Promoting creativity in elementary mathematics education

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Promoting creativity in elementary mathematics education

Het bevorderen van creativiteit
in het primair reken-wiskundeonderwijs
(met een samenvatting in het Nederlands)

Proefschrift

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Contents

Chapter 1

General introduction

Fostering creativity is high on the educational agenda. Different factors can be identified that possibly explain the growing attention for creativity in education. One of these factors is related to the current major societal developments. We live in a rapid changing society with fast technological developments and a growing amount of available information. As a consequence, in educational debates it is discussed what competencies are needed to prepare students for this rapidly changing society. Creativity is proposed as one of the competencies needed in future society, which should, therefore, be fostered in schools (Csikszentmihalyi, 2006; European Parliament and the Council, 2006; OECD, 2018; Puccio, 2017; Thijs, Fisser, & Hoeven, 2014). For example, the growing amount of available information on the internet might give reason to search for a different approach in education in which students not only learn to reproduce knowledge and skills, but especially learn to creatively apply these knowledge and skills to discover and create new possibilities. From this societal point of view, it seems important that students' creativity is fostered in education (Oosterheert & Meijer, 2017). Furthermore, the importance of fostering creativity in education is also in line with new views on learning. Instead of more traditional views of learning in which students learn to reproduce knowledge and skills, new ideas of learning have become increasingly important. In these new views of learning, it is important that students learn to actively construct their own knowledge, create more insight in the world around them, and are able to function in that world (Oosterheert & Meijer, 2017; Volman, 2006). For example, in mathematics education, it is currently considered important that students construct their mathematical knowledge and skills by solving meaningful mathematical problems, which requires creativity (Gravemeijer, 2007). Furthermore, the importance of creativity is also acknowledged by the Dutch government in their vision on future-oriented education. The government believes that it is necessary to promote creativity in educational practice (Platform Onderwijs 2032, 2016). Thus, from several perspectives it is highly supported that students' creativity is nurtured in education. However, as I will elucidate in the upcoming section, more expertise is needed on how to nurture creativity in education, especially in a key discipline as mathematics. Therefore, this dissertation focuses on the promotion of creativity in elementary mathematics education.

1.1. Defining creativity

Creativity is a complex multidimensional construct that refers to the act of creating novel and meaningful ideas, solutions and products within a particular (social) context (Plucker, Beghetto, & Dow, 2004). These ideas, solutions, and products can be novel and meaningful on a large, global scale (also called Big-C creativity), such as a scientific break-through, or on a very small-scale, when a student finds a novel and meaningful solution to a problem in daily life (also called little- or mini-c creativity; Kaufman & Beghetto, 2009). In this dissertation, we refer to creativity on a small scale: creative ideas, solutions or products are novel and meaningful within a specific reference group (e.g., grade level or age group) or for a specific person. Students' creativity mainly relates to novel and meaningful insights and interpretations when they learn new subject matter and is (to a certain degree) related to knowledge construction (Kaufman & Beghetto, 2009). A creative idea, solution, or product is the result of a creative process, in which persons' characteristics (e.g., perseverance, interest, openness), cognitive processes (e.g., divergent and convergent thinking; Guilford, 1967), behavioral actions, and the material and social environment interact (Glăveanu, 2013; Isaksen, Dorval, & Treffinger, 2011).

Still, there remain unanswered questions regarding the construct of creativity. For example, it is still undetermined whether the nature of creativity is domain-specific or domain-general, as support has been found for both views (e.g., Huang, Peng, Chen, & Tseng, 2017; Jeon, Moon, & French, 2011; Plucker, 1999, 2004). Some researchers argue that creative processes are similar across domains and, therefore, consider creativity as a domain-general ability (e.g., Plucker, 1999). Others argue that creativity is domain-specific, because specialized domain skills, knowledge and interests play a crucial role in creativity (e.g., Baer, 2012). Still, others state that both domain-general and domain-specific factors contribute to creative performances in different domains (Baer & Kaufman, 2005; Jeon et al., 2011). The answer to this theoretical question about the nature of creativity has implications for the promotion of creativity in education. It could help us to understand if creativity should be promoted either by a general creativity training or by a specialized training. For example, should creativity be promoted by a divergent thinking training that focuses on a range of situations that minimally take domain differences into account (e.g., Renzulli, 1986)? Or should it rather be promoted by a training tailored to the unique demands of a given performance domain (e.g., Baer, 1996; Scott, Leritz, & Mumford, 2009)?

1.2. Nurturing creativity in education

Although it is important to accommodate and nurture creativity in education, the complexity and unanswered questions regarding the construct of creativity present a challenge for schools: "While it is impossible to standardise creativity, inside or outside of the education system, we still need—indeed, crave—a form, pattern, or plan of action in order to instrumentalize the enhancement of creativity in our approach to education" (Harris, 2016, p.2).

Actions indeed seem to be necessary to enhance creativity, because, there are indications that creative performances of new generations are falling behind: performances on divergent thinking tests of subsequent age cohorts indicated a significant decrease since 1990 (Kim, 2011). Furthermore, although teachers acknowledge the importance of creativity, they seem to refrain from structurally encouraging creativity in educational practice as has been observed in several countries, including the Netherlands (Puccio, 2017; Thijs et al., 2014). The question is: why? First, it is suggested that a predominant culture of standardization and accountability is an obstacle to promote creativity in education (Onderwijsraad, 2013; Sternberg, 2015). Content standards are often closely linked to standardized test-based accountability (Baer & Garett, 2010). Teachers may experience pressure to teach all learning goals stated in the standards and may feel insufficient room to structurally promote creativity and, therefore, limit their instruction to that what will be tested (Baer & Garett, 2010; Craft, 2005; Dobbins, 2009; Harris, 2016; Onderwijsraad, 2013). Researchers often argue that these practices suppress creativity, because most standardized tests may encourage students to a particular kind of learning and thinking. For example, by postulating that there are (only) right and wrong answers (Puccio, 2017; Sternberg, 2007). However, it is also important to teach students to act creatively, and to "respond to problems in fresh and novel ways, rather than allowing themselves to respond mindlessly and automatically" (Sternberg, 2007, p.3). Second, there is limited attention for creativity in Dutch core goals and teaching methods as, for example, reflected in the most used text books in elementary and secondary education (Thijs et al., 2014). Third, most teachers do not feel fully capable to structurally accommodate creativity in education and indicate a need for professional development and good examples of educational activities and teaching materials (Thijs et al., 2014).

Since there are indications that creative performances of new generations are falling behind (Kim, 2011) and teachers seem to refrain from structurally encouraging creativity in educational practice (Thijs et al., 2014), it seems to be important that actions are undertaken to structurally foster students' creativity in education (Platform Onderwijs 2032, 2016). Currently, a committee of teachers are revising the Dutch elementary and secondary school curricula in order to adapt education to the new learning needs in society, amongst which creativity (Curriculum.nu, 2018). This dissertation aims to contribute to current attention in society for creativity in education, by obtaining more insights into how we can accommodate creativity in schools. More specifically, it concentrates on how to promote elementary school students' creativity in the key discipline of mathematics.

The dissertation focuses on the discipline of mathematics, as promoting creativity in this particular discipline may be the greatest challenge for elementary school teachers. Many teachers often depart from the assumption that creativity and mathematics do not get along well. Many people tend to consider the arts, but not mathematics, as a creative domain (Cropley, 2012; Glăveanu, 2014; Pehkonen, 1997), while creativity is also at the heart of mathematics (e.g., Ervynck, 1991). Teachers often think that it is mainly logic that is needed in mathematics (Pehkonen, 1997), and, consequently, may find it difficult to identify ways to encourage and assess creativity in the elementary mathematics classroom (Bolden, Harries, & Newton, 2010).

1.3. Creativity and mathematics

Studies have illustrated that creativity is at the heart of mathematics, because it is closely related to mathematical problem solving and problem posing (Ervynck, 1991; Halmos, 1980; Mann, 2006; Silver, 1994; Silver, 1997; Sriraman, 2005). Mathematicians engage in mathematical problems full of uncertainty which require creative thinking (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016; Sriraman, 2005). Unsurprisingly, the definition of mathematical creativity pertains to mathematical problem solving and posing: it is "a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination" (Sriraman, 2005, p.24).

Research on mathematics education indicates that creativity in mathematics can be promoted with the use of problems that have a certain degree of uncertainty and freedom, such as ill-defined, non-routine and open problems (Hershkovitz, Peled, & Littler, 2009; Silver, 1995; Sriraman, 2005). These are mathematical problems that cannot be solved by using familiar or routine procedures. However, in mathematics classrooms these types of problems are rarely used (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009; Van Zanten & Van den Heuvel-Panhuizen, 2018). Dutch elementary school teachers are to a high degree guided by the guidelines of mathematical textbooks to teach the mathematics curriculum. Mathematical textbooks, therefore, play a decisive role in daily Dutch mathematical teaching practice (Gravemeijer, 2007; Hop, 2012; Meelissen et al., 2012; Van Zanten & Van den Heuvel-Panhuizen, 2018). In these textbooks, students mainly have to solve routine problems in which they have to reproduce and apply a fixed solution procedure in one or two steps (Kolovou et al., 2009; Van Zanten & Van den Heuvel-Panhuizen, 2018).

Research also indicates that creativity can be encouraged in education by creating a pedagogical environment characterized by an open atmosphere in which students have the opportunity to act creatively in interaction with others (Kaufman, Beghetto, Baer, & Ivcevic, 2010; Leikin & Dinur, 2007; Sawyer, 2014; Soh, 2000). Several strategies are recommended in the literature to create such a pedagogical environment. Teachers are, for example, advised to guide students by asking them suitable (open) questions (Bostic, 2011; Levenson, 2011; Shen, 2014), and it seems vital that teachers are open and flexible regarding students' responses (Beghetto & Kaufman, 2010; Davies et al., 2013; Hershkovitz et al., 2009; Leikin & Dinur, 2007), and stimulate students to collaborate (Kaufman et al., 2010; Soh, 2000; Taggar, 2002). However, it is as yet unclear whether these strategies indeed promote creativity within elementary mathematics education, and to what extent Dutch elementary school teachers use these strategies.

Furthermore, to promote mathematical creativity it may be important that a teacher enriches mathematics education by making connections with subjects other than mathematics or with contexts outside school (Colucci-Gray et al., 2017). More generally, this phenomenon can be characterized as boundary crossing (Akkerman & Bakker, 2011a; 2011b). To create something new and meaningful in mathematics, it is important to break away from established mindsets (e.g., Haylock, 1987b). By crossing disciplinary boundaries, it may be easier for students to break away from established mindsets and to think and act in a mathematically creative way. Integrating different conceptual systems, for example from the disciplines of visual arts and mathematics, may activate students to combine familiar concepts in new ways.

Thus, although there has been done some research identifying key principles of promoting creativity in education, creativity in elementary school mathematics is still an understudied topic. More research is needed to know better how to promote students' creativity in mathematics education in elementary schools. *First*, more research is needed to explore how creativity and mathematics are related in an elementary school setting. For example, since there is no consensus on the nature of creativity, it is not clear whether creativity can only be specifically promoted and expressed in the discipline of mathematics, or whether it is a domain-general ability that could be trained by a general creativity training. *Second*, the creativity promoting strategies mentioned above, are often not specifically recommended for elementary mathematics education, but are domain-general or specific for high school (mathematics) education. Furthermore, studies investigating these strategies are rare. Consequently, there is insufficient knowledge on what strategies can promote creativity in elementary school mathematics education. Therefore, empirical research on this topic is highly needed. For example, it is as yet unclear which recommended pedagogical strategies are currently used to promote students' creativity in mathematics education and which should be used. *Third,* there are only few evaluation studies that investigated the effectiveness of creativity promoting mathematics lessons in educational practice.

1.4. Aims and outline of this dissertation

1.4.1. This dissertation

The main aim of this dissertation is to obtain more insight into how we can nurture creativity in elementary school mathematics. To this purpose, I will first explore how creativity and mathematics are related in an elementary school setting. Subsequently, I will examine which pedagogical strategies are needed in elementary mathematics education to promote students' creativity. The studies described in Chapters 2 to 5 focused on students in the upper grades of elementary school and investigated the main questions of this dissertation from different perspectives.

An important part of this dissertation concentrated on the evaluation of the Mathematics Arts Creativity in Education (MACE) program [Meetkunst]. The program aimed to partly teach domain-specific and partly overlapping learning goals and objectives of visual arts and geometry in order to promote students' creative skills in both disciplines, by creating opportunities for students to act creatively in an integrated visual arts and geometry context. To achieve the goals of the program, a lesson series was designed for fourth, fifth and sixth grade students, along with a professional development program for teachers to enhance implementation of the lesson series. The MACE program was designed and evaluated in collaboration with Museum Boijmans van Beuningen, the Freudenthal institute, University of Applied Sciences iPabo, University of Applied Sciences Rotterdam, and two elementary school teachers. The MACE program built on the the Boijmans Language and Mathematics Program in which the museum's educational staff together with two elementary schools and an artist explored opportunities and possibilities for crossovers between art, mathematics and language (Brinkman, Miedema, & Schreuder, 2017; Schreuder, 2013).

1.4.2. Outline of the chapters

To obtain more insight into how we can nurture creativity in mathematics education, it is first desirable to know how mathematics and creativity are related in elementary school. Due to inconclusive results regarding the nature of creativity, it is not yet clear how creativity is related to mathematical ability. Hence, *Chapter 2* describes a study in which the relations between domain-specific mathematical creativity, domain-general creativity and mathematical ability are investigated. This chapter provides further insight into the nature of creativity and may contribute to the literature on mathematical learning and teaching with new insights regarding the role of creativity.

Chapter 3 further explores the relation between creativity and mathematics by reporting on a study that investigates the relation between creativity and performance on different types of mathematical problems. To improve the integration of creativity in mathematics education, it is crucial to find an answer to the question whether commonly used mathematics textbooks with the predominant routine type of problems they contain, provide sufficient support for promoting creativity in mathematics education. Therefore, this study examined the relations of students' creativity as assessed with a standard test of creative thinking with their performance on three types of mathematical problems, controlling for possible confounding factors. The aim was to determine whether the relation between creativity and performance differed between closed routine geometrical problems, geometrical multiple solution problems and non-routine visual-art geometry problems, which assumingly would indicate differences between the problem types in the degree in which they call upon creative higher-order thinking. A better understanding of the relation between creativity and mathematical problem type can inform curriculum and textbook designers.

The next two chapters present studies that were conducted in the context of the MACE program. *Chapter 4* describes a case study of a fourth-grade teacher and her class who took part in the MACE program. More specifically, this study provides in-depth insights into how recommended creativity-promoting pedagogical strategies are implemented in the classroom, how they relate to different types of mathematics lessons (i.e., two MACE lessons and a regular textbook-based mathematics lesson) and how they relate to students' mathematical creativity as expressed in classroom dialogues. This chapter can contribute to our understanding of the processes involved in the teaching of the MACE lessons, whereas the study in *Chapter 5* reports the results of an evaluation study into the effectivity of the MACE program. The study investigates the effects of the integrated MACE approach on students' geometrical ability, geometrical creativity and perception of visual arts in a quasi-experimental design. Chapters 4 and 5 can jointly provide us with a better understanding of how creativity can be encouraged in elementary education.

Finally, *Chapter 6* provides an overall discussion of the findings described in Chapters 2 to 5 and outlines the implications and directions for future research and practice.

Chapter 2

Mathematical creativity: A combination of domain-general creative and domain-specific mathematical skills

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Authors contributions: E.S., and E.K. designed the study. E.S. collected and analyzed the data and wrote the paper. E.K., and M.K. critically reviewed the paper.

Abstract

Creativity is an understudied topic in elementary school mathematics research. Nevertheless, we argue that creativity plays an important role in mathematics, but that more research is needed to understand this relation. Therefore, this study aimed to investigate this relation, specifically between domain-general creativity, domain-specific mathematical creativity, and mathematical ability. Measures for these constructs were administered to 342 Dutch fourth graders. In order to examine the nature of the relation between creativity and mathematics, two competing models were tested by using Structural Equation Modeling. The results indicated that models in which general creativity and mathematical ability both predict mathematical creativity fitted the data better than models in which mathematical and general creativity predict mathematical ability. This study showed that both general creativity and mathematical ability are important to think creatively in mathematics.

2.1. Introduction

Creativity is an increasingly important aspect of personal functioning in several sectors of contemporary society (Sternberg & Lubart, 1999). Creativity seems to be related to mathematics, since it is required when a student or mathematician faces a mathematics problem for which there is no learned solution (Leikin & Pitta-Pantazi, 2013). However, due to a lack of insight into the nature of creativity, in particular the relation between domain-general and domain-specific creativity, it is not yet clear how creativity and mathematics are related. The aim of this study was, therefore, to investigate the relations between domain-general creativity (GC), domain-specific mathematical creativity (MC), and mathematical ability (MA) in a new and integrated way. This provides further insight into the nature of (mathematical) creativity and may inform the current literature on mathematical learning and teaching with new insights regarding the role of creativity.

Creativity is a multidimensional construct, which may or may not be domain-specific. Therefore, we will first shortly discuss the nature, definition, and measurement of creativity. With regard to the nature of creativity, some researchers argue that creativity is a domain-general ability, because creative processes are similar across domains (e.g., Plucker, 1999). Others, however, state that creativity is always related to a specific domain (e.g., mathematics), because a certain degree of knowledge or expertise within a particular content domain is required for creativity (e.g., Baer, 2012). As a result, researchers have begun to investigate domain-related creativity, like MC. Support was found for both views (e.g., Huang et al., 2017; Jeon et al., 2011; Plucker, 1999, 2004), suggesting that creativity may be partly domain-general and domain-specific. In this study, we have investigated both GC and MC in relation to mathematical ability.

