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Oil Price Changes and U.S. Real GDP Growth: Is this Time Different?[☆]

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Abstract

This paper contributes to the large debate regarding the impact of oil price changes on U.S. GDP growth. Firstly, it replicates empirical findings of prominent studies and finds that the proposed oil price measures have a dissipating effect with recent data up to 2016Q4. Secondly, it re-examines the issue and provides evidence that oil price decreases affect the GDP growth, when taking into consideration mixed data sampling technique. Finally, it puts particular focus on nonlinearity and a possible instability and shows that combining Markov switching and mixed data sampling models allows to identify different regimes permanently changing with the Great Moderation.

Keywords: Oil prices, GDP growth, Asymmetry, Nonlinearity, Markov switching models, Mixed Data Sampling

JEL classification: C24; E32; F43; Q43

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1. Introduction

This paper examines the relationship between oil price changes and U.S. real gross domestic product (GDP) growth.

We revisit and extend the findings of five highly quoted articles (Hamilton, 1983, Mork, 1989, Lee et al., 1995, Hamilton, 1996, 2003) and contrast them with more recent advances. We replicate these results on their respective data range.¹ Then, we update the data to 2016Q4 and re-estimate all models. The results suggest that all implemented measures explain the relationship insufficiently with recent data. Applying a novel model that incorporates regime switching and mixing of different resolutions of data, we detect several breakpoints. These breakpoints correspond to major events and we find that the *Great Moderation* is a major change point. By incorporating monthly oil price observations to explain quarterly real GDP growth, we provide evidence that higher frequency data contributes to understanding the oil price-GDP link with regime-specific reactions.

Hamilton (1983) provides evidence that oil shocks between 1948 and 1972 are a contributing factor to U.S. recessions measured as Gross National Product (GNP). The correlation of preceding price shocks and recessions is found to be significant. Mork (1989) extends the data to 1988 and the oil price measure is adjusted to account for increases and decreases separately. It is found that oil price increases are negatively correlated to GNP growth, which is in line with the findings in Hamilton (1983). In addition, an asymmetric response is significant and oil price decreases are found to have little, if not zero, correlation to GNP growth. Lee et al. (1995) review these results and update the data until 1992. The vector autoregressive (VAR) model applied in Hamilton (1983) and Mork (1989) is augmented with a normalized oil shock variable to account for the general variability of real oil prices. It is found that in stable environments, oil price changes have a more pronounced effect on real GNP growth than in times of erratic oil price movements. In response to the findings of Hooker (1996), which are contradicting to Hamilton (1983) and Mork (1989) on more recent data, Hamilton (1996) proposes a new measure of real oil price changes that accounts for a phenomenon observed after 1986:

¹These results are found in the Appendix.

almost all increases of real oil prices are corrections to declines in the preceding quarters. Increases in the proposed net oil price measure are negatively related to GDP growth for the full sample from 1948 to 1994 and on a weaker scale, yet significant, from 1973 to 1994. This relationship is revisited by Raymond & Rich (1997) who employ a two-state Markov switching model for net oil increases. It is found that between 1951 and 1994, oil price shocks have, to a certain extent, an effect on the unconditional mean of low-growth rate regimes, while it is also noted the effect of net oil price increases might be overstated. This net oil measure is adjusted in Hamilton (2003) and tested with a data set spanning 1948 to 2001.

Since the publication of the results in Hamilton (1983), the relationship between oil prices and the U.S. economy has been subject to a large and controversial debate. The presence of a negative and significant impact of oil price *increases* on economic activity is one of the main topics of academic discussion. There seems to be some consensus on the absence of an impact of oil price *decreases* on the the macroeconomy. However, literature questioning an ex ante separation of oil price changes into positive and negative components is constantly growing and challenging findings of earlier studies.

With the introduction of the net oil price increase applied in Hamilton (1996) and further refinements in Hamilton (2003), the GDP growth and oil price shock relationship seems to be rectified on more recent data. This measure is adapted or further tested in a vast amount of literature, e.g. Lee & Ni (2002), Jimenez-Rodriguez & Sanchez (2005), Herrera & Pesavento (2009), Herrera et al. (2015); with the consistent result that the signed or censored measure for oil price increases has a significant relationship to GDP growth declines. Davis & Haltiwanger (2001) find an asymmetric response of employment growth rates to oil price shocks; employment growth declines after oil price increases, whereas there is only little reaction to oil price declines. Other macroeconomic factors are addressed in Kilian (2008). In the same paper, the notion of distinguishing between supply and demand shocks, already mentioned in Barsky & Kilian (2004), is renewed. Barsky & Kilian (2004) also find that oil price shocks are not necessarily the cause of recessions, but are a contribution and oil shocks cannot explain stagflation in real GDP.

Kilian (2009) highlights the importance of including structural demand and supply shocks in modeling any relationship to the GDP as they have different impact on U.S. economy. Adding to the dissent, Kilian & Vigfusson (2011a) challenge the usage of censored oil price measures (such as the net oil price increase or any other ex ante asymmetric data manipulation to that end) and suggest a more encompassing VAR model. Kilian & Vigfusson (2011b) review existing evidence of asymmetries with a refined test and conclude that only abnormally large innovations might have an asymmetric effect, where this could also be of spurious nature. In general, these papers conclude that detected asymmetries could be artifacts of the censoring of oil price data and subsequent slope-based tests. Also, data is restricted to start in 1973 and it is found that—in some contrast to other papers—there are no break or change points in the oil-GDP relationship in the remainder of the data.

In the recent decade, oil prices are extremely volatile compared to earlier data samples. In 2007, oil prices sharply increase to US\$145/bbl just to plummet to under US\$40/bbl in 2008 in the wake of the financial crisis. The reasons for this shock are manifold, however, as analyzed in Hamilton (2009) and Kilian & Hicks (2013); the main contributors to the strong increase are positive demand shocks from emerging countries and their rapid economic growth. The shale oil revolution leads to an increasing spread between the WTI and European Brent between 2011-2014, ending in a price collapse in 2014, presumably caused by an overproduction, high stock levels, and declining demand (Baumeister & Kilian, 2016, Klein, 2018). Non-linearities play an increasing role in recent data (Jimenez-Rodriguez, 2009, Kilian & Vigfusson, 2013) and structural changes in the oil price-GDP relationship seem present (Blanchard & Riggi, 2013), whereas the most recent shocks motivate further research (Bodenstein et al., 2011, Kilian, 2014). However, a possible instability has long been present in literature. It is considered a major contributor to the misspecification of the impact oil price changes on the real GDP growth (Blanchard & Gali, 2007). Other studies present evidence for the existence of structural breaks without questioning the model framework in principal, including Hamilton (1989) and Hamilton (2003). Perron (1989) suggests that the oil price shock of 1973 causes a permanent change

in the growth rate of post-war quarterly U.S. GNP. Blanchard & Gali (2007) show that the volatility of the real GDP growth has significantly decreased since the mid 1980's. The authors find that the first quarter of 1984 corresponds to structural changes in the GDP-oil relationship which is already reported in McConnel & Perez-Quiros (2000). This period is connected to an economic phenomenon labeled *Great Moderation* which affected economic cycles by a stabilization and reduction of volatility (e.g. Herrera & Pesavento, 2009).

We contribute to this discussion by finding evidence of structural changes in the relationship. We show that these effects are still present when including a higher frequency of monthly oil price measures within the Mixed Data Sampling (MIDAS) framework. Literature shows that MIDAS is capable to forecast quarterly GDP growth based on higher frequency information. Clements & Galvão (2008) find that MIDAS is a suitable tool to forecast quarterly GDP growth by using monthly data from common indicators. Kuzin et al. (2011) compare the MIDAS approach with the mixed frequency VAR and conclude that both approaches are helpful to forecast the GDP depending on the horizon. We compare the classical model frameworks with regime-switching modifications of quarterly and monthly oil prices. With periods of both strong increases and decreases, the most recent years provide an interesting foundation to test the different price measures and the postulated theories on extreme price movements compared to the original literature.

The remainder of this article is structured as follows. The methodological framework and different oil price measures are defined in Section 2. Section 3 introduces our data sets and preliminary tests thereof. In Section 4, empirical results are presented and discussed. Section 5 concludes this article.

2. Empirical Methodology

2.1. Oil Price Measures

In addition to the linear oil price measure applied in Hamilton (1983), we implement the four nonlinear oil price measures proposed by Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003). These nonlinear measures are motivated by the assumption

that oil price changes have an asymmetric impact on the macroeconomy in general and on GDP growth rates in particular. Let O_t denote oil price changes defined by the following log-difference

$$O_t = 100 * (\log p_t - \log p_{t-1}),$$

where p_t denotes an oil price at time t , and its resolution—quarterly or monthly—depends on the applied model. Based on O_t , the nonlinear oil price measures are defined as follows. Note that Hamilton (1983, 1996, 2003) uses nominal oil price changes (denoted by O_t^{nom}), whereas Mork (1989) and Lee et al. (1995) use real oil price changes (denoted by O_t^{real}).

2.1.1. Mork's asymmetric approach

The asymmetric measure proposed by Mork (1989) is based on the observation that the significant relationship between oil prices and the macroeconomy presented in Hamilton (1983) pertains to a period of oil price increases and that the large oil price declines of 1985–1986 do not lead to a proportional impact on the macroeconomy as in the case of previous oil price increases. Consequently, Mork (1989) assumes that the impact of oil price changes on the macroeconomy cannot be symmetric and suggests two new measures. The measure for price increases is given by

$$O_{\text{Mork},t}^+ = \max(O_t^{\text{real}}, 0),$$

and analogously for oil price decreases, defined as

$$O_{\text{Mork},t}^- = \min(O_t^{\text{real}}, 0),$$

where O_t^{real} denotes the real oil price changes of the producer price index (PPI) and $O_{\text{Mork},t}^+$ and $O_{\text{Mork},t}^-$ are the positive and negative censored parts of the real oil price changes, respectively.

2.1.2. Volatility scaling of Lee et al.

An alternative measure of oil price changes is proposed by Lee et al. (1995), which is based on a similar assumption presented in Hamilton (1996). Oil price increases are expected to have greater impact on the macroeconomy during periods where oil prices are stable than during periods characterized by high volatility. In these volatile periods, oil price increases are assumed to be an adjustment to previous price decreases and hence, these increases might not affect the macroeconomy on a greater scale. The measure of Lee et al. (1995) bases on the estimation of an AR(p)-GARCH(1,1) model for oil price returns. Then, both oil price increases and decreases are scaled by their volatility to obtain the measures. The AR(p)-GARCH(1,1) model is defined as

$$O_t^{\text{real}} = \mu + \sum_{i=1}^p \alpha_i O_{t-i}^{\text{real}} + e_t,$$

where μ is an unconditional mean, the AR order lag is set to $p = 4$, and α_i are the parameters of the AR model. The error term e_t is modeled as GARCH(1,1) process with variance h_t and reads

$$\begin{aligned} e_t &= \sqrt{h_t} \zeta_t, \\ h_t &= \gamma_0 + \gamma_1 e_{t-1}^2 + \gamma_2 h_{t-1}, \end{aligned}$$

where γ_0 , γ_1 , and γ_2 refer to GARCH parameters and $\zeta_t \sim \mathcal{N}(0, 1)$ i.i.d. for all $t = 1, \dots, n$. The standardized residual $e_t^* = e_t / \sqrt{h_t}$ is then censored and we obtain the positive and negative oil price measures of Lee et al. (1995) by

$$\begin{aligned} O_{\text{LNR},t}^+ &= \max(0, e_t^*), \quad \text{and} \\ O_{\text{LNR},t}^- &= \min(0, e_t^*). \end{aligned}$$

2.1.3. Hamilton's adjusted measures

The measure of Hamilton (1996) is based on the empirical observation that since 1985, most of the oil price increases have been followed by oil price decreases in the subsequent one to four quarters. Consequently, the oil price measure is defined as the difference of the increase of *nominal* oil prices and the maximum of increases during the previous year. This measure is denoted by $O_{\text{Ham1},t}^+$ and formally defined as

$$O_{\text{Ham1},t}^+ = \max \{0, O_t - \max \{O_{t-1}, O_{t-2}, O_{t-3}, O_{t-4}\}\}. \quad (1)$$

In this study, we also consider the case of net oil price decreases which then reads

$$O_{\text{Ham1},t}^- = \min \{0, O_t - \min \{O_{t-1}, O_{t-2}, O_{t-3}, O_{t-4}\}\}.$$

Notably, this negative part is neither introduced nor tested in the original source. It is taken into consideration in later studies such as Kilian & Vigfusson (2011b).

Hamilton (2003) observes that net oil price increases that follow the massive price declines during the Asian crisis of 1997–1998 do not cause consumers and firms to postpone their spending plans. In response to this observation, Hamilton (2003) suggests an adjustment of the measure $O_{\text{Ham1},t}^+$ defined in Eq. (1) to account for longer reaction times within a three year window. This adjusted measure is denoted $O_{\text{Ham3},t}^+$ and defined analogously as

$$O_{\text{Ham3},t}^+ = \max \{0, O_t - \max \{O_{t-1}, O_{t-2}, \dots, O_{t-12}\}\},$$

which now spans twelve quarters. The net decrease reads

$$O_{\text{Ham3},t}^- = \min \{0, O_t - \min \{O_{t-1}, O_{t-2}, \dots, O_{t-12}\}\},$$

which is—again—not tested in the original article. Note that the notation of the above formulas refer to quarterly data. When using *monthly* observations, the one and three year oil price measures correspond to 12 and 36 months.

2.2. Testing for nonlinearity in the oil price-real GDP relationship

Two important issues regarding the oil-GDP relationship arise from empirical literature, in particular in more recent research. Firstly, is the relationship between oil price changes and the real GDP growth linear or nonlinear? And secondly, if the assumption of linearity is ruled out, then how does one decide which nonlinear functional form should be used? To answer these two questions, a new framework is outlined in Hamilton (2001, 2003). We use this approach to examine nonlinearity (question 1) and to determine the choice of an appropriate functional form (question 2). Moreover, we use the test extensions developed in Dahl & González-Rivera (2003a,b) for robustness checks. The test framework is defined by

$$y_t = \alpha_0 + \alpha'x_t + \delta'z_t + \lambda m(g \odot x_t) + \epsilon_t, \quad (2)$$

where y_t is the real GDP growth, x_t is a k -dimensional vector of oil price changes with $k = 4$ yielding $x_t = (O_{t-1}, \dots, O_{t-4})'$, z_t is a p -dimensional vector which contains lags in GDP growth and we set $p = 4$ to obtain $z_t = (y_{t-1}, \dots, y_{t-4})'$. Both x_t and z_t are assumed to be stationary and ergodic processes. With this definition, the conditional mean of y_t consists of a linear component given by $\alpha_0 + \alpha'x_t + \delta'z_t$ and the nonlinear component $m(g \odot x_t)$, where $m(\cdot)$ is a realization of a Gaussian random field (see Hamilton (2001, 2003) for more details). The contribution of the nonlinear part in the conditional mean is scaled by λ . The elementwise matrix product, also referred to as Hadamard product, is denoted by \odot .

In order to test for nonlinearity, Hamilton (2001) proposes to test the null hypothesis $H_0 : \lambda = 0$ against the alternative $H_1 : \lambda \neq 0$ using the ν^2 test statistic (Hamilton, 2003). Notably, by using $x_t = (O_{t-1}, \dots, O_{t-4})'$ in the linear part of Eq. (2) testing whether $\lambda = 0$ is the test of the null hypothesis of linearity against the alternative of nonlinearity. For determining an appropriate functional form if H_0 is rejected, we use the four nonlinear oil price measures defined above in the vector x_t in the linear part. However, we use the oil price changes in the nonlinear part of the conditional mean (we refer to Hamilton (2001, 2003) for more details). Under this new specification of Eq. (2), the hypothesis of $\lambda = 0$

captures whether the nonlinear oil price measure used in the linear part of conditional mean is appropriate and covers the nonlinearity between y_t and O_{t-i} for $i = 1, \dots, k$. These tests are carried out in the replication of previous studies and when using Markov switching autoregressive specifications.²

2.3. Econometric models

2.3.1. An extended version of Hamilton's model

Investigating the impact of oil price shocks on the U.S. real GDP growth, we adapt a univariate autoregressive model, outlined in Hamilton (2003), with p lags of the real GDP growth and q lags of the selected oil price measures. The ARX(p) model reads

$$y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_i O_{\{M\},t-i}^+ + u_t, \quad (3)$$

where y_t denotes the real GDP growth and $O_{\{M\},t}^+$ proxies the positively censored oil price changes with $\{M\}$ as a placeholder for the different positive measures defined in the previous subsections, namely $O_{\text{Mork},t}^+$, $O_{\text{LNR},t}^+$, $O_{\text{Ham1},t}^+$, and $O_{\text{Ham3},t}^+$. The parameters ϕ_i are restricted to ensure stationarity while $\delta_i \in \mathbb{R}$. The error term u_t is Gaussian i.i.d.

In a recent study, Kilian & Vigfusson (2011a) show that the use of only positively censored data leads to an overestimation of the impact of oil price changes on the real GDP growth in VAR models. To overcome this problem, we include both positively and negatively censored data of oil prices changes. Hence, we augment Eq. (3) with negative measures and obtain

$$y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_i O_{\{M\},t-i}^+ + \sum_{i=1}^q \gamma_i O_{\{M\},t-i}^- + u_t, \quad (4)$$

where p , q , and $O_{\{M\}}^{+/-}$ are defined as above.

