ZDM (2019) 51:991–1003
https://doi.org/10.1007/s11858-019-01048-6

ORIGINAL ARTICLE

Promoting active citizenship in mathematics teaching

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Accepted: 28 March 2019 / Published online: 3 April 2019
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Abstract
Mathematical and scientific knowledge are integral to preparing our population to be actively engaged and responsible citizens. Science and mathematics education, however, has mainly focused on concepts and skills detached from societal implications. In this paper we present an interdisciplinary international design research study in which we developed teaching and professional development materials connecting mathematics and science education to citizenship education. We outline the design research process, its theoretical basis as well as the design products. The study shows that it is indeed possible to develop such approaches supporting active citizenship and thereby the development of 21st century skills in mathematics education, thus strengthening the role of mathematics education in the STEM field.

Keywords Mathematical modelling · Inquiry-based learning · Socio-scientific issues · Numeracy · Teachers' professional development · Design research

1 Introduction

In recent years, Europe has changed: We have an increasingly technology driven society, industries are moving to low-income countries, and people are migrating from conflict areas or for economic reasons. Such changes affect fundamental values of freedom, democracy, equality, human rights and dignity. In response, the European Commission emphasizes the necessity of ensuring that young people acquire social, civic and intercultural competences, by promoting democratic values and fundamental rights, social inclusion and active citizenship, and by enhancing critical thinking and media literacy (Eurydice 2016).

These competences are closely connected to the notion of twenty first century skills, which include creativity, critical thinking, problem-solving, decision-making, communicating, collaborating, and information literacy. The OECD has proposed that twenty first-century skills, rather than being distinct from the traditional school curriculum, are instead relevant to effective learning in all knowledge domains (Schleicher 2012). Naturally, this also includes science, technology, engineering and mathematics (STEM) subjects (cf. Bybee 2010). STEM knowledge is fundamental for being an actively engaged and responsible citizen and for becoming fully aware of the complex challenges that our society faces. It helps to explain and understand the world, to guide technological development and innovation and to plan for the future (Hazelkorn et al. 2015).

Conventionally, however, science (including mathematics) education has focused on the 'learning of science' (Hazelkorn et al. 2015), on pure science detached from societal implications. This focus can be contrasted with learning 'of and about science' (Osborne and Dillon 2008). It is often neglected that science has social, cultural and ethical dimensions (e.g., decision-making in genetic engineering, water management, energy production and consumption, and usage and storage of big data). Learning of and about science also fosters young people's understanding of nature and of applications and implications of science. Consequently, by learning of and about science, they learn principles and

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competences vital in democratic, pluralistic and increasingly multi-cultural European societies. In this sense, science and mathematics education also have become part of citizenship education.

This paper explores the contribution of mathematics education to the development of such civic competences, active citizenship, and personal and social responsibility as part of twenty first century skills in the STEM area. The aim of this study is to investigate the possibilities of enhancing learning 'about' mathematics (including its social, cultural and ethical aspects), promoting active citizenship in mathematics lessons, and supporting teachers in extending their practices in these directions.

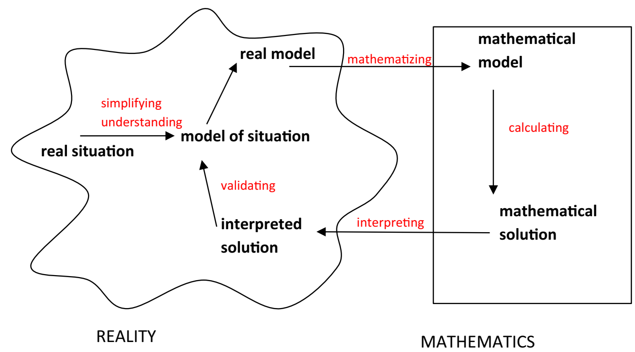

Fig. 1 An idealized scheme of the modelling process (according to Blum and Leiss 2007)

2 Theoretical background

The idea of connecting mathematics education and citizenship education is not new. Already in the twentieth century, scholars discussed the notion of mathematical modelling and bringing extra-mathematical contexts into the classrooms (e.g., Burkhardt 2018a, b). Since then, learning environments involving modelling real-life problems and providing effective support for the learning processes, have entered education, as they can create opportunities to develop content knowledge as well as skills such as creativity and critical thinking. Thus, the modelling perspective provides basic arguments for including authentic situations in the mathematics classrooms. Furthermore, in this section we extend that perspective, with attempts to address socio-scientific issues in education, and inquiry-based teaching methods for supporting teachers in implementing these issues in their classrooms.

2.1 Mathematical modelling

We first explore mathematical modelling, in order better to understand the potential of this perspective for fostering twenty first century skills in teaching practice. Mathematical modelling has a variety of different definitions (Kaiser and Sriraman 2006). We define mathematical modelling as the solving of an extra-mathematical problem from the real world by carrying out a modelling process (Niss et al. 2007). Based on the formulation of Blum and Leiss (2007), the modelling process can be conceptualized as outlined in Fig. 1.

Knowledge about modelling processes as shown in Fig. 1 can be regarded as meta-knowledge about modelling, and thus forms a basis for metacognitive modelling competences (Maass 2007). Within the discussion of modelling, metacognitive modelling competences are considered to be important for the development of modelling competences and have become an object of systematic study (Vorhölter 2018).

