaanalysis models. *Communications in Statistics - Theory and Methods*, *41*, 953–969. doi:10.1080/03610926.2010.535631

Dormann, C., & Griffin, M. A. (2015). Optimal time lags in panel studies. *Psychological Methods*, *20*, 489. doi:10.1037/met0000041

Fabio Di Narzo, A., Aznarte, J. L., & Stigler, M. (2009). tsdyn: Time series analysis based on dynamical systems theory (R package version 0.7) [Computer software manual]. Retrieved from https://cran.r-project.org/package=tsDyn/vignettes/tsDyn.pdf

Finkel, S. E. (1995). *Causal analysis with panel data*. Thousand Oaks, CA: Sage.

Gollob, H. F., & Reichardt, C. S. (1987). Taking account of time lags in causal models. *Child Development*, *58*, 80–92. doi:10.2307/1130293

Granger, C. W. J. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, *37*, 424–438. doi:10.2307/1912791

Hamaker, E. L., Kuiper, R. M., & Grasman, R. P. P. P. (2015). A critique of the cross-lagged panel model. *Psychological Methods*, *20*, 102–116. doi:10.1037/a0038889

Hamerle, A., Nagl, W., & Singer, H. (1991). Problems with the estimation of stochastic di□erential equations using structural equations model. *Journal of Mathematical Sociology*, *16*, 201–220. doi:10.1080/0022250X.1991.9990088

Hamilton, J. D. (1994). *Time series analysis*. Princeton, NJ: Princeton University Press.

Innstrand, S. T., Langballe, E. M., Espnes, G. A., Falkum, E., & Aasland, O. G. (2008). Positive and negative work family interaction and burnout: A longitudinal study of reciprocal relations. *Work & Stress*, *22*, 1–15. doi:10.1080/02678370801975842

Jacobson, N. C., & Newman, M. G. (2017). Anxiety and depression as bidirectional risk factors for one another: A meta-analysis of longitudinal studies. *Psychological Bulletin*, *1*, 1–46. doi:10.1037/bul0000111

Kuiper, R. M. (2018a). *CT meta-analysis on lagged effects [shiny app software]*. Retrieved from https://www.uu.nl/staff/RMKuiper/Websites%20%2F%20Shiny%20apps

Kuiper, R. M. (2018b). *Phi-and-psi-plots and find deltaT [shiny app software]*. Retrieved from https://www.uu.nl/staff/RMKuiper/Websites%20%2F%20Shiny%20apps

Kuiper, R. M. (2018c). *Standardizing and/or transforming lagged regression estimates [shiny app software]*. Retrieved from https://www.uu.nl/staff/RMKuiper/Websites%20%2F%20Shiny%20apps

Kuiper, R. M. (2019). *CTmeta: Continuous-time meta-analysis on lagged effects [r package]*. Retrieved from https://github.com/rebeccakuiper/CTmeta

Kuiper, R. M., & Ryan, O. (2018). Drawing conclusions from cross-lagged relationships: Re-considering the role of the time-interval. *Structural Equation Modeling*, *25*, 809–823. doi:10.1080/10705511.2018.1431046

Maricuţoiu, L. P., Sulea, C., & Iancu, A. (2017). Work engagement or burnout: Which comes first? a meta-analysis of longitudinal evidence. *Burnout Research*, *5*, 35–43. doi:10.1016/j.burn.2017.05.001

Masselink, M., van Roekel, E., Hankin, B., Keijsers, L., Lodder, G., Vanhalst, J., … Old-ehinkel, A. (2018). The longitudinal association between self-esteem and depressive symptoms in adolescents: Separating between-person e□ects from within-person e□ects. *European Journal of Personality.* doi:10.1002/per.2179

Mayer, L. (1986). On cross-lagged panel models with serially correlated errors. *Journal of Business and Economic Statistics*, *4*, 347–357. doi:10.1080/07350015.1986.10509531

Moberly, N. J., & Watkins, E. R. (2008). Ruminative self-focus and negative affect: An experience sampling study. *Journal of Abnormal Psychology*, *117*, 314–323. doi:10.1037/0021-843X.117.2.314

Moler, C., & Van Loan, C. (2013). Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later. *SIAM Review*, *45*, 3–49. doi:10.1137/S00361445024180

Nohe, C., Meier, L. L., Sonntag, K., & Michel, A. (2015). The chicken or the egg? a meta-analysis of panel studies of the relationship between work–Family conflict and strain. *Journal of Applied Psychology*, *100*, 522–536. doi:10.1037/a0038012

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2009). A hierarchical Ornstein Uhlenbeck model for continuous repeated measurement data. *Psychometrika*, *74*, 395–418. doi:10.1007/s11336-008-9106-8

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. *Psychological Methods*, *16*, 468–490. doi:10.1037/a0024375.supp

Oud, J. (2002). Continuous time modeling of the cross-lagged panel design. *Kwantitatieve Methoden*, *69*, 1–27.

Oud, J. (2007). Continuous time modeling of reciprocal relationships in the cross-lagged panel design. In S. M. Boker & M. J. Wenger (Eds.), *Data analytic techniques for dynamic systems in the social and behavioral sciences* (pp. 87–129). Mahwah, NJ: Lawrence Erlbaum Associates.