²Note that these two tests are then carried out for each regime as determined by probability smoothing.

2.3.2. Markov switching model for the oil price-GDP growth relationship

Outlined in the literature review, empirical findings based on more recent data suggest that the oil price - real GDP growth relationship is unstable and subject to the presence of structural breaks and changes. We control for this instability by implementing an extension of the original Markov switching model proposed by Hamilton (1989) who shows that the real GNP growth is better described by an MS-AR(4) model where only the intercept switches between regimes. Raymond & Rich (1997) employ a similar model with a two-state mean where transition probabilities are time-varying depending on an AR(4) structure of net increases. We allow the intercept, the oil price measure coefficients, and also the variance of the error term to be regime-dependent, with fixed probabilities, however. We aim to simultaneously disentangle the effects of price changes which might differ over regimes. The extension of Eq. (4) then reads

$$y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_{i,s_t} O_{\{M\},t-i}^+ + \sum_{i=1}^q \gamma_{i,s_t} O_{\{M\},t-i}^- + u_{s_t,t}, \quad (5)$$

where $s_t \in \{0, 1\}$ indicates the regime at time t and $u_{s_t,t} = \sigma_{s_t} \epsilon_t$ with $\epsilon_t \sim N(0, 1)$ i.i.d.

2.3.3. Mixed data sampling regression with Markov switching

Since oil price data is available at higher frequencies than GDP estimates, we consider MIDAS originating from Ghysels et al. (2004). The main advantage of these models is their ability to make use of the information updates within the quarters of a year. The Augmented Distributed Lag (ADL) regression with MIDAS is presented by Andreou et al. (2013) as

$$y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \theta_1 B(L^{1/m}; \boldsymbol{\omega}_1) O_{\{M\},t}^{(m)} + u_t,$$

where $B(L^{1/m}; \boldsymbol{\omega}_1) = \sum_{k=1}^K b(k; \boldsymbol{\omega}_1) L^{(k-1)/m}$ is a lag polynomial with a weighting scheme $b(k; \boldsymbol{\omega}_1)$, weighting parameters $\boldsymbol{\omega}_1$, and $L^{(i)/m} O_t^{(m)} = O_{t-i/m}^{(m)}$ as lag operator with coefficient $\theta_1 \in \mathbb{R}$. Hence, the quarterly data y_t is explained by monthly data $O_{t-i/m}^{(m)}$ with $m = 3$. There are several options for the weighting scheme. We choose the flexible beta distribution with $\boldsymbol{\omega}_1 = (\omega_{11}, \omega_{12})' \in \mathbb{R}_{>0}^2$ introduced in Ghysels et al. (2007),

$$b(k; \boldsymbol{\omega}_1) = \frac{\left(\frac{k}{K-1}\right)^{\omega_{11}-1} \left(1 - \frac{k}{K-1}\right)^{\omega_{12}-1}}{\sum_{i=0}^{K-1} \left(\frac{i}{K-1}\right)^{\omega_{11}-1} \left(1 - \frac{i}{K-1}\right)^{\omega_{12}-1}},$$

which allows for declining, increasing, and hump-shaped weighting schemes for K monthly lags of oil price measures. We add a second MIDAS regressor to account for the asymmetric oil price measures in analogy to Eq. (4) yielding

$$y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \theta_1 B(L^{1/m}; \boldsymbol{\omega}_1) O_{\{M\},t}^{+, (m)} + \theta_2 B(L^{1/m}; \boldsymbol{\omega}_2) O_{\{M\},t}^{-, (m)} + u_t, \quad (6)$$

with $\boldsymbol{\omega}_2 = (\omega_{21}, \omega_{22})' \in \mathbb{R}_{>0}^2$ analogously to $\boldsymbol{\omega}_1$ defined above and $(\theta_1, \theta_2)' \in \mathbb{R}^2$. By using $K = 12$ months, we implement the exact time scale of four quarters as used in the previous models. However, due to the MIDAS approach the intra-quarter information at monthly frequency are available.

Finally, we apply the Markov switching variant of the MIDAS framework as in Guérin & Marcellino (2013). The equation for the censored oil price measures including both measures reads

$$y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \theta_{1,s_t} B(L^{1/m}; \boldsymbol{\omega}_{1,s_t}) O_{\{M\},t}^{+, (m)} + \theta_{2,s_t} B(L^{1/m}; \boldsymbol{\omega}_{2,s_t}) O_{\{M\},t}^{-, (m)} + u_{t,s_t}, \quad (7)$$

which is the regime-switching equivalent to Eq. (6).

3. Data set

For reason of comparison, we use both nominal oil price changes as in Hamilton (1996, 2003), and the real oil price changes as in Mork (1989), Lee et al. (1995), and Kilian & Vigfusson (2011b) for example. The *initial* data set covers a period of 70 years ranging from 1947Q2 to 2016Q4 and provides a total number of $T = 279$ quarterly observations. At a glance, there is a change in the behavior of oil prices in the early 1970s and again in the 1980s, which should be taken into consideration for choosing the respective data range. In the early years prior to 1973, the oil price behaves more like a step function

rather than a freely fluctuating price.³ Fig. 1 plots real oil price changes and NBER recessions between the 1947 and 2016. A more detailed history with contributing factors can be found in Barsky & Kilian (2004) and Hamilton (2011).

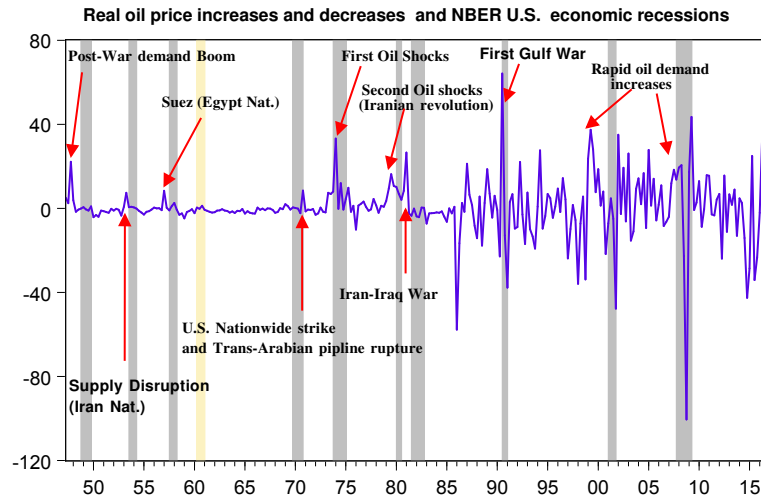


Figure 1: Crude oil price increases and decreases (solid lines) and U.S. recessions (shaded areas). Source: Authors calculations and NBER data.

We begin with a stationary test of the initial sample with the assumption of breaks under the null hypothesis. To this end, we use the unit root test of Kapetanios (2005) which tests the null hypothesis of the existence of a unit root against the alternative of stationarity with $\tilde{k} \leq 4$ unspecified breaks. The test results are reported in Tab. 1. As the real GDP growth does not show any evidence of a trend, we report the results where only the intercept is subject to changes. We draw several important conclusions from the results obtained. Firstly, for all tested $\tilde{k} = 1, \dots, 4$, the most important change point is 1973Q2. Secondly, the results hint towards stationarity under the presence of breaks. This result is of particular interest in the context of the following MS-ARX and MS-MIDAS models, as it shows that the real GDP growth is stationary and subject to structural breaks, which further motivates the application of Markov switching variants. Interestingly, the second most important break corresponds to the *Great Moderation* beginning in 1984. When applying regime-switching models in subsequent sections, we again identify these change

³We thank Lutz Kilian for additional insight to this issue, which is also extensively addressed in Kilian & Vigfusson (2011b).

Table 1: Results of the unit root test of Kapetanios (2005) with structural breaks for the real GDP growth for the period 1947Q2 to 2016Q4.

	Number of Breaks (m)			
	$\tilde{k} = 1$	$\tilde{k} = 2$	$\tilde{k} = 3$	$\tilde{k} = 4$
Kapetanios statistic	-5.592***	-6.966***	-6.966***	-6.966***
Critical value (5%)	(-4.354)	(-5.036)	(-5.234)	(-5.367)
Break points	[1973Q2]	[1973Q2; 1983Q3]	[1973Q2; 1983Q3; 2000Q3]	[1958Q2; 1973Q2; 1983Q3; 2000Q3]

Note: Given the sample size $T = 279$ and a trimming value of 15%, it is possible to test for the case $\tilde{k} \leq 4$ only. *, **, *** refer to the 10%, 5% and 1% levels of significance.

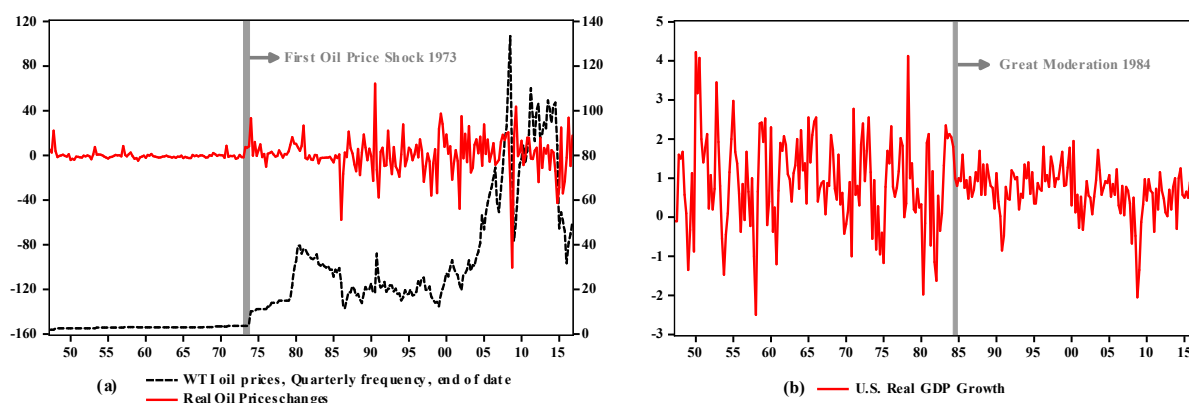


Figure 2: (a) Crude oil price in level and in first differences (changes), and (b) U.S. real GDP growth. Shaded areas correspond to the first oil shock of 1973 and the U.S. Great Moderation of 1984.

points.

Given the behavior of oil prices prior to 1973, the important break point in GDP growth in the same year, and findings and suggestions from recent literature, we restrict our sample period to 1973Q1 to 2016Q4 visualized in Fig. 2. This truncation yields 176 quarterly observations. Fig. 3 depicts the evolution of the different oil price measures in our restricted sample starting 1973. In addition we find all oil price measure with quarterly and monthly frequency to be stationary. The results can be found in the Appendix.

We use quarterly data for the standard linear $ARX(p)$ model and $MS(k)$ - $ARX(p)$ with $p = 4$ and $k = 2$.⁴ For the standard $MIDAS(m)$ and $MS(k)$ - $MIDAS(m)$, we mix quarterly frequency for the real GDP growth and monthly frequency for oil price changes. The selected range 1973Q1 to 2016Q4 provides $T = 531$ monthly observations. The quarterly growth rate of chain-weighted real GDP is collected from the U.S. Bureau of Economic Analysis (BEA) and the nominal crude oil producer price index (PPI) is collected from

⁴For explanation of the optimal values of k and p see sections 4.2 and 4.4

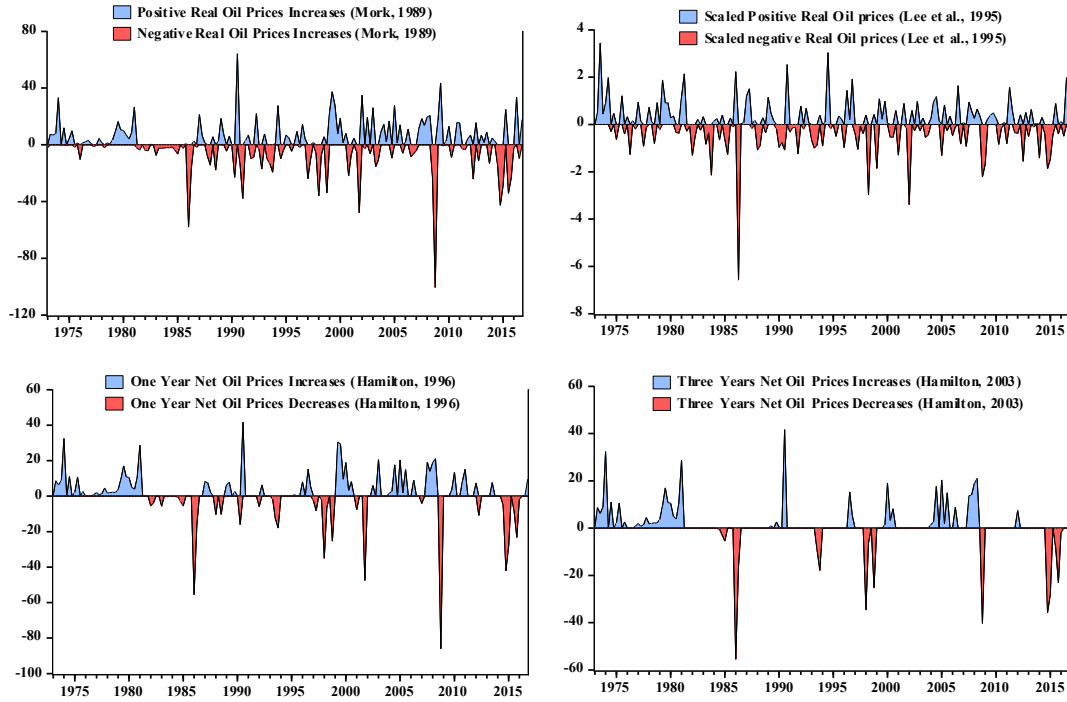


Figure 3: Nonlinear oil price measures from 1947Q2 to 2016Q4 of Mork (1989) (*top left*), Lee et al. (1995) (*top right*), Hamilton (1996) (*bottom left*), and Hamilton (2003) (*bottom right*) applied throughout the paper.

U.S. Bureau of Labor Statistics (BLS). For the $ARX(p)$ and $MS(k)$ - $ARX(p)$ models the PPI is seasonally unadjusted and obtained by converting the monthly data to quarterly data by using end-of-period values. Similar to Hamilton (2003), we use the percentage change of the GDP deflator to convert nominal quarterly changes in oil prices to real ones. For the monthly oil price changes, we use the Consumer Price Index obtained from the St. Louis FED at a monthly frequency to convert the nominal prices to the real prices.⁵

4. Results

4.1. Replication of previous studies

The objective of this section is to replicate the results of some prominent studies (such as the articles of Hamilton (1983, 1996, 2003), Mork (1989) and Lee et al. (1995)) regarding the oil prices changes - real GDP growth relationship using an identical sample

⁵Data accessed via bea.gov/national/index.htm#gdp (GDP) on 02/16/2017 and fred.stlouisfed.org/series/CPIAUCSL (CPI) on 07/20/2017, respectively. We selected the seasonally unadjusted oil price time series WPU0561 via data.bls.gov/cgi-bin/dsrv?wp on 02/16/2017. The original data, the constructed data, the code, and outputs are available upon request.

period *and* by augmenting recent data up to 2016Q4.⁶ The corresponding replication results are obtained from slightly different data than in the original studies where data is sampled from differing sources. In addition, we examine whether the non-inclusion of negative oil prices changes (decreases) in these equations leads to an overestimation of the impact oil price increases on the real GDP growth (see Kilian et al., 2009).

All replication results of Hamilton (2003) are reported in Tab. B.16 in the supplementary document for the two cases with and without negative oil prices measures. We also report the results of the nonlinearity tests of Hamilton (2001) and Dahl & González-Rivera (2003a) in Tab. B.15 and the adequacy test of an appropriate functional form to capture the nonlinearities for the period 1949Q2 to 2001Q3 in Tab. B.17. Overall, the obtained results are very similar to those reported in Hamilton (2003) in term of statistical significance, signs and magnitude. The test results of nonlinearity and adequacy of the functional form are indifferent to those reported in Hamilton (2003).

Regarding the impact of the inclusion of negative (decreases) oil price changes on the real GDP growth, the results show that the resulting difference due to the omission of the negative oil price measures is very weak and not statistically significant. For reasons of readability, detailed results regarding the replication are presented in Supplementary Material B and Supplementary Material C for the original and augmented sample period, respectively.

4.2. Linear ARX(p) Model

In order to analyze the impact of oil price changes on the real GDP growth for the period 1973Q1 to 2016Q4, we begin by testing the linearity of the oil price - real GDP relationship. We follow the same methodology as in Hamilton (2001) and Dahl & González-Rivera (2003a). The results, reported in Tab. 2, provide clear evidence of a nonlinear relationship between the oil price measures and real GDP growth. Both asymptotic and bootstrapped p -values for the three statistics ν^2 , λ^A , and λ^E , are lower than 1% indicat-

⁶As all the results of the above mentioned papers are summarized in Hamilton (2003), we limit our analysis to replicating the results of Eq. (1.5), Eq. (1.6), Eq. (1.8), Eq. (3.2), and Eq. (3.8) in Hamilton (2003).

ing that the null hypothesis of linearity is overwhelmingly rejected. For the g^A the null hypothesis of linearity is rejected only at 10%.