Modelling practices in the school context need problems that engage students in modelling processes. In our interpretation, modelling tasks are required to have an authentic, extra-mathematical, character (cf. Kaiser et al. 2011). This requirement means that they are problems of a certain professional discipline, which are only a little simplified, and that experts working in this discipline recognize them as problems they might meet in their daily work (Niss 1992). In a similar direction, Palm (2007) defines authenticity of problems as 'being true' in relation to whether the problem, taken from a situation in the real world, has occurred or might happen. Current approaches to authentic modelling activities can be found, for example, in publications by Vos (2015) and Kaiser et al. (2013).

The implementation of modelling in mathematics lessons has the potential to motivate students to develop their talents fully (EC 2013; Kaiser 1995), support an appropriate view on mathematics (Kaiser 1995), foster mathematical and scientific literacy (Steen 2001), develop an in-depth understanding of mathematical content (Gravemeijer 2007; English 2016), and to foster students' development of civic competences (Artigue and Blomhøj 2013).

The importance of including modelling activities in mathematical education is generally acknowledged as they play an important role in the PISA studies. Furthermore, modelling activities have a positive impact on students' competence in applying mathematics to complex situations (see, e.g., Maass 2007), on students' attitudes towards mathematics (e.g., Mischo and Maass 2013), on mathematical competences (e.g., English and Watson 2018) as well as on transversal competences (e.g., Ärlebäck and Doerr 2018). Modelling is also considered to be "a powerful vehicle for bringing features of twenty first century problems into the mathematics classroom" (English 2016, p. 362).

The definition of modelling given above, and the link with civic and transversal competences, emphasize the role of problems taken from the real world and thereby also include issues and decisions that have ethical, moral, social or cultural aspects. However, often these aspects are not explicitly mentioned in the discussion about mathematical modelling. In contrast, within natural sciences it has become common also to use contexts involving controversy. The related ethical, moral, social or cultural aspects in these contexts are explicitly discussed with respect to the scientific problem, and called socio-scientific issues.

2.2 Socio-scientific issues (SSIs)

SSIs have a basis in science education and require students to engage in dialogue, discussion, and debate. They are mainly controversial in nature but also require forming opinions and making decisions including moral, ethical or social reasoning issues (Zeidler and Nichols 2009). Most of the time, people have to deal with these issues through incomplete information because of conflicting or incomplete scientific evidence and incomplete reporting. Often these issues involve a cost–benefit analysis in which risk interacts with ethical reasoning (Ratcliff and Grace 2003). Consequently, such contexts especially serve the purpose of educating for scientific citizenship (Owen et al. 2009).

An example of an SSI in the area of biology is the question whether vaccination against measles should be obligatory or not. Opponents of vaccination ignore scientific evidence on vaccination and epidemics, and tend to refer to their own evidence and experts. In order to follow the discussion on this issue as an active citizen, young people need to learn about such issues and how they are influenced by ethical, moral and cultural issues.

Research has shown that SSIs can be used as contexts for learning scientific content (Applebaum et al. 2006; Walker 2003; Zohar and Nemet 2002) and for understanding the nature of science (learning 'about science', see part 1) and for citizenship education (Herman et al. 2018; Radakovic 2015; Sadler et al. 2007). In this respect, the authors highlight the following important aspects when dealing with SSIs: (1) recognising the inherent complexity of SSIs, (2) examining issues from multiple perspectives, (3) appreciating that SSIs are subject to ongoing inquiry, (4) exhibiting scepticism when presented with potentially biased information.

Within mathematics education as well, aspects such as decision-making, controversy and critical thinking are included in the concept of numeracy or mathematical literacy (Geiger et al. 2015): "Numeracy is a concept used to identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life, and to participate in society as informed, reflective, and contributing citizens" (p. 531). Consequently, numeracy includes making socially conscious decisions based upon quantitative information and the ability to develop arguments that support or challenge quantitative reasoning by some kind of authority (Geiger et al. 2015). Within this discussion, mathematics education is allocated a role in building democratic societies (D'Ambrosio 2003; Skovsmose 1998).

The considerations thus far focus on mathematical modelling and the use of socio-scientific issues. But how exactly should modelling and dealing with SSIs be implemented in mathematics classroom practices? One approach that has proven to be helpful in science education is inquiry-based learning (Knippels and van Dam 2017). Consequently, combining modelling and inquiry-based teaching approaches with SSIs seems to have the potential to promote active citizenship in mathematics education.

2.3 Inquiry-based learning (IBL)

By IBL, we refer to a student-centred learning paradigm in which students are involved in inquiry-related processes like observing phenomena and creating their own questions, selecting mathematical approaches, creating representations to clarify relationships, seeking explanations, interpreting and evaluating solutions, and communicating their solutions (Dorier and Maass 2014).

On the teacher's part, pedagogies evolve from a 'transmission' orientation, in which teacher explanations, illustrative examples and exercises dominate and are not questioned, towards a more collaborative orientation. The teacher's role includes making constructive use of students' prior knowledge, challenging students through probing questions, managing small group and whole class discussions, encouraging alternative viewpoints, learning from mistakes and helping students to make connections between their ideas (Swan 2005, 2007).