GC is defined as "the interaction among aptitude, process, and environment by which an individual or group produces a perceptible product that is both novel and useful as defined within a social context" (Plucker, Beghetto & Dow, 2004, p. 90). Measures that are often used to measure GC are, for example, the Torrance Test of Creative Thinking (TTCT; Torrance, 2008) and the Test of Creative Thinking – Drawing Production (TCT-DP; Urban & Jellen, 1996). However, it is questionable whether the TTCT and TCT-DP indeed measure GC, since domain-specific features are involved (e.g., verbal or figural features). Furthermore, these tests may not measure the whole construct of GC. Therefore, it is recommended to use different measurements instead of relying on a single score when measuring students' creativity (Cropley, 2010; Kim, 2006; Treffinger et al., 2002). MC is defined as

> "(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination" (Sriraman, 2005, p. 24).

MC is most often assessed by using a multiple-solution task, in which students can provide several solutions to a mathematical problem (Leikin, 2009).

2.1.1. Relations Between GC, MC, and MA

From the current literature on creativity and mathematics, it can be inferred that GC, MC, and MA are related. However, it is still ambiguous how they are related. Two different patterns can be hypothesized. First, it could be hypothesized that MA and GC both predict MC. The definition and assessment of MC suggests that MA is related to MC because a certain level of mathematical knowledge is necessary to be creative in mathematics (Sak & Maker, 2006; Weisberg, 1999). This positive relation is indeed supported by several studies (Huang et al., 2017; Mann, 2005; Sak & Maker, 2006; Weisberg, 1999). GC is expected to be related to MC since general creative processes are similar across domains (Plucker, 1999). Some studies have indeed found a connection between GC and MC (Hwang, Lee, & Seo, 2005; Jeon et al., 2011; Kattou & Christou, 2013; Kroesbergen & Schoevers, 2017). It should be noted that MC was measured by a multiple-solution task or math teachers' ratings of math creativity and GC by divergent thinking tasks and the TCT-DP.

Few have investigated the hypothesis that both GC and MA influence MC, simultaneously in a single study (Huang et al., 2017; Jeon et al., 2011; Kattou & Christou, 2013). However, the findings of these studies are not univocal. The study by Jeon et al. (2011), using regression analysis, found that both GC, measured by divergent thinking, and mathematical performance predicted MC. Nevertheless, mathematical performance explained more variance (10%) in MC than GC (3%), indicating that in a structured domain like mathematics, domain knowledge is more important for MC than GC is. Kattou and Christou (2013) found similar results, although in their study MA and GC, measured by divergent thinking, were equally strong predictors of MC. In contrast, Huang et al. (2017) found that only MA, and not divergent thinking, predicted MC.

Second, it could also be argued that MC influences MA, and that MC in turn is influenced by GC, which indirectly influences MA. Recent research suggests that MC is a prerequisite for the development of high levels of MA (Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2013). Several other studies also showed that MC predicted MA: the more creative a child is in mathematics, the higher his/her performance in mathematics is (Bahar & Maker, 2011; Kattou et al., 2013; Leikin, 2007). Indeed, as an individual tries to find multiple solutions, she/he considers mathematical ideas from different perspectives, which leads to deeper mathematical knowledge (Leikin, 2007). Furthermore, it can be expected that GC influences MC (Hwang et al., 2005; Jeon et al., 2011; Kattou & Christou, 2013; Kroesbergen & Schoevers, 2017) and thus indirectly MA. It is not expected that GC is directly related to MA since these general processes may not be intrinsically related to MA (Baran et al., 2011; Livne & Milgram, 2006).

2.1.2. Research Goals and Hypotheses

Given that there have been few previous studies and their results are mixed and inconclusive, the question remains of how GC, MC, and MA are related. To deepen the insight into these relations, we used more than one measure for GC to better estimate the latent construct;

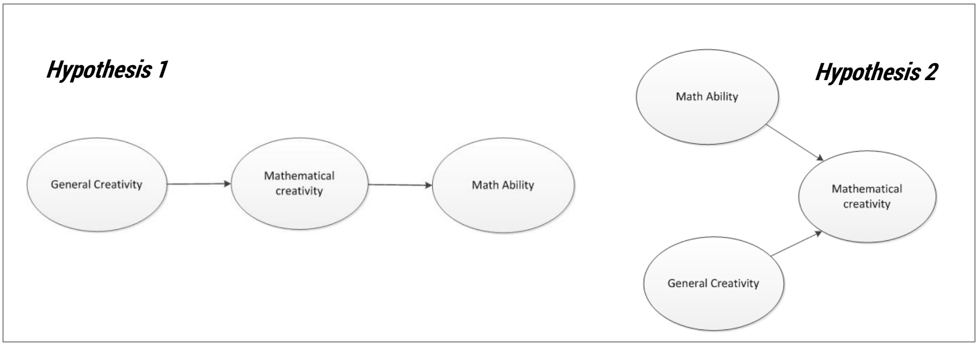

Figure 1. Simplified visual representations of hypotheses 1 & 2

furthermore, two competing models derived from the literature were tested. We considered the following two possible competing hypotheses.

First, it was hypothesized that MC influences MA (e.g., Kattou et al., 2013) and that GC in turn directly influenced MC but only indirectly influenced MA (see Hypothesis 1, Figure 1). Second, it was hypothesized (see Hypothesis 2, Figure 1) that MC was influenced by both MA and GC (e.g., Bahar & Maker, 2011; Hong & Milgram, 2010; Jeon et al., 2011).

2.2. Method

2.2.1. Participants

In this study, 342 fourth-grade students participated, coming from 18 classes of 12 elementary schools in medium- to large-sized towns in the Netherlands. Schools differed with regard to their policies and teaching methods used in class, and were located in various districts containing citizens of low, middle, and high socioeconomic status. Students in this sample were 50% boys and had a mean age of 9.68 years (*SD* = 0.45). Prior to the data collection, a power analysis, performed with online software called Sloper (2015), indicated that this study required a sample size of at least 305, with an anticipated effect size of 0.15, desired power of 0.8, 10 latent variables and 34 observed variables, and a probability level of 0.05. Furthermore, Kline (2010) indicates that a minimum of 10 cases per variable are required for Structural Equation Modeling (SEM). This indicates that with 34 observed variables we have a large enough sample size for SEM (Kline, 2010; Sloper, 2015).

2.2.2. Instruments

Intelligence Quotient (IQ). Raven's Standard Progressive Matrices (SPM; Raven, 1998) was used to get an indication of the non-verbal intelligence of the students. In each of the 60 test items, the subject is asked to identify the missing element that completes a pattern. The test measures the reasoning ability of students and is a measure of non-verbal intelligence. The mean score on Raven's SPM in this study was 101.50 (SD = 14.82), which was based on Dutch norms (Van de Weijer-Bergsma, Kroesbergen, Prast, & Van Luit, 2014). A sample question is shown in Figure 2. With regard to predictive validity, Raven's SPM predicted

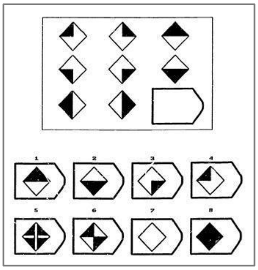

Figure 2. A sample question of Raven's Standard Progressive Matrices

mathematical ability (r = .53) in this study, which is comparable to other studies (e.g., Neisser et al., 1996).

Mathematical Creativity Test (MCT). The MCT, developed by Kattou et al. (2013), was translated into Dutch for this study by following the steps for a good translation process described by Beaton, Bombardier, Cullemin, and Bos Ferraz (2002). The MCT took a maximum of 45 minutes and consisted of five questions, which were open-ended and could have multiple solutions. Students were required to provide multiple solutions, original and distinct from each other (Kattou et al., 2013). A sample question is displayed in Figure 3. Scores were obtained for fluency (number of correct solutions), flexibility (number of different types or categories of correct solutions), and originality for each question. For detailed scoring guidelines, see Kattou et al. (2013). The internal consistency for the MCT is high (α = .80; Kline, 1999).

Look at this number pyramid. Each cell contains only one number. Each number in the pyramid can be calculated by always performing the same operation with the two numbers that appear below it. Complete the missing numbers in the pyramid, by keeping number 35 on the top cell of the pyramid. Find as many solutions as possible.

Figure 3. A sample question of the MCT.

Tests of domain-general creativity. The TCT-DP (Urban & Jellen, 1996) and the Dutch version of the TTCT (Torrance, 2008) were used to measure students' general creative potential.

TCT-DP. The TCT-DP Form A was used and took 15 minutes. This test mirrors a more holistic concept of creativity. Students had to complete a drawing using certain figural fragments, such as a half circle and a half square, which was scored according to the guidelines (Urban & Jellen, 1996). A total score was obtained by adding the scores of 13 categories and transforming them into z-scores. The TCT-DP has good inter-rater reliability: α = .81 – .99 for the total score and α ≥ .89 for test criteria (Urban & Jellen, 2010).

TTCT. The TTCT measures divergent thinking with words and pictures. Each activity took 10 minutes. Activities 5 (unusual uses) and 7 (just suppose) were used from the verbal test (version A). Activity 5 requires students to write down as many alternative uses for a cardboard box that they can think of. Activity 7 requires students to hypothesize about an improbable situation. Activities 2 (picture completion) and 3 (repeated lines) were used from the figural test (version A). Activity 2 requires students to draw pictures using ten incomplete figures as a starting point, to which titles are added. Activity 3 consists of three pages of sets of parallel lines, and students must draw something using these parallel lines as part of their picture. These four activities were used because they require different forms of divergent thinking. Activity 7, for example, requires more use of imagination than activity 5 (Torrance, 2008). Both tests were scored according to the guidelines (Torrance, 2008). For all activities, scores were obtained for fluency, flexibility, and originality. Additionally, for activities 2 and 3 (TTCT Figural), scores were obtained for elaboration, abstractness of titles, and resistance to premature closure. Raw scores were transformed into z-scores. The internal consistency was good for the TTCT Verbal in this sample (α = .75), but questionable for the TTCT Figural (α = .61; Kline, 1999).

Test of MA. Scores from a widely used standard Dutch mathematical achievement test (Janssen, Scheltens, & Kraemer, 2007) were used as a measure for MA. We used the test that was designed for grade 4. All subscales from the math test were used: 'number and number relations', 'mental arithmetic', 'estimation arithmetic', 'arithmetical operations', 'geometry', 'arithmetic with time and money', and 'proportions, fractions and percentages'. For each student, the percentage correct on the subscales was calculated. The questions on the math test are mainly math word problems. A sample question from the MA test is the following: "Cycle racers have to cycle 5 rounds of 18 kilometers. How many kilometers are that in total?" Cronbach's alpha for the math test was excellent (α = .94) in this study (Kline, 1999).

2.2.3. Procedure

Data were collected in the fall of 2014 by four master's students, each with a bachelor's degree in special education, supervised by the first author. Informed consent was obtained from the parents or guardians of all children involved. Information about students' age, gender, and socioeconomic status was obtained from school records. The measures of creativity were part of a larger test battery, administered in two sessions, each lasting 90 minutes. All tests were administered in a classroom setting by one or two proctors. Test instructions were read aloud. Students were not allowed to copy the work of their fellow students or to talk during test sessions.

All tests were scored by the same master's students. For the MCT, TTCT, and TCT-DP, the inter-rater reliability (IRR) was determined. For all variables, sufficient to good agreement was reached (Cicchetti, 1994). Unfortunately, no agreement was reached on the variable resistance to premature closure of the TTCT Figural activity 2 (ICC = .28). This variable was excluded from the analyses.

2.2.4. Analyses

Prior to the data analyses, assumptions for SEM were checked in SPSS Statistics (IBM corporation, 2013). Next, data were analyzed by testing different models, using SEM in M*plus* version 7.2 (Muthén & Muthén, 2012). As a result of checking our assumptions, we decided to use the MLM estimator in M*plus* to take non-normality into account (Tabachnick & Fidell, 2013).

First, separate confirmatory factor analyses (CFAs) were conducted for MA and MC. Second, an exploratory factor analysis (EFA) was conducted to test whether one latent construct for GC could be created. Third, correlations between MC, MA, and GC were computed to get insight into the existence and strength of the relations. Fourth, the two hypotheses were tested by examining the two competing models (Model 1 & 2); comparative model fit was evaluated. In Model 1, GC influenced MC and MC influenced MA. In Model 2, it was hypothesized that both MA and GC influenced MC. In these models, we controlled for IQ and gender since we expected that these variables could influence the relations between GC, MC, and MA. The covariate gender was added to the models because boys and girls score significantly differently on mathematical ability tests (Preckel, Goetz, Pekrun, & Kleine, 2008) and mathematical creativity tests (Mann, 2005). Regarding the influence of gender on domain-general creativity tests, research has found inconsistent results (Baer & Kaufman, 2008). The covariate IQ was added to the models because IQ is positively related with school performance (Laidra, Pullman, & Allik, 2007) and creativity (Kim, 2005).

2.3. Results

Descriptive statistics of all variables used can be found in the Appendix.

2.3.1. MA

A CFA with one factor was examined for MA, using the seven subscales of mathematical performance as observed variables to test the unidimensionality of MA. The CFA indicated that the model fitted well (CFI = .99; TLI = .99; χ^2 = 33.83, *df* = 14, *p* = .002; RMSEA = 0.06, SRMR = 0.01).

2.3.2. MC

A second-order CFA was conducted for MC, with the 15 subscales of MC as observed variables, and with fluency, flexibility, and originality as first-order latent variables. Covariances between the error variances of the abilities (fluency, flexibility, and originality) were added because the variables were highly correlated per question. This was expected because flexibility and originality scores are likely to be higher when more answers are provided (high fluency score; Torrance & Safter, 1999). Furthermore, we obtained a negative residual variance for the latent variable originality in the model. This negative variance is probably caused by outliers in the data (Bollen, 1987). Outliers were not deleted since they were deemed realistic scores. Since a negative variance is not possible, we scaled the variance to zero, which is the most closely related possible value. Results indicated that the model fitted well (CFI = .98; TLI = .98; χ^2 = 106.22, *df* = 73, *p* = .007; RMSEA = .04; SRMR = 0.04).

2.3.3. GC

For the latent variable GC, an EFA was applied using all the variables of the TTCT (Figural and Verbal) and the total score of the TCT-DP. The default setting was used (Geomin oblique rotation). An EFA was chosen because it was not clear how the TTCT and TCT-DP were related. The EFA indicated that a five-factor model would fit best with the following factors, which theoretically made sense: (1) TTCT Verbal activity 5, (2) TTCT Verbal activity 7, (3) TTCT Figural activity 2 (only fluency and originality), (4) TTCT Figural activity 3 (only fluency and originality), and (5) TCT-DP total score and TTCT Figural title and elaboration activities 2 and 3. However, not all factors were significantly correlated. After GC was added as a second-order factor, only factors 1, 2, 3, and 4 were significant indicators of GC. The insignificant factor (i.e., factor 5) was deleted from the model, and the negative residual variances of fluency (TTCT Verbal activity 7) and originality (TTCT Figural activity 2) were scaled to zero. This model had a good fit (CFI = .97; TLI = .96; χ^2 = 70.87, *df* = 33, *p* < .001; RMSEA = .06; SRMR = .04). Theoretically, it makes sense to have factors of each activity because fluency, flexibility, and originality are often highly correlated per activity and, therefore, measure almost the same; it is more likely that flexibility and originality scores are higher when more answers are provided (i.e., a high fluency score; Torrance & Safter, 1999). Currently, each activity gives an indication of divergent thinking. In fact, the factor GC represents "generating ideas" rather than the more complex construct of creativity.

The other factor, measured by the TCT-DP and by "abstractness of title" and "elaboration" of the TTCT Figural activities 2 and 3, represents another measure of GC, which is, however, correlated with divergent thinking (see Table 1). This factor might be related to "deeper digging into ideas" and the "openness and courage to explore ideas" (Treffinger et al., 2002). Models 1 and 2 were examined with both measures of GC separately since adding both measures of GC in one model would reduce the power of the study. An overview of the inter-correlations of the obtained factors and their relations with the covariates are given in Table 1, which provide an indication of the existence and strength of the relationships. Regarding the existence of the relation between gender and the other variables, we used an independent *t*-test and Mann-Whitney's U-test. The results indicated that there are no significant differences on the factor scores of MC (*t* = 1.77, *p* = .08), MA (*t* = -1.24, *p* = .22), and IQ (*t* = 1.27, *p* = .20) for boys and girls. There are, however, significant differences for gender on "generating ideas" (GC1; *U* = 8719, *p* < .01, *r* = -0.17) and "explore and dig deeper into ideas" (GC2; *U* = 7605.50, *p* < .001, *r* = -0.26), with girls scoring higher than boys.

To study how MC, GC, and MA are related, we tested two different models, namely Model 1 and Model 2. Because the EFA indicated that GC was not a unitary construct, but represented two different constructs, namely "Generating ideas (GC1)" and "Explore and dig deeper into ideas (GC2)," we decided to test Models 1 and 2 separately with GC1 and GC2. Furthermore, we added the covariates IQ and gender in the models, predicting the observed variables of MA, MC, and GC. Insignificant paths between the covariates and observed variables were deleted.

Table 1.

Spearman correlations between the factor scores of MC, GC and mathematical performance and the covariates gender and IQ.

	1	2	3	4	5
1. MC total (factor score)	-				
2. GC1 ('generating ideas')	.64*	-			
3. GC2 ('explore and dig deeper into ideas')	.55*	.64*	-		
4. MA total (factor score)	.70*	.15	.17	-	
5. IQ (Raven SPM)	.55*	.25*	.25*	.57*	-

Note .* p <.005 (Bonferroni correction applied (0.05 / ((5*4)/2)).

2.3.4. Model 1 & 2 with GC1

It was tested whether creative thinking in mathematics predicted mathematical performance, and whether divergent thinking in general (GC1) predicted MC but not MA directly (Model 1) or whether divergent thinking in general (GC1) and MA predicted MC (Model 2). In Model 1, MC was a significant predictor of MA (r = .27, p < .001) and GC1 a significant predictor of MC (r = .35, p = < .01). This model had a good fit (CFI = .96; TLI = .95; χ^2 = 714.48, df = 481, p < .001; RMSEA = .04; SRMR = .06). In Model 2, GC1 was a significant predictor of MC (r = .40, p = < .01) and MA a significant predictor of MC (r = .34, p < .001). This model also had a good fit (CFI = .96; TLI = .96; χ^2 = 706.01, df = 481, p < .001; RMSEA = .04; SRMR = .06). Model 2 had a lower Akaike information criterion (AIC) and Bayesian information criterion (BIC; AIC = 296.65; BIC = 826.30) compared to Model 1 (AIC = 301.85; BIC = 831.49), indicating that Model 2 fitted best (Muthén & Muthén, 2012). This model is shown in Figure 4.