Table 2: Results of the nonlinearity tests of Hamilton (2001) and Dahl & González-Rivera (2003a) for the most recent period considered in our study, 1973Q1 to 2016Q4.

Tests		Test statistics	Asymp. p -value	Boots. p -value
Hamilton	ν^2	84.343	(0.000)	[0.001]
	λ^A	71.640	(0.000)	[0.003]
Dahl & González-Rivera	λ^E	75.481	(0.000)	[0.001]
	g^A	24.662	(0.038)	[0.093]

This finding motivates the use of nonlinear measures of oil price changes. The results of estimating the oil price - GDP growth relationship with recent data based on the different measures, O_t , $O_{\text{Mork},t}^{-/+}$, $O_{\text{LNR},t}^{-/+}$, $O_{\text{Ham1},t}^{-/+}$, and $O_{\text{Ham3},t}^{-/+}$, are reported in Tab. 3. Columns 2 and 3 refer to results when estimating the ARX(4) model using a linear functional form for the exogenous oil price changes variable O_t without any censoring. The results show that, in line with previous studies, only the fourth lag of oil price changes is significant at the 5% level.

The results of applying the oil price measures of Mork (1989), $O_{\text{Mork},t}^{-/+}$, with the extended model defined in Eq. (4) are reported in Tab. 3, columns 4 and 5. Our results show that the oil price increases measure of Mork (1989) is unable to capture any effect of oil price changes on the GDP growth as all coefficients are insignificant at the 5% level for the period 1973Q1 to 2016Q4. Unexpectedly, the results show that both the first and fourth lag, γ_1 and γ_4 , of the negative real oil price changes, $O_{\text{Mork},t}^-$, are significant at 10% and 5% respectively.

The results obtained with the measures introduced in Lee et al. (1995) provide strong evidence that the third normalized positive oil price changes, $O_{\text{LNR},t-3}^+$, has a highly significant impact on real GDP growth as the associated p -values is smaller than 1%.

The results of applying the net oil price measure of Hamilton (1996), $O_{\text{Ham1},t}^{-/+}$, are reported in columns 8 to 9 of Tab. 3. Regarding the net oil price increases, the results are in line with the findings of other oil price proxies; only the fourth lag is significant with an estimated coefficient of $\delta_4 = -0.0201$. These empirical results show that the 1y net oil price increases have lower impact for recent data. For instance the estimated δ_4 is

lower in absolute terms than the reported coefficient of -0.0310 in Hamilton (2003). The behavior of consumers and their reactions to price, demand, and supply changes might have changed in recent period. By taking into consideration the volatility of oil price changes, they become less sensitive to fluctuation of oil prices. This could hold true for the last 15 years in particular as these years are characterized by extremely erratic prices compared to earlier subsets. Hamilton (2003) already suggests that the 1y net oil price measure is not consistent with recent oil price fluctuations relative to 2003. The results obtained with the 3y net measures $O_{\text{Ham3},t}^{-/+}$ are reported in columns 10 and 11 in Tab. 3. The fourth lag of the positively censored measure, δ_4 is highly significant. Allowing for a 10% level of significance, we find the loads of δ_1 and δ_2 also to become significant.

Table 3: Regression results from the linear extended Hamilton (1989) model using different oil price measures for the period 1973Q1 to 2016Q4. The applied model reads $y_t = \mu_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_i O_{\{M\},t-i}^+ + \sum_{i=1}^q \gamma_i O_{\{M\},t-i}^- + u_t$. We report robust standard errors.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value
μ_0	0.4032	(0.0000)	0.6156	(0.0007)	0.6521	(0.0002)	0.6686	(0.0000)	0.7064	(0.0000)
ϕ_1	0.3153	(0.0001)	0.2797	(0.0004)	0.2962	(0.0002)	0.2607	(0.0010)	0.2206	(0.0046)
ϕ_2	0.1135	(0.1690)	0.0849	(0.4713)	0.0767	(0.3557)	0.0766	(0.3557)	0.0648	(0.4248)
ϕ_3	0.0248	(0.7614)	0.0161	(0.8756)	-0.0066	(0.9367)	0.0186	(0.8210)	0.0076	(0.9245)
ϕ_4	0.0105	(0.8928)	-0.0069	(0.9453)	0.0229	(0.7736)	-0.0082	(0.9162)	-0.0132	(0.8616)
δ_1	0.0005	(0.8908)	-0.0082	(0.2892)	-0.0437	(0.6805)	-0.0088	(0.3030)	-0.0164	(0.0933)
δ_2	-0.0028	(0.4033)	-0.0086	(0.1662)	-0.1636	(0.1332)	-0.0133	(0.1341)	-0.0174	(0.0820)
δ_3	-0.0043	(0.2056)	-0.0095	(0.1978)	-0.2843	(0.0090)	-0.0088	(0.3218)	-0.0151	(0.1367)
δ_4	-0.0080	(0.0211)	-0.0068	(0.3147)	-0.1309	(0.2330)	-0.0201	(0.0241)	-0.0264	(0.0094)
γ_1	—	—	0.0076	(0.0504)	-0.0341	(0.6952)	0.0065	(0.2828)	0.0102	(0.2109)
γ_2	—	—	-0.0005	(0.8761)	0.0242	(0.7817)	0.0009	(0.8834)	-0.0048	(0.5600)
γ_3	—	—	-0.0016	(0.6754)	0.0332	(0.7020)	-0.0004	(0.9434)	-0.0014	(0.8584)
γ_4	—	—	-0.0113	(0.0366)	-0.0194	(0.8234)	-0.0021	(0.7235)	-0.0044	(0.5853)
σ_0	0.7769	(0.0000)	0.7745	(0.0000)	0.7775	(0.0000)	0.7688	(0.0000)	0.7458	(0.0000)
Log-likelihood and information criteria										
<i>LL</i>	-200.6984		-198.018		-198.697		-196.709		-191.356	
<i>AIC</i>	2.3943		2.4093		2.4170		2.3944		2.3335	
<i>SIC</i>	2.5744		2.6615		2.6692		2.6466		2.5879	
<i>HQC</i>	2.4673		2.5116		2.5193		2.4967		2.4359	
<i>RMSE</i>	0.7568		0.7454		0.7483		0.7399		0.7177	
<i>MAE</i>	0.5535		0.5387		0.5476		0.5350		0.5260	

Note: The oil price measures refer to the uncensored O_t of Hamilton (1983), the asymmetric measures $O_{\text{Mork},t}^{-/+}$ of Mork (1989), the volatility scaled measures $O_{\text{LNR},t}^{-/+}$ of Lee et al. (1995), and Hamilton's measures $O_{\text{Ham1},t}^{-/+}$ (Hamilton, 1996) as well as $O_{\text{Ham3},t}^{-/+}$ (Hamilton, 2003).

Finally, we examine whether the four nonlinear oil price measures are able to capture present nonlinearity. We apply the adequacy test outlined in Subsection 2.2 and report the results in Tab. 4. We reject the hypothesis that the four nonlinear oil price measures are able to capture all nonlinearity in the oil - GDP link; all *p*-values are smaller than

5%. This is due to several possible reasons. Firstly, it might be caused by the employed linear model specification and structural breaks. Secondly, it is possible that the nonlinear measures are not of an appropriate functional form to capture the nonlinearity, and thirdly, it is possible that by converting monthly oil price data into quarterly data one may lose viable information.

Table 4: Results of the adequacy test of an appropriate functional form to capture nonlinearities for the period 1973Q2 to 2016Q4.

Tested measure	Test statistic	Asymptotic p -value
Mork (1989)	7.535	(0.000)
Lee et al. (1995)	8.196	(0.000)
Hamilton (1996)	8.394	(0.000)
Hamilton (2003)	8.335	(0.000)

In order to statistically assess whether the oil price changes have a significant impact on the real GDP growth, we conduct two exclusion tests for each oil price measure. Test (a) has the null hypothesis that all parameters related to the oil measure are zero, i.e. $H_0 : \delta_1 = \dots = \delta_4 = \gamma_1 = \dots = \gamma_4 = 0$. Test (b) attempts to capture the importance of measures for oil price increases and decreases separately. The results of these two tests for each model (as in Tab. 3) are reported in column 2 and 3 of Tab. 5. For the symmetric linear ARX(4) model with the O_t measure, the null hypothesis cannot be rejected at the 10% level. For the four censored oil price measure, we reject the null hypothesis only for the $O_{\text{Ham3},t}$ at the 1% level of significance and for the $O_{\text{Ham1},t}$ at 10% level of significance. Test (b) goes into more detail and we find that the rejection of test (a) is mainly attributed to the measure capturing the increase of oil prices for the $O_{\text{Ham1},t}$ and to both $O_{\text{Ham3},t}^+$ and $O_{\text{Ham3},t}^-$ for the Hamilton (2003) oil price changes measures. The null hypothesis that oil price increases are not relevant is not rejected for Mork (1989) even at 10%, it is rejected only at 10% for Lee et al. (1995), at 5% for Hamilton (1996) and at 1% for Hamilton (2003). However, while the results show that the hypotheses that oil price decreases are not relevant cannot be rejected for Lee et al. (1995) and Hamilton (1996) at all levels of significance, the null hypothesis is rejected at 10% for Mork (1989) and at 5% for Hamilton (2003). An important issue that might arise when using long periods of data is the possibility of structural changes. This instability of the relationship between

oil prices and GDP growth is tested in literature using the Chow test and the tests of Andrews (1993) and Andrews & Ploberger (1994). The results of using the Sup F , Avg F , and Exp F (Andrews & Ploberger, 1994) tests are reported in columns 3-5 of Tab. 5 with their p -values in parentheses.⁷ The results show that the null hypotheses of stability of the oil price changes - GDP growth relationship are rejected for all oil price measures at the 1% level for most of the tests. For all the specification used, the date of break corresponds to the third quarter of 1982. This date may corresponds to the structural changes in the real GDP growth which is associated with the Great Moderation.

Table 5: Exclusion tests for linear models with data from 1973Q1 to 2016Q4. Exclusion test (a) restricts oil price increases and decreases. Test (b) restricts increases and decreases separately. The Andrews and Ploberger test has the null hypothesis that the model parameters are stable over the whole sample.

	Exclusion tests		Andrews and Ploberger (1994) tests		
	(a)	(b)	Sup F	Avg F	Exp F
Panel A: Hamilton (1983)					
O_t	1.7551 (0.1403)	-	4.7035 (0.0001)	0.7605 (0.2333)	1.3185 (0.1580)
Panel B: Mork (1989)					
$O_{\text{Mork},t}^+$	1.4797 (0.1683)	1.2719 (0.2832)	3.6848 (0.0002)	0.7962 (0.1683)	1.4785 (0.047)
$O_{\text{Mork},t}^-$		2.0600 (0.0885)			
Panel C: Lee et al. (1995)					
$O_{\text{LNR},t}^+$	1.5020 (0.1602)	2.1829 (0.0732)	4.2624 (0.0000)	0.9319 (0.0631)	1.5389 (0.0334)
$O_{\text{LNR},t}^-$		0.4065 (0.8038)			
Panel D: Hamilton (1996)					
$O_{\text{Ham1},t}^+$	1.9142 (0.0611)	3.2996 (0.0125)	2.2794 (0.0633)	0.6550 (0.3399)	1.2327 (0.1944)
$O_{\text{Ham1},t}^-$		0.3312 (0.8567)			
Panel E: Hamilton (2003)					
$O_{\text{Ham3},t}^+$	4.0821 (0.0004)	4.3858 (0.0002)	3.7228 (0.0027)	0.6588 (0.3767)	1.2443 (0.2056)
$O_{\text{Ham3},t}^-$		2.6728 (0.0339)			

⁷The F -statistics of the Andrews (1993), Andrews & Ploberger (1994) test are calculated as log ratios $F_\tau = (LL_1 - LL_0)$ where LL_0 and LL_1 are the log-likelihoods from the full model and the model with implemented break point at $t = \tau$. The F_τ -statistics are calculated for each possible break point between $t_1 = \lfloor 0.15T \rfloor$ and $t_2 = \lceil 0.85T \rceil$, where T is the total number of observations. Then, the statistics are $\text{Sup } F = \max_{t_1 < \tau < t_2} F_\tau$, $\text{Avg } F = \frac{1}{t_2 - t_1 - 1} \sum_{\tau=t_1}^{t_2} F_\tau$, and $\text{Exp } F = \ln \left(\frac{1}{t_2 - t_1 - 1} \sum_{\tau=t_1}^{t_2} \exp \left(\frac{1}{2} F_\tau \right) \right)$. The p -values are calculated following the procedure of Hansen (1997). We are thankful for the provided MatLab code on www.ssc.wisc.edu/~bhansen/progs/jbes_97.html.

Table 6: MIDAS regression results for 1973Q1-2016Q4 and $n = 176$ observations. The p -values are based on robust standard errors.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	p -value	Coef.	p -value	Coef.	p -value	Coef.	p -value	Coef.	p -value
μ_0	0.4059	(0.0003)	0.6343	(0.0000)	0.6386	(0.0000)	0.4869	(0.0000)	0.5216	(0.0000)
ϕ_1	0.3016	(0.0004)	0.2747	(0.0009)	0.2698	(0.0017)	0.2782	(0.0004)	0.2617	(0.0029)
ϕ_2	0.1510	(0.2094)	0.1411	(0.1998)	0.0908	(0.3989)	0.1324	(0.1683)	0.1537	(0.1504)
ϕ_3	-0.0192	(0.8842)	-0.0250	(0.7967)	0.0105	(0.9337)	0.0012	(0.8465)	-0.0398	(0.6825)
ϕ_4	-0.0178	(0.8864)	-0.0377	(0.7036)	-0.0182	(0.8559)	-0.0290	(0.7671)	-0.0201	(0.8505)
θ_1	0.0129	(0.5171)	-0.0244	(0.0868)	-0.5075	(0.0171)	-0.0525	(0.0354)	-0.2075	(0.0077)
ω_{11}	8.0335	(0.8234)	225.3834	(0.6501)	29.4944	(0.6722)	195.9361	(0.8588)	2.4273	(0.2307)
ω_{21}	42.4645	(0.8572)	288.7746	(0.6148)	3.7463	(0.7109)	240.6860	(0.8539)	3.5975	(0.3267)
θ_2	—	—	0.0405	(0.0602)	0.1606	(0.2644)	0.0881	(0.0145)	0.1321	(0.0333)
ω_{12}	—	—	2.2889	(0.1243)	8.4523	(0.3737)	199.4558	(0.3428)	84.1579	(0.8800)
ω_{12}	—	—	6.6508	(0.1757)	24.3482	(0.4212)	236.0402	(0.3427)	298.3815	(0.8893)
σ_0	0.7484	(0.0000)	0.7294	(0.0000)	0.7205	(0.0000)	0.7304	(0.0000)	0.7238	(0.0000)
Log-likelihood and information criteria										
LL	-198.7172		-194.1929		-192.0369		-194.4446		-192.8317	
AIC	2.3604		2.3431		2.3186		2.3460		2.3276	
SIC	2.5225		2.5593		2.5348		2.5621		2.5438	
HQC	2.4262		2.4308		2.4063		2.4336		2.4153	
$RMSE$	0.7484		0.7294		0.7205		0.7304		0.7238	
MAE	0.5370		0.5241		0.5148		0.5293		0.5247	

4.3. Linear MIDAS

In order to access the information gain of using monthly data, we apply the MIDAS framework defined in Eq. (6). For the MIDAS model, we use 12 monthly lags on the oil price measures corresponding to the four quarterly lags in the ARX(4). Firstly, we compare the regression results. Secondly, we analyze the results from the exclusion and instability tests. The regression results are given in the Appendix, Tab. 6.

Focusing on the significance of the different oil price measures on GDP growth, we firstly review the coefficients θ_1 and θ_2 of the MIDAS regressors with monthly oil price changes. For the linear measure, O_t , oil price changes on a monthly base are insignificant. For all asymmetric measures, oil price increases with coefficient θ_1 are significant at least at 10% level. Oil price decreases, which have so far been insignificant on a quarterly basis in the ARX(4), become a significant contributor to the oil price - GDP relationship, when oil data is included on a monthly basis. For the asymmetric measures $O_{\text{Mork},t}^-$, $O_{\text{Ham1},t}^-$, and $O_{\text{Ham3},t}^-$, θ_2 is positive and statistically significant. With the estimated weighting scheme, this results in the surprising effect that oil price decreases also decrease the GDP, whereas this effect is of smaller magnitude compared to oil price increases. Fig. 4 visualizes the weighting scheme for the different measures.

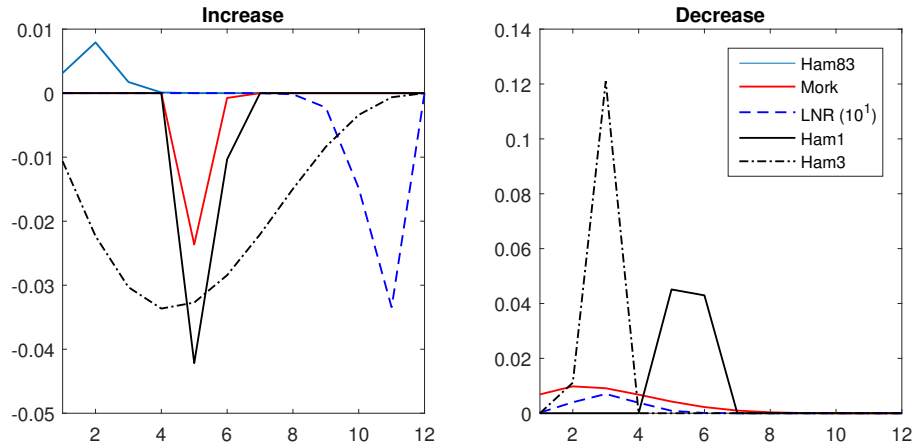


Figure 4: Lag distribution of the MIDAS model for oil price increases (left plot) and for oil price decreases (right plot).