Definitions of IBL, however, differ in the degree of autonomy given to students in the selection of problems and in the responsibility for inquiry processes (Artigue and Blomhøj 2013). In our approach to IBL, we refer to a socio-cultural approach in which learning needs to happen in interactive social classroom settings (Radford 2010) and the teacher takes an active role by creating learning situations inspired by inquiry-related processes. Teachers who take these active roles in guiding their students are more effective than those who take passive roles and let students discover on their own (Askew et al. 1997; Swan 2006).

For the purpose of promoting citizenship education, students need to have an active role, similar to that in IBL, for developing critical thinking and decision making, for learning to take into account ethical, social and cultural aspects, and for learning to deal with controversy (Zeidler and Nichols 2009; Geiger et al. 2015). Already Dewey (1916) emphasized the connection between IBL and education serving democracy.

The path of connecting IBL to SSIs was also followed in the EU-project Promoting Attainment of Responsible

Research and Innovation in Science Education (Parrise). SSIs were connected to IBL by defining “Socio-Scientific Inquiry Based Learning” (SSIBL) (Knippels and van Dam 2017). In these SSIBL lessons three main stages can be distinguished: Raising authentic questions (ask), performing research by integrating social and scientific inquiry (enact), and formulating solutions (act). Important activities in the second stage ‘enact’ are modelling as well as discussing values from personal, global and social perspectives, and mapping controversies. The aim of this study is to investigate possibilities and limitations of implementing citizenship education in mathematics by transferring the concept of SSIs in connection with IBL to mathematics. We intend to do so by developing related classroom materials and a professional development course.

2.4 Professional development of teachers

The term teachers’ professional development (PD) relates to growth in teachers’ content knowledge (knowledge about the subject), pedagogical content knowledge (knowledge about how to teach the subject), and pedagogical knowledge (Shulman 1986). PD includes teachers’ classroom practice (Clarke and Hollingsworth 2002), as well as their beliefs, motivation, and competence in self-reflection (Baumert and Kunter 2013). Following on from the so-called “interconnected model” (Clarke and Hollingsworth 2002) teachers’ professional growth can start through changes in teachers’ knowledge and beliefs, through professional experimentation; through salient outcomes of experimentation or through external stimulus.

In the last decades, much research has been carried out on PD. Following on from the results of these studies, it is now commonly accepted that PD courses should:

- take into account all facets of teachers competences and practices (Barzel and Selter 2015);
- take into account teachers’ needs (Guskey 2000) and challenges they face (Maass 2011);
- combine phases of learning in seminars and at school (Lipowsky and Rzejak 2012);
- stimulate cooperation between teachers (McLaughlin and Talbert 2006);
- be relevant to teaching practice (Clarke 1994) and
- foster teachers’ reflection on their beliefs about mathematics teaching and their teaching experiences (Tirosh and Graeber 2003).

These criteria for PD courses have proven efficiency in practice (Maaß and Engeln this volume; Maass and Engeln 2018), and provide guidelines for the design of PD courses on a general level. However, they do not answer the question of what a PD course preparing mathematics teachers for citizenship education using SSIs and IBL should look like.

2.5 Research questions

Based on our discussion in Sects. 2.1 to 2.4, the following research questions frame our study:

- How can citizenship education using SSIs and IBL be addressed in mathematics education?
- How can mathematics teachers be supported in implementing active citizenship education through a PD course?

3 Methodology

In order to find answers to these research questions we needed to design classroom and PD materials connecting mathematics education to citizenship education. Therefore, we followed a design research approach. The aim of design research is to find solutions for open questions and problems in complex, real contexts. The design research study was carried out in the project MaSDiV.

3.1 Context: the Masdiv project

MaSDiV (Supporting mathematics and science teachers in addressing diversity and promoting fundamental values, 2017–2020) answered a call within the Erasmus + Key Action 3, which has the goal of “Promoting fundamental values through Education and Training addressing diversity in the learning environment”. MaSDiV aimed to tackle the societal challenges mentioned in the introduction by developing practice-based classroom and PD materials and delivering related PD courses suited to linking science and mathematics education with citizenship education. In the project we intended to (1) introduce IBL as a teaching approach for students of all achievement levels, (2) enhance active scientific citizenship and (3) promote intercultural learning. Accordingly, the PD materials consist of three modules. In this paper we focus on module 2 in relation to mathematics and thus on enhancing active citizenship through using SSIs.

In MaSDiV, partners from six European countries (Cyrus, Germany, Malta, Netherlands, Spain, Turkey) worked together. The consortium included a university and the Ministry of Education from each country. MaSDiV is an interventionist design research study as it aims to design an intervention for real classrooms, and it is utility-orientated as all the MaSDiV materials needed to be usable for other users in local teaching contexts, as otherwise countries across Europe would not be able to implement them.

3.2 Overview of the research phases

In Sects. 2.1 to 2.3 we outlined the challenges of and possibilities for connecting mathematics education to citizenship education. In order to develop concrete materials of high quality that were usable across the project consortium, we decided to address our research questions within an iterative, cyclic process of design, consultation, re-design and try-out (Gravemeijer and Cobb 2006). In each design research cycle a variety of experts was involved to support the design of materials in an area where, so far, no proven solutions exist. As we had to (1) design materials on an international level and then (2) adapt them for use in the individual national contexts, we split the design into research phases 1 and 2 respectively.