2.3.5. Model 1 & 2 with GC2

It was tested whether MC predicted MA, and whether "Explore and dig deeper into ideas" in general (GC2) predicted MC but not MA directly (Model 1) or whether "Explore and dig deeper into ideas" (GC2) and MA predicted MC (Model 2). In Model 1, MC was a significant predictor of MA (r = .38, p = < .001) and GC2 a significant predictor of MC (r = .27, p = < .01). This model had a good fit (CFI = .97; TLI = .96; χ^2 = 452.34, df = 330, p < .001; RMSEA = .04; SRMR = .06). In Model 2, GC2 was a significant predictor of MC (r = .31, p = < .001) and MA a significant predictor of MC (r = .41, p = < .001). This model also had a good fit (CFI = .97; TLI = .96; χ^2 = 448.86, df = 330, p < .001; RMSEA = .04; SRMR = .06). Model 2 had a lower AIC and BIC (AIC = -1857.89; BIC = -1383.16) compared to Model 1 (AIC = -1854.78; BIC = -1380.04), indicating that Model 2 fitted best (Muthén & Muthén, 2012). Model 2 is shown in Figure 5.

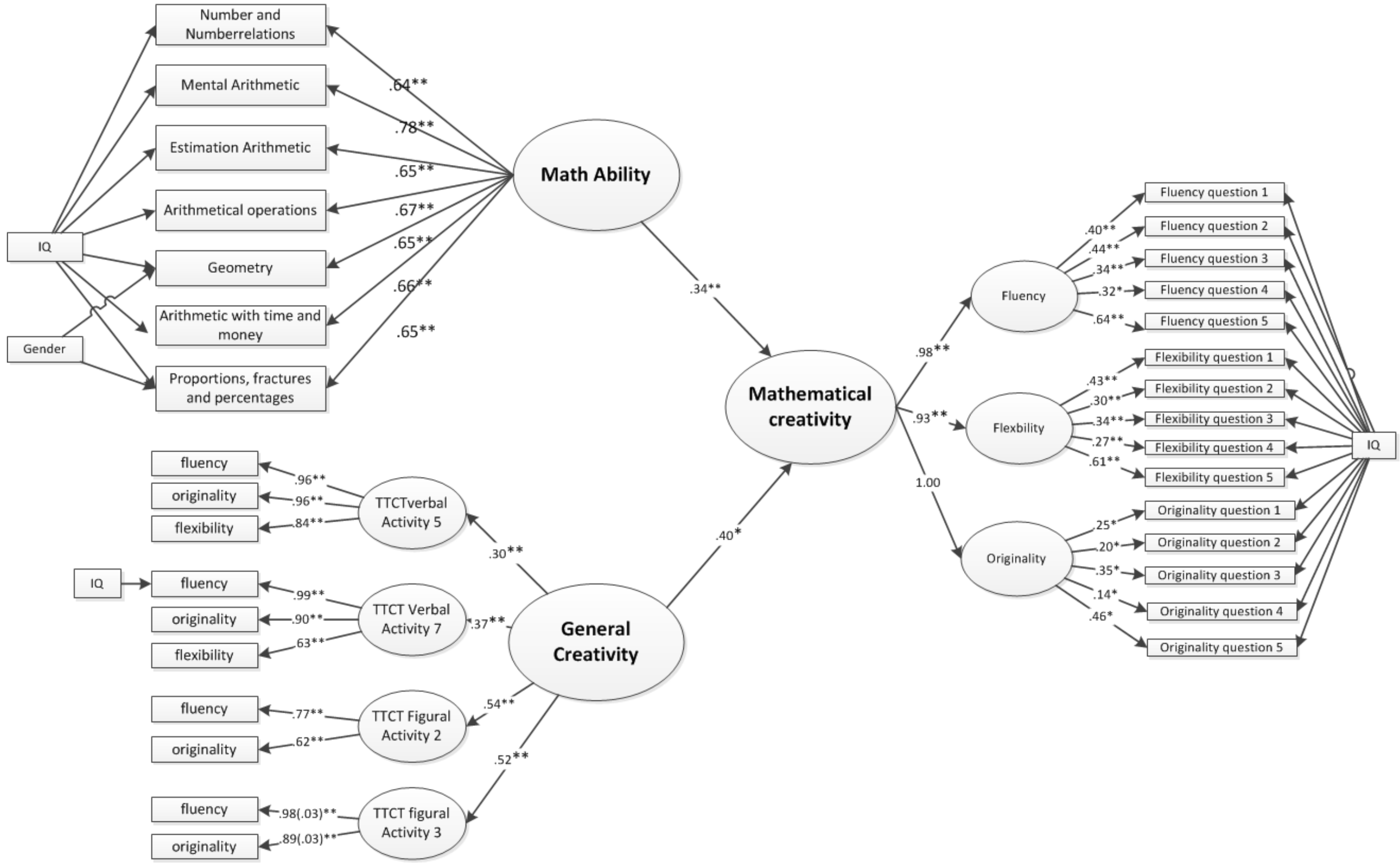

Figure 4. Standardized factor loadings of Model 2 (with GC representing 'generating ideas') with covariates gender and IQ. *Note.* Covariances of the observed variables of MC are not visualized and IQ is visualized multiple times in the model to make the image more clear. **p<.001, *p<.05

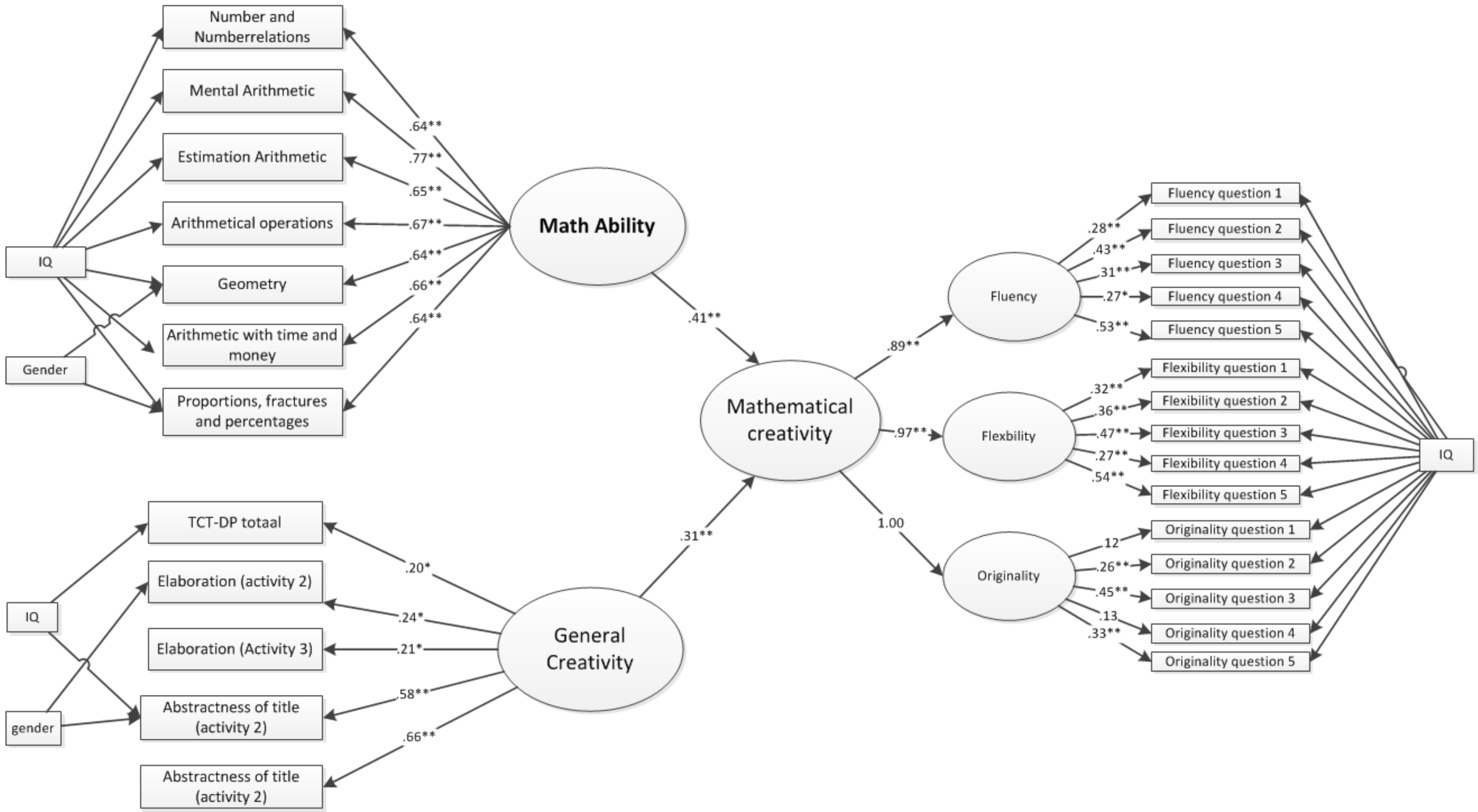

Figure 5. Standardized factor loadings of Model 2 (with GC representing 'explore and dig deeper into ideas') with covariates gender and IQ. *Note*. Covariances of the observed variables of MC are not visualized and IQ and gender are visualized multiple times in the model to make the image more clear. $^{**}p < .001$, $^{*}p < .05$

2.4. Discussion

This study aimed to provide insight into the role of creativity in mathematical ability in fourth-grade students by examining the relations between MC, GC, and MA. Two competing hypotheses were tested using SEM.

A crucial first result is that GC was not an unitary construct, but consisted of two different constructs, namely "Generating ideas (GC1)" and "Explore and dig deeper into ideas (GC2)." This result suggests that either one instrument cannot capture the general measure of creativity or one of these instruments might not measure (an element of) GC. This finding highlights the importance of careful use of instruments that attempt to assess a "general creative ability." This result is in line with recommendations of other researchers to use multiple measures of creativity (Cropley, 2010; Kim, 2006; Treffinger et al., 2002). Since analyses indicated that GC was not a unitary construct, relations between MC, GC1, and MA, and those between MC, GC2, and MA are discussed separately.

Regarding the relationship between GC1 ("Generating ideas"), MC, and MA, we found most support for the second hypothesis, that MA and GC1 both influence MC. Although no causal direction could be established with this study, the results suggest that divergent thinking and mathematical knowledge are almost equally important for MC. In order to think divergently, students need to combine and reorganize existing concepts to generate new concepts and ideas (Mumford, Baughman, Maher, Constanza, & Supsinski, 1997). This requires cognitive flexibility, which is also an important capacity in MC. Existing mathematical concepts are combined and reorganized to generate new and multiple mathematical solutions. This result is in line with the studies of Kattou and Christou (2013) and Jeon et al. (2011), although there were some small differences regarding the strength of the predictors. For example, the variance accounted for by MA (r^2 = 10%) appeared to be larger compared to that accounted for by GC (r^2 = 3%) in the study by Jeon et al. compared to our study. Our result is in contrast to the findings of Huang et al. (2017), which showed that MA was a strong predictor of MC, but that GC was not a significant predictor of MC. These small and large differences between our study and others may be caused by the diverse measures used. We used a multiple-solution task as a measure of MC that mainly required knowledge of arithmetical operations and number relations – knowledge already mastered by most students. Therefore, GC1 ("generating ideas") may have played a slightly stronger role in MC than MA. Huang et al. (2017) used an MC task that was rather more difficult and required a higher level of MA. For future studies, it would be interesting to study in more depth how MC and GC are related to MA, by focusing, for example, on specific task aspects.

Concerning the relation between GC2 ("Explore and dig deeper into ideas"), MC, and MA, we also found most support for the second hypothesis: MA and GC2 both influenced MC. This result is in agreement with the findings of Kroesbergen & Schoevers (2017), which similarly showed that GC (measured by the TCT-DP) was a predictor of MC. However, this is the first study that used this instrument simultaneously with measures of MC and MA in one model. This finding could therefore be significant for the literature. Contrary to our result regarding GC1, MA was a stronger predictor of MC than GC2. The reason for this may be that "Explore and dig deeper into ideas" requires both divergent and convergent thinking (Treffinger et al., 2002). Since the measurement of MC (i.e., a multiple solution task) is more closely related to divergent thinking than to convergent thinking, it may explain why GC2 had a smaller influence on MC than MA.

When interpreting the results of this study, the reader should also take into account that the sample size used in this study was rather small for SEM analyses. Therefore, it was not possible to take the multilevel structure of the data into account (Hox, 2010). We recommend that future studies do so, which would require larger samples.

In conclusion, despite the crucial finding that GC is not a unitary construct, both our results regarding the relation between GC (1 and 2), MC, and MA give more support to the hypothesis that both GC (1 and 2) and MA predict MC. The amount of influence of a component of GC (i.e., "generating ideas" or "explore and dig deeper into ideas") and MA on MC seems to depend on the instruments that are used. This result is important to take into account in (designing) research on mathematical learning and MC. Careful use of tests that attempt to measure GC is recommended.

With regard to the implication of this research for educational practice, this study suggests that in order to creatively solve mathematical problems, both mathematical knowledge and general creative thinking skills are needed. Teachers should be aware of both components when promoting students' mathematical creativity.

2.5. Appendix

Table 1.

Means and standard deviations of the MCT

	Question 1 *M(SD)*	Question 2 *M(SD)*	Question 3 *M(SD)*	Question 4 *M(SD)*	Question 5 *M(SD)*
Fluency	.26 (.15)	.18 (.19)	.20 (.19)	.17 (.16)	.13 (.11)
Flexibility	.54 (.24)	.26 (.19)	.52 (.20)	.36 (.21)	.26 (.18)
Originality	.50 (.32)	.35 (.30)	.35 (.19)	.57 (.28)	.36 (.27)

Note. MCT: Mathematical Creativity Test

Table 2.

Means and standard deviations of the subscales of the MA task

	M(SD)
Numbers & number relations	.76 (.19)
Mental Arithmetic	.70 (.21)
Estimation arithmetic	.73 (.22)
Arithmetical operations	.63 (.21)
Geometry	.67 (.17)
Arithmetic with money and time	.70 (.20)
Proportions, fractions & percentages	.64 (.24)

Table 3.

Means and standard deviations of the subscales of the TTCT and final score of the TCT-DP

	Fluency *M(SD)*	Flexibility *M(SD)*	Originality *M(SD)*	Elaboration *M(SD)*	Abstractness of title *M(SD)*	Final score *M(SD)*
TTCT verbal (Act. 5)	18.48 (10.30)	8.35 (3.09)	10.94 (6.73)	-	-	-
TTCT verbal (act. 7)	6.06 (5.36)	4.20 (2.98)	3.92 (4.87)	-	-	-
TTCT figural (act. 2)	8.71 (1.83)	-	5.43 (2.22)	1.16 (.42)	2.46 (2.73)	-
TTCT figural (act. 3)	10.54 (5.17)	-	7.19 (4.24)	1.13 (.41)	1.16 (2.03)	-
TCT-DP	-	-	-	-	-	20.44(7.41)

Chapter 3

The relation between creativity and students' performance on different types of geometrical problems in elementary education

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Abstract

This study investigated the relation between creativity and geometrical problem solving in elementary school. We examined whether the predictive value of creativity was different for students' performance on three types of geometrical problems, while controlling for several covariates at the student and class level, including students' visual-spatial working memory, age, gender, socio-economic status, teachers' experience, and type of mathematical textbook. A sample of 1665 Dutch students from 3th – 6th grade participated. Students had to solve four closed routine problems, six non-routine problems (related to a visual artwork) and four geometrical multiple solutions problems. The Test of Creative Thinking-Drawing Production (TCT-DP) was used to measure students' creativity. Multivariate multilevel analyses were conducted to take the nested structure of the data into account. Creativity was a significant predictor of students' performance on all types of problems in the domain of geometry, but most strongly associated with performance on geometrical multiple solution problems, suggesting that more creative students perform better in solving geometry problems in general, but even more so in geometry problems asking for multiple solutions. Possible implications of this finding for mathematics education are discussed.

3.1. Introduction

In current elementary school mathematics education, textbooks largely determine what students learn as most teachers strongly rely on textbooks to structure their lessons (Gravemeijer, 2007; Hop, 2012; Meelissen et al., 2012; Stein & Smith, 2010). In these textbooks, students predominantly have to solve routine problems in which they have to reproduce and apply a fixed solution procedure in one or two steps (Kolovou, 2011; Van Zanten & Van den Heuvel-Panhuizen, 2018). However, within mathematics education it is increasingly considered important that students also learn to solve problems that are not straightforward and for which they do not have a learned solution immediately available (Schoenfeld, 1983), and which, therefore, may elicit other, more complex cognitive processes, such as creative thinking (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). Complex higher order thinking and creativity in mathematical problem solving are at the heart of functional mathematical competence (Halmos, 1980; Schoenfeld, 1983) and, thus, vital to teach to students. A key question is whether commonly used mathematics textbooks with the predominant routine type of problems they contain, provide sufficient support for these aspects of functional mathematical competence. Research suggests that in order to promote higher-order creative thinking in mathematics, other types of problems should be offered to students, such as non-routine and open-ended problems (Carlson & Bloom, 2005; Levav-Waynberg & Leikin, 2012a; Silver, 1997), but strong evidence that these types of problems call upon, and thereby foster, higher-order creative thinking is rare. Moreover, different types of non-routine and open problems exist (Leikin, 2018), and it is an open question if all types of non-routine problems call upon creative thinking equally. To provide initial answers to these questions, we investigated the relations of students' creativity as assessed with a standard test of creative thinking with their performance on three types of mathematical problems in the upper grades of elementary school, controlling for possibly confounding factors. A better understanding of the relation between creativity and mathematical problem type can inform curriculum and textbook designers.