Facing the situation that we obtain a significant impact of both increases and decreases measured by θ_1 and θ_2 while the respective MIDAS weighting hyperparameters $(\omega_{11}, \omega_{12})'$ and $(\omega_{21}, \omega_{22})'$ are (partly) insignificant might indicate that the proposed weighting scheme is unable to completely depict the distribution of weights of the monthly lags. This suggests that we are confronted with a more complex relationship of monthly oil price changes and the quarterly reported GDP growth.

Comparing the MIDAS framework to the ARX counterpart reported in Tab. 3, we highlight the following findings. The intercept μ_0 and the standard deviation of the error terms σ_0 are of similar level while for most models, the estimates in the MIDAS framework are slightly lower. Except for $O_{\text{Ham3},t}^{+/-}$, the linear MIDAS features a better fit in comparison to the ARX(4) model in term of log-likelihood. With a reduced number of parameters, most of the estimated MIDAS models feature better information criteria relative to their ARX equivalent. Moreover, the slightly better fit is transferred to lower loss function results for RMSE and MAE.

The results of the exclusion and instability tests are given in detail in the Appendix, Tab. 7. In contrast to the ARX(4) model, the MIDAS model reveals the usefulness of oil price decreases. We reject the null hypothesis for all oil price decrease measures at least at a 10% level, except $O_{\text{Mork},t}^-$. Hence, we conjecture that including the information of higher frequencies shows the importance and difference between oil price increases and decreases. As previously reported for the ARX framework, the Andrews & Ploberger

Table 7: Exclusion tests for non-switching MIDAS regressions for 1973Q1-2016Q4. Exclusion test (a) restricts oil price increases and decreases. Test (b) restricts increases and decreases separately.

	Exclusion tests		Andrews and Ploberger (1994) tests		
	(a)	(b)	Sup F	Avg F	Exp F
Panel A: Hamilton (1983)					
O_t	0.2696 (0.6043)		31.4656 (0.0062)	18.2003 (0.0058)	12.7987 (0.0036)
Panel B: Mork (1989)					
$O_{\text{Mork},t}^+$	2.6618 (0.0728)	1.6050 (0.2070)	36.5141 (0.0074)	16.7499 (0.0830)	14.3454 (0.0083)
$O_{\text{Mork},t}^-$		2.6779 (0.1037)			
Panel C: Lee et al. (1995)					
$O_{\text{LNR},t}^+$	3.3638 (0.0370)	5.5953 (0.0192)	27.8553 (0.0975)	17.6822 (0.0553)	11.5324 (0.0548)
$O_{\text{LNR},t}^-$		2.8378 (0.0940)			
Panel D: Hamilton (1996)					
$O_{\text{Ham1},t}^+$	4.2877 (0.0153)	4.1881 (0.0423)	31.3548 (0.0368)	16.6729 (0.0858)	12.3172 (0.0332)
$O_{\text{Ham1},t}^-$		3.0601 (0.0821)			
Panel E: Hamilton (2003)					
$O_{\text{Ham3},t}^+$	8.8683 (0.0002)	8.0086 (0.0052)	40.2783 (0.0021)	19.3290 (0.0257)	16.4808 (0.0017)
$O_{\text{Ham3},t}^-$		2.9587 (0.0873)			

(1994) test shows parameter instability for all MIDAS models based on all three statistics. This suggests that the relationship is time-varying or disrupted by a structural break. In order to cope with the parameter instability of the models, we suggest implementing a regime-switching modification of both the ARX and the MIDAS models which is analyzed in the next subsection.

4.4. Markov switching models for real GDP growth

As outlined in the literature review, two important features characterize the oil price - GDP relationship. Firstly, several studies have highlighted that the year 1984, labeled *The Great Moderation*, is associated with a permanent change in the DGP process of the real GDP growth (McConnel & Perez-Quiros, 2000, Blanchard & Gali, 2007). Secondly, several others studies show that the effect of oil prices on the real GDP growth has changed with recent data. In order to control for the instability in both the real GDP growth and the estimated oil coefficients of the oil - GDP relationship, we propose to use the Markov

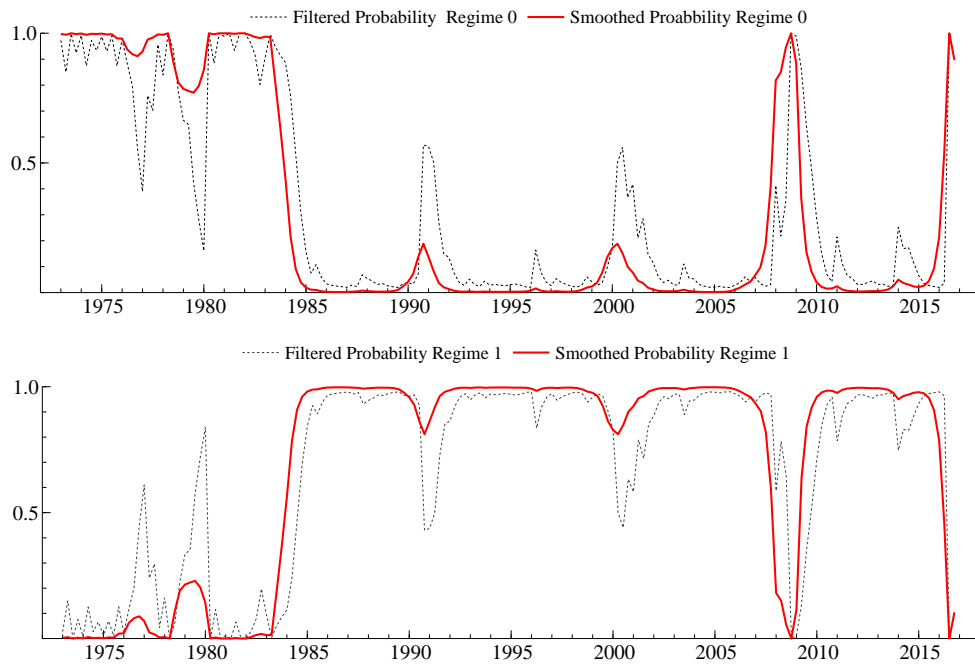


Figure 5: Probability smoothing in solid red line and filtered probabilities in dashed line of the MS(2)-AR(4) model with switches in intercept and variance.

switching modifications of models introduced in the previous sections.

We firstly investigate which Markov switching specification is more appropriate to describe the evolution of the U.S. real GDP growth. Introducing a regime switch to the AR(4) model, the bound testing approach proposed by Davies (1977, 1987) supports a MS(2)-AR(4) specification. The two-states Markov switching specification ($k = 2$) outperforms all other higher order Markov switching autoregressive specifications.⁸

Testing between the different MS(2)-AR(4) specifications and the linear AR(4) model, our results reported in the Appendix, Tab. A.14 show that the null hypothesis of the linear model is rejected against all three alternative MS(2)-AR(4) models. Testing the MS(2)-AR(4) with a switch in the intercept only, with switching in the intercept and variance, and finally the variant with switching the intercept, the autoregressive coefficients, and variance against each other; results provide strong evidence on the superiority of the MS(2)-AR(4) with Markov switch in intercept and variance. These results are emphasized

⁸We use the log-likelihood ratio (LR) test to assess MS(k)-AR(p) specifications. Despite the fact that the LR test has differing distributional properties in the presence of a nuisance parameter under the null hypothesis, several studies (e.g. Garcia, 1998, Carrasco et al., 2014) show that critical values of the test statistic are greater than those obtained with the χ^2 test statistic. The results of the different specifications are available upon request.

by the AIC, SIC, and HQC information criteria and are in line with findings of McConnel & Perez-Quiros (2000) for earlier data.

In summary, we confirm that models with a Markov switch better describe the U.S. real GDP growth evolution, which supports the findings of Hamilton (1989) and Hansen (1992) among many others studies. Regarding the identified break or switching dates, visualized in the regime probabilities in Fig. 5, we find three significant break points. The first one, 1984Q1, corresponds to a permanent switch in the real GDP behavior and the two other breaks correspond to the beginning and ending of the 2007 financial crisis.

4.4.1. $MS(2)$ - $ARX(4)$ regression results

Before reviewing the $MS(2)$ - $ARX(4)$ estimates for the nonlinear measures, we conduct the nonlinearity test on the respective regimes. The results are reported in Tab. 8 which show strong evidence for nonlinearity in the first regime as the asymptotic and bootstrapped p -values are all virtually zero. These results indicate that until 1984Q1, oil prices contribute to the real GDP growth and that this impact is nonlinear as found in Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003). For the second regime, the results are mixed and evidence for nonlinearity is very weak. Only one test, the g^A test of Dahl-Gonzalez-Rivera, shows evidence of nonlinearity at the 1% level of significance when using the asymptotic p -values and at 5% when using the bootstrapped p -values.

Table 8: Results of the nonlinearity tests for Hamilton (2001) and Dahl & González-Rivera (2003a) for each regime period with the total period 1973Q1 to 2016Q4.

Tests		Regime 0			Regime 1		
		Test statistic	Asymp. p -value	Boots. p -value	Test statistic	Asymp. p -value	Boots. p -value
Hamilton	ν^2	15.159	(0.000)	[0.001]	0.069	(0.793)	[0.779]
Dahl & González-Rivera	λ^A	34.960	(0.002)	[0.033]	21.330	(0.127)	[0.097]
	λ^E	5.586	(0.018)	[0.033]	414	(0.520)	[0.636]
	g^A	19.652	(0.142)	[0.071]	37.096	(0.001)	[0.032]

The results of estimating the $MS(2)$ - $ARX(4)$ model, with inclusion of the exogenous oil price measures as defined in Eq. (5), are reported in Tab. 9. The estimated tran-

sition probabilities p_{00} and p_{11} are close to 1,⁹ whereas they are slightly greater in the second regime compared to the first regime. The second regime that spans from 1984Q1 to 2007Q4, and from 2009Q1 to 2016Q4 has the longest duration. Only the first two autoregressive coefficients, ϕ_1 and ϕ_2 , are significant. This is, as already reported in previous subsections, in line with empirical literature. The results also show that some of the coefficients associated with the positive price measures, estimated in δ_i , are significant in the first regime. For negative oil price measures, only the first, third and fourth lags for the Mork regressor and the third lag for the Hamilton 3y net measure are significant in the first regime. For the second regime, we fail to obtain evidence of a significant impact of oil price changes on GDP growth, independent of their direction. Notably, the 3y net oil price increase carries the only statistically significant factor load.

In a similar fashion to the previous subsection, we report the results of tests of excluding coefficients within the presented Markov switching framework. In particular, we report the test results of the following exclusions with the hypotheses that (1) positive (increases) oil price changes; (2) negative (decreases) oil price changes in each regime or both regimes; (3) positive and negative oil price changes in the two regimes do not have a significant impact on the real GDP growth. The results of these tests are reported in Tab. 10 under columns (a), (b), (c), and (d), respectively.

The results of the exclusion tests for the linear oil price measures (Hamilton, 1983) with a Markov shifting MS(2)-ARX(4) model are reported in Panel A, columns (a) of Tab. 10. The tests provide evidence that the oil price changes have a significant impact on the real GDP at the 1% level in the full set from 1973Q1 to 2016Q4.¹⁰ However, by testing the effect of an exclusion separately for regime 0 and 1, reported in column (c₀) and (c₁), the test statistics show that only the oil price changes in the regime 0 are of significance and hence, determine the real GDP growth. In regime 1, the impact of oil price changes is neutral and an inclusion of the oil price measure has insignificant effect

⁹The probabilities p_{00} and p_{11} denote the likelihood to remain in the current regime whereas the switch to the other regime are the respective complementary probabilities.

¹⁰The difference in the test results compared to Tab. 5, with a reported p -value of 0.1403, is due to the fact that the presumably time-varying impact of oil price changes is well captured under the Markov switching model where both the intercept and variance are regime dependent.

Table 9: Regression results from the Markov switching models using different oil price measures for the period 1973Q1 to 2016Q4. The applied model reads $y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_{i,s_t} O_{\{M\},t-i}^+ + \sum_{i=1}^q \gamma_{i,s_t} O_{\{M\},t-i}^- + u_{s_t,t}$.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value
Non-switching variables										
ϕ_1	0.1789	(0.0178)	0.1835	(0.0164)	0.2053	(0.0160)	0.1068	(0.1327)	0.1480	(0.0283)
ϕ_2	0.1918	(0.0186)	0.1765	(0.0213)	0.1734	(0.0409)	0.1767	(0.0210)	0.1709	(0.0166)
ϕ_3	-0.0725	(0.3431)	-0.0632	(0.5548)	-0.0656	(0.4308)	-0.0495	(0.5551)	-0.0270	(0.7255)
ϕ_4	0.0315	(0.7031)	-0.0053	(0.9546)	0.0666	(0.4498)	0.0064	(0.9351)	0.0079	(0.9177)
Switching variables										
μ_0	0.7033	(0.0002)	0.6974	(0.0092)	1.2155	(0.0353)	1.0288	(0.0000)	1.0252	(0.0001)
δ_{10}	0.0008	(0.9222)	-0.0149	(0.1194)	0.0037	(0.9843)	-0.0260	(0.0861)	-0.0125	(0.5576)
δ_{20}	-0.0056	(0.7567)	-0.0252	(0.0017)	-0.6021	(0.0020)	-0.0349	(0.0291)	-0.0252	(0.1634)
δ_{30}	-0.0340	(0.0113)	-0.0052	(0.7188)	-0.6222	(0.0395)	-0.0198	(0.2436)	-0.0340	(0.0501)
δ_{40}	-0.04168	(0.0003)	-0.0109	(0.5795)	-0.3694	(0.0660)	-0.0422	(0.1144)	-0.0511	(0.0114)
γ_{10}	—	—	0.0161	(0.0728)	-0.0055	(0.9883)	0.0115	(0.1060)	0.0088	(0.4629)
γ_{20}	—	—	0.0098	(0.5476)	0.2804	(0.4368)	0.0071	(0.9020)	0.0004	(0.9905)
γ_{30}	—	—	-0.0327	(0.0703)	0.1070	(0.7826)	-0.0496	(0.1902)	-0.0777	(0.0054)
γ_{40}	—	—	-0.0573	(0.0000)	0.0518	(0.9042)	-0.0272	(0.2711)	-0.0065	(0.6579)
σ_0	1.0557	(0.0000)	1.0079	(0.0000)	1.0524	(0.0000)	0.9948	(0.0000)	1.0063	(0.0000)
μ_1	0.4965	(0.0000)	0.5616	(0.0323)	0.3914	(0.0026)	0.5403	(0.0000)	0.5493	(0.0000)
δ_{11}	-0.0024	(0.4489)	-0.0043	(0.6936)	-0.0095	(0.9030)	0.0117	(0.1851)	-0.0262	(0.0019)
δ_{21}	-1.27E-05	(0.9958)	0.0079	(0.4340)	0.1609	(0.1608)	0.0037	(0.7733)	-0.0106	(0.2625)
δ_{31}	0.0002	(0.9232)	-0.0054	(0.5823)	-0.046	(0.4974)	-0.0006	(0.9493)	0.0074	(0.2720)
δ_{41}	-0.0024	(0.2888)	-0.0029	(0.5824)	0.063	(0.3840)	-0.0046	(0.5772)	-0.0033	(0.6692)
γ_{11}	—	—	-0.0008	(0.8815)	-0.0054	(0.9064)	-0.0027	(0.4983)	0.0035	(0.2526)
γ_{21}	—	—	-0.0027	(0.3261)	-0.025	(0.6009)	-0.0014	(0.6346)	-0.0052	(0.1218)
γ_{31}	—	—	0.0028	(0.5192)	0.0040	(0.3117)	-0.0005	(0.8539)	-0.0002	(0.9753)
γ_{41}	—	—	-0.0009	(0.7809)	-0.0536	(0.1791)	-0.0015	(0.6537)	-0.0039	(0.4433)
σ_1	0.4589	(0.0000)	0.3898	(0.0000)	0.4535	(0.0000)	0.4281	(0.0000)	0.4215	(0.0000)
Transition probabilities										
p_{00}	0.9619	(0.0000)	0.9029	(0.0000)	0.9670	(0.0000)	0.9414	(0.0000)	0.9297	(0.0000)
p_{11}	0.9747	(0.0000)	0.9338	(0.0000)	0.9757	(0.0000)	0.9609	(0.0000)	0.9594	(0.0000)
Information criterion and Likelihood ratio statistic										
<i>LL</i>	-171.656		-169.90		-170.651		-169.268		-164.023	
<i>AIC</i>	2.1552		2.2262		2.2347		2.2189		2.1594	
<i>SIC</i>	2.4794		2.6945		2.7030		2.6873		2.6277	
<i>HQC</i>	2.2867		2.4161		2.4246		2.4089		2.3493	
<i>RMSE</i>	0.8045		0.7735		0.7235		0.7909		0.7526	
<i>MAE</i>	0.5516		0.5390		0.5100		0.5425		0.5221	

with a reported *p*-value of 0.6976 in column (c_1).