During research phase 1, three cycles of design, review and optimization of the international MaSDiV course took place. Design and optimization were carried out by the Dutch design team. The experts reviewing the designed materials in the three cycles differed from a geographical perspective (diverse EU countries), from a disciplinary perspective (experts in science, mathematics and general education), and from a stakeholder perspective (designers and researchers, teacher educators as well as policy makers and representatives from ministries).

A Delphi method was used for collecting and processing the expert opinions (Clayton 1997). This method is considered to be a relatively strong method using opinions of experts to reach a common agreement in a systematic way (Gupta and Clarke 1996). Purposive sampling was used (Denscomb 2008) in order to obtain in-depth qualitative information on the designed materials and how to refine them. Our main criteria for evaluation in research phase I (cycles 1–3) were the theoretical basis (see part 2), consistency (the intervention is logically designed and supports the intended learning goals) and practicality.

In research phase 2, the international course was adapted to the national context and then piloted to investigate usability and implementation of the underlying theoretical ideas in practice (van den Akker et al. 2006). The respective national project teams discussed necessary changes with teacher educators, representatives of high-level school authorities, colleagues from universities and teachers (institutionalized in MaSDiV as so-called National Impact Boards) and then adapted the materials accordingly before piloting them. Our main criteria for the evaluation in research phase II (cycles 4–5) were practicality and usability within the given national context (i.e., to check that the intervention is usable in the settings for which it has been designed) (Nieveen 2007). Figure 2 provides an overview of the research phases.

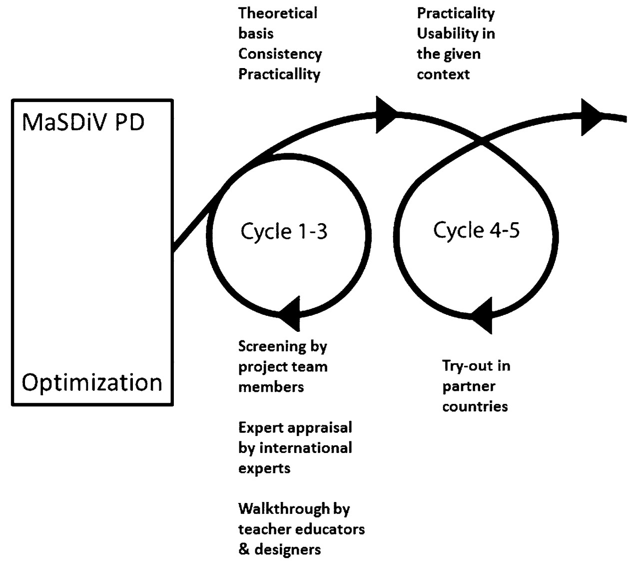

Fig. 2 Iterative cycles of improvement of the MaSDiV model

3.3 Research phase 1: Towards the international MaSDiV course design

Research phase 1 consisted of three cycles. *In the first cycle* the Dutch team, who were responsible for the design, started the process by formulating the initial aims of the PD module. These were as follows:

- to understand the value of using modelling tasks in science and mathematics to support the learning process by making connections between context and concepts, and to apply this in classroom teaching;
- to understand how the use of modelling tasks in science and mathematics can support inclusive education and intercultural learning, and to apply this in classroom teaching;
- to understand the nature, applications and implications of science and mathematics for societies.

The design team developed the first version of the module based on these aims and the theoretical background. The module contained activities for 4 h including a theoretical introduction and examples of classroom tasks. As a final activity, teachers were expected to prepare a lesson plan based upon the content of the course, which they could try out in class, and then reflect upon the results.

As a result of the screening by consortium members (review of cycle 1, May 2017), the design team emphasized the role of contexts by adapting the aims to focus more on SSIs. The following quotations may illustrate this: “Module 2 is supposed to enhance citizenship education and it is not on intercultural learning” and “You focus too strongly on general education whilst we intend to focus on contexts relevant for society, critical thinking and decision-making". The design team was also asked to address drawbacks of using modelling tasks in class and how to overcome them. Suggestions for potential contexts included the production and availability of drinking water, fabrication of sugar pills, building human towers, and designing parking space for different housing areas. Furthermore, we were asked to integrate classroom materials in the PD that are really ready-to-use in day-to-day practice, and to give suggestions concerning how to adjust the module to different target groups (such as teachers with different teaching experiences). All these suggestions were processed in a second version of the module.

In the second cycle the module was presented to an international expert group consisting of representatives from five further countries (again consisting of teams from universities and ministries) as well as representatives from relevant international networks in STEM education (such as ESERA, ICMI and Scientix).

In the second cycle the experts completed a form commenting on the emerging materials (October 2017). The form contained questions about the proposed topics in the PD course, the content, their order and what was found missing. For each of the activities in the course the experts were asked whether the design team should keep, change or delete the activity and to provide (research-based) arguments for their choice.

The qualitative data gathered with the forms were analyzed by first grouping answers of all experts for each question. Comments were added to the PD course documents, containing every core idea of each separate answer. The most important feedback from the experts again related to a stronger focus on citizenship education, as illustrated in the following feedback: "Make a clearer distinction between the sort of contexts used in module 2—focusing on SSI as to promote active citizenship—and module 3—focusing on culture-related contexts" and "Please focus more strongly on citizenship education in module 2 and involve more tasks involving reasoning, critical thinking and decision-making on contexts relevant to society". The designers were also asked to design the classroom materials in such a way that they would be more ready-to-use materials. All suggestions were processed into changes in the structure of activities within the course.