The current study was conducted in the mathematical domain of geometry. Geometry can be defined as grasping the concept of space and the mathematization of space. Geometry education has the aim to teach students to understand, explain and predict geometric phenomena, to reason spatially, and to order and organize spatial situations. For example, it is considered important that students are able to draw a map or to reason about the effect of the height of the sun on the length of the shadow (Gravemeijer et al., 2007; Jones, 2002). Typical topics in geometry in the upper grades of elementary school are (1) 'spatial sense', which is about localizing, taking a standpoint and navigation, (2) 'plane and solid figures', which is about spatial properties and relations between figures, operations, transformations and constructions, and (3) 'visualization and representation', which is about the representation of two-dimensional and three-dimensional reality (Gravemeijer et al., 2007).

In educational practice, especially in the Netherlands, most teachers use mathematical textbooks to teach the mathematics curriculum (Gravemeijer, 2007; Hop, 2012; Meelissen et al., 2012; Stein & Smith, 2010). The geometry content and geometrical problems presented in these textbooks determine to a large extent students' opportunities to learn in geometry (Stein, Remillard, & Smith, 2007). Although several types of problems can be used in teaching geometry, Dutch mathematics textbooks mainly offer closed-ended routine problems (Kolovou, 2011). A typical example is a multiple choice test item where students have to choose the correct picture among distractors of how a block construction would look like from another point of view. Another example is an item where students have to choose the correct picture representing the paper model of a cube (Van Grootheest et al., 2011). Problems like these are considered closed-ended problems, because there is only one correct solution (Mihajlović & Dejić, 2015). Furthermore, in the upper grades of elementary school, problems like these can be considered routine problems as well, because they are already familiar to students and students may have developed sufficient experience and routine-based strategies to solve these types of problems (Schoenfeld, 2013).

In addition to closed-ended routine problems, also non-routine geometrical problems could be offered to students. However, generally, in most textbooks only few of these more complex problems are included (i.e., 0-8%; Van Zanten & Van den Heuvel-Panhuizen, 2018). Non-routine problems are problems that are unfamiliar to a student and cannot be solved by using routine or familiar procedures (Carlson & Bloom, 2005; Schoenfeld, 1983). Non-routine problems require more complex cognitive processing than routine problems. Students need to recall, use and combine facts, skills, procedures and ideas in a new and meaningful way to solve the problem, calling in particular upon creative and flexible thinking (Chapter 4; Lester, 2013; Liljedahl et al., 2016; Mayer & Dow, 2004; Warner, Alcock, Coppolo Joseph, & Davis, 2003). Although there are different types of non-routine problems, most are characterized by a degree of openness. A problem can be open "with respect to interpretation, perhaps with different solutions afforded by each interpretation, or which otherwise accommodate a range of plausible solutions'' (Silver, 1995, p.68), but also when the problem invites different solution methods or different solutions (Mihajlović & Dejić, 2015; Silver, 1995). An example of a geometrical non-routine open problem is a multiple solution task in which students have to compare three different plane figures (e.g., isosceles triangle, right-angled triangle, square) and have to give multiple answers on the question in which respects one plane figure differs from the other two. In this way, students obtain insight into the classification of plane figures based on comparison, reasoning and making connections (Gravemeijer et al., 2007). Another example of a non-routine problem is when students have to look closely at a picture of a still-life painting and then have to make a floor map of what is depicted in the painting. Although this problem is basically similar to a routine block construction problem, making a floor map is unfamiliar to students and it is a less closed task because there are several options to make the map, while in making the map students need to combine new ideas with existing knowledge in a new context to solve the problem.

Research suggests that solving non-routine and open problems requires students to think and act creatively (Liljedahl et al., 2016). Creativity can be defined as a process that results in a novel and meaningful product (Kaufman & Beghetto, 2009; Runco & Jaeger, 2012;

Sriraman, 2005). In elementary geometry education, this product can, for example, be an idea that is new to the student, a newly posed problem or a solution to a non-routine problem which is novel and meaningful for a specific age group or student (e.g., Leikin, 2009). In order to create something novel and meaningful, it is important to be flexible and to overcome fixation on common ideas (Haylock, 1987b). Multiple (cycles of) creative processes are thought to be involved, such as divergent (idea generation) and convergent (idea evaluation) thinking (Guilford, 1967). Several researchers argue that non-routine problems — both more closed (i.e., with more constraints; Bokhove & Jones, 2018) and more open problems (e.g., Kwon, Park, & Park, 2006) — have the potential to elicit creative thinking. More specifically, researchers have argued, and indeed shown, that multiple solution tasks where students have to provide multiple solutions for a problem, elicit creative thinking (Chapter 2; Kroesbergen & Schoevers, 2017; Kwon, Park, & Park, 2006; Leikin, 2018; Leikin, 2009; Levav-Waynberg & Leikin, 2012b; Silver, 1995). Being provided with the opportunity to think about more than one answer, students can overcome a fixation on particular deeply entrenched ideas and are challenged to think divergently to generate new, meaningful answers (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012).

Although the role of creativity has been investigated in multiple solution tasks (e.g., Chapter 2), to the best of our knowledge, no study to date investigated whether creativity plays a similar role in other non-routine problems or, for that matter, in routine problems. Only one study examined simultaneously the relations between independently assessed creativity and students' performance in solving mathematical routine word problems and multiple solutions problems, revealing that creativity was a significant predictor of performance on both types of problems, next to working memory and number sense (Kroesbergen & Schoevers, 2017). However, whether the associations between creativity and performance on the different problem types differed in strength was not explicitly tested. Therefore, the current study examined whether the relations between students' domain-general creativity and their mathematical performance differed between three problem types: routine, non-routine and multiple solution problems. In doing so, we adopted the basic assumption that the association between general creativity and mathematical performance indicates the extent to which solving the mathematical problem calls upon general creative thinking processes.

When studying the relation between creativity and performance on different mathematical problem types, the possible effects of other factors, correlated with domain-general creativity, that can at least partly explain the observed relation, should be taken into account. This holds in particular for students' visual-spatial working memory (VSWM), general mathematical ability, gender, and socio-economic status (SES). Especially in solving geometrical problems, students' VSWM is likely involved (Giofrè, Mammarella, Ronconi, & Cornoldi, 2013). VSWM refers to the ability to temporarily hold visual-spatial information activated for processing (Baddeley, 2013; Kroesbergen & Van Dijk, 2015). When geometrical problems are related to spatial visualization and spatial sense, students may need to temporarily store and manipulate visual-spatial information. For example, a target figure, such as a block construction, needs to be temporarily stored in memory in order to mentally rotate the construction to see how it looks like from another point of view. In addition, general mathematical ability should be taken into account, since basic arithmetical and mathematical procedures and knowledge about, for example, numbers, proportions and measurement can be used to solve geometrical problems (Gravemeijer et al., 2007). Gender might also play a role. Although male students often score higher on routine mathematical problems than girls (Frost, Hyde, & Fennema, 1994; Reilly, Neumann, & Andrews, 2015), girls have been found to outperform boys on multiple solution tasks (Mann, 2006), while they often also score higher on creativity tasks (Baer & Kaufman, 2006). Finally, also SES should be taken into account since a low SES is related to lower educational performance in general, and in mathematics in particular, possibly due to disadvantages in financial, cultural and social resources, and lower parental involvement in students' education (Crane, 1996; OECD, 2016; Sirin, 2005).

Besides considering the role of student characteristics, also class level factors may influence the relation between creativity and performance, such as teachers' experience and the type of mathematical textbook that is used. Research has shown a positive effect of teachers' years of experience on students' mathematical performance (Clotfelter, Ladd, & Vigdor, 2007). More experienced teachers may be more effective, because they have improved their teaching performance over the years (Darling-Hammond, 2000). Furthermore, the contents of mathematical textbooks, in particular the type of problems they provide, may influence students' performance as well (Van Zanten & Van den Heuvel-Panhuizen, 2018).

3.1.1. The present study

The present study investigated the relations between students' independently assessed domain-general creativity and their performance on closed routine geometrical problems, geometrical multiple solution problems and non-routine visual arts-geometry problems, involving geometrical reasoning in relation to visual arts. The aim of the study was to determine whether the relation between creativity and performance differed between the three problem types, which assumingly would indicate differences between the problem types in the degree to which they call upon creative higher-order thinking. VSWM, gender, age, low SES and general mathematical ability, teachers' experience and the type of mathematical textbooks were included as covariates to control for spurious relations between creativity and mathematical performance.

We hypothesized that students' creativity had stronger predictive relations with performance on non-routine visual-arts geometry problems and multiple solution problems than on the routine problems. Both non-routine visual-arts and multiple solution problems were expected to be unfamiliar for students compared to routine problems and, therefore, to require stronger involvement of higher-order creative thinking. We expected that routine tasks required no or little creative thinking of students, since these problems were familiar and closed-ended, and, therefore, would show no or a weak relation between creativity and performance. Moreover, as there is consensus that in addition to unfamiliarity openness of a task puts additional demands on higher-order creative thinking (Leikin, 2018), we also expected that students' domain-general creativity would be strongest associated with their performance on the multiple solution problems.

3.2. Methods

For the present purpose, pre-intervention data of a large-scale evaluation study of the Mathematics, Arts and Creativity in Education (MACE) program were used (see Chapter 5) The MACE program was developed to support elementary schools in the Netherlands to meet the partly overlapping learning goals and objectives of the disciplines visual arts and geometry, and to promote students' creative skills in both disciplines. A lesson series for Grades 4 to 6 (ages 9 to 12 years) was designed in which students could engage in open activities and classroom discussions in an integrated visual arts and geometry context. The evaluation study investigated the effects of the MACE program on students' ability in geometry and visual arts in the upper grades of elementary school. The results are reported in Chapter 5 of this dissertation. In the present study, only the students who took all the tests relevant for the present purpose were included.

3.2.1. Participants

Participants were 1665 students in grade three (*n* = 18), four (*n* = 409), five (*n* = 669) and six (*n* = 563), in 92 classes at 50 schools in the Netherlands. Schools were recruited by sending flyers of the MACE program to 428 regular elementary schools in all regions of the Netherlands; 11.68% of the schools were willing to participate. Schools differed regarding their educational vision and teaching methods, and were located in both rural and urban areas spread across the Netherlands. The sample consisted of students with low, medium and high socio-economic background, of which 48.3% boys, and a mean age of 10.91 years (*SD* = 0.93).

3.2.2. Instruments

Geometric Ability Test (GAT). The GAT took between 20–30 minutes and was stopped after 30 minutes. The test consisted of 11 geometry problems, of which four closed-ended routine geometry problems (see the left image in Figure 1), and seven non-routine visual arts-geometry problems (see the right image in Figure 1). All routine problems called upon spatial sense and spatial visualization (Gravemeijer et al., 2007). In the non-routine problems, students mainly had to reason geometrically in relation to a painting. In one problem, students had to draw a floor plan of a painting. The test started with four routine problems and ended with the non-routine art-geometry problems. Within problem type, the order of the problems was randomized and the same for all students; there was no ascending order of difficulty.

Scoring. A separate score for, respectively, the routine and non-routine art-geometry problems was calculated. The routine problems were relatively straightforward with clearly one correct answer. Therefore, one point was given for a correct answer and zero points for an incorrect answer. An average score for the four routine problems was calculated; a total score between 0–1 could be obtained.

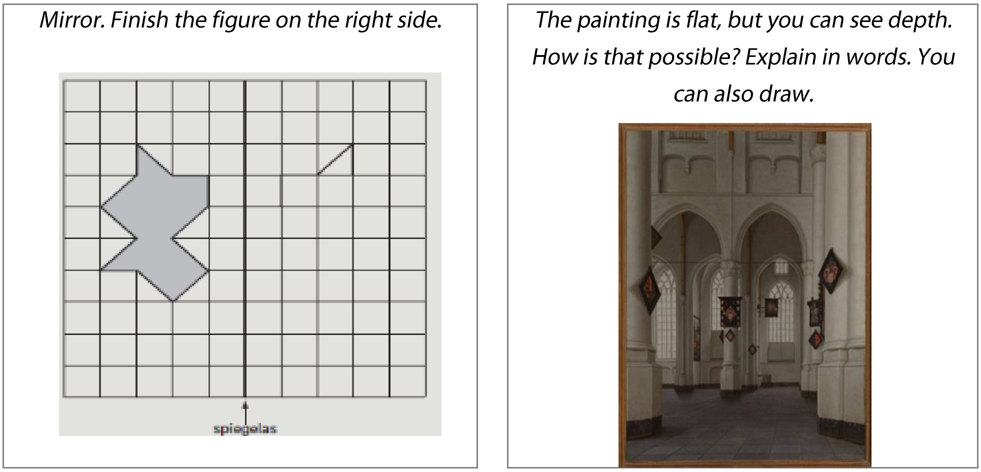


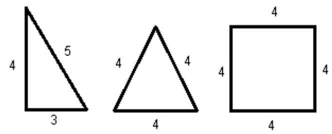
Figure 1. An example of respectively a routine and art-geometry problem of the GAT

The geometry problems related to visual arts were more complex and could also yield partially correct solutions. Therefore, two points were given for an answer in which students showed to be able to reason correctly about geometric phenomena, for example when students explained why persons or buildings in front of the painting are bigger than in the back of the painting while referring to and explaining perspective. One point was given for an answer in which the reasoning was not complete (e.g., 'by painting smaller', without a specific explanation). Zero points were given for answers in which the question was merely repeated or there was no reasoning about the question involved (e.g. 'it is a painting'). The last visual arts-geometry problem of the task was considered too difficult for grade 4 and 5; only 9% of the students scored one or two points (Hopkins & Antes, 1978). Therefore, this problem was not used in this study. Furthermore, 27.1% of the students were not able to finish the GAT in the maximum time of 30 minutes. Therefore, we calculated an average final score for the visual arts-geometry problems based on the number of problems the students were able to finish. For the visual arts-geometry problems a final score between 0–2 could be obtained. Since items were not in an ascending order of difficulty, this was the most valid way to calculate the score. Furthermore, the 27.1% students who did not complete the task scored highly similar if all visual arts-geometry problems would be used to create a total score. The correlation between the average score for only the completed visual arts-geometry problems and the score on all visual arts-geometry problems, in which a missing answer was given zero points, was high (r = .88). Also, correlations between the routine and non-routine visual arts-geometry problems are similar for students who completed all problems (r = .36) and did not complete all problems (r = .42), z = -1.28, p = .20.

The test-retest reliability of the GAT was acceptable (r = .66). Tests at the pre- and posttest were scored by four raters. The interrater reliability was sufficient to excellent for all items (κ = .81–1.00, ICC = .67 – 1.00, based on 25 tests). In line with our expectations, the internal consistency of the GAT was not very high (α = .62), because the GAT represents a heterogenous set of knowledge and skills.

Geometrical multiple solution task (GMST). The GMST, based on the mathematical multiple solution task (see Chapter 2; Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2013), took between 20–30 minutes and was stopped after 30 minutes. The task consisted of four geometry questions and one problem posing question which were open-ended and could have multiple correct answers. Students were instructed to provide multiple, but distinct solutions which, moreover, had to be original. In the problem posing task students were asked to pose mathematical questions based on a photo. A sample question of the test is depicted in Figure 2.

Look carefully at the three given shapes. Which shape does not belong in the same group as the other two? Explain your answer. Are there more than one possible answers? If so, write down as many answers as possible.



Shape _____ does not belong in this group of shapes because ....

Figure 2. A sample question of the GMST

Scoring. For the scoring of the GMST, we used the scoring scheme of Leikin for creativity in the individual solution space (Leikin, 2009; Levav-Waynberg & Leikin, 2012a). Within this scheme a distinction between fluency, flexibility and originality is made, which is explained in more detail in Leikin (2009), Levav-Waynberg and Leikin (2012a) and Leikin, Koichu and Berman (2009). Fluency was calculated by adding the number of correct answers for each question. With the use of the scheme, each solution of a student was scored regarding flexibility and originality. Next, a final score per solution was calculated as a product of $Flexibility_i$ x $Originality_i$. Afterwards, a score per question was concluded as: Fluency x ($\sum$ ($Flexibility_i$ x $Originality_i$). Last, the scores of all questions were added into a total creativity score.

The test-retest reliability of the GMST was good (r = .84; Schoevers, 2018). In this study, the GMST was scored by seven raters. The interrater reliability was sufficient to excellent for all scores per solution (ICC = .72 – .99, based on 25 tests). The internal consistency of the GMST was not very high (α = .68; Schoevers, 2018).

Creativity. The TCT-DP was used to obtain an indication of students' creativity and took 15 minutes. Students had to complete a drawing using given figural fragments, such as a half circle and a half square (Urban & Jellen, 1996). Version A was used. According to the authors, "the test was designed to mirror a more holistic concept of creativity than the mere quantitatively oriented, traditional divergent thinking tests" (Urban, 2005, p.272). However, according to a recent study, the TCT-DP addresses mainly one aspect of creativity, namely 'explore and dig deeper into ideas'. As this aspect involves both divergent and convergent thinking processes (see Chapter 2), it can be used as an indicator of students' domain-general creativity. Note, however, that it does not fully capture the complex multidimensional construct of creativity (Chapter 2).

Scoring. The TCT-DP was scored according to the guidelines in the manual (Urban & Jellen, 1996). Scores were obtained for 13 categories: 'continuations', 'completions', 'new elements', 'connections made with a line', 'connections made to produce a theme', 'boundary breaking that is fragment dependent', 'boundary breaking that is fragment independent', 'humor and affectivity' and four categories of 'unconventionality'. A total score was obtained by adding the scores of the 13 categories. If the total was more than 25, a score for speed was added to the total score. The TCT-DP was scored by four raters and had a good interrater reliability in this study (ICC = .80, based on 25 tests) for the total score.