In addition to the linear oil price measure, Tab. 10 reports the results for restricting the asymmetric measures in panels B, C, D, and E. Panel A shows that all oil price measures have significant impact on the GDP growth at the 1% level except for the Mork oil price measures at 10%, see column (a). Consistently for all oil price measures, we find that price increases have a significant impact on the GDP growth whereas the inclusion of decreases has no effect for the Lee et al. (1995) and Hamilton (1996) oil price measures, see column (b). For the Mork (1989) and Hamilton (2003), oil price decreases have a

significant impact at the 1% and 5% level of significance. In order to determine if the impact is variable across regimes, we test the joint hypothesis that both the positive and negative part of the respective oil price measures are insignificant in each regime. The results are reported in columns (c_0) and (c_1) . When distinguishing between the regimes, the tests indicate that only for the first regime, the oil price changes have an effect on the GDP growth for all the Mork (1989), Lee et al. (1995) and Hamilton (1996). However, for the Hamilton (2003), we found evidence for significant impact in both regimes, with 1% level of significance in the first regime and 5% level of significance in the second regime. This is evidence for a time-varying impact of oil price changes.

To further understand this time variability in the relationship of oil price changes and the GDP growth, we distinguish between regimes *and* positive or negative part of the respective oil price measure. This yields four separate tests for each type of price measure which are reported in column (d_0) for the first regime (regime 0) and in column (d_1) for the second regime (regime 1). Firstly, the results show that both positive and negative oil price changes are insignificant in the second regime except for the positive Hamilton (2003) oil price measure which is significant at the 5% level. For the first regime, all positive oil prices measures have a significant impact on the real GDP growth at 5% where the two net oil measures are significant at 1%. Surprisingly, we find that Mork's, Hamilton's 1y and 3y decrease measures, $O_{\text{Mork},t}^-$, $O_{\text{Ham1},t}^-$ and $O_{\text{Ham3},t}^-$ are significant in the first regime at conventional levels.

Results of the regime-specific exclusion tests confirm the results of the nonlinearity test. Under the first regime there is strong evidence of nonlinearity in the oil - GDP relationship, while in the second regime oil price changes seem to have no impact on real GDP growth.

Allowing for instability in the relationship, the results can be summarized as follows. Firstly, we find that the Markov switching ARX(4) outperforms the linear ARX(4) model. Secondly, we find that both positive and negative parts of oil price changes determine the U.S. real GDP growth in the first regime whereas the decreases are weakly significant for $O_{\text{Mork},t}^-$, $O_{\text{Ham1},t}^-$, and $O_{\text{Ham3},t}^-$. Lastly, neither increases nor decreases of the oil price have

Table 10: Exclusion tests for Markov switching regression models for 1973Q1-2016Q4. Exclusion test (a) restricts both measures in both regimes, test (b) restricts the measure in regime 0 and 1 separately, tests (c) restrict both measures only in regime 0, reported in (c_0), or regime 1, reported in (c_1), and tests (d_0) and (d_1) restrict the oil price measure separately only in regime 0 or 1. p -values are given in parentheses.

Exclusion tests						
	(a)	(b)	(c_0)	(c_1)	(d_0)	(d_1)
Panel A: Hamilton (1983)						
O_t	6.5626 (0.0000)		11.3848 (0.0000)	0.5523 (0.6976)		
Panel B: Mork (1989)						
$O_{\text{Mork},t}^+$	5.4963 (0.0000)	2.6409 (0.0098)	7.4592 (0.0000)	1.0529 (0.3993)	3.2949 (0.0128)	1.1021 (0.3578)
$O_{\text{Mork},t}^-$		5.4088 (0.0000)			8.4436 (0.0000)	0.5835 (0.6751)
Panel C: Lee et al. (1995)						
$O_{\text{LNR},t}^+$	1.6184 (0.0702)	1.9377 (0.0058)	2.4313 (0.0169)	0.7066 (0.6853)	3.1878 (0.0151)	0.7076 (0.5879)
$O_{\text{LNR},t}^-$		0.4504 (0.8887)			0.1716 (0.9526)	0.7590 (0.5535)
Panel D: Hamilton (1996)						
$O_{\text{Ham1},t}^+$	4.1858 (0.0000)	3.2027 (0.0022)	7.1186 (0.0000)	0.5140 (0.8445)	5.7925 (0.0002)	0.7556 (0.5558)
$O_{\text{Ham1},t}^-$		1.5228 (0.1537)			2.7708 (0.0294)	0.2655 (0.8998)
Panel E: Hamilton (2003)						
$O_{\text{Ham3},t}^+$	7.2780 (0.0000)	4.4239 (0.0001)	10.6637 (0.0000)	2.1715 (0.0327)	5.5913 (0.0000)	3.0800 (0.0180)
$O_{\text{Ham3},t}^-$		2.2010 (0.0303)			3.0977 (0.0175)	0.8475 (0.4972)

a significant impact on GDP growth in the second regime.

4.4.2. MS(2)-AR(4)-MIDAS(12) regression results

Having identified different periods in the oil change - GDP growth relationship by Markov switching in Subsection 4.4.1, we extend the model with the MIDAS structure applied in Subsection 4.3. The resulting MS(2)-AR(4)-MIDAS(12), defined in Eq. (7), combines switching regimes and taking into account a higher frequency of oil price changes on a monthly basis. In analogy to the previous subsection, we allow for switching in the exogenous variables, including the hyperparameters in the weighting schemes, whereas the AR(4) part in Eq. (7) is non-switching.

For the non-switching parameters, the estimation results and their significances are similar to those for the MS(2)-ARX(4) which models oil price changes on a quarterly basis. The same parameter instability caused by different regimes in the oil price - GDP relationship is identified. For the first regime, we obtain significant estimates for both oil price increases and decreases (θ_{10} and θ_{20}). Again, the parameter scaling the oil price increases, θ_{10} , is negative whereas for the oil price decreases, the parameter θ_{20} is positive for all models, except for $O_{\text{Mork},t}^-$. This is in line with the estimates presented in Tab. 9, where the parameters of the fourth lag for the Mork oil price decrease turns negative.

Additionally, the hyperparameters ω_{110} , ω_{120} , ω_{210} , and ω_{220} are significant for $O_{\text{Ham}1,t}^+$ and $O_{\text{LNR},t}^-$ and partly for $O_{\text{Ham}3,t}^+$ and $O_{\text{Ham}1,t}^-$. For the second regime, only the oil price increases $O_{\text{Ham}1,t}^+$, $O_{\text{LNR},t}^+$, and $O_{\text{Ham}3,t}^+$ remain significant. The impact of oil price increases is of smaller magnitude relative to the first regime, but remain negative. In the second regime, the parameters for oil price decreases are significant for Mork's and Hamilton's one and three years measures. Again, they are much smaller than their counterparts in the first regime. However, for Hamilton's one year measure, we observe a negative parameter, while we find positive association with Mork's and the three year measure.

The value of additional information introduced by monthly oil prices also becomes apparent when comparing log-likelihood and information criteria with the MS(2)-ARX(4) models. With MS-MIDAS, all log-likelihoods, information criteria, and loss function results are improved. This provides evidence of a better fit and hence, a better depiction of the oil price - GDP relationship in this Markov switching setting.

The results of the exclusion tests are provided in Tab. 12. From these tests we conclude that the additional information from higher frequency changes some of the results we obtain with the MS-ARX framework. Comparing Tab. 12 with Tab. 10, we see that—in addition to Mork and Hamilton's three year measure—including 1y net decreases in both regimes becomes important (column (b), at 1%), which is not the case for quarterly sampled oil price decrease measures. Separating between regimes in exclusion test (c), it is evident that all measures play an important role in the relationship in both regimes. Only for Hamilton's linear measure O_t , this does not hold. Including monthly data, the depiction of the oil price - GDP relationship in the second regime, test (c₁), is improved. For quarterly data, only $O_{\text{Ham}3,t}^{+/-}$ is significant at the 5% level. Analyzing this finding in more detail in tests (d₀) and (d₁), we find that with monthly data, oil price increases and decreases are equally important in all models in the first regime (regime 0) and should not be excluded. In the second regime (regime 1), the use of measures for oil price increases remains important for all models but for $O_{\text{Mork},t}^+$. Except for $O_{\text{LNR},t}^-$, all oil price decreases should be included. This is in some contrast to the MS-ARX formulation with quarterly data where the second regime indicates no impact of the oil price measures on the GDP

Table 11: Regression results from MS-MIDAS models using different oil price measures for 1973Q1-2016Q4 with $n_1 = 176$ quarterly and $n_2 = 531$ monthly observations. The models reads $y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \theta_{1,s_t} B(L^{1/m}; \omega_{1,s_t}) O_{\{M\},t}^{+,(m)} + \theta_{2,s_t} B(L^{1/m}; \omega_{2,s_t}) O_{\{M\},t}^{-,(m)} + u_{t,s_t}$.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value
Non-switching variables										
ϕ_1	0.2129	(0.0195)	0.2685	(0.0006)	0.1866	(0.0402)	0.1840	(0.0104)	0.1395	(0.0666)
ϕ_2	0.2033	(0.0092)	0.2032	(0.0086)	0.1764	(0.0403)	0.2086	(0.0034)	0.2228	(0.0068)
ϕ_3	-0.0533	(0.4763)	-0.0788	(0.3002)	-0.0472	(0.4661)	-0.0430	(0.5194)	-0.0426	(0.5784)
ϕ_4	0.0294	(0.7148)	0.0805	(0.3030)	-0.0037	(0.9431)	0.0055	(0.4955)	0.0060	(0.9579)
Switching variables										
μ_0	0.6394	(0.0162)	0.4103	(0.0607)	1.1635	(0.0238)	0.7745	(0.0004)	0.6588	(0.0042)
θ_{10}	-0.1840	(0.0412)	-0.2445	(0.0070)	-0.8908	(0.0009)	-0.8440	(0.0000)	-0.8600	(0.0092)
ω_{110}	26.5548	(0.4648)	27.9445	(0.5614)	226.8732	(0.8802)	34.7295	(0.0000)	1.8484	(0.1465)
ω_{210}	4.5411	(0.4594)	4.4714	(0.6046)	33.9312	(0.8821)	6.9266	(0.0000)	1.0173	(0.0000)
θ_{20}	—	—	-0.4985	(0.0067)	1.7406	(0.0859)	0.8821	(0.0092)	0.5776	(0.0000)
ω_{120}	—	—	37.1259	(0.6484)	0.9823	(0.0000)	0.9326	(0.0000)	4.2424	(0.4601)
ω_{120}	—	—	230.2021	(0.6647)	1.9964	(0.0600)	1.1564	(0.3592)	10.8991	(0.4903)
σ_0	1.0723	(0.0000)	1.0269	(0.0000)	1.0022	(0.0000)	0.9776	(0.0000)	1.0516	(0.0000)
μ_1	0.4446	(0.0000)	0.5423	(0.0000)	0.5227	(0.0000)	0.4739	(0.0000)	0.5450	(0.0000)
θ_{11}	0.0019	(0.7454)	-0.0110	(0.1484)	-0.2686	(0.0152)	-0.0332	(0.0000)	-0.1569	(0.0003)
ω_{111}	13.5477	(0.7151)	4.2168	(0.4672)	170.4443	(0.5658)	172.4069	(0.6805)	1.9954	(0.0000)
ω_{211}	203.2756	(0.7819)	140.4261	(0.7498)	299.6133	(0.5580)	297.4627	(0.6894)	3.9684	(0.0027)
θ_{21}	—	—	0.0432	(0.0194)	-0.0772	(0.5274)	-0.0185	(0.0409)	0.0615	(0.0134)
ω_{121}	—	—	1.0644	(0.0000)	193.0154	(0.9540)	297.5750	(0.9251)	5.9573	(0.3083)
ω_{221}	—	—	9.6870	(0.1417)	201.2365	(0.9521)	170.2899	(0.9255)	27.7219	(0.4033)
σ_1	0.4622	(0.0000)	0.4939	(0.0000)	0.4419	(0.0000)	0.4424	(0.0000)	0.4376	(0.0000)
Transition probabilities										
p_{00}	0.9671	(0.0000)	0.9908	(0.0000)	0.9212	(0.0000)	0.9653	(0.0000)	0.9656	(0.0000)
p_{11}	0.9847	(0.0000)	0.9950	(0.0000)	0.9698	(0.0000)	0.9838	(0.0000)	0.9835	(0.0000)
Log-likelihood and information criteria										
LL	-166.4963		-161.1320		-158.9605		-156.7851		-159.2306	
AIC	2.0738		2.0810		2.0564		2.0316		2.0594	
SIC	2.3620		2.4774		2.4527		2.4280		2.4557	
HQC	2.1907		2.2418		2.2171		2.1924		2.2202	
$RMSE$	0.7035		0.6774		0.6478		0.6537		0.6912	
MAE	0.5034		0.4873		0.4694		0.4785		0.4949	

growth at all. With monthly data, we reinstate the impact for both regimes.

As indicated by the regression results above, the use of higher frequency data allows to measure the impact of oil price changes in both regimes, even though models with quarterly oil price data neglect their importance in the second regime. Given the identified time-varying nature of the relationship, the impact is of different magnitude yet statistically significant. To further examine the differences across the regimes and the effect of oil price decreases—now a significant contributor to the oil - GDP relationship—we turn to the weights each monthly lag is associated within the MIDAS part of Eq. (7). Fig. 6 plots the different lag distributions over twelve months, corresponding to four quarters in the ARX(4) structure, for all applied measures. Several conclusions can be drawn from

Table 12: Exclusion tests for Markov Switching MIDAS regression models for 1973Q1-2016Q4. Exclusion test (a) restricts both measures in both regimes, test (b) restricts the measure in regime 0 and 1, tests (c₀) and (c₁) restrict both measures only in regime 0 or 1, and tests (d₀) and (d₁) restrict the oil price measure only in regime 0 or 1.

Exclusion tests						
	(a)	(b)	(c ₀)	(c ₁)	(d ₀)	(d ₁)
Panel A: Hamilton (1983)						
O_t	2.2971 (0.1039)		4.2297 (0.0413)	0.1058 (0.7454)		
Panel B: Mork (1989)						
$O_{\text{Mork},t}^+$	3.9548 (0.0044)	4.4631 (0.0131)	4.8780 (0.0088)	3.2363 (0.0420)	7.4391 (0.0071)	2.1078 (0.1486)
$O_{\text{Mork},t}^-$		6.6352 (0.0017)			7.5340 (0.0068)	5.5662 (0.0196)
Panel C: Lee et al. (1995)						
$O_{\text{LNR},t}^+$	3.2906 (0.0128)	6.3913 (0.0022)	6.0399 (0.0030)	3.0465 (0.0504)	11.4747 (0.0009)	6.0143 (0.0153)
$O_{\text{LNR},t}^-$		1.5420 (0.2172)			2.9833 (0.0861)	0.4010 (0.5275)
Panel D: Hamilton (1996)						
$O_{\text{Ham1},t}^+$	70.1569 (0.0000)	123.0547 (0.0000)	25.2691 (0.0000)	125.7838 (0.0000)	50.5374 (0.0000)	217.4114 (0.0000)
$O_{\text{Ham1},t}^-$		5.8700 (0.0035)			6.9303 (0.0093)	4.2447 (0.0411)
Panel E: Hamilton (2003)						
$O_{\text{Ham3},t}^+$	11.6686 (0.0000)	9.6295 (0.0001)	17.3114 (0.0000)	10.5816 (0.0000)	6.9451 (0.0093)	13.4680 (0.0003)
$O_{\text{Ham3},t}^-$		20.0579 (0.0000)			34.6071 (0.0000)	6.2415 (0.0135)

the distribution plot which are discussed in what follows.

Firstly, oil price increases have a negative load (θ_1) for both regimes. This translates to a decrease in GDP growth which is in line with literature. Generally speaking, all oil price increase measures spike around similar lags. In the first regime, lags 9 to 11 representing the third and fourth quarter, carry the highest weights forming a significant spike for all measures in Fig. 6, top left plot. This is in line with our findings for the ARX(4) framework as well as empirical literature (Mork, 1989, Hamilton, 2001, 2003). In the second regime, weights for oil price increases peak in lags two to five corresponding to the first and second quarter (bottom left plot of Fig. 6). In the second regime for increases, the weight distribution is shifted to an earlier impact that dies out in subsequent months. This leads to the aforementioned effects that increases in oil prices have a negative effect on GDP growth, although with different magnitude and temporality. The magnitude of the impact is scaled down by approx. factor 10. We observe a short termed and immediate reaction to increases in the second regime. From the plot, we follow that the instability and change points in the oil - GDP relationship are clearly visible in the plot as the distributions across the regimes are fundamentally different.