Oud, J., & Delsing, M. (2010). Continuous time modeling of panel data by means of *SEM*. In K. van Montfort et al. (Ed.), *Longitudinal research with latent variables* (pp. 201–244). Berlin: Springer-Verlag. doi:10.1007/978-3-642-11760-27

Pelz, D., & Lew, R. (1970). Heise's causal model applied. In E. Borgatta (Ed.), *Sociological methodology* (pp. 28–37). San Francisco, CA: Wiley.

Pfaff, B. (2008). Var, svar and svec models: Implementation within R package vars. *Journal of Statistical Software*, *27*. Retrieved from http://www.jstatsoft.org/v27/i04/

R Core Team. (2019). R: A language and environment for statistical computing [Computer software manual]. Vienna, Austria. Retrieved from https://www.R-project.org/

Rogosa, D. (1979). Causal models in longitudinal research: Rationale, formulation, and interpretation. In J. R. Nesselroade, & P. R. Baltes (Eds.), *Longitudinal Research in the Study of Behavior and Development*, 263–302. New York, NY: Academic Press.

Rogosa, D. (1980). A critique of cross-lagged correlation. *Psychological Bulletin*, *88*, 245. doi:10.1037/0033-2909.88.2.245

Shannon, C. E. (1984). Communication in the presence of noise. *Proceedings of the IEEE*, *72*, 1192–1201. doi:10.1109/PROC.1984.12998

Usami, S., Murayama, K., & Hamaker, E. L. (2019). A unified framework of longitudinal models to examine reciprocal relations. *Psychological Methods*. doi:10.1037/met0000210

van Montfort, K., Oud, J. H. L., & Voelkle, M. C. (2018). *Continuous time modeling in the behavioral and related sciences*. New York, NY: Springer.

Viechtbauer, W. (2010). Conducting meta-analyses in R with the metafor package. *Journal of Statistical Software*, *36*, 1–48. Retrieved from http://www.jstatsoft.org/v36/i03/

Viswesvaran, C., & Ones, D. S. (2007). Theory testing: Combining psychometric meta- analysis and structural equations modeling. *Personnel Psychology*, *48*, 865–885. doi:10.1111/j.1744-6570.1995.tb01784.x

Voelkle, M. C., & Oud, J. H. L. (2013). Continuous time modelling with individually varying time intervals for oscillating and non-oscillating processes. *British Journal of Mathematical and Statistical Psychology*, *66*, 103–126. doi:10.1111/j.2044-8317.2012.02043.x

Voelkle, M. C., Oud, J. H. L., Davidov, E., & Schmidt, P. (2012). An SEM approach to continuous time modeling of panel data: Relating authoritarianism and anomia. *Psychological Methods*, *7*, 176–192. doi:10.1037/a0027543

Wolf, F. M. (1986). *Meta-analysis: Quantitative methods for research synthesis*. Thousand Oaks, CA: Sage.

APPENDIX A. STANDARDIZED LAGGED EFFECT MATRICES VIA (MEAN) CORRELATIONS

In this article, we are interested in meta-analyzing standardized lagged effects. However, sometimes studies report contemporaneous and lagged correlations between the variables (see Figure 5b) instead of lagged effects (see Figure 5a). One possibility is to meta-analyze (the Fisher's transformation of) these correlations, as Jacobson and Newman (2017) do. Another possibility is to transform these to standardized lagged effects and meta-analyze these. How one should do this (in general) will be shown in this appendix.

To show how one can obtain $\boldsymbol{\Phi}(\Delta t)$ via contemporaneous and lagged correlations, we rewrite the DT-VAR(1) model as a regression model, where the predictors $\boldsymbol{X}$ are the lagged (and mean-centered) observations (i.e., $\boldsymbol{y}_{m-1}$; e.g. Stress and Anxiety on measurement occasion $m-1$) and the outcomes $\boldsymbol{Y}$ the (mean-centered) current observations (i.e., $\boldsymbol{y}_m$; e.g. Stress and Anxiety on measurement occasion m). Note that both variables have either $T_s - 1$ observations (if single subject and T_s measurement occasions) or N_s observations (if N_s persons and two waves). Evidently, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are study-specific and, moreover, will differ when another time interval is used. For example, if one measures every 2 hours instead of each hour, every second 'observation' is not measured. For readability, we will leave out the study-specific subscript s and sometimes the dependency on Δt. Given this notation, the transpose of the DT-VAR(1) lagged effects matrix can be estimated by

$$(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y}.$$

Here, we are interested in the standardized lagged effect parameter matrix, in this article denoted by $\boldsymbol{\Phi}(\Delta t)$. In that case, both $\boldsymbol{X}$ and $\boldsymbol{Y}$ are not only mean-centered but even standardized (i.e., also have a variance of one). Let the correlation matrix of $\boldsymbol{Y}$ and $\boldsymbol{X}$ consists of four block matrices (each of size $q \times q$):

$$\begin{bmatrix} \boldsymbol{R}_{YY} & \boldsymbol{R}_{XY}(\Delta t) \\ \boldsymbol{R}_{XY}(\Delta t)' & \boldsymbol{R}_{XX} \end{bmatrix},$$

with $\boldsymbol{R}_{YY}$ the correlation matrix of the $\boldsymbol{Y}$s (i.e., the contemporaneous correlations of the current variables); $\boldsymbol{R}_{XX}$ the correlation matrix of the $\boldsymbol{X}$s (i.e., the contemporaneous correlations of the lagged variables); and $\boldsymbol{R}_{XY}(\Delta t)$ the matrix with correlations between the outcome $\boldsymbol{Y}$ and each predictor $\boldsymbol{X}$ (i.e., between the current and lagged variables). Using this, the transpose of the DT-VAR(1) standardized lagged effect parameter matrix is calculated by

$$\boldsymbol{\Phi}(\Delta t)' = \boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(\Delta t). \tag{5}$$