Finally, *in the third cycle*, this version of the PD course was jointly checked by designers and PD providers from the partner universities (walkthrough by the designers and teacher educators, January 2018) (Nieveen 2007). They were asked to complete the same form as in cycle 2. Their main concern in this cycle was evaluating the changes and checking usability in the respective countries. Therefore, the Dutch design team also included suggestions for national adaptations (e.g., as two one-day courses, or as a longer sequence of 2-h sessions in the evening) in the final version of the international PD course.

3.4 Research phase 2: Try-out of PD in partner countries

Research phase II consisted of adapting the international MaSDiV PD course to national requirements (cycle 4) and piloting the resulting PD course in partner countries (cycle 5). In the following we focus on the pilot studies in Germany and the Netherlands.

First, both national project teams screened the final version of the international MaSDiV course and reflected on what changes were needed in relation to their respective educational contexts and the teachers' needs (see Sect. 4.2). In the next step, the national teams discussed necessary changes with their National Impact Boards (see Sect. 3.2) These experts commented on the international version of the PD course and suggested changes based on their expertise. Based on these discussions, both country teams adapted the international course for their national purposes and prepared it for piloting.

In Germany, the piloting of all three modules was conducted in a course of three one-day-meetings with 25 teachers starting in the spring of 2018. After every meeting, we collected participants' opinions of the course through a questionnaire. In the questionnaire, participants were asked what they considered relevant for their teaching, what they considered irrelevant, and suggestions for optimization. These suggestions were then implemented in the next version of the German PD course.

In the Netherlands, the course was piloted with a group of 20 experienced teachers who were connected to Utrecht University in a regional school network. The PD course consisted of four 3.5 h meetings s over 3 months. Data were collected through minutes of the meetings, homework tasks designed by teachers, and 'tips & tops' provided by participants at the end of the final session of the course. The feedback gathered will allow optimization of the Dutch version of the course.

The results of research phase II are (1) adapted national versions for further implementations in the respective countries and (2) an optimized international course (based on the feedback from all six partner countries, see part 5).

4 Results

By the nature of the ongoing design process, the versions of the MaSDiV PD course are preliminary results, as a substantial design process should never end, and optimization should go on after each try-out. Nevertheless, the major design work has been achieved, and therefore its products will be outlined here. In part 5 we elaborate on the further processing.

4.1 Result of research phase I: The international MaSDiV course

4.1.1 The classroom teaching approach

In relation to our first research question (2.5), we first had to elaborate on the form citizenship education in mathematics using SSIs and IBL can take. Based on the theoretical background as outlined in Sects. 2.1–2.3, we searched for scientific fields relevant to our society that also involve forming opinions, ethical or social reasoning, or where decision-making and mathematical modelling are needed (see 2.2). One of the issues we came across was the production of chocolate. In the following we show a condensed excerpt of the worksheet we developed.

The chocolate production task does not only involve modelling a real-life context, but it does include forming an opinion and decision-making (Zeidler and Nichols 2009), in this case, on buying fair trade chocolate or not. It may involve a cost–benefit analysis (Ratcliff and Grace 2003) for students, as fair trade chocolate is more expensive than other chocolate. The task also creates opportunities for learning to deal with incomplete information (Ratcliff and Grace 2003). Consequently, this task can be regarded as serving citizenship education in mathematics for younger students (aged 13 and older). In relation to mathematical topics, it involves calculating percentages, reading statistical data, drawing graphics and understanding large numbers.

In a classroom setting using the IBL approach (see Sect. 2.3), one could first start a discussion on students' individual chocolate consumption and then introduce the concept of fair trade chocolate. At this stage, the discussion on buying fair trade chocolate or not would be informed by

Chocolate production

Find out more about the chocolate production process. Where is it produced; what steps are needed, and which persons are involved?

You may have found out a lot about chocolate production, but some things are not told. There are 2.26 million children working on the more than 2.5 million cocoa farms in Ghana and Ivory Coast. More than 90% (2.1 million) of these children are victims of child labour. One of the main reasons for the use of child labour is poverty. Can you imagine these numbers? In small groups: think of measures the chocolate industry can take to solve the problems of child labour and poverty. What can customers do? What can you do? Discuss this in class.

Fair trade chocolate

In several countries there are chocolate factories that take measures to prevent poverty. Often, they have a Fairtrade label. Some of these factories show information on how the money earned from chocolate is distributed between the retailer, ingredients, the production process, VAT and the cacao workers. How much money from the chocolate you buy would go to the farmer? Would you buy this chocolate and why?

students' personal opinion rather than by data. There would be different opinions. A brainstorm session using the method think-pair-share could follow, in which students could reflect on what information they could gather in order to make a more informed decision. They could, for example, collect information on chocolate consumption in their class (their country and around the world), chocolate production and the delivery process, how the price of chocolate is distributed among people involved in production and delivery, what measures industry could take to change the situation, what extra costs a household buying fair trade chocolate would have or where to buy fair trade chocolate. In the next step, students could work in groups to collect this information. Each group would have to present the information found in a mathematical way using posters. In a gallery walk all students would be informed about the information gathered. Afterwards, the question of buying fair trade chocolate could be dealt with again using data evidence. Here students would have to deal with controversy and ethical aspects (see Sect. 2.2), for example balancing child work (ethical aspect) with extra costs and efforts for buying fair trade chocolate. They would have to make a decision, despite the controversial aspects. Here also the questions of why so many people still buy normal chocolate, or why many chocolate producers still accept child labour, could be discussed. Proceeding in this way would serve the purpose of citizenship education as envisaged in the discussion on SSI and IBL (see Sects. 2.2 and 2.3).