VSWM. The online computerized task 'the Lion game' was administered to measure students' VSWM. The Lion game is a visual-spatial complex span task, in which students have to search for colored lions presented on a computer screen (Van de Weijer-Bergsma, Kroesbergen, Prast, & Van Luit, 2015). Students were presented with a 4 x 4 matrix containing 16 cells. In each trial, eight lions of different colors (red, blue, green, yellow, purple) were consecutively presented at different locations in the matrix. Students had to remember the last location where a lion of a certain color had appeared, and they had to use the mouse button to click on that location after the sequence has ended. The task started with two practice trials in which they had to remember the location of the last red and the last blue lion. Students received feedback on their performance after each trial. After the trials, the assessment started which consisted of 20 items at five difficulty levels with increasing WM load due to the increase in the number of colors that had to be remembered. At Level 1, students had to remember the location of the last red lion. At Level 2, students had to remember the locations of the last red and the last blue lion, and so on (Level 3: red, blue, and yellow; Level 4: red, blue, yellow, and green; Level 5: red, blue, yellow, green, and purple). The sequences of the location and color of the items were based on randomization, with one constraint: items never ended with a red lion, since the first response required the location of the last red lion. Students could complete the task individually (Kroesbergen & Schoevers, 2017; Van de Weijer-Bergsma et al., 2015).

Scoring. The proportion of items recalled in the correct location was used as a measure of VSWM. The Lion game is reported to have excellent internal consistency (Cronbach's α between .86 and .90), satisfactory test-retest reliability (ρ = .71), and good concurrent and predictive validity (Van de Weijer-Bergsma et al., 2015).

Questionnaire. A questionnaire for teachers was used to obtain information on student, teacher and class characteristics. Teachers were asked to provide information from school records about students' date of birth, gender, general mathematical ability, and the educational attainment of both parents. The educational attainment of both parents was used as an indicator of students' SES. Scores from the national standard mathematics test (Janssen, Scheltens, & Kraemer, 2007), developed by the national institute of educational testing, were used as measures of students' general mathematical ability. Different, age-appropriate versions of this test are administered twice a year by the teacher in most elementary schools. Norm-referenced ability scores obtained at the end of the previous school year were used in this study. The test consisted primarily of word problems that covered a wide range of mathematical domains such arithmetic operations, geometry, measurement, time, and proportions. The Cito mathematics test has been shown to be highly reliable; the reliability coefficients of different versions range from .91 to .97 (Janssen, Verhelst, Engelen, & Scheltens, 2010). Furthermore, teachers were asked to provide information about their own gender, years of experience and the mathematic textbook(s) they used to teach mathematics.

3.2.3. Procedure

The pre-intervention data were collected in September 2017 by the first author and twelve research assistants with a bachelor's or master's degree in (special) education. Tests were administered individually in one session in a quiet classroom by a research assistant who read aloud the test-instructions. Next to the GMST, GAT and TCT-DP, also a visual arts assignment was administered to the students, which is not reported on in this study. In addition, teachers had to fill out the questionnaire digitally or on paper. In the weeks after the test administration in the classroom, students individually completed the Lion Game on a computer in the school, supervised by their teacher. Research assistants were intensively trained beforehand to administer the tests in the classrooms and to code the tests. Sufficient interrater agreement had to be reached before students could start with the data collection. Passive informed consent of parents was obtained before the start of the study for almost all students; 0.8% of the parents did not consent to the participation of their child in this study. The study was approved by the Ethical Committee of the Faculty of Social and Behavioural Sciences of Utrecht University (FETC15-083).

3.2.4. Analyses

Multivariate multilevel analyses were conducted in MLwiN 3.02 to take the nested structure of the data into account, and because the three outcome measures used in this study were expected to be correlated (Charlton, Rasbash, Browne, Healy, & Cameron, 2017; Goldstein, 2011). A major advantage of using a multivariate approach instead of a series of univariate analyses is the larger power and lower risk of Type 1 errors (Tabachnick & Fidell, 2013). A three-level model was used in which the three types of problems (level 1) were nested within students (level 2), which were nested within classes (level 3; Hox et al., 2018). We did not take the school level into account as a fourth level, as preliminary analyses revealed little variance located at this level (2.0% – 4.8% of the variance), and no school-level variables were available in the data set that could be expected to be related to students' performance on the geometrical problems. All variables used in the multivariate multilevel analyses were z-standardized on the grand mean.

In the first model, second- and third-level predictors were added for each outcome variable: VSWM, students' age, students' gender, low SES, general mathematical ability, creativity (level 2), teachers' years of work experience, and dummy variables representing the four most frequently used mathematical textbooks (i.e., *Wereld in Getallen (WiG), Alles Telt, Pluspunt* and *Rekenrijk*, level 3). Effects of all predictors could vary between the different outcome measures: predictors had separate coefficients for the three outcomes measures. Next, for each predictor a new model was estimated in which coefficients for a predictor were constrained to be equal (i.e. common coefficients) to test whether effects were similar for all outcome measures. The order in which coefficients for each predictor were constrained to be equal was random and conducted in the order of the predictors described above. Based on the deviance and a chi-square test, we tested whether the new model, with common coefficients for a predictor, fitted the data. If this was true, common coefficients of a predictor were kept in the next model. This procedure led to a total of 12 models that were compared. Subsequently, if the effects of creativity on the three different problems were not similar, we tested if creativity differed significantly between the different problems by using contrasts. The coefficients of creativity were compared in this way and results were obtained about whether creativity was a stronger predictor in one problem compared to two other problems, in which effects were similar. If necessary, coefficients of creativity were constrained according to the results of the contrasts in a final, 13th model (Hox et al., 2018).

Before the multilevel analyses were conducted, data were screened and prepared. A missing data analysis indicated that between 0% and 7.2% of the data were missing. The main reasons for missing data were that students were ill or not present for other reasons in the class during test administration. Furthermore, the assumptions for multilevel analysis were checked (Hox et al., 2018). With the use of SPA-ML (Moerbeek, 2015), we calculated the required sample size. Sample size was calculated for a univariate model with two levels (i.e., students in classes), because the program could not calculate the required sample size for a multivariate model. With a desired power of .80 and expected effect size of .25 in a two-level model with 15% variance located at class level, and 85% located at student level, this study required a sample of 61 classes and 1220 students. The sample size requirement was met in this study. Furthermore, the assumptions of linearity and absence of outliers were checked by inspecting scatterplots at the student and class level. The assumption of linearity was met for all variables. In addition, the assumption of normally distributed residuals at all levels was tested. The assumption of normally distributed residuals was violated for the residuals at student level for the GMST. Therefore, robust standard errors are reported (Hox et al., 2018).

3.3. Results

3.3.1. Descriptive statistics

Descriptive statistics can be found in Table 1. Teachers in this sample had on average 17.11 years of experience (*SD* = 10.36). Regarding the mathematical textbooks, we found that four textbooks were predominantly used for mathematics education (see Table 2). Four other mathematical textbooks were used much less frequently and some classes did not use a textbook at all. To represent the main mathematical textbooks, four dummy variables were computed and included in the multilevel analyses. The other textbooks and no text book were pooled in one rest category which served as reference category. Table 3 presents the Spearman correlations between students' creativity and their performance on the three types of problems.

Table 1

Means and standard deviations of students' measurements

Measurements	*M (SD)*
Routine geometrical problems	.54 (.32)
Non-routine art-geometry problems	.61 (.37)
Geometrical multiple solution problems	957.52 (985.25)
Creativity (TCT-DP)	19.67 (8.32)
VSWM (Lion Game)	.73 (.15)

Table 2

Percentage of mathematical textbooks used

Mathematical Textbooks	Percentage
Wereld in Getallen (WiG)	32.4
Alles Telt	11.9
Pluspunt	23.0
Rekenrijk	15.7
Other	17.1

Table 3

Spearman correlations between variables used in this study

	1.	2.	3.	4.
1. Creativity (TCT-DP)	-	*.15***	*.19***	*.23***
2. Routine geometry problems	.17**	-	*.37***	*.14***
3. Non-routine art-geometry problems	.20**	.38**	-	*.25***
4. Geometrical multiple solution problems	.24**	.19**	.27**	-

**Significant at $p < .01$. *Note*. Correlations above the diagonal are corrected for age

3.3.2. Multivariate multilevel results

First, an intercept-only model was estimated to calculate the percentage of variance located at student and class level. The variance components were nearly equal for each type of problem; 15% was located on class level for the routine problems, 14% for the visual arts-geometry problems and 12% for the multiple solution problems. Subsequently, student- and class-level predictors were added for each outcome variable; effects of all predictors could vary between the different outcome measures. Next, for each predictor we tested in a new model whether the effects were similar for all outcome measures. We found that the effects of age, low SES, teachers' experience and the four mathematical textbooks were similar for all three types of problems. Effects of students' VSWM, gender, mathematical achievement and creativity were different for the three types of problems (see Appendix). Subsequently, with the use of contrasts, we found that the effect of creativity was similar for the routine and non-routine visual arts-geometry problems, but significantly different for the multiple solution problems tested with the GMST ($\chi^2(1) = 0.68$, $p = .41$). The final multivariate multilevel model can be found in Table 4.

The standardized results show that creativity was a significant predictor of all types of problems; students who scored higher on creativity, also performed better on the three types of mathematical problems. However, creativity was a significantly stronger predictor of students' performance on the multiple solution problems than on the non-routine visual arts-geometry and routine geometry problems.

With regard to the covariates, we found that VSWM was only a significant predictor of performance for the routine problems and non-routine visual arts-geometry problems, but not for the multiple solution problems. Students with a high VSWM performed better on the routine problems and non-routine visual arts-geometry problems. Furthermore, age and general mathematical ability were significant predictors of all three problems types; older students and students with a higher general mathematical ability performed better on the three problems than younger students and students with lower mathematical ability. Gender was only a significant predictor for the non-routine visual arts-geometry problems and multiple solution problems, but not for the routine problems; girls performed better on visual arts-geometry problems and multiple solution problems than boys. Also, low SES was a significant predictor of performance on the three types of problems; students with parents with a lower educational level performed worse on the three types of problems. Teachers' experience was not related to students' performance on none of the problems. Furthermore, the textbooks *WiG*, *Pluspunt* and *Rekenrijk* did not significantly predict performance on the three types of problems, compared to the reference category. However, the mathematical textbook *Alles Telt* was a significant negative predictor of performance on all three types of problems: students in classes that used this mathematical textbook performed worse on all types of problems compared to the reference category and the other text books.

3.4. Discussion

The present study investigated the relations between students' independently assessed domain-general creativity and their performance on closed routine geometrical problems, non-routine visual arts-geometry problems and geometrical multiple solution problems. The aim of the study was to determine whether the relation between creativity and performance differed between the three problem types. Visual-spatial working memory (VSWM), gender, age, low socio-economic status (SES) and general mathematical ability, teachers' experience and the type of mathematical textbooks were included as covariates

The main results of this study show that, although independently measured domain-general creativity was a significant positive predictor of students' performance on all three problem types, it was a significantly stronger predictor of students' performance on multiple

Table 4

Standardized fixed effects on the routine, art-geometry and multiple solution problems

	Routine geometry problems		Non-routine art-geometry problems		Geometrical multiple solution problems	
	B [95% CI]	*SE*	*B* [95% CI]	*SE*	*B* [95% CI]	*SE*
Intercept	-.01 [-.09, .06]	.04	-.00 [-.08, .08]	.04	.00 [-.07, .08]	.04
Level 2 (student)						
VSWM	.13 [.07, .20]**	.03	.09 [.04, .15]*	.03	-.03 [-.08, .02]	.03
Age	.18 [.14, .22]**	.02	.18 [.14, .22]**	.02	.18 [.14, .22]**	.02
Gender	.02 [-.04, .08]	.03	.16 [.10, .21]**	.03	.11 [.06-.16]**	.02
Low SES	-.07 [-.10, -.04]**	.02	-.07 [-.10, -.04]**	.02	-.07 [-.10, -.04]**	.02
General math ability	.37 [.31, .44]**	.03	.26 [.19, .33]**	.04	.21 [.14, .29]**	.04
Creativity	.09 [.06, .13]**	.02	.09 [.06, .13]**	.02	.17 [.12, .22]**	.03
Level 3 (class)						
Teacher experience	.02 [-.04, .08]	.03	.02 [-.04, .08]	.03	.02 [-.04, .08]	.03
Textbook_WiG	-.05 [-.13, .04]	.04	-.05 [-.13, .04]	.04	-.05 [-.13, .04]	.04
Textbook_Alles Telt	-.07 [-.13, -.02]*	.03	-.07 [-.13, -.02]*	.03	-.07 [-.13, -.02]*	.03
Textbook_Pluspunt	-.02 [-.10, .06]	.04	-.02 [-.10, .06]	.04	-.02 [-.10, .06]	.04
Textbook_Rekenrijk	.02 [-.06, .11]	.03	.02 [-.06, .11]	.03	.02 [-.06, .11]	.03

Note. Robust standard errors are reported for all problems. * Significant at $p < .01$, ** Significant at $p < .001$

solution problems. The basic assumption of the current study was that the association between domain-general creativity and mathematical performance indicates the extent to which solving particular mathematical problems calls upon general creative thinking processes. If this assumption is justified, the current results imply that multiple solution problems, compared to other types of problems, trigger creative thinking of students most and, for that reason, can be used in mathematics education to foster higher-order creative thinking as a key component of functional mathematical competence. However, before concluding this, alternative explanations need to be considered.

One alternative explanation is that not so much creativity but general intelligence explains the pattern of findings, as creativity measures are often found to be moderately to strongly related to measures of general intelligence (e.g., Silvia, 2008). Note, however, that in the present study a measure of students' visual-spatial working memory was included as a control covariate and that the associations of creativity with mathematical performance we found were net of the shared variance with working memory. Research has shown that working memory underlies the correlation between creativity and intelligence (Benedek, Jauk, Sommer, Arendasy, & Neubauer, 2014). Other studies have shown that working memory is a stronger predictor of mathematical achievement in elementary school than intelligence (e.g., Alloway & Alloway, 2010). Therefore, explaining the pattern of findings as an effect of intelligence instead of indicating involvement of creativity seems less plausible. Also, the inclusion of other covariates as control measures, in particular general mathematical ability, render an interpretation of the observed differential association of creativity with performance on the three problem types less likely: the involvement of general cognitive processes and mathematical knowledge was adequately controlled. A specific interpretation, therefore, is more likely: especially multiple solution problems call upon creative thinking.

Other findings confirm this pattern and suggest that different geometrical problem types involve different cognitive processes and problem solving strategies. VSWM and mathematical ability were relatively strong predictors of performance on both routine and non-routine problems, but not of performance on multiple solution problems. A possible explanation is that both the routine and non-routine type of problems essentially require a single solution and are, in that sense, more closed than multiple solution problems, possibly triggering the use of strategies in students such as the retrieval of knowledge of facts and procedures, and activating memory of previous experiences with similar problems to find the correct answer or procedure.

The finding that creativity was the strongest predictor of performance on the multiple solution problems was in line with our hypotheses, because this type of problem was not familiar to the students and most open. Openness of a problem was expected to trigger creative thinking as it stimulates students to find answers beyond what is already known and readily retrievable from memory (Leikin, 2018). The finding is in line with previous research that also showed that creativity predicted performance on mathematical multiple solution tasks (Chapter 2; Kroesbergen & Schoevers, 2017). Contrary to our hypotheses, the association of creativity with performance did not differ between routine and non-routine visual arts-geometry problems. We expected that the relative unfamiliarity of non-routine visual arts-geometry problems compared to routine-problems would require more creative thinking and, thus, would show a stronger association between creativity and performance, which was not the case. A possible explanation is, as was already mentioned above, that both types of problems trigger knowledge-retrieval strategies rather than construction of new ideas and procedures. The question-type, closed versus open, then, would be a more decisive feature with regard to creativity than the relative familiarity of the problem. Although we classified the visual arts-geometry problems used in this study as non-routine, students may not have experienced the problems as such and may still have known how to solve the problems by applying familiar procedures (Schoenfeld, 1983; Schoenfeld, 2013).

Although creativity was a stronger predictor of performance on the multiple solution problems, creativity significantly predicted performance on all three types of problems: more creative students were better in solving geometrical problems whatever the type. This finding is compatible with the study of Kroesbergen and Schoevers (2017), which also found that creativity predicted performance on routine mathematical word problems and on a mathematical multiple solution task. Two explanations seem possible. Despite the inclusion of VSWM and mathematical ability as controlling covariates, our measure of creativity may still have shared variance with general intelligence (Silvia, 2008) and other generic abilities such as verbal skills, motivation and work attitude, which were not included as covariates. The overall association of creativity with problem solving performance regardless the type of problem may indicate that in addition to type-specific cognitive processes (memory retrieval, creative-divergent thinking), also general cognitive processes, abilities and attitudes were involved.

Regarding the other covariates, we found no relations of teachers' work experience, three of the four mathematical textbooks (*WiG, Pluspunt* and *Rekenrijk*), and the rest category of other textbooks and no textbook with students' performance on all three problem types. Based on the studies of Van Zanten et al. (2018) and Kolovou et al. (2009), the results for the textbooks were not surprising. In a content analysis conducted by these authors, the textbook series *WiG, Alles Telt, Pluspunt* and *Rekenrijk* were found to contain ample routine problems and a similar, though very small proportion of non-routine problems. Students may have similar experiences with problem solving, and, therefore, the types of textbooks used in classes did not differentially affect performance on the different types of problems. However, we found a negative relation for the textbook *Alles Telt* with students' performance on geometrical problems. A likely explanation for this result may be that, in this textbook, students were provided with even much less opportunities to learn geometry, which negatively affected their performance on all three types of geometrical problems.

Furthermore, we found that age and low SES were similarly related to students' performance on the three types of geometry problems; older students performed better than younger students and students from a low SES background performed worse on the geometry problems than students with a medium or high SES. Girls performed better on non-routine visual arts-geometry and multiple solution problems than boys, but gender was not significantly related to performance on the routine problems. These findings fit in well with the extant research literature.

3.4.1. Limitations

The present study has several limitations. In interpreting the results, it should be kept in mind that the study examined the role of creativity in only three types of geometrical problems. There are other types of open and non-routine geometrical problems (Leikin, 2018), which were not included in this study. Moreover, geometry is only one of the several domains constituting the discipline of mathematics in elementary school. The present findings cannot be generalized to these other types of problems or mathematical domains. Future research could extend the present approach to other types of problems and other mathematical domains. Nevertheless, to our knowledge this is the first study that investigated whether the predictive value of creativity differed between performance on different types of mathematical problems.