Note that $\boldsymbol{R}_{YY}$ and $\boldsymbol{R}_{XX}$ are independent of the time interval Δt, since they are the contemporaneous correlations.[6] In contrast, $\boldsymbol{R}_{XY}(\Delta t)$ does depend on the time interval used, since it contains the lagged correlations which depend on the time interval. This time-interval dependency can also be seen from the following:

$$\begin{aligned}
\boldsymbol{\Phi}(1)' &= \boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(1);\\
\boldsymbol{\Phi}(2)' &= \boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(2);\ \text{and}\\
\boldsymbol{\Phi}(2)' &= (\boldsymbol{\Phi}(1)')^2\\
&= (\boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(1))\,(\boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(1))\\
&= \boldsymbol{R}_{XX}^{-1}\,(\boldsymbol{R}_{XY}(1)\boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(1));\ \text{hence}\\
\boldsymbol{R}_{XY}(2) &= \boldsymbol{R}_{XY}(1)\boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{XY}(1).
\end{aligned}$$

As an example, when looking at the first DT-VAR(1) equation of a bivariate DT-VAR(1) model, Equation 5 states that the two parameters can be written as:

$$\phi_{11}(\Delta t) = \frac{r_{y_1,x_1}(\Delta t) - r_{x_1,x_2}\, r_{y_1,x_2}(\Delta t)}{1 - r_{x_1,x_2}^2},$$

$$\phi_{12}(\Delta t) = \frac{r_{y_1,x_2}(\Delta t) - r_{x_1,x_2}\, r_{y_1,x_1}(\Delta t)}{1 - r_{x_1,x_2}^2}.$$

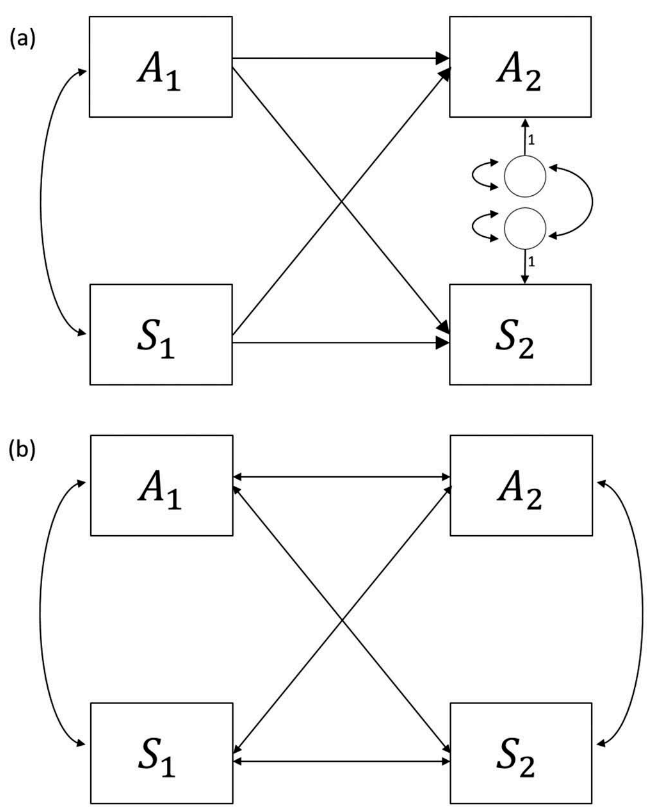


FIGURE 5 A simplified graphical representation of a bivariate process with two waves: (a) the cross-lagged panel model and (b) the contemporaneous and lagged (cross-)correlations.

[6] In case of longitudinal single-subject data, $\boldsymbol{R}_{YY}$ and $\boldsymbol{R}_{XX}$ will asymptotically equate the covariance matrix of the standardized data including all T_s measurements (i.e., the stationary/contemporaneous covariance matrix).

Thus, a meta-analysis combing standardized lagged effect parameters from multiple longitudinal studies can also be based on studies reporting correlations between the current and lagged observations. In terms of panel data, one needs the correlation for the variables themselves on the first wave and also that of the second wave, and the correlations for each variable on wave 1 with another variable on wave 2. Notably, in case of a bivariate model (i.e., a 2×2 lagged effects matrix), one needs six correlations (cf. Figure 5b), that is, the number of lower-triangular elements in a 4×4 correlation matrix of the four variables (i.e., the two on the second wave and the two on the first wave).