4.1.2 The professional development approach

The aims of the PD module as phrased after research phase 1 were as follows:

- understanding the value of using a modelling/IBL approach in science and mathematics and apply this in classroom teaching;
- understanding how SSI/IBL in science and mathematics can support active citizenship;
- being able to use SSI/IBL in daily science and mathematics teaching;
- understanding the nature, applications and implications of science and mathematics for societies;
- making students understand that scientific decisions, based on science/mathematics, are also influenced by moral, ethical and social reasons.

These aims were elaborated in the module with five main activities. Activity 1 is a warm-up activity, in which teachers reflect on their experiences on using contexts and the reasons for using modelling tasks (starting off from teachers' competences and needs, see Sect. 2.4). The main reason for the latter was to give teachers insight into the different reasons for using modelling tasks (see Sect. 2.1) as they often tend to see only students' motivation as a reason, which in turn leads them to neglect modelling when time is too short (Maass 2011).

In activity 2 teachers work on the topic "Can the earth feed us?" They are given, for example, information on the loss of calories and protein when grain passes through animals to produce meat, as compared to when grain is directly used as food for humans. In this context, the controversy arises between eating preferences and availability in rich countries on the one hand and feeding all humans across the world on the other hand. The decision on reducing personal meat consumption clearly involves ethical and social aspects (see Sect. 2.2). Consequently, this task serves the purpose of citizen education and can be used by teachers in their day-to-day teaching (for relevance of PD to practice, see Sect. 2.4). In the PD course, it also served the purpose of fostering teachers' beliefs on connecting mathematics and science education to citizenship education. To facilitate teachers' use of the task (relevance to teaching practice), they are asked to discuss the subject-specific content that can be related to this context and the benefits of using of this context in relation to scientific citizenship.

In activity 3, we introduce SSIs by giving a definition and asking teachers to relate the definition to the example dealt with in activity 2. Additionally, they are asked to relate SSIs to the OECD notion of global competency, which aims at preparing future European citizens for dealing with global challenges. This task aimed at deepening teachers' knowledge about SSI. In activity 4, participants are asked to design a lesson using an SSI (to connect to practice and combine phases of learning in seminar and school, see Sect. 2.4). Finally, in activity 5, teachers have to reflect on drawbacks of the use of contexts and how to overcome the drawbacks (taking into account their needs, and the challenges they face, see Sect. 2.4).

4.2 Results of research phase II: National adaptation and try out in the countries

4.2.1 The case of Germany

We adapted the international MaSDiV course to the needs of German teachers to have the highest possible impact. Our course addressed mathematics teaching only, and so we could not use tasks designed for science teaching. Furthermore, experience and research show that German teachers prefer ready-to-use classroom materials they can try out directly in the next lesson (e.g., Maass 2018). Last but not least, they appreciate direct connections to the curriculum.

Therefore, we did not connect the aspect of SSIs to the OECD notion of global competency (see Sect. 4.1), but to the curriculum of Baden-Württemberg, and we did this right at the beginning of the course. We also skipped activity 1, in which teachers were supposed to reflect on the use of contexts, because we knew that most participants were familiar with this aspect, and we had to shorten the module due to time limitations.

Instead of starting with the issue of "Can the earth feed us?" (see Sect. 4.1), we were looking for an example which would possibly be easier to understand for their students at lower secondary level (responding to teachers' needs and relevance to practice, see Sect. 2.4). We started with the context of plastic waste, and more concretely, with shops selling unpackaged food. In Baden-Württemberg, these shops can be found everywhere, and teachers and students would be able to visit them. As the German teachers appreciate concrete tasks, we connected the context with the following questions for students:

- What amount of waste can be saved in a household/in our town, if all people buy their food in shops selling unpackaged food?
- What amount of waste can be saved in our class if we all skipped buying drinks in "to-go-cups"?
- Consider the waste produced in your town. If you took the waste and distributed it in town (1 m high), which area would it take?

Teachers were asked to reflect on the following questions: What do students learn when dealing with the problems? Would you use the task in your class and why? Teachers' answers to these questions ranged from "There is too much context in the task and only a little calculation, I would not use it," to "Students have to deal with statistics, measures and geometry," "Students become aware of the amount of waste they produce and how easily waste can be reduced. The task therefore combines learning mathematics with reflecting about our society. It is important." These different answers provided a platform for a fruitful discussion among teachers, and thus made them reflect on their beliefs (see Sect. 2.4).

We then turned to more contexts as foreseen in the module. However, we selected only the contexts suitable for mathematics (can the earth feed us, breast cancer, chocolate production) and added further contexts (distribution of refugees in Germany, fair pricing of trainers). To comply with teachers' wishes of having tasks, we gave them some examples of questions for students to deal with, but also asked them to develop their own questions for all these contexts. As foreseen, we then dealt with the definition and theoretical background on SSIs. Due to lack of time, we skipped designing a lesson during the PD course. Instead, we turned to drawbacks and difficulties in relation to SSIs and how to overcome them.