Another limitation relates to our measure of creativity. The TCT-DP provided only a limited assessment of students' creativity, with an emphasis on the aspect 'explore and dig deeper into ideas', and did not fully capture the complex multidimensional nature of creativity as it is defined nowadays (Chapter 2). Note, however, that this aspect covers both divergent and convergent thinking processes (Chapter 2) and can, for that reason, be regarded as best single indicator of students' domain-general creativity. Nonetheless, it is recommendable to take multiple measures of creativity into account in future research.

Although we controlled for several relevant covariates in this study, it cannot be ruled out that other characteristics, such as students' attitudes towards new types of problems, students' intelligence or metacognition, are also important factors that can, at least partly, explain the relations we found between creativity and geometrical problem solving performance (Carlson & Bloom, 2005; Elia, van den Heuvel-Panhuizen, & Kolovou, 2009). Unfortunately, measures of these characteristics were not available in the current data set. Future research could include a wider set of covariates when addressing the role of creativity in mathematical problem solving.

Finally, it should be kept in mind that the current study was cross-sectional. Consequently, no firm conclusions can be drawn regarding the causal direction of the relations that were observed between creativity and problem solving performance. Longitudinal and experimental research is needed to strengthen the evidence base regarding the role of creativity in mathematical problem solving. Nonetheless, the present study provides important first ideas on the possible involvement creativity in geometrical problem solving as related to the type of problem that may inspire new research and curriculum development.

3.4.2. Conclusion and implications

Students' domain-general creativity was positively associated with their performance on three different types of geometry problems. However, students' creativity was significantly stronger associated with performance on multiple solution problems than with performance on routine and non-routine visual arts-geometry problems. As several covariates were included to control for spuriousness, we may cautiously conclude that the remaining effects of creativity reflect true involvement of creative thinking processes in geometrical problem solving, especially in the more open multiple solution problems. Creativity is considered an important skill in current society in general and an essential component of functional mathematical competence in particular. Providing students with appropriate problems in mathematics education is probably an important way to promote creative thinking. Multiple solution problems in geometry, due to their open character, are likely to serve this goal well and should be prominently included in mathematical textbooks.

3.5. Appendix

	Model 1		Model 2		Model 3		Model 4	
Fixed part	*B*	*SE*	*B*	*SE*	*B*	*SE*	*B*	*SE*
Intercept_problem1	0.003	0.038	0.003	0.038	0.003	0.038	0.004	0.038
Intercept_problem2	-0.001	0.040	-0.001	0.041	-0.001	0.040	0.001	0.039
Intercept_problem3	-0.013	0.036	-0.013	0.037	-0.013	0.037	-0.015	0.038
Visual-spatialWM_ problem1	-0.019	0.025	0.064**	0.019	-0.020	0.026	-0.018	0.026
Visual-spatialWM_ problem2	0.090*	0.029	0.064**	0.019	0.088*	0.029	0.094*	0.030
Visual-spatialWM_ problem3	0.126**	0.032	0.064**	0.019	0.127**	0.032	0.121**	0.032
Gender_problem1	0.106**	0.024	0.099**	0.025	0.106**	0.024	0.087**	0.020
Gender_problem2	0.153**	0.028	0.155**	0.028	0.154**	0.028	0.087**	0.020
Gender_problem3	0.023	0.032	0.028	0.031	0.021	0.032	0.087**	0.020
General math ability_problem1	0.213**	0.038	0.190**	0.040	0.214**	0.038	0.210**	0.038
General math ability_problem2	0.246**	0.036	0.253**	0.034	0.249**	0.036	0.234**	0.036
General math ability_problem3	0.380**	0.031	0.397**	0.030	0.376**	0.031	0.390**	0.031
Creativity_problem1	0.172**	0.027	0.171**	0.027	0.172**	0.027	0.173**	0.027
Creativity_problem2	0.106**	0.025	0.106**	0.025	0.105**	0.026	0.110**	0.026
Creativity_problem3	0.081**	0.022	0.081**	0.022	0.082**	0.022	0.078**	0.022
Age_problem1	0.173**	0.027	0.167**	0.028	0.183**	0.020	0.183**	0.020
Age_problem2	0.157**	0.033	0.158**	0.033	0.183**	0.020	0.183**	0.020
Age_problem3	0.211**	0.032	0.214**	0.032	0.183**	0.020	0.183**	0.020
LowSES_problem1	-0.037	0.024	-0.029	0.023	-0.038	0.024	-0.037	0.024
LowSES_problem2	-0.094**	0.018	-0.097**	0.018	-0.096**	0.018	-0.094**	0.018
LowSES_problem3	-0.081**	0.022	-0.087**	0.023	-0.080**	0.022	-0.081**	0.023
Teachers' experience_problem1	-0.005	0.041	-0.007	0.040	-0.005	0.041	-0.006	0.041
Teachers' experience_problem2	-0.028	0.050	-0.026	0.051	-0.029	0.050	-0.033	0.049
Teachers' experience_problem3	0.064	0.040	0.068	0.042	0.066	0.041	0.070	0.042
WiG_problem1	-0.015	0.055	-0.020	0.056	-0.016	0.055	-0.017	0.054
WiG_problem2	-0.011	0.072	-0.010	0.073	-0.014	0.072	-0.017	0.070
WiG_problem3	-0.103	0.054	-0.100	0.057	-0.100	0.054	-0.097	0.056
Alles Telt_problem1	-0.108*	0.039	-0.116*	0.040	-0.109*	0.039	-0.110*	0.038
Alles Telt_problem2	-0.003	0.052	-0.001	0.052	-0.003	0.053	-0.005	0.052
Alles Telt_problem3	-0.068	0.039	-0.063	0.040	-0.067	0.038	-0.065	0.039
Pluspunt_problem1	0.008	0.047	0.009	0.049	0.008	0.047	0.006	0.047
Pluspunt_problem2	-0.027	0.068	-0.029	0.068	-0.027	0.068	-0.034	0.065
Pluspunt_problem3	-0.048	0.055	-0.051	0.057	-0.049	0.054	-0.043	0.056
Rekenrijk_problem1	0.084	0.063	0.086	0.063	0.084	0.062	0.084	0.062
Rekenrijk_problem2	0.042	0.064	0.041	0.065	0.043	0.064	0.042	0.062
Rekenrijk_problem3	-0.054	0.052	-0.056	0.054	-0.056	0.052	-0.055	0.053
Random part class level								
$\sigma^2_{Constant\ problem1}$	0.056		0.056		0.055		0.055	
$r_{Problem1\_problem2}$	0.017		0.013		0.015		0.015	
$\sigma^2_{Constant\ problem2}$	0.074		0.075		0.071		0.068	
$r_{Problem3\_problem1}$	0.006		0.004		0.006		0.006	
$r_{Problem3\_problem2}$	0.032		0.035		0.031		0.031	
$\sigma^2_{Constant\ problem3}$	0.055		0.059		0.059		0.062	
Random part student level								
$\sigma^2_{Constant\ problem1}$	0.803		0.808		0.803		0.804	
$r_{Problem1\_problem2}$	0.126		0.125		0.127		0.129	
$\sigma^2_{Constant\ problem2}$	0.756		0.755		0.757		0.762	
$r_{Problem3\_problem1}$	0.023		0.020		0.023		0.022	
$r_{Problem3\_problem2}$	0.166		0.166		0.166		0.161	
$\sigma^2_{Constant\ problem3}$	0.689		0.691		0.688		0.691	
Deviance	10228.240		10.244.234		10.230.466		10.249.602	

Note. Problem 1 is the GMST, problem 2 in the non-routine art-geometry problems, and problem 3 is the routine problem. *p<.05., **p<.001

	Model 5		Model 6		Model 7		Model 8	
Fixed part	*B*	*SE*	*B*	*SE*	*B*	*SE*	*B*	*SE*
Intercept_problem1	0.004	0.037	0.004	0.038	0.003	0.038	0.004	0.038
Intercept_problem2	-0.001	0.040	-0.001	0.041	-0.000	0.040	-0.003	0.041
Intercept_problem3	-0.014	0.037	-0.013	0.037	-0.012	0.037	-0.012	0.037
Visual-spatialWM_ problem1	-0.023	0.025	-0.043	0.026	-0.022	0.025	-0.024	0.025
Visual-spatialWM_ problem2	0.091*	0.029	0.083*	0.029	0.091*	0.029	0.090*	0.028
Visual-spatialWM_ problem3	0.128**	0.033	0.154**	0.032	0.128**	0.032	0.130**	0.033
Gender_problem1	0.107**	0.024	0.120**	0.024	0.110**	0.024	0.108**	0.024
Gender_problem2	0.154**	0.028	0.160**	0.029	0.153**	0.028	0.155**	0.028
Gender_problem3	0.021	0.032	0.004	0.032	0.019	0.032	0.019	0.032
General math ability_problem1	0.212**	0.038	0.285**	0.027	0.217**	0.039	0.213**	0.038
General math ability_problem2	0.251**	0.036	0.285**	0.027	0.249**	0.036	0.253**	0.035
General math ability_problem3	0.377**	0.031	0.285**	0.027	0.373**	0.031	0.375**	0.031
Creativity_problem1	0.170**	0.027	0.163**	0.027	0.120**	0.016	0.171**	0.027
Creativity_problem2	0.106**	0.026	0.102**	0.026	0.120**	0.016	0.108**	0.025
Creativity_problem3	0.083**	0.022	0.092**	0.023	0.120**	0.016	0.080**	0.022
Age_problem1	0.183**	0.020	0.184**	0.020	0.183**	0.020	0.183**	0.020
Age_problem2	0.183**	0.020	0.184**	0.020	0.183**	0.020	0.183**	0.020
Age_problem3	0.183**	0.020	0.184**	0.020	0.183**	0.020	0.183**	0.020
LowSES_problem1	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
LowSES_problem2	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
LowSES_problem3	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
Teachers' experience_problem1	-0.005	0.040	-0.000	0.041	-0.011	0.041	0.017	0.031
Teachers' experience_problem2	-0.029	0.050	-0.028	0.052	-0.028	0.050	0.017	0.031
Teachers' experience_problem3	0.066	0.041	0.059	0.038	0.069	0.040	0.017	0.031
WiG_problem1	-0.014	0.055	-0.019	0.057	-0.015	0.056	-0.016	0.055
WiG_problem2	-0.016	0.072	-0.018	0.075	-0.015	0.072	-0.020	0.069
WiG_problem3	-0.101	0.055	-0.095	0.051	-0.100	0.053	-0.096	0.057
Alles Telt_problem1	-0.111*	0.039	-0.117*	0.040	-0.113*	0.040	-0.111*	0.039
Alles Telt_problem2	-0.002	0.053	-0.005	0.055	-0.001	0.052	-0.003	0.051
Alles Telt_problem3	-0.066	0.038	-0.055	0.037	-0.065	0.038	-0.064	0.041
Pluspunt_problem1	0.007	0.048	0.001	0.050	0.010	0.049	0.006	0.048
Pluspunt_problem2	-0.027	0.068	-0.030	0.071	-0.027	0.068	-0.030	0.065
Pluspunt_problem3	-0.049	0.054	-0.040	0.050	-0.050	0.054	-0.045	0.056
Rekenrijk_problem1	0.082	0.063	0.073	0.063	0.082	0.063	0.082	0.063
Rekenrijk_problem2	0.046	0.064	0.042	0.066	0.046	0.063	0.047	0.064
Rekenrijk_problem3	-0.055	0.052	-0.044	0.051	-0.056	0.052	-0.057	0.053
Random part class level								
$\sigma^2_{Constant\ problem1}$	0.055		0.056		0.057		0.055	
$r_{Problem1\_problem2}$	0.015		0.019		0.017		0.016	
$\sigma^2_{Constant\ problem2}$	0.073		0.076		0.071		0.075	
$r_{Problem3\_problem1}$	0.006		0.006		0.006		0.005	
$r_{Problem3\_problem2}$	0.032		0.027		0.029		0.029	
$\sigma^2_{Constant\ problem3}$	0.059		0.057		0.058		0.061	
Random part student level								
$\sigma^2_{Constant\ problem1}$	0.805		0.809		0.806		0.805	
$r_{Problem1\_problem2}$	0.126		0.127		0.125		0.126	
$\sigma^2_{Constant\ problem2}$	0.757		0.756		0.758		0.757	
$r_{Problem3\_problem1}$	0.023		0.018		0.021		0.023	
$r_{Problem3\_problem2}$	0.166		0.165		0.167		0.166	
$\sigma^2_{Constant\ problem3}$	0.688		0.695		0.690		0.688	
Deviance	10.233.808		10.256.639		10.239.963		10.238.614	

Note. Problem 1 is the GMST, problem 2 in the non-routine art-geometry problems, and problem 3 is the routine problem. *p<.05., **p<.001

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	Model 9		Model 10		Model 11		Model 12	
Fixed part	*B*	*SE*	*B*	*SE*	*B*	*SE*	*B*	*SE*
Intercept_problem1	0.005	0.038	0.004	0.038	0.003	0.038	0.001	0.038
Intercept_problem2	-0.002	0.041	-0.003	0.041	-0.000	0.041	-0.001	0.041
Intercept_problem3	-0.014	0.037	-0.014	0.038	-0.014	0.038	-0.012	0.038
Visual-spatialWM_ problem1	-0.023	0.025	-0.024	0.025	-0.025	0.025	-0.026	0.026
Visual-spatialWM_ problem2	0.091*	0.028	0.091*	0.028	0.093*	0.028	0.092*	0.028
Visual-spatialWM_ problem3	0.129**	0.033	0.129**	0.033	0.129**	0.033	0.130**	0.033
Gender_problem1	0.107**	0.024	0.107**	0.024	0.107**	0.024	0.107**	0.024
Gender_problem2	0.155**	0.028	0.155**	0.028	0.156**	0.028	0.156**	0.028
Gender_problem3	0.020	0.032	0.020	0.032	0.020	0.032	0.020	0.032
General math ability_problem1	0.213**	0.038	0.210**	0.038	0.211**	0.038	0.212**	0.037
General math ability_problem2	0.253**	0.035	0.256**	0.035	0.256**	0.035	0.256**	0.035
General math ability_problem3	0.374**	0.032	0.375**	0.032	0.375**	0.032	0.373**	0.032
Creativity_problem1	0.171**	0.027	0.172**	0.027	0.173**	0.027	0.174**	0.027
Creativity_problem2	0.107**	0.025	0.107**	0.025	0.105**	0.025	0.105**	0.025
Creativity_problem3	0.081**	0.022	0.081**	0.022	0.081**	0.022	0.081**	0.022
Age_problem1	0.183**	0.020	0.183**	0.020	0.183**	0.020	0.183**	0.020
Age_problem2	0.183**	0.020	0.183**	0.020	0.183**	0.020	0.183**	0.020
Age_problem3	0.183**	0.020	0.183**	0.020	0.183**	0.020	0.183**	0.020
LowSES_problem1	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
LowSES_problem2	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
LowSES_problem3	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015	-0.069**	0.015
Teachers' experience_problem1	0.017	0.031	0.017	0.031	0.017	0.031	0.017	0.031
Teachers' experience_problem2	0.017	0.031	0.017	0.031	0.017	0.031	0.017	0.031
Teachers' experience_problem3	0.017	0.031	0.017	0.031	0.017	0.031	0.017	0.031
WiG_problem1	-0.047	0.044	-0.047	0.044	-0.047	0.044	-0.047	0.044
WiG_problem2	-0.047	0.044	-0.047	0.044	-0.047	0.044	-0.047	0.044
WiG_problem3	-0.047	0.044	-0.047	0.044	-0.047	0.044	-0.047	0.044
Alles Telt_problem1	-0.126**	0.036	-0.073*	0.030	-0.073*	0.030	-0.074*	0.030
Alles Telt_problem2	-0.016	0.043	-0.073*	0.030	-0.073*	0.030	-0.074*	0.030
Alles Telt_problem3	-0.041	0.037	-0.073*	0.030	-0.073*	0.030	-0.074*	0.030
Pluspunt_problem1	-0.013	0.044	0.001	0.043	-0.020	0.040	-0.020	0.040
Pluspunt_problem2	-0.046	0.055	-0.060	0.054	-0.020	0.040	-0.020	0.040
Pluspunt_problem3	-0.016	0.051	-0.023	0.051	-0.020	0.040	-0.020	0.040
Rekenrijk_problem1	0.066	0.061	0.078	0.060	0.073	0.060	0.022	0.042
Rekenrijk_problem2	0.033	0.057	0.021	0.056	0.030	0.056	0.022	0.042
Rekenrijk_problem3	-0.031	0.050	-0.038	0.049	-0.037	0.049	0.022	0.042
Random part class level								
$\sigma^2_{\text{Constant problem1}}$	0.056		0.058		0.059		0.060	
$r_{\text{Problem1\_problem2}}$	0.016		0.014		0.013		0.013	
$\sigma^2_{\text{Constant problem2}}$	0.076		0.078		0.080		0.080	
$r_{\text{Problem3\_problem1}}$	0.004		0.003		0.003		0.000	
$r_{\text{Problem3\_problem2}}$	0.029		0.030		0.030		0.030	
$\sigma^2_{\text{Constant problem3}}$	0.062		0.062		0.062		0.066	
Random part student level								
$\sigma^2_{\text{Constant problem1}}$	0.805		0.805		0.805		0.805	
$r_{\text{Problem1\_problem2}}$	0.126		0.126		0.126		0.126	
$\sigma^2_{\text{Constant problem2}}$	0.757		0.757		0.756		0.757	
$r_{\text{Problem3\_problem1}}$	0.023		0.023		0.023		0.023	
$r_{\text{Problem3\_problem2}}$	0.166		0.166		0.166		0.166	
$\sigma^2_{\text{Constant problem3}}$	0.688		0.688		0.688		0.688	
Deviance	10.240.473		10.245.384		10.246.804		10.251.280	

Note. Problem 1 is the GMST, problem 2 is the non-routine art-geometry problems, and problem 3 is the routine problem. **p*<.05., ***p*<.001

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Chapter 4

Promoting students' creative thinking in elementary school mathematics: A case study

Schoevers, E. M., Leseman, P. P. M., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking Skills and Creativity, 31,* 323–334. doi:10.1016/j.tsc.2019.02.003

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Abstract

The importance of promoting mathematical creativity in education is increasingly acknowledged. Several strategies have been recommended to foster mathematical creativity such as creating an open atmosphere in the classroom, offering open lesson, and possibly enriching mathematics education with ideas from other disciplines and experiences from out-of-school contexts. However, it is not yet clear in what way recommended pedagogical strategies promote students' mathematical creativity. Therefore, the purpose of this case study was to gain in-depth understanding of promoting mathematical creativity in educational practice. To this end, interactions between a teacher and her 22 fourth-grade students in three different types of mathematics lessons were investigated. An 'open' in-school mathematics lesson, an 'open' out-of-school mathematics lesson and a regular ('closed') mathematics lesson were video recorded and interactions were transcribed verbatim. Subsequently, dialogic episodes were identified in the video transcripts and were coded on mathematical creative expressions of students and strategies used by the teacher. Furthermore, after each lesson the teacher was interviewed regarding her experiences with the given lesson. Interviews were audiotaped, transcribed verbatim and analyzed by using constant comparison analyses. Findings indicate that mathematical creativity was only promoted in the two open mathematics lessons. More specifically, mathematical creative expressions were related to longer whole class dialogues in which the teacher created an open atmosphere; she created opportunities for students to express their ideas and took these ideas seriously. Although in some episodes of the regular mathematics lesson this open atmosphere was also created, no mathematical creativity occurred.