Secondly, for oil price decreases the results hint towards a mixed impact on GDP growth. In the first regime, Hamilton's decrease measures have a positive and significant

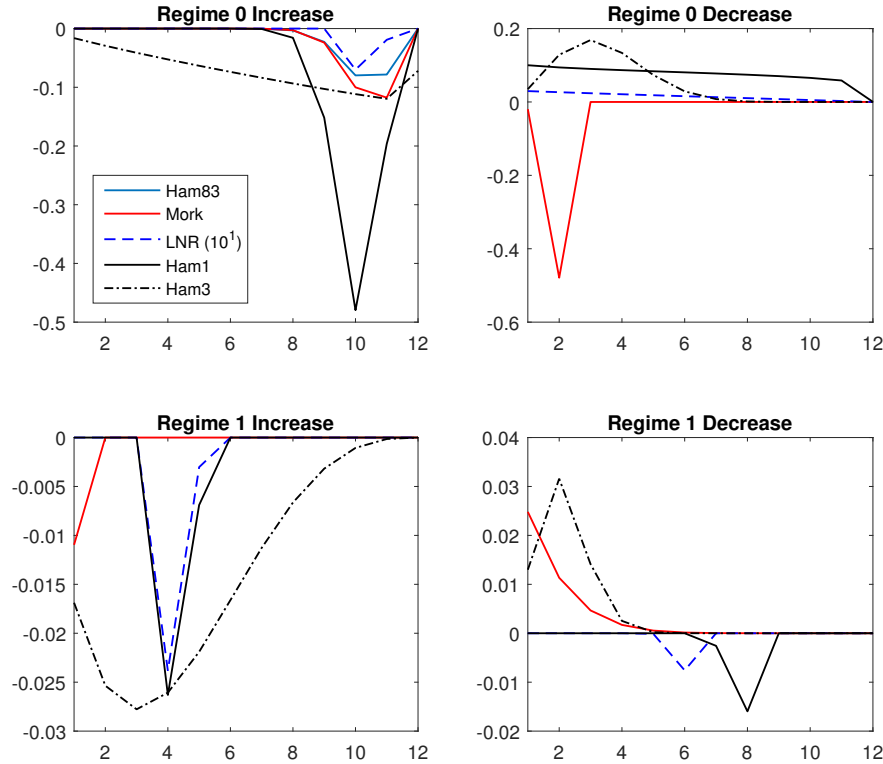


Figure 6: Lag distribution of the MS-MIDAS model for oil price increases (left side) and for oil price decreases (right side) in regime 0 (upper row) and regime 1 (lower row). The weights are scaled with respective parameter translating to the full weights vectors $[\theta_{1,s_t} B(L^{1/m}; \omega_{1,s_t})] \cdot O_{\{M\},t}^{+,(m)}$ for increases and $[\theta_{2,s_t} B(L^{1/m}; \omega_{2,s_t})] \cdot O_{\{M\},t}^{-,(m)}$ for decreases.

load, indicating that price decreases again have a lowering effect on GDP growth. We emphasize the statistical significance of these estimates. Mork's measure, on the other hand, has a significant negative load that spans one month in the first quarter only. In the second regime, the decrease measures of Mork and Hamilton's three years have an immediate decreasing impact on GDP, Hamilton's one year measure is influencing the GDP in the third quarter into positive direction. This changing sign over different measures might be due to limitations of the MIDAS weight function, which only allows either positive or negative impact, and points towards a reversal of GDP growth in the subsequent months after an initial decreasing impact.

Lastly, the second regime only depicts a weak impact for both negative and positive measures compared to the first regime.

In conclusion, the first regime appears to be the default regime in line with empirically verified results regarding the effect of oil price increases on real GDP growth. By including

monthly data, we provide evidence that oil price decreases also have an impact on this growth. The second regime appears to model short-lived and immediate reactions (relative to the applied time scales) of the GDP growth to oil price changes whereas these reactions are much weaker than in the first regime. This novel result is obtained by including monthly oil prices. For the quarterly data, we identify no significant impact in the second regime indicating a decoupling of oil prices and GDP growth with clear break points. We improve the understanding of this change towards a second regime with short-term reactions.

Finally, we compare all models from Tab. 3, 6, 9, and 11 by means of their in-sample loss functions. To this end, we use the Model Confidence Set proposed by Hansen et al. (2011) to derive the models with superior predictive ability. The results suggest that using the information within the quarterly observations of the GDP from the oil prices leads to more accurate models. In particular, we find that the linear MIDAS model using $O_{\text{LNR},t}^{+/-}$ and all MS-MIDAS models except O_t are included in the Model Confidence Set for the RMSE and MAE loss function.¹¹

5. Conclusions

We replicate the findings of Hamilton (2003) and apply different oil price measures proposed throughout the last three decades of research. We use more recent data up to 2016Q4 and investigate whether including negative and positive oil price measures simultaneously, as recommended by Kilian & Vigfusson (2011a), overcome the problem of a likely overestimation of effects of oil price increases on GDP growth. We find that for recent data, the estimated impact of price increases is downscaled compared to earlier studies, and that including oil price decreases has no significant effect, except for the three year net decreases measure of Hamilton (2003). This indicates that the linear framework and measures might be misspecified, especially on more recent data. Hence, we test the parameters for stability and reject the hypothesis of stable parameters within our

¹¹Note that we use the T_R statistic with a level of significance of 10% and 10 000 bootstraps for the Model Confidence Set. The results are available upon request.

dataset and conclude that there might be breaks in the relationship between oil and GDP growth. In order to account for this instability of the regression parameters, we augment the models with a Markov switching approach to allow for data driven regime switches. With quarterly oil prices, we find that only in the first regime until 1984 positive oil price measures influence the GDP, while in the second regime, no measure shows a significant contribution to GDP growth rates.

Using monthly oil price information, we additionally find that negative oil measures negatively influence the GDP growth. Again the parameters are found to be unstable over our regression period and we apply these monthly prices in a MS-MIDAS framework which is novel to the research on the oil - GDP link. The results obtained contradict our results for quarterly data and literature on linear models. Firstly, some positive oil price measures become statistically significant in the first *and* second regime, hinting towards an ongoing effect of price increases on real GDP growth across regimes. Secondly, even negative oil price measures show significant effects in both regimes. Surprisingly, this effect is negative in some cases. Generally speaking, we detect two fundamentally different regimes which change at the pivotal 1984. Hence, we find the *Great Moderation* to permanently change the estimated relationship. One regime is best described as norm regime which is in line with findings in literature. The second regime depicts a fast but short lived reaction of GDP growth rates to oil price changes, albeit on a smaller magnitude. Hence, we conclude that the aggregated oil price measures, which have a dissipating effect for quarterly data, are gaining some usefulness in explaining GDP growth rates when mixed data sampling on monthly measures is applied.

Future research could address three areas. Firstly, the lag structure of the regime-switching MIDAS weights could be analyzed in more detail and developed towards the unrestricted MIDAS variant of Foroni et al. (2015). Secondly, using monthly data, our findings could be tested against a multi-frequency VAR framework (Mariano & Murasawa, 2003, Ghysels, 2016). Lastly, the promising $MS(k)$ -MIDAS(m) employed in this paper should be tested for forecasting.

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Appendix A. Additional Tables

Table A.13: Unit root test statistics for all time series with quarterly and monthly frequencies for the period 1973Q1 to 2016Q4.

		quarterly			monthly		
		ADF	PP	KPSS	ADF	PP	KPSS
GDP change	y_t	−6.3513***	−8.9263***	0.1257*	—	—	—
Nominal Oil Price change	O_t	−12.8309***	−12.8628***	0.0588	−18.3460***	−18.3482***	0.0766
Real Oil Price change	O_t^{real}	−12.9100***	−12.8852***	0.0629	−18.5903***	−18.5466***	0.0688
Mork increase	$O_{\text{Mork},t}^+$	−9.8467***	−13.2734***	0.0643	−13.9504***	−16.8760***	0.2099**
Mork decrease	$O_{\text{Mork},t}^-$	−9.9186***	−11.9690***	0.0606	−22.4435***	−22.9967***	0.0296
Lee increase	$O_{\text{LNR},t}^+$	−10.3978***	−13.8158***	0.0154	−17.4108***	−21.0268***	0.2368***
Lee decrease	$O_{\text{LNR},t}^-$	−11.0258***	−13.8953***	0.0347	−13.8596***	−15.4640***	0.1439*
Hamilton 1y increase	$O_{\text{Ham1},t}^+$	−8.8306***	−10.5221***	0.1371*	−18.2775***	−18.9532***	0.0447
Hamilton 1y decrease	$O_{\text{Ham1},t}^-$	−11.2149***	−12.1869***	0.0377	−22.6744***	−22.9499***	0.0434
Hamilton 3y increase	$O_{\text{Ham3},t}^+$	−9.4379***	−10.6979***	0.1850**	−16.2346***	−16.5582***	0.0680
Hamilton 3y decrease	$O_{\text{Ham3},t}^-$	−10.1632***	−10.7296***	0.0721	−19.0108***	−18.9965***	0.2505***

Note: The ADF test and the PP test have the null hypothesis of an existing unit root. The KPSS test has the null hypothesis of no unit root. *, **, *** refer to the 10%, 5% and 1% levels of significance.

Table A.14: Regression results for the linear model and the Markov switching alternatives with switching in only intercept/variance and switching in all parameters. I, A, and V indicate intercept, autoregressive coefficients and variance of the model, respectively.

	Linear		MS in I		MS in I and V		MS I, A, and V	
	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value
μ_0	0.3915	(0.0001)	0.3819	(0.0006)	0.3670	(0.0082)	0.3254	(0.2205)
μ_1	—	—	3.5117	(0.0000)	0.4326	(0.0000)	0.4911	(0.1588)
ϕ_{10}	0.3383	(0.0000)	0.3219	(0.0001)	0.2524	(0.0003)	0.4410	(0.0577)
ϕ_{11}	—	—	—	—	—	—	0.1543	(0.5893)
ϕ_{20}	0.1089	(0.1890)	0.1684	(0.1471)	0.2192	(0.0002)	−0.0121	(0.9612)
ϕ_{21}	—	—	—	—	—	—	0.2609	(0.0536)
ϕ_{30}	0.0116	(0.8876)	−0.0528	(0.5711)	−0.0661	(0.3380)	0.1097	(0.6375)
ϕ_{31}	—	—	—	—	—	—	−0.0810	(0.4678)
ϕ_{40}	−0.0003	(0.9971)	−0.0294	(0.7629)	0.0252	(0.7540)	−0.0417	(0.8196)
ϕ_{41}	—	—	—	—	—	—	0.0177	(0.9098)
σ_0	0.7851	(0.0000)	0.6940	(0.0000)	1.2272	(0.0000)	1.1860	(0.0000)
σ_1	—	—	—	—	0.4628	(0.0000)	0.4510	(0.0000)
p_{00}	—	—	0.9884	(0.0000)	0.9507	(0.0000)	0.9386	(0.0000)
p_{11}	—	—	2.47E − 9	(0.4546)	0.9698	(0.0000)	0.9634	(0.0000)
Log-likelihood, likelihood ratio (LR), and information criteria								
<i>LL</i>	−204.61		−193.61		−180.274		−178.29	
<i>LR</i>	—		22.000		26.672		3.968	
<i>AIC</i>	2.3933		2.3024		2.1622		2.1851	
<i>SIC</i>	2.5014		2.4645		2.3423		2.4373	
<i>HQC</i>	2.4371		2.3681		2.2353		2.2874	
<i>RMSE</i>	0.7739		0.7730		0.7806		0.7729	
<i>MAE</i>	0.5548		0.5543		0.5543		0.5501	

Supplementary Material B. Replication of previous studies with identical data

We replicate the results reported in Hamilton (2003)¹² which allows to verify previous results. For reasons of comparability with earlier studies and replication of their findings, we later repeat all estimations on the full sample from 1947 to 2016. Results are reported in Supplementary Material C. Secondly, we assess the impact of omitting the negative parts in the linear ARX(4) model. *In this subsection only, all references to equations refer to the original article*, however, the model framework is defined in Eq. (3) of this paper.

Supplementary Material B.1. The role of positive oil price changes

The results of the replication with and without negative oil price measures are reported in Tab. B.16. Columns 2, 3, 4, 6, and 8 present results obtained by applying an identical model framework and data periods as in Hamilton (2003). The results of estimating Eq. (1.5) show that only the fourth lag of nominal oil changes has a significant negative impact (coefficients of -0.0647). With the fully estimated model, this means that a 10% increase of nominal oil prices leads to an approx. 1.4% decrease of real GDP growth four quarters later (Hamilton, 2003, p. 369). When replicating Eq. (1.6), the only statistically significant lag of oil price changes is lag four, δ_4 , with an estimated coefficient of -0.0146 which is close to the reported -0.0160 in Hamilton (2003). Both coefficients are significant at the 5% level.

Before applying the nonlinear price measures, we conduct the nonlinearity test outlined in Subsection 2.2. The results of all the four test statistics are reported in Tab. B.15, which are very similar to those in Hamilton (2003, Tab. 1, p. 373). The test statistics lead to a rejection of the null hypothesis of linearity for the alternative of nonlinearity.

Table B.15: Results of the nonlinearity tests for Hamilton (2001) and Dahl & González-Rivera (2003a) for an identical period 1949Q2 to 2001Q3 as in Hamilton (2003) in Tab. 1 on p. 373.

Tests		Test value	Asymptotic p -value	Bootstrap p -value
Hamilton	ν^2	68.230	(0.000)	[0.001]
	λ^A	40.592	(0.000)	[0.011]
Dahl & González-Rivera	λ^E	51.022	(0.000)	[0.001]
	g^A	17.226	(0.244)	[0.295]

Having ruled out the hypothesis of linearity between oil price changes and real GDP growth, we continue our discussion and analysis of the results of the estimation of Eq. (1.8) in column (a) of Tab. B.16 with the measure of Mork (1989), $O_{\text{Mork},t}^+$. Again, the only significant positive oil price change is δ_4 estimated at -0.0213 which is similar to -0.023 estimated in Hamilton (2003). Both standard errors are identical at 0.009.

The results of using Hamilton's 1y net oil price increases, $O_{\text{Ham1},t}^+$ are presented in column Eq. (3.2), side (a) in Tab. B.16. The results mirror those reported in Hamilton

¹²In particular, results obtained with the models Eq. (1.5), Eq. (1.6), Eq. (1.8), Eq. (3.2), and Eq (3.8) in Hamilton (2003) are replicated.

(2003). Yet again, the only significant coefficient is at lag four, estimated at -0.0306 . Finally, by using the 3y net oil price increases, $O_{\text{Ham3},t}^+$, only δ_4 is found to be significant at the 5% level. We estimate the same magnitude (-0.0427) and level of significance (standard error 0.014). These results are given in the Eq. (3.8) column, side (a), of Tab. B.16.

Supplementary Material B.2. Combining positive and negative oil price changes

Kilian & Vigfusson (2011a, p. 427) suggest that if “both energy price increases and decreases matter for the real GDP but at a different extend, then the censored regressor model is likely to overestimate the effect of an energy price increase.” Following this argument, one may expect that all results reported in side (a) of the columns in Tab. B.16 are biased by overestimating the impact of price changes on real GDP since all of them are obtained with positively censored price measures only. To assess the impact of omitting the negative (decreases) part, we re-estimate all models of the previous subsection including both positive and negative measures, $O_{\{M\},t}^+$ and $O_{\{M\},t}^-$, as defined in Eq. (4).

The estimation results are reported in side (b) for each column of Tab. B.16. The estimated coefficients associated with negative (decreasing) oil price measures are all insignificant at the 10% level. Moreover, the results show that the inclusion of the negative oil price measures neither impacts the magnitude of positive oil price measures nor their significance. For example, the estimated coefficient associated to the fourth lag for Eq. (1.8(b)) is -0.0207 , whereas for Eq. (1.8(a)), the coefficient is estimated at -0.0213 . This holds true for estimates in Eq. (3.2) as well as Eq. (3.8). Moreover, in terms of significance there is no difference between including or omitting the negative measures as all lag coefficients of the oil price proxies keep their level of significance for this data set.

In order to determine which nonlinear measure is most suitable to capture nonlinearity, we carry out the second test introduced in Subsection 2.2. As noted, the specificity of this test compared to the first nonlinearity test is that it allows to examine whether the nonlinearity in the relationship is well captured. It also allows for comparison of the different price measures. The results are reported in Tab. B.17 and show that all measures applied are able to capture a certain degree of nonlinearity in the oil-GDP relationship as all the p -values are higher than the 5% level of significance.

In summary, empirical results show that for the data set 1949Q2 to 2001Q3, we obtain identical results to those in Hamilton (2003). Motivated by the theoretical and empirical findings of Kilian & Vigfusson (2011b), we include both positive and negative oil price measures. For this particular data and model set, omitting the negatively censored data has no impact on the results. Factor loads of the negative proxies are statistically insignificant throughout all models and proxies.

Table B.16: Parameter estimates for the replication of the results in Hamilton (2003) using identical periods from 1949Q2 to 1980Q4 for Eq. (1.5), and 1949Q2 to 2001Q3 for the rest of equations. Where applicable, column (a) refers to positively censored data only (parameters δ_1 to δ_4), while column (b) presents parameters for models including both positive and negative oil price change proxies (with additional parameters γ_1 to γ_4). p -values are given in parentheses. The information criteria applied throughout this study are the Akaike Information Criterion (AIC), the Schwarz Information Criterion (SIC, also known as Bayesian Information Criterion), and the Hannan-Quinn Criterion (HQC), defined as $AIC = (-2LL + 2k) / T$, $SIC = (-2LL + k \log(T)) / T$, $HQC = (-2LL + 2k \log(\log(T))) / T$.