As we gathered from the feedback questionnaires (see Sect. 3.4), the course apparently raised teachers' interest in dealing with SSIs in mathematics education and met their needs. This can be illustrated by the following quotations: "We got a lot of interesting tasks", "I liked how the task connected mathematics and contexts relevant for society," and "I like that we got tasks we can use right away". However, it also became clear that they apparently had difficulties in identifying the mathematics 'hidden' in the contexts, which indeed can be difficult, and that lack of time would prevent them from dealing with such issues more often.

4.2.2 The case of the Netherlands

We adapted the international MaSDiV course to fit a group of experienced teachers of all STEM-subjects. We decided to focus on SSIs that have an interdisciplinary character to present opportunities to share experiences between subjects (see Sects. 2.2 and 2.3). In the Netherlands, both science and mathematics teachers are familiar with the use of modelling tasks and have insights into the different reasons for doing so.

For this reason, we decided to skip the first activity targeting this topic (see Sect. 2.4: connecting to the teachers' expected needs). Instead, we started discussing the use of contexts across subjects, which was highly appreciated by the teachers. The opportunity to share experiences and stimulate cooperation among teachers of all STEM-subjects is not very common in Dutch PD.

Instead of discussing the example 'can the earth feed us' (see Sect. 4.1), we used an interdisciplinary project on water, designed and presented during the course meeting by a student of one of the schools, as an example, in order to value and build on teachers' own contributions. This project involved IBL activities related to issues about (drinking) water and safety (a large part of the Netherlands is below sea level). In this way, teachers were shown an exemplary successful SSI teaching activity.

We proceeded with an activity in which the teachers discussed this in connection with other examples of contexts, to let them start talking about the challenges they face with context-rich science education and to better connect the implementation of SSIs to their practices (see Sect. 2.4). For this activity, we gave the teachers a handout specifically designed for this mixed group, in which we presented seven examples of contexts, all but one with the focus on SSIs. The contexts—listed below—are situated in different subjects, and some of them are interdisciplinary.

- Budget for 'defense': based on the same data, argue for a peace organization that lowers the budget and for a military school that raises the budget.
- Meat or vegetables (based on: can the earth feed us).
- Plastic soup: explore the method used by the Ocean Clean up and discuss statements on raising tax on plastic and on banning buying (and selling) products packaged in plastic.
- Fisheries: explore dynamite-fishing (or pulse fishing), and discuss if and how fisheries can be made more sustainable.
- Health risk and choices: Why do teenagers smoke? What scientific arguments could help them to stop?
- Chocolate and child labor: Can we still eat chocolate?
- Rope puzzles—as an example of a non-relevant, but learning-related context.

We had teachers discuss these contexts in small interdisciplinary groups. They were asked to discuss the subject-specific content that can be related to this context, the benefits of using this context in relation to scientific citizenship, and the challenges they might meet when implementing them. Having interdisciplinary groups was an advantage, since it enriched the discussion about the actual use and the potential of the contexts. Then we provided a worksheet with characteristics and examples of SSIs. Teachers were asked—as homework—to think of an SSI that would fit in their teaching, to let them learn to apply SSI-characteristics and have experience with searching for and evaluating SSIs. Not all teachers succeeded in doing the homework, but in the next meeting we had enough input of examples of SSIs to again start the meeting with discussing and reflecting on the ideas. Contexts the teachers had used for their design included xeno-transplantation, reimbursement of PET-bottles, plastic soup and daylight saving time.

All teachers were asked to send in evaluation forms at the end of the meeting, so that we could provide them with written and personal feedback on their chosen context, the relation with content, and the relation with IBL (in many cases more responsibility for one or more processes of inquiry could be given to the students). Almost all teachers did indeed share their experiences before the follow-up meeting.

From the 'tips & tops' notes at the end of the final session of the course, we concluded that many of them appreciated the "inspiring" examples of classroom materials (e.g., the tops "nice to discuss so many examples", "good to let us experiment with example tasks in our lessons"). Some explicitly noted that they were surprised about the possibilities of addressing citizenship education in mathematics (e.g., the top "after 30 years of teaching I now realized the variety of possibilities in mathematics"). Some would have appreciated more group exchange on experiences (e.g., the tip "nice to hear the ideas from the group, they help you how to adapt tasks for your lessons"). About one-third mentioned the challenge of how to involve all students in these kinds of activities.

5 Summary and discussion

Based on the identified need that scientific knowledge is integral to preparing our population to be actively engaged and responsible citizens, and consequently to evolve science education (including mathematics) towards 'of and about science,' including its social, cultural and ethical dimensions (Hazelkorn et al. 2015), we raised two research questions in relation to mathematics:

- How can citizenship education using SSIs and IBL be addressed in mathematics education?
- How can mathematics teachers be supported in implementing active citizenship education through a PD course?

Following on from the intention of designing concrete materials for citizenship education, we combined theoretical knowledge from mathematics (discussion on modelling, numeracy) and science education (SSI) and thus followed an interdisciplinary approach (see part 2). We based our innovative design work on this interdisciplinary approach.

Our study shows that it is indeed possible to develop concrete mathematics and science teaching materials which connect mathematical modelling and IBL with SSIs, numeracy and critical thinking. Our materials aim to involve students in controversial issues connected to ethical, moral or social reasoning and decision-making (Zeidler and Nichols 2009; Ratcliff and Grace 2003), and can thus potentially contribute to the development of twenty first century skills in students. Here, further data collection on the effects on students is needed.