As an example, consider the longitudinal study of Innstrand, Langballe, Espnes, Falkum, and Aasland (2008), which is included in the meta-analysis study of Nohe et al. (2015). One can deduce from their Table 1 (and using $\Delta t = 2$ years) that for the two variables of interest (WFC and FWC) the lower triangular correlation matrix with two current and four lagged correlations is:

$$\begin{bmatrix} 1 & & & \\ r_{FWC_{T2},WFC_{T2}} & 1 & & \\ r_{WFC_{T1},WFC_{T2}} & r_{WFC_{T1},FWC_{T2}} & 1 & \\ r_{FWC_{T1},WFC_{T2}} & r_{FWC_{T1},FWC_{T2}} & r_{FWC_{T1},WFC_{T1}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ 0.40 & 1 & & \\ 0.63 & 0.31 & 1 & \\ 0.34 & 0.63 & 0.41 & 1 \end{bmatrix}$$

In the example above, we had only two waves and many persons. There is also the type of longitudinal study where there are multiple persons and a few waves (i.e., a few measurement occasions). In that case, the mean correlations averaged over pairs of consecutive waves are used in the formulas above to determine the standardized lagged effect parameters. Thus, in a trivariate model with three waves, one obtains the six correlations for waves 1 and 2 and the six for waves 2 and 3. Then, one takes the average of these two values for each of the six correlations. Note that one should not include the correlations based on wave 1 with that of wave 3. Only consecutive pairs should be taken into account, because of the time-interval problem. From these six mean correlations (or better, the 3×3 mean correlation matrix), the standardized lagged effect parameters can be calculated using Equation 5. These equations are incorporated in the R package *CTmeta* and in a Shiny applications, see Appendix C.

APPENDIX B. COVARIANCE MATRIX OF STANDARDIZED LAGGED EFFECTS

In this appendix, we will derive what the expression is for the covariance matrix of a standardized lagged effects matrix.

B.1 Covariance Matrix of a DT Lagged Effect

To perform a meta-analysis we also need, besides the (transformed) standardized lagged effects, the variances (or covariance matrix) of the (transformed) standardized lagged effect parameters; namely, as a weight for the overall standardized lagged effect parameters but also for the standard errors (or covariance matrix) of the overall standardized lagged effect parameters. In case a DT-VAR(1) model is conducted, the standard errors (i.e., the square root of the variances) of the standardized lagged effect parameters will be reported. In contrast, in case a study reports (mean) contemporaneous and lagged correlations which can be transformed to standardized lagged effect parameter (see Appendix A), the standard errors or variances of the standardized lagged effect parameters are not directly available. However, one can calculate them.

As will be shown in the next subsection, in general, the variance expression for a standardized lagged effect estimate in Study s (i.e., for $\hat{\phi}_{jk,s}(\Delta t_s)$) is: with $\boldsymbol{R}_{..,s}$ a matrix of (mean) correlations for Study s between two vectors/matrices mentioned in the subscript—where $\boldsymbol{Y}$ refers to the current observation and $\boldsymbol{X}$ to the lagged ones—and $(\boldsymbol{M})_{dd}$ the dth diagonal element of matrix $\boldsymbol{M}$. The square root of $var(\hat{\phi}_{jk,s}(\Delta t_s))$ is the standard error of $\hat{\phi}_{jk,s}(\Delta t_s)$. Note that for the transformed parameters, one should use $var(\hat{\phi}_{jk,s}(\Delta t^*))$.

B.1.1 Derivation of Variance Expression

In this subsection, we derive the expression for $var(\hat{\phi}_{jk,s}(\Delta t_s))$ and $var(\hat{\phi}_{jk,s}(\Delta t^*))$. For Study s, the q parameters in the jth DT-VAR(1) equation can be determined by:

$$\begin{aligned} \hat{\phi}_{j.,s}(\Delta t_s) &= (\hat{\phi}_{j1,s}(\Delta t_s), \ldots, \hat{\phi}_{jq,s}(\Delta t_s))' \\ &= \boldsymbol{R}_{XX,s}^{-1}(\boldsymbol{R}_{XY,s}(\Delta t_s))_j, \end{aligned}$$

where $(\boldsymbol{R}_{XY,s}(\Delta t_s))_j$ denotes column j in $\boldsymbol{R}_{XY,s}(\Delta t_s)$ and contains elements $(\boldsymbol{R}_{XY,s}(\Delta t_s))_{1j}$ to $(\boldsymbol{R}_{XY,s}(\Delta t_s))_{qj}$. In regression, the covariance matrix of the parameter estimates for one outcome is $\sigma_\varepsilon^2\ (\boldsymbol{X}'\boldsymbol{X})^{-1}$, where σ_ε^2 is the residual variance. In terms of correlations, $(\boldsymbol{X}'\boldsymbol{X})^{-1}$ equals $\boldsymbol{R}_{XX,s}^{-1}/(N_s - 1)$, where N_s is the number of observations in the regression (where N_s should be replaced by $T_s - 1$ in case of longitudinal single-subject data). Using this, it follows that the sampling covariance matrix of $\hat{\phi}_{j.,s}(\Delta t_s)$ is:

$$var(\hat{\phi}_{j.,s}(\Delta t_s)) = (\sigma_{\varepsilon,s,j}(\Delta t_s))^2\ \frac{\boldsymbol{R}_{XX,s}^{-1}}{N_s - 1},$$