	Eq. (1.5)	Eq. (1.6)	Eq. (1.8)		Eq. (3.2)		Eq. (3.8)	
			(a)	(b)	(a)	(b)	(a)	(b)
μ	1.1808 (0.0000)	0.7287 (0.0000)	0.8801 (0.0000)	0.8752 (0.0000)	0.8995 (0.0000)	0.9126 (0.0000)	1.0022 (0.0000)	1.0137 (0.0000)
ϕ_1	0.2078 (0.0222)	0.2860 (0.0000)	0.2669 (0.0002)	0.2692 (0.0002)	0.2568 (0.0002)	0.2549 (0.0003)	0.2201 (0.0014)	0.2209 (0.0015)
ϕ_2	0.0593 (0.5166)	0.1336 (0.0640)	0.1251 (0.0820)	0.1205 (0.0971)	0.1178 (0.0984)	0.1167 (0.1048)	0.1078 (0.1238)	0.1052 (0.1376)
ϕ_3	-0.1046 (0.2633)	-0.0784 (0.2764)	-0.0863 (0.2298)	-0.0817 (0.2601)	-0.0841 (0.2375)	-0.0843 (0.2404)	-0.0974 (0.1635)	-0.0954 (0.1771)
ϕ_4	-0.1813 (0.0456)	-0.1077 (0.1192)	-0.1252 (0.0697)	-0.1306 (0.0611)	-0.1230 (0.0715)	-0.1225 (0.0754)	-0.1414 (0.0352)	-0.1417 (0.0366)
δ_1	-0.0079 (0.7635)	-0.0042 (0.5074)	-0.0106 (0.2561)	-0.1179 (0.2317)	-0.0115 (0.3452)	-0.0116 (0.2328)	-0.0259 (0.0551)	-0.0263 (0.0547)
δ_2	-0.0269 (0.3165)	-0.0064 (0.3064)	-0.0079 (0.4008)	-0.0075 (0.4526)	-0.0146 (0.2440)	-0.0156 (0.2328)	-0.0234 (0.0876)	-0.0238 (0.0854)
δ_3	-0.0317 (0.2403)	-0.0018 (0.7786)	-0.0047 (0.6197)	-0.0058 (0.5675)	-0.0069 (0.5833)	-0.0076 (0.5491)	-0.0140 (0.3068)	-0.0144 (0.2996)
δ_4	-0.0647 (0.0188)	-0.0146 (0.039)	-0.0213 (0.0240)	-0.0207 (0.0389)	-0.0306 (0.0134)	-0.0305 (0.0151)	-0.0427 (0.0021)	-0.0431 (0.0021)
γ_1	-	-	-	0.0054 (0.6150)	-	0.0043 (0.7355)	-	0.0076 (0.5624)
γ_2	-	-	-	-0.0051 (0.6285)	-	0.0039 (0.7572)	-	-0.0036 (0.7853)
γ_3	-	-	-	0.0053 (0.6109)	-	0.0029 (0.8210)	-	0.0048 (0.7189)
γ_4	-	-	-	-0.0098 (0.3595)	-	-0.0046 (0.7201)	-	0.0022 (0.8666)
Log-likelihood and information criteria								
LL	-185.284	-285.749	-284.245	-283.597	-282.363	-282.142	-277.170	-276.876
AIC	3.0753	2.8166	2.8023	2.8343	2.7844	2.8204	2.7349	2.7702
SIC	3.2993	2.9760	2.9617	3.0574	2.9438	3.0435	2.8943	2.9934
HQC	3.1663	2.8811	2.8667	2.9244	2.8488	2.9106	2.7994	2.8604

Table B.17: Results of the adequacy test of an appropriate functional form to capture nonlinearities for the period 1949Q2 to 2001Q3.

Tested measure	Test value	Asymp. p -value
Mork (1989)	1.778	(0.1823)
Lee et al. (1995)	0.1532	(0.6954)
Hamilton (1996)	3.1135	(0.0776)
Hamilton (2003)	1.5921	(0.2070)

Supplementary Material C. Replication with data up to 2016Q4

Turning to the results of replicating previous analysis when adding newer data up to 2016Q4. The results are reported in Tab. C.18 to Tab. C.30. Several important conclusions can be drawn from these results.

1. First, for the linear relationship (in parameter) between oil prices and the real GDP growth (ARX(p) and linear MIDAS) we found that:

a. The linear specification as in Hamilton (1989) where oil prices changes have a symmetric impact on the real GDP growth is not an appropriate as it suffers from several weakness (see the results of the nonlinearity tests of Hamilton (2001) and Dahl & González-Rivera (2003a) reported in Tab. C.19).

b. The asymmetric oil prices measures are unable to capture all the nonlinearity in the oil price - real GDP relationship (see the results of the test of adequacy of the different nonlinear oil price measures reported in Tab. C.22).

c. The analysis using monthly oil price observations in the MIDAS framework, firstly, confirms the results of the ARX(p) model. Secondly, the linear MIDAS models reveals an impact of oil price decreases on the GDP (see Tab. C.23).

d. The relationship between oil price and the real GDP is instable as shown by the *Sup - F*, *Avg - F* and *Exp - F* tests (see Tab. C.21 and C.24).

2. For the nonlinear (in parameter) oil prices-real GDP relationship (ARX(p) and MIDAS):

a. The results of the nonlinearity tests of the oil price - real GDP relationship show strong evidence for the presence of nonlinearity in the first regime where all the p-values of the ν^2 , λ^A , λ^E , and g^A are lower than 1% in the first regime (from 1948Q2 to 1984Q2). In the second regime, from 1984Q3 to 2016Q4, the results show weak evidence for nonlinearity as only two over four tests statistics have a p-values lower than 5%.

b. Allowing for a instability in the real GDP growth and the oil price measures by using a Markov switching model, the results of estimation of the different models specifications show that in the first regime, up to 1984Q2, positive (increases) oil prices have a significant impact on the real GDP growth. In the second regime, from 1984Q3 to 2016Q4, positive (increases) oil prices have significant impact only when using the Mork and 3-years Hamilton oil price measures. For the negative (decreases) oil prices, we found that they have no effect for almost all the cases considered except for the case of Mork specification (see Tab. C.26).

c. The results of coefficients exclusions show that in the first regime all the positive (increases) oil prices have a significant impact on the real GDP growth at the 1% level except for the Mork's specification at the 5%. In this same regime, regime 1, negative (decreases) oil prices seems that have also a significant impact but at 10% level of significance for the 1-year and 3-years net oil price increases of Hamilton (1996, 2003). In the second regime, all positive (increases) and negative (decreases) oil prices have no impact on the real GDP growth except for the 3-years net oil prices increases at 10% level of significance.

d. Finally, the results of the tests of adequacy of the four oil prices measures reported in Tab. C.28 show that only the Mork's specification and 3-years Hamilton specification capture all the nonlinearity in the oil price - real GDP growth relationship in regime 1 as the p-values of the test statistics are greater than 5%.

e. The MIDAS regression shows evidence that in the first regime, price increases and decreases negatively affect the GDP in all models. In the second regime, the effect remains only for $O_{\text{Ham3},t}^{+/-}$.

Table C.18: Unit root test statistics for all time series with quarterly and monthly frequencies for the period 1948Q2 to 2016Q4.

		quarterly			monthly		
		ADF	PP	KPSS	ADF	PP	KPSS
GDP change	y_t	-7.8062***	-11.2707***	0.0468	—	—	—
Nominal price change	O_t	-16.1344***	-16.1604***	0.0349	-23.1557***	-23.1559***	0.0448
Real price change	O_t^{real}	-16.2197***	-16.1645***	0.0358	-23.4838***	-23.4404***	0.0378
Mork increase	$O_{\text{Mork},t}^+$	-12.4552***	-16.4293***	0.0688	-17.6470***	-21.2464***	0.2350***
Mork decrease	$O_{\text{Mork},t}^-$	-12.4723***	-15.0188***	0.0678	-28.3841***	-29.0508***	0.0468
Lee increase	$O_{\text{LNR},t}^+$	-13.7145***	-17.7522***	0.0579	-23.7423***	-26.9642***	0.1548**
Lee decrease	$O_{\text{LNR},t}^-$	-13.9958***	-17.7465***	0.0437	-17.7007***	-20.2205***	0.2407***
Hamilton 1y increase	$O_{\text{Ham1},t}^+$	-11.2427***	-12.9703***	0.1091	-23.1464***	-23.9979***	0.0398
Hamilton 1y decrease	$O_{\text{Ham1},t}^-$	-14.1514***	-15.3666***	0.0357	-28.6361***	-28.9786***	0.0516
Hamilton 3y increase	$O_{\text{Ham3},t}^+$	-11.9757***	-13.0029***	0.1495**	-20.5757***	-20.9835***	0.0559
Hamilton 3y decrease	$O_{\text{Ham3},t}^-$	-12.8232***	-13.5377***	0.0524	-25.6388***	-25.6402***	0.1110

Note: The ADF test and the PP test have the null hypothesis of an existing unit root. The KPSS test has the null hypothesis of no unit root. *, **, *** refer to the 10%, 5% and 1% levels of significance.

Table C.19: Results of the nonlinearity tests of Hamilton (2001) and Dahl & González-Rivera (2003a) for the most recent period considered in our study, 1947Q2 to 2016Q4.

Tests		Test statistics	Asymp. p -value	Boots. p -value
Hamilton	ν^2	333.269	(0.000)	[0.001]
	λ^A	184.045	(0.000)	[0.001]
Dahl & González-Rivera	λ^E	277.278	(0.000)	[0.001]
	g^A	91.348	(0.000)	[0.002]

Table C.20: Regression results from the linear extended Hamilton (1989) model using different oil price measures for the period 1948Q2 to 2016Q4.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value
μ_0	0.5671	(0.0000)	0.8002	(0.0000)	0.9753	(0.0000)	0.8224	(0.0000)	0.8299	(0.0000)
ϕ_1	0.3288	(0.0000)	0.2978	(0.0000)	0.2808	(0.0000)	0.2804	(0.0000)	0.2608	(0.0000)
ϕ_2	0.1433	(0.0269)	0.1198	(0.0632)	0.0764	(0.2425)	0.1177	(0.0665)	0.1165	(0.0654)
ϕ_3	-0.0623	(0.3336)	-0.0801	(0.2131)	-0.0495	(0.4507)	-0.0748	(0.2416)	-0.0789	(0.2102)
ϕ_4	-0.0742	(0.2253)	-0.1084	(0.0783)	-0.0512	(0.3980)	-0.1007	(0.0983)	-0.1035	(0.0832)
δ_1	-0.0003	(0.9308)	-0.0086	(0.2434)	-0.0948	(0.3033)	-0.0111	(0.2509)	-0.0199	(0.0725)
δ_2	-0.0036	(0.3559)	-0.0085	(0.2421)	-0.2478	(0.0082)	-0.0119	(0.2181)	-0.0150	(0.1685)
δ_3	-0.0049	(0.2199)	-0.0132	(0.0743)	-0.4011	(0.0000)	-0.0125	(0.1976)	-0.0180	(0.1009)
δ_4	-0.0089	(0.0250)	-0.0127	(0.0818)	-0.1124	(0.2434)	-0.0271	(0.0050)	-0.0341	(0.0002)
γ_1	-	-	0.0071	(0.2369)	-0.0173	(0.8432)	0.0064	(0.3498)	0.0098	(0.2992)
γ_2	-	-	-0.0012	(0.8362)	0.0941	(0.2814)	0.0011	(0.8738)	-0.0053	(0.5837)
γ_3	-	-	0.0007	(0.9148)	0.1127	(0.1963)	0.0021	(0.7604)	0.0003	(0.9759)
γ_4	-	-	-0.0089	(0.1509)	0.0581	(0.5054)	0.0004	(0.9537)	-0.0011	(0.9096)
σ_0	0.9099	(0.0000)	0.9023	(0.0000)	0.8469	(0.0000)	0.8947	(0.0000)	0.88047	(0.0000)
Log-likelihood and information criteria										
<i>LL</i>	-359.689		-355.282		-324.083		-352.945		-348.541	
<i>AIC</i>	2.6886		2.6857		2.4588		2.6687		2.6366	
<i>SIC</i>	2.8202		2.8698		2.6429		2.8528		2.8208	
<i>HQC</i>	2.7414		2.7596		2.5327		2.7426		2.7106	

Note: The oil price measures refer to the uncensored O_t of Hamilton (1983), the asymmetric measures $O_{\text{Mork},t}^{+/-}$ of Mork (1989), the volatility scaled measures $O_{\text{LNR},t}^{+/-}$ of Lee et al. (1995), and Hamilton's measures $O_{\text{Ham1},t}^{+/-}$ (Hamilton, 1996) as well as $O_{\text{Ham3},t}^{+/-}$ (Hamilton, 2003).

Table C.21: Exclusion tests for linear models with data from 1948Q2 to 2016Q4. Exclusion test (a) restricts oil price increases and decreases. Test (b) restricts increases and decreases separately. The Andrews and Ploberger test has the null hypothesis that the model parameters are stable over the whole sample.

	Exclusion tests		Andrews and Ploberger (1994) tests		
	(a)	(b)	Sup F	Avg F	Exp F
Panel A: Hamilton (1983)					
O_t	1.8123 (0.1267)		3.3181 (0.0018)	2.3673 (0.0009)	1.2733 (0.0106)
Panel B: Mork (1989)					
$O_{\text{Mork},t}^+$		3.3181 (0.0113)			
$O_{\text{Mork},t}^-$	1.9883 (0.0482)	0.7954 (0.5291)	2.7424 (0.0160)	1.8571 (0.0044)	0.9760 (0.0446)
Panel C: Lee et al. (1995)					
$O_{\text{LNR},t}^+$		5.9678 (0.0001)			
$O_{\text{LNR},t}^-$	3.1229 (0.0022)	0.7642 (0.5491)	3.0700 (0.0040)	1.6327 (0.0189)	0.8554 (0.1116)
Panel D: Hamilton (1996)					
$O_{\text{Ham1},t}^+$		4.9776 (0.0007)			
$O_{\text{Ham1},t}^-$	2.5837 (0.0099)	0.2645 (0.9006)	3.2016 (0.0022)	1.7776 (0.0074)	0.9638 (0.0492)
Panel E: Hamilton (2003)					
$O_{\text{Ham3},t}^+$		6.9907 (0.0000)			
$O_{\text{Ham3},t}^-$	3.7337 (0.0004)	0.2995 (0.8781)	3.1735 (0.0000)	1.6593 (0.0112)	0.9836 (0.0347)

Table C.22: Results of the adequacy test of an appropriate functional form to capture nonlinearities for the period 1947Q2 to 2016Q4.

test (2)		
Tested measure	Test statistic	Asymptotic p -value
Mork (1989)	4.683	(0.030)
Lee et al. (1995)	14.701	(0.000)
Hamilton (1996)	16.454	(0.000)
Hamilton (2003)	22.182	(0.000)

Table C.23: MIDAS regression results for 1948Q2-2016Q4 and $n = 275$ observations.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	p -value	Coef.	p -value	Coef.	p -value	Coef.	p -value	Coef.	p -value
μ_0	0.5283	(0.0000)	0.7550	(0.0000)	0.8753	(0.0000)	0.7040	(0.0000)	0.6439	(0.0000)
ϕ_1	0.3304	(0.0000)	0.2998	(0.0001)	0.2780	(0.0006)	0.2969	(0.0001)	0.2983	(0.0001)
ϕ_2	0.1391	(0.0950)	0.1348	(0.1007)	0.1289	(0.0980)	0.1303	(0.1094)	0.1449	(0.0803)
ϕ_3	-0.0486	(0.4995)	-0.0694	(0.3360)	-0.0574	(0.4633)	-0.0669	(0.3504)	-0.0665	(0.3570)
ϕ_4	-0.0687	(0.3692)	-0.0884	(0.2332)	-0.0854	(0.2140)	-0.0752	(0.2902)	-0.0812	(0.2588)
θ_1	-0.0097	(0.1143)	-0.0499	(0.0492)	-0.9268	(0.1371)	-0.2661	(0.0045)	-0.2916	(0.0004)
ω_{11}	232.5065	(0.8317)	0.8896	(0.4780)	2.0984	(0.7466)	0.7624	(0.1022)	1.4694	(0.1416)
ω_{21}	14.3970	(0.7941)	0.9759	(0.0000)	1.0194	(0.0000)	0.9683	(0.0000)	1.9748	(0.1441)
θ_2	-	-	0.0351	(0.0575)	0.2045	(0.0894)	0.2032	(0.0614)	0.1437	(0.0177)
ω_{12}	-	-	3.0767	(0.4527)	3.4883	(0.5636)	0.4050	(0.3333)	84.0082	(0.3959)
ω_{12}	-	-	12.8570	(0.5058)	8.9215	(0.5788)	0.9538	(0.0000)	293.9155	(0.4358)
σ_0	0.8947	(0.0000)	0.8789	(0.0000)	0.8631	(0.0000)	0.8780	(0.0000)	0.8743	(0.0000)
Log-likelihood and information criteria										
LL	-359.598		-354.700		-349.714		-354.437		-353.265	
AIC	2.6807		2.6669		2.6306		2.6650		2.6565	
SIC	2.7991		2.8247		2.7885		2.8228		2.8143	
HQC	2.7282		2.7302		2.6940		2.7283		2.7198	

Table C.24: Exclusion tests for non-switching MIDAS regressions for 1948Q2-2016Q4. Exclusion test (a) restricts oil price increases and decreases. Test (b) restricts increases and decreases separately.