Consequently, the gains of our design research study are concrete classroom and PD materials which are based on an interdisciplinary STEM approach. These new gains are to be placed in the field of research transfer or research-based practice, an area which is still considered to be underrepresented in education (Burkhardt and Schoenfeld 2003; Burkhardt 2018b).

By designing materials which connect mathematics education to citizenship education, based on an interdisciplinary basis, our study provides a new pivotal example on shifting up the role of mathematics in the STEM discussion, where it is often underrepresented (English 2016): "Not only do we need to ensure that mathematics receives the attention it deserves within our STEM climate, but also that our students

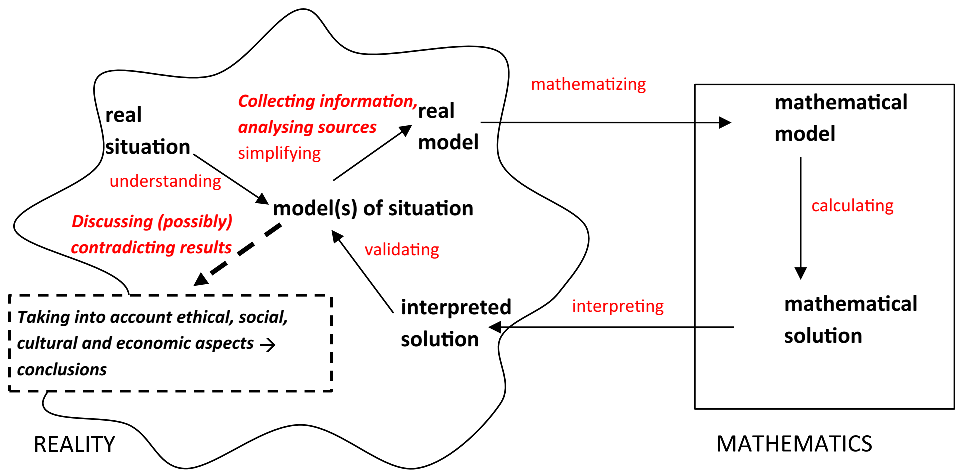

Fig. 3 Extended modelling process for socio-scientific issues in relation to mathematics education

are provided equitable opportunities to develop the mathematical literacy for successful participation in their current and future worlds" (English 2016, p. 358).

The iterative design research approach (Gravemeijer and Cobb 2006) and the involvement of numerous experts (see Sect. 3.2) also show that it is time consuming to find contexts that can serve for citizenship education within mathematics and science. Based on the experiences of the design team, as well as on reactions of experts and participating teachers, we noticed that it can be difficult to identify and address the mathematics involved in the contexts (e.g., "The topic of plastic waste is more connected to biology", "The mathematical outcomes of dealing with the chocolate task are too few compared with the time needed to understand the context"). From our perspective, this difficulty in identifying the mathematics in contexts precisely highlights the need to include such tasks in mathematics education to make future citizens familiar with these issues and the inherent mathematics. This demand is in line with the view of English and Gainsburg (2016), who emphasize that mathematical knowledge needs to be used and applied far more fluently than it is today. According to the authors, there seems to be a need to enrich students' understanding of, for example, algebra, geometry, and statistics, and to develop their skills in applying this understanding to a variety of authentic problems, a need for which our design provides an answer.

Our study also shows that it is possible to design a PD course that raises teachers' interest in citizenship education and SSIs. According to the feedback we collected, teachers considered most topics relevant to their teaching practice and appreciated the course.

Our design research study also contributes new insights to the understanding of mathematical modelling and related modelling schemes (Blum and Leiss 2007). We suggest that when dealing with SSIs, the steps as outlined in the modelling process (see Sect. 2.1) need to be extended to fully take into account the specific features of SSIs. These extensions could include: search for information and (risk) analysis of sources of information, discourse about (possibly) contradicting scientific results and ethical, social, cultural reasoning (Zeidler and Nichols 2009). Particularly the difference between scientific results and conclusions has to be made clear (Ratcliff and Grace 2003). A possible resulting modelling process is shown in Fig. 3.

In relation to the *design research process* and our iterative cycles of improvement, all the steps carried out proved to be necessary. In particular, the involvement of the different experts from different areas of expertise and different working backgrounds (universities, ministries, teacher education) proved to be valuable, as they all contributed different aspects (see Sect. 3.2). For the same reason, the design on an international level was very fruitful. However, we also saw that in these international design processes national adaptation is important. The different approaches as illustrated by the Dutch and German team (see Sect. 4.2) show that, although the international principles remained the same, the national adaptation included definitely more than translation.

As the study presented here is a design research study, the main results of the study are design products. This paper reported on the design process of the classroom und PD materials, and its theoretical basis, and presented the design products, therefore contributing important insights into the transfer process of research into practice. In the next steps, we intend to evaluate effects of the PD course in more detail. The steps presented in this paper helped us to move forward in understanding how to address citizenship education in mathematics classrooms.

Acknowledgements The project MaSDiV received funding from the European Union Erasmus + Programme under grant agreement no. 582943-EPP-1-2016-2-DE-EPPKA3-PI-POLICY. This paper reflects only the authors' views and the European Union is not liable for any use that may be made of the information contained herein.

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