with $(\sigma_{\varepsilon,s,j}(\Delta t_s))^2$ the residual variance for the jth outcome in Study s, which also depends on the used time interval Δt_s. Note that $var(\hat{\phi}_{j.,s}(\Delta t_s))$ is a $q \times q$ matrix for the jth outcome. The sampling variance of $\hat{\phi}_{jk,s}(\Delta t_s)$ is the kth diagonal of this matrix (i.e., $(\sigma_{\varepsilon,s,j}(\Delta t_s))^2\ (\boldsymbol{R}_{XX,s}^{-1})_{kk}/(N_s - 1)$); and, evidently, its square root is the the standard error of $\hat{\phi}_{jk,s}(\Delta t_s)$. Furthermore, the vector of residual variances is determined by:

$$\begin{aligned} (\boldsymbol{\sigma}_{\in,\mathbf{s}}(\Delta t_s))^2 &= ((\sigma_{\in,s,1}(\Delta t_s))^2, \ldots, (\sigma_{\in,s,q}(\Delta t_s))^2) \\ &= \frac{diag(\boldsymbol{R}_{YY,s} - \boldsymbol{R}_{YX,s}(\Delta t_s)\boldsymbol{R}_{XX,s}^{-1}\boldsymbol{R}_{XY,s}(\Delta t_s))\ (N_s - 1)}{N_s - q}, \end{aligned}$$

where $diag(\boldsymbol{M})$ denotes the diagonal elements of matrix $\boldsymbol{M}$. These two equations render:

$$var(\hat{\phi}_{jk,s}(\Delta t_s)) = \frac{\sigma_{\varepsilon,s,j}(\Delta t_s))^2\ (\boldsymbol{R}_{XX,s}^{-1})_{kk}}{N_s - q},\ \text{with}$$

$$\sigma_{\varepsilon,s,j}(\Delta t_s))^2 = ((\boldsymbol{R}_{YY,s} - \boldsymbol{R}_{YX,s}(\Delta t_s)\boldsymbol{R}_{XX,s}^{-1}\boldsymbol{R}_{XY,s}(\Delta t_s))_{jj}.$$

$$var(\hat{\phi}_{jk,s}(\Delta t_s)) = \frac{(\boldsymbol{R}_{YY,s} - \boldsymbol{R}_{YX,s}(\Delta t_s)\boldsymbol{R}_{XX,s}^{-1}\boldsymbol{R}_{XY,s}(\Delta t_s))_{jj}\ (\boldsymbol{R}_{XX,s}^{-1})_{kk}}{N_s - q},$$

Thus, in case the standard errors of the standardized lagged effect parameters are not given but the correlation matrix is, one can determine the standard errors or variances of the elements in $\boldsymbol{\Phi}_s(\Delta t_s)$. From the formula above, it is clear that the variances of the elements in the DT-VAR (1) standardized lagged effect parameter matrix do not (asymptotically) equate to $1/(N_s - 3)$ – an expression used by some – and that they depend on the time interval as well.

In case one wants to calculate the variances of the transformed standardized lagged effect parameters, one should make use of $\boldsymbol{R}_{XY,s}(\Delta t) = \boldsymbol{R}_{XX,s}\boldsymbol{\Phi}_s(\Delta t)'$. When transforming to $\boldsymbol{\Phi}_s(\Delta t^*)$, this results in:

$$var(\hat{\phi}_{jk,s}(\Delta t^*)) = var(\hat{\phi}_{jk,s}(\Delta t_s))\ \frac{(\sigma_{\varepsilon,s,j}(\Delta t^*))^2}{\sigma_{\varepsilon,s,j}(\Delta t_s))^2}.$$

Note that the expression for the covariance matrix of the vectorized $\boldsymbol{\Phi}_s(\Delta t_s)$ (i.e., $vec\ \hat{\phi}_{j.,s}(\Delta t_s)$) is:

$$var(vec\ \boldsymbol{\Phi}_s(\Delta t_s)) = \frac{\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s)\ \otimes\ R^{-1}_{XX,s}}{N_s - q},\ \text{with}$$

$$\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s) = \boldsymbol{R}_{YY,s} - \boldsymbol{R}_{YX,s}(\Delta t_s)\boldsymbol{R}^{-1}_{XX,s}\boldsymbol{R}_{XY,s}(\Delta t_s),\ \text{stated otherwise}$$

$$cov(\hat{\phi}_{jk,s}(\Delta t_s), \hat{\phi}_{lp,s}(\Delta t_s)) = \frac{(\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s))_{jl}\ (\boldsymbol{R}^{-1}_{XX,s})_{kp}}{N_s - q},$$

where $\otimes$ denotes a Kronecker product:

$$\boldsymbol{\Sigma}\ \otimes\ \boldsymbol{\Gamma}^{-1} = \begin{bmatrix} \sigma_{11}\ \boldsymbol{\Gamma}^{-1} & \cdots & \sigma_{1q}\ \boldsymbol{\Gamma}^{-1} \\ \vdots & \ddots & \vdots \\ \sigma_{q1}\ \boldsymbol{\Gamma}^{-1} & \cdots & \sigma_{qq}\ \boldsymbol{\Gamma}^{-1} \end{bmatrix}.$$