4.1. Introduction

Promoting creative thinking is widely regarded as an important goal of present-day elementary mathematics education (e.g., Leikin & Pitta-Pantazi, 2013; Leikin & Sriraman, 2017; National Governors Association Center for Best Practices, 2010). To promote mathematical creativity, a supportive pedagogy is considered essential: creating an open atmosphere in the classroom (e.g., Beghetto & Kaufman, 2011), offering open lessons (e.g., Hershkovitz, Peled, & Littler, 2009) and possibly enriching mathematics education with ideas from other disciplines and experiences from out-of-school contexts (e.g. Davies et al., 2013). Although national standards seem to provide opportunities in the mathematics curriculum for such a pedagogy, time constraints seem to limit Dutch teachers, as many other teachers around the world, to use such a creativity-supportive pedagogy (Harris, 2016; Platform Onderwijs 2032, 2016). Dutch teachers are expected to teach the complete mathematics curriculum, and they typically strongly rely on mathematical textbooks to structure their lessons (Gravemeijer, 2007; Harris, 2016; Platform Onderwijs 2032, 2016). These textbooks incorporate the entire mathematics curriculum, but do not provide many opportunities for students to be creative (Gravemeijer, 2007; Van Zanten & Van den Heuvel-Panhuizen, 2018). Currently, it is unclear which strategies, if at all, teachers use in elementary mathematics education to promote mathematical creativity, and how these strategies are related to students' emerging mathematical creativity. More insight into these processes can contribute to a theory on the promotion of mathematical creativity in educational practice. This article describes a case study of a teacher and her 4th grade class in Dutch elementary education who applied a new pedagogy for promoting mathematical creativity in her classroom. The aim of the case study was to obtain an in-depth understanding of how students' mathematical creativity can be promoted in educational practice and, more specifically, to uncover how different strategies may stimulate mathematical creativity in students.

4.1.1. Defining mathematical creativity

Mathematical creativity is commonly defined as the creation of something new and meaningful by breaking away from established mindsets (Haylock, 1987b; Runco & Jaeger, 2012; Sriraman, 2005). In defining mathematical creativity, a distinction is often made between student level and professional level creativity (Kaufman & Beghetto, 2009; Sriraman, 2005). Sriraman (2005) has provided a working definition of school-level mathematical creativity in which mathematical creativity is related to problem solving and problem posing. He defined mathematical creativity as:

> ''(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination'' (p. 24).

This definition seems to limit mathematical creativity to the process of mathematical problem posing and problem solving. However, within a classroom context, mathematical creativity might also occur in other ways (e.g., in whole-class dialogues about a mathematical subject); this can be the creation of a new and meaningful mathematical idea or conception, which is not necessarily a solution or a problem. Therefore, we considered the definition of Sriraman (2005) to be incomplete for studying mathematical creativity in educational practice in a broader sense. In this study, we propose an adapted view on mathematical creativity, in which mathematical creativity (also) refers to the cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student's (correct) understanding of mathematics (cf. Ervynck, 1991; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). For example, a student may have a limited understanding of shapes (e.g., a triangle has three angles, a circle is round) and not relate this to his or her understanding of infinity. We consider it an instance of creativity in the subject mathematics, when the student combines the two aforementioned concepts into a new and deeper conceptualization of 'shape', in which a circle is conceived as a polygon and thus as similar to the triangle, but with infinitely many sides of infinitely small magnitude. Although this is not a new idea in mathematics, it is new (and useful) for the student and deepens his or her mathematical understanding.

4.1.2. Promoting mathematical creativity

To promote mathematical creativity a pedagogical environment is needed characterized by an open atmosphere in which students have the opportunity to develop new mathematical concepts in interaction with others (Colucci-Gray et al., 2017; Kaufman, Beghetto, Baer, & Ivcevic, 2010; Leikin & Dinur, 2007; Sawyer, 2014; Soh, 2000). Teachers are advised to guide students by asking them suitable (open) questions and by giving them opportunities to reflect on mathematical ideas (Bostic, 2011; Hong & Milgram, 2008; Levenson, 2011; Shen, 2014). Furthermore, it is considered vital that teachers are open and flexible with regard to students' responses (Beghetto & Kaufman, 2011; Davies et al., 2013; Hershkovitz, Peled, & Littler, 2009; Leikin & Dinur, 2007), encourage students to share (multiple) ideas, and stimulate students to collaborate (Kaufman et al., 2010; Soh, 2000; Taggar, 2002). In this way, multiple perspectives can be discussed which could help students to step back from fixed cognitive schemas (Bakker, Smit, & Wegerif, 2015). Moreover, it is important that teachers stimulate students' intrinsic motivation; teachers should create opportunities for choice and discovery, and should not use extrinsic motivators (Beghetto & Kaufman, 2014; Beghetto & Kaufman, 2011; Davies et al., 2013; Schacter, Thum, & Zifkin, 2006).

In addition to the teachers' strategies to create an open atmosphere in the classroom, it is advised that teachers offer 'open' mathematics lessons to stimulate students' mathematical creativity (Beghetto & Kaufman, 2011; Davies et al., 2013; Hershkovitz et al., 2009). A lesson is open if it invites different solutions, methods of solution or are open for interpretation (Silver, 1995). Creating new and useful mathematical concepts requires a certain degree of freedom in the lesson. This means that the lessons should enable students to search, explore, make conjectures, hypothesize, examine, refute, adapt strategies, devise plans, conclude, reason and/or justify their conclusions and/or reflect on them (Nadjafikhah et al., 2012). Lessons that do not enable these features and require students to solely apply rules and arithmetical procedures (i.e., ‘closed’ lessons), are expected to restrain students’ mathematical creativity. Regular mathematical teaching practices and textbooks mainly contain this closed type of learning lessons (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009)

Furthermore, to promote mathematical creativity it may be important that a teacher enriches mathematics education by making connections with subjects other than mathematics or with contexts outside school (Colucci-Gray et al., 2017). More generally, this phenomenon can be characterized as boundary crossing (Akkerman & Bakker, 2011a). To create something new and meaningful in mathematics, it is important to break away from established mindsets (e.g., Haylock, 1987a; 1987b). It could be expected that by crossing disciplinary boundaries it is easier for students to break away from established mindsets and to think and act in a mathematically creative way. Integrating different conceptual systems, for example from the disciplines of visual arts and mathematics, could activate students to combine familiar concepts in new ways. This principle might also apply to the location of the mathematics lesson. Teaching mathematics in an outdoor environment might activate different conceptual systems, which could stimulate students to combine concepts into new, enriched or deeper mathematical understanding or to discover unknown relations between mathematical concepts (Bancroft, Fawcett, & Hay, 2009; Davies et al., 2013).

4.1.3. The aim of this study

To conclude, several pedagogical strategies seem to be important for promoting mathematical creativity: strategies for creating an open environment, offering open mathematics lessons, and enrichment of mathematics education with ideas from other disciplines and out-of-school experiences. It is yet unclear how these strategies can be implemented in the classroom and how they relate to students’ mathematical creativity as expressed in classroom dialogues (Kazak, Wegerif, & Fujita, 2015). To address these questions, we conducted a case study of a teacher and her class. To be able to distinguish the role of creativity promoting strategies, we asked the teacher to conduct three different mathematics lessons: a regular mathematics lesson following the textbook, an open in-school interdisciplinary lesson enriched with visual arts reception and production, and an open out-of-school interdisciplinary lesson in which artworks in the neighborhood were discussed. Subsequently, classroom dialogues were recorded, transcribed and analyzed to answer the following research questions:

- Which strategies for promoting mathematical creativity in students’ thinking are used by the teacher in the three lessons and do lessons differ in this regard?
- How are the mathematical creativity promoting strategies of the teacher related to students’ creativity as expressed in classroom dialogues?

4.2. Method

A single case study (Creswell, 2007) was conducted in the context of the Mathematics Arts Creativity in Education (MACE) program.

4.2.1 Research context

The MACE program. The MACE program aims to increase students' creative problem-solving skills in geometry and visual arts, and to increase students' geometrical ability in the upper grades of elementary school. To achieve these goals, a teaching sequence for students in Grades 4–6 (aged 9–12) was designed in which geometry and visual arts education were integrated. The teaching sequence consisted of nine lessons, which were related to the theme 'space' and 'patterns' and took about 60–90 minutes to complete (Meetkunst projectteam, 2018b). To support implementation of the teaching sequence, a professional development (PD) program for teachers was developed (Keijzer, Oprins, De Moor, & Schoevers, 2018; Meetkunst projectteam, 2018c). The PD program consisted of five sessions (2.5 hours each). After each PD session teachers had to teach one or two lessons of the MACE teaching sequence in their own schools. Lessons 2 and 5 of this teaching sequence were observed in the current study, in addition to a regular lesson. The content of the PD sessions was focused on how to teach an integrated visual arts and mathematics curriculum and how to promote students' creative thinking within visual arts and mathematics education. In these sessions exchanging experiences with other teachers, reflection and discussion of the background theory and didactics related to MACE was important. Furthermore, (hands-on) activities were carried out in which the teachers could practice with and experience activities from the teaching sequence. An overview of the MACE program can be found in Figure 1.

School and class context. The school that participated in the current study was situated near Amsterdam, the Netherlands. The school and 12 teachers participated in the MACE program because they had the aim to improve their mathematics education. The aims of the MACE program were in line with the vision of the school.

A female teacher, Anna (pseudonym), and her fourth-grade class (9- to 10-year-olds) participated in this study. Her class consisted of 22 students, all from a middle to high socioeconomic background. On average, these students had above average mathematical achievement on standard tests. The teacher used a mathematical textbook for the regular mathematics education, which was inspired by the concept of Realistic Mathematics Education. Other mathematical teaching materials were used to supplement the textbook.
Anna had 18 years of experience as a teacher in the upper grades of elementary school. She was very open and enthusiastic about the MACE program. Her motivation to take part in this program was to learn how to make geometry more meaningful to her students. She believed that, for many students, this domain of mathematics was rather difficult to grasp. We consider Anna to be representative for the other eleven teachers in the program.

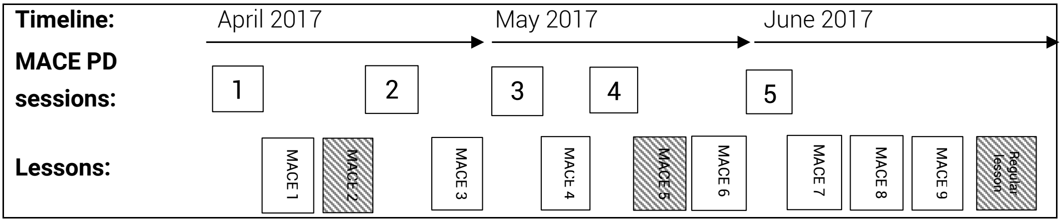

Figure 1. Timeline of the data collection and overview of the MACE program. The grey lessons are observed for this study.

4.2.2. Data collection

Data were collected from April until June 2017. The third author video-recorded the three selected instruction lessons: a regular lesson, an open in-school arts and mathematics lesson, and an open partially out-of-school arts and mathematics lesson. After the first PD session, the out-of-school MACE lesson was conducted. Six weeks later, the in-school MACE lesson was conducted. At that time, the teacher had attended four PD sessions. Three weeks after the in-school MACE lesson, the regular mathematics lesson was conducted. At that time the PD trajectory was already finished. To record the lessons without disturbing the classroom processes the researcher positioned herself in the back of the classroom using a video camera mounted on a tripod. The video-recordings were transcribed verbatim to enable coding of the data on a creativity promoting dialogues.

Furthermore, after each lesson, the teacher was interviewed. The interviews always started with an open question about her experiences with the lesson and took 10–15 minutes. Subsequently, questions were asked to invite the teacher to elaborate on issues that arose. The three interviews were audio-recorded and also transcribed verbatim. Findings from the interviews were solely used to interpret and corroborate the findings of the analyses of the classroom dialogues.

4.2.3. Establishing trustworthiness

The main researcher (first author) was involved in the design of the MACE program. To decrease possible bias and to assure the credibility of the findings, a second coder (i.e., third author) was involved in analyzing the data as described in the former paragraph. Similar to the first author, she was trained as an education researcher, but she was not involved in the MACE program. To assure transparency, a log was kept during the collection and analyses of the data to keep track of the decisions made.

Within this research we tried to act in an ethical way by trying to keep the burden low for the teacher. She participated voluntarily and did not have any problems with us videotaping three lessons and with having a short interview after each lesson. We always discussed whether time and date of the video-observations and interviews were convenient for the teacher. We also asked and received permission of the teacher to use her quotes in this article. For the whole MACE project ethical approval was obtained from the ethics committee of the Faculty of Social and Behavioural Sciences of Utrecht University (FETC15-083).

4.2.4. Data analysis

Analysis of the video transcripts. To examine how pedagogical actions can influence students' mathematical creativity, the video transcripts were analyzed by the first author. A first step in the analysis was to identify dialogic teaching episodes in which mathematical creativity could occur, to ensure that the co-occurrence of particular teaching strategies and expressions of creativity could be meaningfully interpreted. Episodes were identified based on the definition of Muhonen, Rasku-Puttonen, Pakarinen, Poikkeus and Lerkkanen (2016):

> A dialogic teaching episode was identified as an extended exchange in which the topic continued essentially unchanged between the teacher and child or between children and which manifested three of the five principles of dialogic teaching described by Alexander (2006): purposefulness (teachers plan and steer classroom talk with specific educational goals in mind), collectiveness (teachers and children address learning tasks together as a small group or as a the whole classroom) and reciprocity (teachers and children listen to each other, share ideas and consider alternative viewpoints; p. 146)

In total, 35 episodes were identified over the three lessons, of which 14 concerned the out-of-school lesson (of which 10 took place in the classroom), 14 the in-school lesson and 7 the regular mathematics lesson. Regarding the out-of-school lesson we noted that a large part of the dialogues that took place outside the school were not considered as dialogic teaching episodes since often no collectiveness or purposefulness was involved (Muhonen et al., 2016). In these cases, the teacher walked around while the students were working on their assignment. The teacher occasionally said something to students, but this often concerned one student or only a very short exchange.

A second step in analyzing the video transcripts was to identify dialogic episodes in which students expressed mathematical creativity. An operational definition of the code for expressing mathematical creativity is presented in Table 1, based on our conceptual definition in the introduction. For each episode, we coded if students expressed mathematical creativity or not, without taking into account how often mathematical creativity was expressed during an episode.

A third step in the analysis was to identify the strategies used by the teacher to promote mathematical creativity. This part of the analysis was applied to all observed dialogic episodes to be able to evaluate which strategies used by the teacher co-occurred with expressions of mathematical creativity by the students, and which did not. Based on the literature discussed in the introduction about teachers' strategies to promote mathematical creativity, a codebook was established (see Table 1). For each dialogic episode, codes were assigned indicating per strategy if the teacher used that particular strategy to promote mathematical creativity, resulting in multiple codes per episode. Furthermore, the number of utterances per episode was counted. In Appendix A an example is given of how a dialogic episode was coded. Appendix B gives an overview of all codes in all episodes. As the last step of our analyses, we examined to what degree particular strategies co-occurred, focusing again on all episodes.

Furthermore, we calculated the number of episodes in which the strategies from Table 1 were used, broken down by type of lesson and the occurrence of mathematical creativity in students' expressions.

Table 1.

Operationalization of the codes

1. Students' expressed mathematical creativity	1_mathematical creativity	During the episode, an expression of a student was considered as mathematically creative if an idea, solution or problem was expressed in which two or more concepts were integrated, if the idea, solution or problem could be considered new to the student, and if at least one of the integrated concepts was related to mathematics. A student's mathematical creative utterance could occur in a single utterance or in a short interaction sequence with the teacher or a peer.
2.Teaching for creativity	2A_ask students to connect concepts	During the episode, the teacher asked students to connect a mathematical concept to another concept. It was only coded as teaching for creativity when it could be assumed that the students used to consider these concepts as unrelated (e.g., The teacher asked students to look for shapes in their environment).
	2B_activating questions	During the episode, the teacher asked one or several activating questions that elicited multiple answers, ideas, reasons et cetera (e.g., 'What shapes do you see in your environment?'). The questions started a new (sub)dialogue and were not mere follow-up questions (code 2F).
	2C_support or encourage creative thinking	During the episode, the teacher provided at least one suggestion that supported students' creative thinking (e.g., 'you could try to do X, this may help you') or encouraged students to think creatively (e.g., 'try to think out of the box', 'try to think creatively').
	2D_opportunties express ideas	<u>2D1_students' ideas central</u>: Ideas of students were central in the episode. The teacher aimed to elicit students' ideas about a specific subject and did not predominantly instruct or explain, or give his or her own ideas or opinion. <u>2D2_sub-dialogue allowed:</u> Within the episode, one or more students started a sub-dialogue (with the teacher). The teacher allowed this by asking more about what student is saying. This sub-dialogue was not directly related to a question asked by the teacher in the main dialogic episode.
	2E_react with respect	<u>2E1_idea personal:</u> Within the episode, the teacher emphasized and valued that students' ideas are personal. The teacher implicitly or explicitly stated that there are no wrong ideas, for example by saying 'So in your opinion' or 'everyone can have his own opinion'. <u>2E2_answers heard:</u> Within the episode, the teacher reacted with respect and interest to all ideas and gave no explicit feedback (e.g. correct/incorrect). She let the students know their answers are heard (e.g., by saying

4

	'Ok', 'thank you', or by repeating or elaborating upon the student's answer).
2F_teacher asks more about students' ideas	During the episode, the teacher asked a least one follow-up question about students' ideas to learn more about them, to support students' thinking or to obtain clarification (e.g., 'What do you exactly mean by that?').
2G_students share ideas	The teacher encourages students to share (multiple) ideas and to let students collaborate to hear different points of view. 2G1_students react on each other: The teacher stimulates students to react on ideas of each other in a dialogic episode. 2G2_students collaborate: Within the episode students are collaborating. 2G3_exchange ideas: The teacher asks students to exchange ideas in dyads or (small) groups.
2H_autonomy	The teacher stimulates students' autonomy in the episode: the teacher stimulates students to make their own choices and/or the teacher uses language such as 'you may', 'you can'.