	Exclusion tests		Andrews and Ploberger (1994) tests		
	(a)	(b)	Sup F	Avg F	Exp F
Panel A: Hamilton (1983)					
O_t	2.5095 (0.1143)		43.0740 (0.0001)	22.3164 (0.0005)	17.6201 (0.0001)
Panel B: Mork (1989)					
$O_{\text{Mork},t}^+$		3.9049 (0.0492)			
	4.6342 (0.0105)	3.6392 (0.0575)	47.5534 (0.0001)	20.8002 (0.0124)	19.3429 (0.0002)
$O_{\text{Mork},t}^-$					
Panel C: Lee et al. (1995)					
$O_{\text{LNR},t}^+$		2.2234 (0.1371)			
	2.1955 (0.1133)	2.9061 (0.0894)	40.2661 (0.0021)	21.3799 (0.0092)	16.6499 (0.0015)
$O_{\text{LNR},t}^-$					
Panel D: Hamilton (1996)					
$O_{\text{Ham1},t}^+$		8.2209 (0.0045)			
	6.9957 (0.0011)	3.5288 (0.0614)	43.4775 (0.0007)	22.6814 (0.0047)	17.9483 (0.0006)
$O_{\text{Ham1},t}^-$					
Panel E: Hamilton (2003)					
$O_{\text{Ham3},t}^+$		12.9337 (0.0004)			
	9.2446 (0.0001)	5.6922 (0.0177)	53.0824 (0.0000)	24.9514 (0.0013)	22.1713 (0.0000)
$O_{\text{Ham3},t}^-$					

Table C.25: Results of the nonlinearity tests for Hamilton (2001) and Dahl & González-Rivera (2003a) for each regime period for the sample 1948Q2 - 2016Q4.

Tests		Regime 0			Regime 1		
		Test statistic	Asymp. p -value	Boots. p -value	Test statistic	Asymp. p -value	Boots. p -value
Hamilton	ν^2	195.276	(0.000)	[0.001]	2.122	(0.145)	[0.107]
Dahl & González-Rivera	λ^A	146.373	(0.000)	[0.001]	9.271	(0.863)	[0.495]
	λ^E	54.486	(0.000)	[0.001]	5.169	(0.023)	[0.092]
	g^A	194.993	(0.000)	[0.001]	24.028	(0.045)	[0.090]

Table C.26: Regression results from the Markov switching models using different oil price measures for the period 1948Q2 to 2016Q4. The applied model reads $y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \delta_{i,s_t} O_{\{M\},t-i}^+ + \sum_{i=1}^q \gamma_{i,s_t} O_{\{M\},t-i}^- + u_{s_t,t}$.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value	Coef.	p-value
Non-switching variables										
ϕ_1	0.2214	(0.0009)	0.2585	(0.0043)	0.2151	(0.0006)	0.1802	(0.0103)	0.2068	(0.0035)
ϕ_2	0.1834	(0.0096)	0.1645	(0.0239)	0.1500	(0.0293)	0.1767	(0.0098)	0.1747	(0.0112)
ϕ_3	-0.0951	(0.3845)	-0.1176	(0.1680)	-0.0637	(0.3594)	-0.0929	(0.1548)	-0.0721	(0.2414)
ϕ_4	0.0262	(0.6729)	-0.0794	(0.2512)	-0.0019	(0.8527)	-0.0533	(0.3882)	-0.0478	(0.4558)
Switching variables										
μ_0	0.8042	(0.0000)	0.7643	(0.0000)	1.4118	(0.0000)	0.9371	(0.0000)	0.8893	(0.0000)
δ_{10}	-0.0005	(0.9516)	-0.0043	(0.8231)	-0.1174	(0.2394)	-0.0192	(0.2156)	-0.0124	(0.5900)
δ_{20}	-0.00087	(0.6696)	-0.0195	(0.2009)	-0.5246	(0.0000)	-0.0261	(0.1480)	-0.0151	(0.4067)
δ_{30}	-0.0328	(0.0051)	-0.0122	(0.4634)	-0.6418	(0.0001)	-0.0222	(0.1252)	-0.0322	(0.0203)
δ_{40}	-0.0499	(0.0000)	-0.0210	(0.3235)	-0.2441	(0.1173)	-0.0516	(0.0248)	-0.0539	(0.0005)
γ_{10}	—	—	0.0101	(0.2913)	0.0328	(0.8936)	0.0055	(0.3915)	0.0028	(0.7944)
γ_{20}	—	—	0.0091	(0.7444)	0.3915	(0.1591)	0.0418	(0.2264)	-0.0006	(0.8649)
γ_{30}	—	—	-0.0284	(0.2529)	0.3355	(0.2449)	-0.0525	(0.1776)	-0.0722	(0.2789)
γ_{40}	—	—	-0.0482	(0.0109)	0.3216	(0.1980)	-0.0355	(0.2654)	-0.0109	(0.5900)
σ_0	1.0569	(0.0000)	1.0865	(0.0000)	0.9876	(0.0000)	1.0401	(0.0000)	1.0460	(0.0000)
μ_1	0.5349	(0.0000)	0.7555	(0.0033)	0.4541	(0.0001)	0.5667	(0.0000)	0.5776	(0.0000)
δ_{11}	-0.0026	(0.4012)	-0.0150	(0.0384)	0.0019	(0.9784)	0.01325	(0.1519)	-0.0249	(0.0372)
δ_{21}	0.0002	(0.9937)	-0.0032	(0.6284)	0.1667	(0.1195)	0.0029	(0.8497)	-0.0082	(0.4292)
δ_{31}	0.0004	(0.8737)	-0.0056	(0.4204)	-0.0528	(0.4250)	-0.0006	(0.9560)	0.0077	(0.3224)
δ_{41}	-0.0019	(0.3845)	-0.0054	(0.6391)	0.0608	(0.4030)	-0.0055	(0.5248)	-0.0049	(0.5182)
γ_{11}	—	—	0.0041	(0.6391)	-0.0115	(0.7877)	-0.0029	(0.4547)	0.0033	(0.3103)
γ_{21}	—	—	-0.0021	(0.3481)	-0.0214	(0.6457)	-0.0019	(0.4647)	-0.0006	(0.9034)
γ_{31}	—	—	0.0020	(0.6095)	0.0417	(0.2969)	0.0003	(0.9123)	0.0006	(0.9034)
γ_{41}	—	—	-0.0020	(0.5301)	-0.0438	(0.2489)	-0.0003	(0.9213)	-0.0027	(0.6033)
σ_1	0.4660	(0.0000)	0.4363	(0.0000)	0.4568	(0.0000)	0.4374	(0.0000)	0.4311	(0.0000)
Transition probabilities										
p_{00}	0.9854	(0.0000)	0.9676	(0.0000)	0.9853	(0.0000)	0.9846	(0.0000)	0.9734	(0.0000)
p_{11}	0.9770	(0.0000)	0.9568	(0.0000)	0.9766	(0.0000)	0.9755	(0.0000)	0.9666	(0.0000)
Information criterion and Likelihood ratio statistic										
LL	-320.1835		-320.993		-293.102		-319.526		-314.615	
LR	79.0114		68.585		61.9744		64.380		67.605	
AIC	2.4595		2.5235		2.4174		2.5129		2.4772	
SIC	2.6962		2.8655		2.7696		2.8549		2.8191	
HQC	2.5545		2.6608		2.5589		2.6502		2.6144	

Table C.27: Exclusion tests for Markov switching regression models for 1948Q2-2016Q4. Exclusion test (a) restricts both measures in both regimes, test (b) restricts the measure in regime 0 and 1 separately, tests (c) restrict both measures only in regime 0, reported in (c₀), or regime 1, reported in (c₁), and tests (d₀) and (d₁) restrict the oil price measure separately only in regime 0 or 1. *p*-values are given in parentheses.

Exclusion tests					
	(a)	(b)	(c ₀)	(c ₁)	(d ₀) (d ₁)
Panel A: Hamilton (1983)					
O_t	3.4091 (0.0010)		6.3227 (0.0001)	0.5083 (0.7297)	
Panel B: Mork (1989)					
$O_{\text{Mork},t}^+$	2.7788 (0.0004)	2.2926 (0.0219)	1.9553 (0.0527)	1.2617 (0.2641)	2.5247 (0.0415) 1.3566 (0.2496)
$O_{\text{Mork},t}^-$		1.3501 (0.2193)			1.9121 (0.1089) 0.5660 (0.6875)
Panel C: Lee et al. (1995)					
$O_{\text{LNR},t}^+$	3.0076 (0.0001)	5.0857 (0.0000)	5.2092 (0.0000)	0.8133 (0.5915)	9.3523 (0.0000) 0.8749 (0.4796)
$O_{\text{LNR},t}^-$		0.8817 (0.5325)			0.7276 (0.5739) 0.9535 (0.4338)
Panel D: Hamilton (1996)					
$O_{\text{Ham1},t}^+$	3.5916 (0.0000)	3.2219 (0.0017)	6.4103 (0.0000)	0.5323 (0.8317)	5.9834 (0.0000) 0.8806 (0.4761)
$O_{\text{Ham1},t}^-$		1.2134 (0.2914)			2.1257 (0.0782) 0.3532 (0.8417)
Panel E: Hamilton (2003)					
$O_{\text{Ham3},t}^+$	5.3940 (0.0000)	7.6678 (0.0000)	7.6678 (0.0000)	2.9265 (0.0566)	7.4071 (0.0000) 1.9952 (0.0958)
$O_{\text{Ham3},t}^-$		1.9265 (0.0566)			2.3802 (0.0523) 1.2180 (0.3036)

Table C.28: Results of the adequacy test of an appropriate functional form to capture nonlinearities for each regime.

	Regime 0		Regime 1	
	Test value	Asymp. <i>p</i> -value	Test value	Asymp. <i>p</i> -value
Mork (1989)	2.379	(0.123)	0.193	(0.661)
Lee et al. (1995)	4.661	(0.008)	0.021	(0.886)
Hamilton (1996)	3.849	(0.049)	0.136	(0.713)
Hamilton (2003)	3.409	(0.065)	0.256	(0.613)

Table C.29: Regression results from MS-MIDAS models using different oil price measures for 1948Q2-2016Q4 with $n_1 = 275$ quarterly and $n_2 = 828$ monthly observations. The models reads $y_t = \mu_{s_t} + \sum_{i=1}^p \phi_i y_{t-i} + \theta_{1,s_t} B(L^{1/m}; \omega_{1,s_t}) O_{\{M\},t}^{+,(m)} + \theta_{2,s_t} B(L^{1/m}; \omega_{2,s_t}) O_{\{M\},t}^{-,(m)} + u_{t,s_t}$.

	O_t		$O_{\text{Mork},t}^{+/-}$		$O_{\text{LNR},t}^{+/-}$		$O_{\text{Ham1},t}^{+/-}$		$O_{\text{Ham3},t}^{+/-}$	
	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value	Coef.	<i>p</i> -value
Non-switching variables										
ϕ_1	0.2530	(0.0004)	0.2351	(0.0007)	0.2683	(0.0001)	0.2251	(0.0014)	0.1957	(0.0048)
ϕ_2	0.1785	(0.0085)	0.1761	(0.0104)	0.1604	(0.0152)	0.1613	(0.0123)	0.2006	(0.0029)
ϕ_3	-0.0758	(0.1991)	-0.0768	(0.1952)	-0.0785	(0.1728)	-0.0761	(0.1856)	-0.0703	(0.2399)
ϕ_4	-0.0226	(0.7203)	-0.0324	(0.5691)	-0.0115	(0.8283)	-0.0248	(0.6642)	-0.0434	(0.4464)
Switching variables										
μ_0	0.7066	(0.0000)	0.7748	(0.0000)	0.8936	(0.0000)	0.8074	(0.0000)	0.8062	(0.0000)
θ_{10}	-0.1823	(0.0207)	-0.1905	(0.0245)	-0.8626	(0.0000)	-0.6555	(0.0010)	-0.9062	(0.0610)
ω_{110}	16.6387	(0.5349)	12.3901	(0.2117)	294.2053	(0.3671)	9.0580	(0.0569)	2.3752	(0.4725)
ω_{120}	2.4278	(0.6190)	1.3051	(0.0000)	43.4185	(0.3809)	1.2176	(0.0000)	1.0869	(0.0000)
θ_{20}	—	—	0.0570	(0.0083)	0.6063	(0.0681)	0.9295	(0.0001)	1.3069	(0.0051)
ω_{210}	—	—	0.7516	(0.5584)	1.5153	(0.5583)	0.9405	(0.0000)	0.9481	(0.0000)
ω_{220}	—	—	61.5215	(0.7909)	203.5914	(0.8234)	1.5128	(0.0315)	1.6907	(0.4194)
σ_0	1.0914	(0.0000)	1.0664	(0.0000)	1.0464	(0.0000)	1.0529	(0.0000)	1.0615	(0.0000)
μ_1	0.4877	(0.0000)	0.6007	(0.0000)	0.5590	(0.0000)	0.5640	(0.0000)	0.5742	(0.0000)
θ_{11}	-0.0082	(0.2038)	-0.0173	(0.0275)	-0.2200	(0.0037)	-0.0350	(0.2137)	-0.1533	(0.0002)
ω_{111}	170.1968	(0.9731)	160.3108	(0.7483)	153.4703	(0.1544)	130.0460	(0.7428)	2.0052	(0.0001)
ω_{121}	299.4396	(0.9727)	292.1936	(0.7375)	268.3854	(0.1753)	209.7350	(0.7717)	3.9575	(0.0019)
θ_{21}	—	—	0.0106	(0.2558)	0.0878	(0.2325)	0.0547	(0.0001)	0.0590	(0.0099)
ω_{211}	—	—	25.3868	(0.7305)	18.0283	(0.4041)	188.1104	(0.8115)	7.7060	(0.4545)
ω_{221}	—	—	185.2650	(0.6866)	1.0056	(0.0000)	1.4995	(0.5025)	36.4772	(0.5440)
σ_1	0.4609	(0.0000)	0.4550	(0.0000)	0.4556	(0.0000)	0.4412	(0.0000)	0.4439	(0.0000)
Transition probabilities										
p_{00}	0.9867	(0.0000)	0.9873	(0.0000)	0.9863	(0.0000)	0.9874	(0.0000)	0.9872	(0.0000)
p_{11}	0.9858	(0.0000)	0.9861	(0.0000)	0.9846	(0.0000)	0.9861	(0.0000)	0.9859	(0.0000)
Information criterion and Likelihood ratio statistic										
<i>LL</i>	-317.347		-311.5077		-308.906		-307.132		-308.858	
<i>LR</i>	84.4320		84.4675		80.9608		94.6105		87.9174	
<i>AIC</i>	2.4243		2.4255		2.4066		2.3937		2.4062	
<i>SIC</i>	2.6348		2.7149		2.6959		2.6830		2.6956	
<i>HQC</i>	2.5088		2.5416		2.5227		2.5098		2.5224	

Table C.30: Exclusion tests for Markov Switching MIDAS regression models for 1948Q2-2016Q4. Exclusion test (a) restricts both measures in both regimes, test (b) restricts the measure in regime 0 and 1, tests (c₀) and (c₁) restrict both measures only in regime 0 or 1, and tests (d₀) and (d₁) restrict the oil price measure only in regime 0 or 1.

Exclusion tests						
	(a)	(b)	(c ₀)	(c ₁)	(d ₀)	(d ₁)
Panel A: Hamilton (1983)						
O_t	4.2471 (0.0153)		5.4117 (0.0208)	1.6228 (0.2038)		
Panel B: Mork (1989)						
$O_{\text{Mork},t}^+$	11.6774 (0.0000)	4.7236 (0.0097)	21.9863 (0.0000)	3.1362 (0.0451)	5.1181 (0.0245)	4.9132 (0.0275)
$O_{\text{Mork},t}^-$		4.5822 (0.0111)			7.0769 (0.0083)	1.2969 (0.2559)
Panel C: Lee et al. (1995)						
$O_{\text{LNR},t}^+$	13.8418 (0.0000)	24.1178 (0.0000)	20.9099 (0.0000)	5.5820 (0.0042)	39.2954 (0.0000)	8.5971 (0.0037)
$O_{\text{LNR},t}^-$		2.2092 (0.1119)			3.3539 (0.0682)	1.4321 (0.2325)
Panel D: Hamilton (1996)						
$O_{\text{Ham1},t}^+$	10.6806 (0.0000)	6.7583 (0.0014)	12.7715 (0.0000)	8.5311 (0.0003)	11.1292 (0.0010)	1.5532 (0.2138)
$O_{\text{Ham1},t}^-$		16.9127 (0.0000)			16.7285 (0.0001)	15.7994 (0.0001)
Panel E: Hamilton (2003)						
$O_{\text{Ham3},t}^+$	8.3959 (0.0000)	8.7553 (0.0002)	5.6107 (0.0041)	11.8527 (0.0000)	3.5389 (0.0611)	13.8565 (0.0002)
$O_{\text{Ham3},t}^-$		6.8147 (0.0013)			7.9854 (0.0051)	6.7452 (0.0100)