Bear in mind that the diagonal elements of the covariance matrix $var(vec\ \boldsymbol{\Phi}_s(\Delta t_s))$ equals the variance of $\hat{\phi}_{jk,s}(\Delta t_s)$, that is, $var(\hat{\phi}_{jk,s}(\Delta t_s))$ and note that $\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s)_{jj} = (\sigma_{\varepsilon,s,j}(\Delta t_s))^2$. Similar to the univariate case, one can transform the covariance matrix $var(vec\ \boldsymbol{\Phi}_s(\Delta t_s))$ to obtain the covariance matrix of the vectorized transformed standardized lagged effect parameters:

$$cov(\hat{\phi}_{jk,s}(\Delta t^*), \hat{\phi}_{lp,s}(\Delta t^*)) = cov(\hat{\phi}_{jk,s}(\Delta t_s), \hat{\phi}_{lp,s}(\Delta t_s))\ \frac{(\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t^*))_{jl}}{(\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s))_{jl}}.$$

Stated otherwise,

$$var(\hat{\boldsymbol{\Phi}}_s(\Delta t^*)) = var(\hat{\boldsymbol{\Phi}}_s(\Delta t_s))(\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t^*) \oslash \boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s)) \otimes \boldsymbol{1}_q,$$

with $\oslash$ the elementwise division and $\boldsymbol{1}_q$ a $q \times q$ matrix of 1s.

Note that $\boldsymbol{R}_{XX,s}$ and $\boldsymbol{R}_{YY,s}$ will asymptotically equate, and they will be equal to the contemporaneous correlations matrix, that is, the stationary correlation matrix ($\boldsymbol{\Gamma}$). In that case and using $\boldsymbol{R}_{XY,s}(\Delta t_s) = \boldsymbol{R}_{XX,s}\boldsymbol{\Phi}_s(\Delta t_s)'$, the matrix $\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s)$ reduces to $\boldsymbol{\Gamma} - \boldsymbol{\Phi}_s(\Delta t_s)\ \boldsymbol{\Gamma}\ \boldsymbol{\Phi}_s(\Delta t_s)'$, which is the residual covariance matrix in the DT-VAR(1) model. Consequently, when knowing or $\boldsymbol{R}_{YY,s}$, $\boldsymbol{R}_{XY,s}(\Delta t_s)$, and $\boldsymbol{R}_{XX,s}$ or $\boldsymbol{\Phi}_s(\Delta t_s)$ and $\boldsymbol{\Sigma}_{\varepsilon,s}(\Delta t_s)$, or $\boldsymbol{\Phi}_s(\Delta t_s)$ and $\boldsymbol{\Gamma}_s$, one can calculate the covariance matrix of $\boldsymbol{\Phi}_s(\Delta t_s)$ and that of its transformation $\boldsymbol{\Phi}_s(\Delta t^*)$. These equations are incorporated in the R package *CTmeta* and in two Shiny applications; for more details see Appendix C.

B.2 Covariance Matrix of the Overall DT Lagged Effect

Until now, we inspected the sampling (co)variances for the standardized lagged effect parameters of one study. We can also inspect the sampling (co)variances for the overall standardized lagged effect parameters. In case the full covariance matrix of the vectorized overall standardized lagged effect parameter is estimated ($var(vec\ \hat{\boldsymbol{\Phi}}(\Delta t^*))$), as in the multivariate/GLS approach, one can also calculate the covariance matrix of the overall standardized lagged effect parameter for other time intervals Δt using the formula above (i.e., without doing a meta-analysis per time interval):

$$var(vec\ \hat{\boldsymbol{\Phi}}(\Delta t)) = \hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t)\ \otimes\ \boldsymbol{\Gamma}^{-1}_*,\ \text{with} \tag{6}$$

$$\hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t) = \boldsymbol{\Gamma}_* - \hat{\boldsymbol{\Phi}}(\Delta t)\boldsymbol{\Gamma}_*\hat{\boldsymbol{\Phi}}(\Delta t)',\ \text{and}$$

$$\boldsymbol{\Gamma}^{-1}_* = var(vec\ \hat{\boldsymbol{\Phi}}(\Delta t^*))[1{:}q, 1{:}q],$$

where the latter means the $q \times q$ upper left submatrix of the full covariance matrix of the vectorized overall standardized lagged effect parameters. This comes down to:

$$var(\hat{\phi}_{jk}(\Delta t)) = var(\hat{\phi}_{jk}(\Delta t^*))\ \frac{(\hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t))_{jj}}{(\hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t^*))_{jj}}.$$

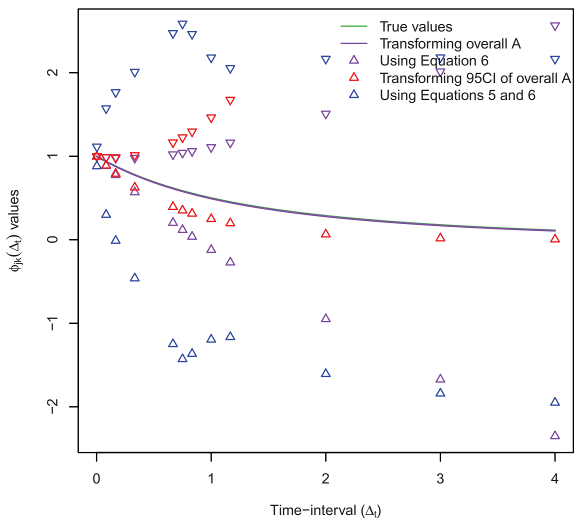
Hence, the correction is the same for the standardized lagged effect parameters within one outcome (i.e., within outcome j), but differ over outcomes. Notably, in case one only has the variances of the overall standardized lagged effect parameters (as is the case when doing a univariate/WLS meta-analysis), one cannot transform the variances of the overall standardized lagged effect parameters as if another time interval was used. In that case, one should do a meta-analysis per time interval.

When trying this out, the overall covariance matrix $var(\hat{\phi}_{jk}(\Delta t^*))$ deviated a bit from the form '$\boldsymbol{\Sigma}\ \otimes\ \boldsymbol{\Gamma}^{-1}$'. For example, the elements in $var(vec\ \hat{\boldsymbol{\Phi}}(\Delta t^*))[1:q,\ q+1:2*q]/var(vec\ \hat{\boldsymbol{\Phi}}(\Delta t^*))[1:q, 1:q]$ resemble a lot, but do not equate. One can take then the average of these elements as value for $(\hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t))_{12}$; and do this for all elements in $\hat{\boldsymbol{\Sigma}}^*_{\varepsilon}(\Delta t)$. More research might be needed and, therefore, we conducted multivariate meta-analyses per time interval in the example and simulation study (discussed in the main text).

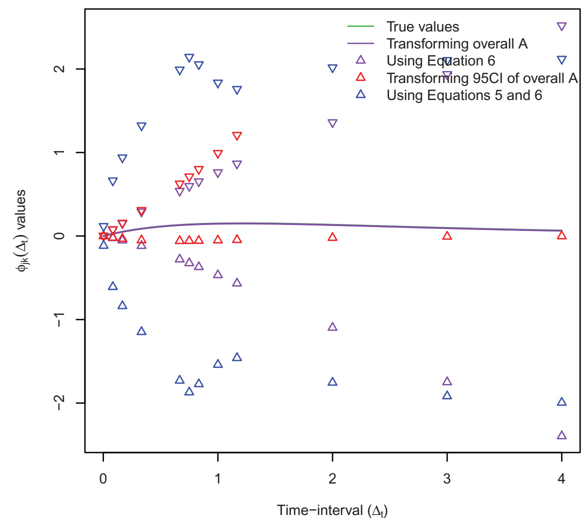
APPENDIX C. ADDITIONAL TOOLS FOR PRACTICAL USE

There are two Shiny web applications which can help the researcher in applying a meta-analysis on transformed standardized standardized lagged effect parameters. The Shiny web application "Standardizing and/or transforming lagged regression estimates" Kuiper (2018c) calculates the standard errors (and full covariance matrix) of the (un)standardized lagged effect parameter and its transformations. It can do this (for one study) based on contemporaneous and lagged correlations but also based on DT-VAR(1) output: standardized or unstandardized lagged effect estimates and either the stationary or residual covariance matrix. This can then serve as input for a meta-analysis.

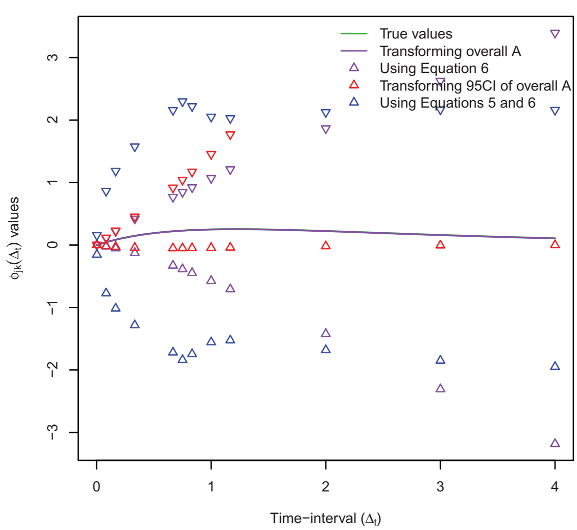
The Shiny application "CT meta-analysis on lagged effects" Kuiper (2018a) can do the same but also directly conducts the meta-analysis on standardized lagged effect parameters (using the *metafor* package) and takes the standardized or unstandardized lagged effect estimates and either the stationary or residual covariance matrix as input but now for multiple studies. You can choose whether you want to conduct a RE of FE model. By default it performs a FE GLS meta-analysis model (using the rma.mv function) on the transformed standardized lagged effect parameters. In case the standardized lagged effect parameters cannot be transformed, it will use the original ones and add dummy variables to the meta-analysis model to account for



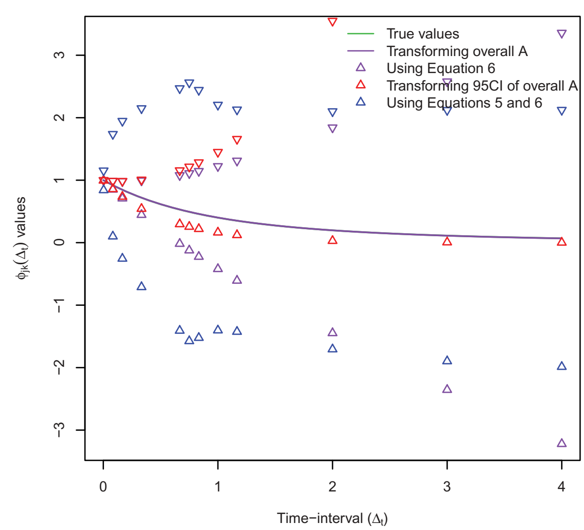
(a) $\phi_{11}(\Delta t)$ and $\hat{\phi}_{11}(\Delta t)$



(b) [$\phi_{12}(\Delta t)$ and $\hat{\phi}_{12}(\Delta t)$



(c) (1,1)$\phi_{21}(\Delta t)$ and $\hat{\phi}_{21}(\Delta t)$



(d) $\phi_{22}(\Delta t)$ and $\hat{\phi}_{22}(\Delta t)$

FIGURE 6 Subfigures (a–d) display one of the four true $\phi_{jk}(\Delta t)$s (green line) and its estimated overall standardized lagged effect $\hat{\phi}_{jk}(\Delta t)$ when transforming the overall drift matrix (purple line) together with the elliptical 95% confidence intervals of $\mathbf{\Phi}(\Delta t)$ for each used time interval (triangles): one based on small time interval approximation of the covariance matrix in Equation 7 (purple), one on transforming the elliptical 95% confidence intervals of $\boldsymbol{A}$ (red), and one based an another approximation of the covariance matrix by combining Equations 6 and 7 (blue).

the time-interval dependency. When the covariance matrix of the standardized lagged effect parameters is not positive definite (comparable to a negative variance), it will conduct a WLS model (using the rma.uni function).

The R package *CTmeta* of Kuiper (2019) also has these functionalities. For example, with the function ‘CTMA’ one can meta-analyze transformed standardized lagged effects and takes the standardized or unstandardized lagged effect estimates, the stationary, and residual covariance matrix for multiple studies as input. In case of correlations, one can first transform these (per study) to the requested input by using the function ‘calc.TransPhi_Corr’.

APPENDIX D. META-ANALYZING THE DRIFT MATRIX

In the main text, we discuss meta-analyzing transformed standardized lagged effects. Alternatively, one can meta-analyze the underlying standardized drift matrix $\boldsymbol{A}$ (i.e., the CT-VAR(1) lagged effects matrix). For this, the meta-analysis model assumes that the vectorized matrix *vec* $\boldsymbol{A}$ is multivariate normally distributed. Since the vectorized DT lagged effect matrix is, vectorized $\boldsymbol{A}$ is not, but according to the Central Limit Theorem it will asymptotically be so if the number of observations in your sample is large enough. This means that the vectorized drift matrix, $vec\,\boldsymbol{A}$, converges in distribution to a normal distribution with a mean equal to the estimated $vec\,\boldsymbol{A}$ and the covariance matrix equal to the covariance matrix of the estimated $vec\,\boldsymbol{A}$ when the sample is large enough.

Furthermore, we need the covariance matrix of $vec\,\boldsymbol{A}$ in each Study s. One can use the following approximation:

$$\begin{aligned} var(vec\,\boldsymbol{A}_s) &\approx var(vec\,((\boldsymbol{\Phi}_s(\Delta t_s) - I)/\Delta t)) \text{ when time interval } \Delta t \text{ goes to } 0 \\ &= var(vec\,(\boldsymbol{\Phi}_s(\Delta t_s)))/(\Delta t)^2. \end{aligned}$$

This approximation is based on the following approximation of $\boldsymbol{\Phi}(\Delta t)$.

$$\begin{aligned} \boldsymbol{\Phi}(\Delta t) &= \exp(\boldsymbol{A}\;\Delta t) \\ &= \boldsymbol{I} + \sum_{k=1}^{\infty} \frac{(\boldsymbol{A}\;\Delta t)^k}{k!} \\ &\approx \boldsymbol{I} + (\boldsymbol{A}\;\Delta t) \text{ when time interval } \Delta t \text{ goes to } 0. \end{aligned}$$

When looking at the overall estimates for the example and simulation used in the main text, the overall estimates are almost equal to the true underlying drift matrix.

As we believe that researchers are primarily interested in lagged effects matrices and how they change depending on the time interval, one would want to transform the overall drift matrix to an overall $\boldsymbol{\Phi}(\Delta t)$ for different values of Δt. This can be straightforwardly done by using $\boldsymbol{\Phi}(\Delta t) = \exp(\boldsymbol{A}\;\Delta t)$. When looking at the example and simulation used in the main text, the overall estimates are then almost equal to the true underlying drift matrix. For example, this is plotted by the purple and green lines, respectively, in Figure 6.

One would also want to obtain the elliptical (i.e., multivariate) 95% confidence intervals of the overall $\boldsymbol{\Phi}(\Delta t)$s. This is unfortunately less straightforward. One can, for example, use Equation 7 for this, assuming that this is true for all time intervals (so, also for not infinitesimally small ones). Or one can transform the elliptical 95% confidence intervals of the drift matrix (and perhaps also make use of other matrices that render the same likelihood). Alternatively, one can use Equation 6 at the end of Appendix B combined with Equation 7. All three methods render very wide elliptical 95% confidence intervals in the example (see purple, red, and blue triangles, respectively, in Figure 6) and simulation used in the main text. Hence, more research is needed.