As described in the previous section, the video transcripts were double-coded by a second coder in three steps. First, the initial identification of dialogical episodes across the three instruction lessons was checked. The majority of dialogical episodes were identified in the same way by the two coders, showing high intercoder agreement. However, regarding the in-school mathematics and arts lesson, the second coder identified 3 episodes less and regarding the out-of-school lesson, she identified one additional episode and considered another episode to last longer than did the first coder. In a discussion with both coders, the definition of dialogic episodes of Muhonen et al. (2016) was used to agree on the final number and lengths of the dialogic episodes. Regarding the regular mathematics lesson, both coders identified the same episodes. Second, the coding scheme for assessing students' mathematical creativity and teachers' creativity promoting teaching strategies was checked. The first and second coder independently applied the first version of the scheme to 9 selected episodes, taken from all three lessons. Based on a comparison of the results, the coding scheme was slightly adjusted, and ambiguous codes were clarified. Third, after consensus was reached, all episodes were double-coded with the adapted coding scheme. With regard to the codes for mathematical creativity, 100% agreement was reached. With regard to all codes for teaching for creativity, interrater agreement was good (κ = .74; Cicchetti, 1994). Differences were discussed afterwards, and a final code was given.

Analysis of the interview transcripts. To analyze the transcripts of the interviews, a constant comparison method was used to identify underlying themes in the data (Leech & Onwuegbuzie, 2007). The starting point of the analysis was the question: What are Anna's experiences in teaching the three mathematics lessons, and why might these experiences differ? Her experiences were used to corroborate and further explain the findings of the video transcript analyses. With this question in mind, the first interview was read and subsequently coded in an open way by the first and second coder separately. The interview transcripts were divided into meaningful parts and labelled with a descriptive code. The fragments and codes selected by the two coders were mostly similar. Differences were discussed, and a final code was assigned. Similar fragments were labelled with the same code. The same coding process was used for the second interview. With regard to the second interview, the second coder segmented the transcripts in smaller fragments than the first coder. Differences were discussed until consensus was reached on the most appropriate segmentation and corresponding codes. For the third interview, the same procedure was used. Most fragments and codes were similarly identified, and differences were discussed and easily resolved. In the end, 66 open codes emerged from the data. After the open coding, axial coding was applied. The obtained codes were grouped according to similarity and for each grouping, a common theme was identified. Each coder made a mind map of the list of grouped open codes to find the axial codes. Both mind maps were discussed, and six final axial codes emerged. Next, the interviews were coded together by both coders again with axial codes. The codes were found satisfactory and well applicable to the identified fragments. Subsequently, selective coding was applied. Both coders analyzed how the axial codes related to each other within each interview and how they related to themes in the scientific literature. It was concluded that there were three main codes under which all axial and open codes could be placed, namely context, teachers' beliefs, and teachers' acting. Axial (core) codes that were related to the research question were used to corroborate the findings of the analyses of the classroom dialogues. In Appendix C an example is given of the coding process.

4.3. Findings

The main purpose of this case study was to gain in-depth insight into how students' mathematical creativity can be promoted in educational practice. The results are reported below in three parts. Since the context of this case study is highly relevant to interpret the findings, we first give a short description of the three lessons that were studied. Next, we report the ways in which the students expressed mathematical creativity in classroom dialogues. Finally, we describe the strategies the teacher, Anna, used to promote mathematical creativity in students' thinking and how they related to students' expressions of mathematical creativity.

4.3.1. Descriptions of the three different settings

Three lessons were observed to study the Anna's strategies in relation to students' mathematical creativity. Anne taught these lessons in the order described below.

The out-of-school interdisciplinary lesson 'Space outside the classroom' .The lesson plan of the MACE program indicated that the teacher had to provide this lesson outside the classroom; this could be a museum or the environment outside the school. Anne chose for the latter. The lesson plan was not prescriptive but provided several suggestions for lessons and questions relating to monuments, statues, buildings, (wall) paintings, open and closed spaces in museums or the environment outside school. Anne started her lesson in the classroom with a discussion about shapes and spaces. Next, the students went outside and walked to an artwork in the neighborhood. During this walk, students were encouraged to name the shapes they encountered. Arrived at the artwork, students were stimulated to discuss whether the artwork had a front side (note: it was a 3-dimensional round artwork) and whether it took space. After this, students had to make a drawing of the artwork from one angle. Next, they had to draw the artwork from another angle of their choice. After this, the class returned to the classroom. Back in the classroom, Anne asked her students whether they had seen particular shapes in the artwork (visualized on the interactive whiteboard) or during their walk to the artwork. Furthermore, students had to discuss from which angle they drew the artwork using pictures of the artwork as memory support, which were taken beforehand by Anne. Finally, Anne discussed with the class how the artist could have made the artwork and what the students had learned from the lesson.

The in-school interdisciplinary lesson: 'What is a pattern?' With regard to the in-school interdisciplinary lesson, Anne mainly followed the lesson plan. Anne started the lesson with an introduction of the concept patterns. She initiated a classroom discussion inviting the students to share where they had seen patterns. After this, Anne further explored the concept of patterns with her class by discussing 3 artworks. Anne used the suggested artworks of the lesson plan. Next, students had to shortly think about the following questions: 'Do patterns arise by coincidence?' and 'Does a pattern always stay a pattern?' Simultaneously, students had to throw a handful of small blocks on their table. Then, the main lesson started (25 minutes). Pairs of students threw the blocks on a sheet of white paper on the table and then marked the location of the blocks on the paper. They discussed whether the blocks formed a pattern or not, and whether they could make it into a pattern and, if so, how. In the second part of the lesson, pairs of students placed the small blocks alternately in a pattern and again marked them on the paper. Next, the students discussed the same questions again. In addition to the lesson plan, Anne also asked the pairs of students to join in groups of four and to combine the patterns they created into a new pattern. After the lesson, Anne reflected on the different products and processes, and discussed students' discoveries.

The regular mathematics lesson. Anne taught a lesson from the mathematics textbook *A world in numbers [Wereld in Getallen]* (Van Grootheest et al., 2011). The regular mathematics lesson we observed was about arithmetic. The teacher discussed arithmetical problems from students' workbooks and asked students to solve these in front of the class on the digital whiteboard. Seven arithmetical problems were discussed in a whole-class setting. In-between and after these whole-class discussions, students had to do exercises in their mathematical workbooks.

4.3.2 Mathematical creativity in the classroom dialogues

Mathematical creativity occurred in 10 out of 14 dialogic episodes during the out-of-school interdisciplinary lesson, in 7 out of 14 episodes during the in-school interdisciplinary lesson, and in 0 out of 7 episodes during the regular mathematics lesson. An expression of a student was considered as mathematically creative, when they posed or solved problems in, for them, novel ways, or in which they combined ideas from mathematics and other conceptual domains into, for them, new ideas, thereby expanding or deepening their understanding of mathematics. The following sub-dialogue illustrates how a student combined the concept of the appearance of shapes with the concept of movement (i.e. jumping):

> Teacher: We go to the statue and observe it from different points of view. It is the statue in front of the museum, not behind the museum. M. would like to say something. It might be wise to listen to him.
> Student M.: When I was there with my dad I saw that the shape changed. When I looked into the water it seems very small and when I looked at it from the front, it was very different.
> Teacher: So, the shapes are very different.
> Student M.: Yes
> Teacher: And what caused this? Why do you think that these shapes were different?
> Student M.: If you jump, you see other shapes. If you stand still you only see the length.
> Teacher: Yes, your perception depends on your point of view.

4.3.3. Promoting mathematical creativity

Used teacher strategies in the three mathematics lessons. In this section, findings are presented about the teaching strategies used by Anne to promote mathematical creativity in the three mathematics lessons and whether the lessons differed in this regard (research question 1). Table 2 shows that all theoretically-derived teaching strategies were used by Anne, but some more than others, while the distribution of the different strategies differed by type of lesson.

Table 2.
Overview of the percentages of episodes in which strategies (2A-2H) occurred in the three lessons.

Code	Lesson 1 $N_{episodes}$= 14	Lesson 2 $N_{episodes}$=14	Lesson 3 $N_{episodes}$=7
2A_Ask students to connect concepts	50%	36%	0%
2B_activating questions	71%	86%	43%
2C_ support or encourage creative thinking	14%	29%	0%
2D1_Student ideas central	57%	71%	29%
2D2_Sub dialogue allowed	43%	14%	14%
2E1_idea personal	36%	71%	43%
2E2_answers heard	64%	43%	14%
2F_aks more about students' ideas	57%	36%	14%
2G1_students react on each other	29%	29%	0%
2G2_students collaborate	0%	57%	0%
2G3_exchange ideas	7%	7%	0%
2H_autonomy	0%	29%	0%

Note. Lesson 1 is the out-of-school interdisciplinary lesson, lesson 2 is the in-school interdisciplinary lesson and lesson 3 is the regular mathematics lesson

Table 3.

Overview of the percentage of episodes in which strategies (code 2A-2H) occurred divided in episodes with and without the occurrence of mathematical creativity and the three lessons

Code	Episodes with mathematical creativity			Episodes without mathematical creativity		
	Lesson 1 $N_{episodes}$= 10	Lesson 2 $N_{episodes}$=7	Lesson 3 $N_{episodes}$=0	Lesson 1 $N_{episodes}$=4	Lesson 2 $N_{episodes}$=7	Lesson 3 $N_{episodes}$=7
2A_Ask students to connect concepts	60%	57%	-	25%	14%	0%
2B_open questions	80%	86%	-	50%	86%	43%
2C_support or encourage creative thinking	10 %	14%	-	25%	43%	0%
2D1_Student ideas central	80%	100%	-	0%	43%	29%
2D2_Sub-dialogue allowed	60%	14%	-	0%	14%	14%
2E1_idea personal	40%	86%	-	25%	57%	43%
2E2_answers heard	80%	71%	-	25%	14%	14%
2F_aks more about students' ideas	70%	71%	-	25%	0%	14%
2G1_students react on each other	40%	43%	-	0%	14%	0%
2G2_ students collaborate	0 %	29%	-	0%	86%	0%
2G3_exchange ideas	0 %	14%	-	25%	0%	0%
2H_autonomy	0 %	0 %	-	0%	57%	0%

Note. Lesson 1 is the out-of-school interdisciplinary lesson, lesson 2 is the in-school interdisciplinary lesson and lesson 3 is the regular mathematics

More strategies and more different strategies were used in both open interdisciplinary lessons compared to the regular mathematics lesson. For example, in the regular mathematics lesson the teacher never asked students to connect two different mathematical concepts (code 2A). The strategy of supporting and encouraging student's creative thinking (code 2C), expressed in questions or suggestions such as 'try to think out of the box', was rather infrequent across all episodes and lessons. This strategy mainly occurred in episodes in which students were collaborating, for example when they were working in pairs on an assignment (code 2G2), and was always used simultaneously with Anne stimulating students' autonomy (code 2H). Anne used these strategies especially when a pair or group of students were stuck in their creative process. Since only the in-school lesson contained such cooperative assignments, this can explain why these strategies were mainly used during this lesson.

Used teacher strategies and students' expressions of mathematical creativity. In this section, findings are presented about how teaching strategies relate to students' expressions of mathematical creativity. In Table 3 the overview of strategies used by Anne during the three lessons is broken down by episodes where students expressed mathematical creativity in educational dialogues and episodes where they did not. The findings show an association between the teaching strategies used and the occurrence of mathematical creativity. Some strategies were clearly more often used in episodes in which students expressed mathematical creativity than in episodes in which no mathematical creativity was observed. This holds in particular for the strategies 2A, 2D1, 2E2, 2F, and 2G1. Other strategies, such as supporting and encouraging creative thinking (code 2C) and students' autonomy (code 2H), mainly occurred in episodes in which students did not express mathematical creativity. For the remaining strategies, for example whether Anne allowed a sub-dialogue (code 2E1), the pattern of use and the relation with the occurrence of mathematical creativity was more diffuse.

A deeper analysis of the dialogic episodes suggested that students' expressions of mathematical creativity occurred more often if Anne combined activating open questions (code 2B) with questions eliciting student's ideas about a specific subject (code 2D1), provided indications that students' answers are heard and respected (code 2E2) and/or asked follow-up questions to learn more about students' ideas (code 2F). If 3 out of 4 of these strategies occurred together, it was related to students' expressions of mathematical creativity. In this study, these aspects together seem to create an open atmosphere in which the teacher created opportunities for students to express their ideas and she was open to these ideas. Anne asked activating open questions, like 'who sees a pattern', 'who else sees a pattern?' and 'what more do we see?', that stimulated students to express their ideas. Several times she repeated these kinds of questions and she gave several students a turn to express their ideas. She often repeated or recast the ideas of students and sometimes added follow-up questions while contingently building upon students' preceding responses. The excerpt below of a dialogic episode, observed during the out-of-school interdisciplinary lesson, may illustrate this.

Student K.: A half O

Teacher: A half O, a half O (teacher makes hand gestures of half O). Okay. What more do you see? What shapes do you see? J., what shapes do you see?

Student J.: A changing shape.

Teacher: Oh, a changing shape. Oh, interesting. Shapes can change. Change in what?

Student J.: Maybe in space. It may grow as well. It can get slightly increasingly longer. An infinite shape!

Teacher: An infinite shape, does it exist? An infinite shape? (B. raises his hand). B.?

Student B.: Yes, a circle never ends, it always continues. It continues, so it is infinite.

Teacher: oh, that is interesting.

Student R.: We also have a rectangle.

Teacher: So, it has been said that we have a circle that is infinite, we have a rectangle. What more do you see more?

Another finding is that mathematical creativity mainly occurred in longer whole-class interactions and at the beginning and at the end of the lessons (see Figure 2 and Appendix A). In episodes where the teacher interacted with small groups and which were often short, hardly any mathematical creativity occurred. This may be related to the fact that the teacher used different strategies in these episodes.

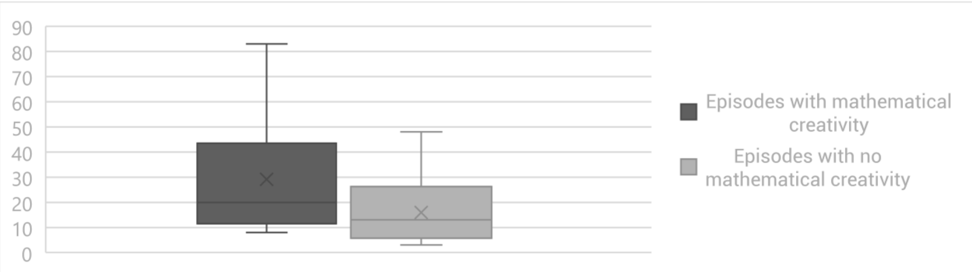

Figure 2. *Boxplots of the numbers of utterances in the dialogic episodes with (N=17) and without (N=18) the occurrence of mathematical creativity.*

Often these dialogues were focused on what the group was thinking, doing or making in relation to their artworks (product).

Used teachers' strategies and students' expressions of mathematical creativity in the three mathematics lessons. In addition to the strategies used by the teacher, the type of mathematics lesson also seemed to play a role in the occurrence of students' expressions of mathematical creativity. We found that students' expressions of mathematical creativity did not occur in the regular mathematics lesson. Although this particular combination of strategies (codes 2B, 2D1, 2E2, 2F) was related to students' mathematically creative expressions in the two open interdisciplinary lessons, it was not in the regular mathematics lesson. This is visible in the following excerpt from the regular mathematics lesson in which the teacher asks students a question that invites them to express their own strategies in adding numbers, while listening with interest to what the students put forth. Although the teacher did not ask any follow-up questions (code 2F), an open atmosphere was created. However, the students express no mathematical creativity as defined in the current study: students only come up with different ways to add instead of constructing new concepts in dialogue.

> **Teacher:** 90 cents! Okay, so I have to add €43.80 and 90 cents. How much is that in total?
> **Student L**: €44,70.
> Teacher: Okay, €44.70. Who says I do it in a different way? F.?
> **Student F:** First add 25 and 55.
> **Teacher:** Okay, so you first add the cents. This one, this one and this one.
> **Student F:** No, not yet the 90.
> **Teacher:** Oh, not yet the 90. So, this one and this one.
> **Student F:** and then you calculate. That is 80. And then you take 17 plus 23.
> **Teacher:** Okay than you take 17 and 23.
> **Student F:** Than 10 plus 20. Than you have 30 and then 7 and 3 and then you get 40.
> **Teacher:** Okay, so plus 40.
> **Student F:** And then I add 90 and 80.
> **Teacher:** So, you add 90.
> **Student F:** Than you have €1,70. And then you do 3 plus the 1 euro and then 4 euro plus 40 and then 70 cents.
> **Teacher:** So, this one plus this one, then I have €4.70. And then you add this to 40. Okay. Yes.
> **Student T:** I added the cents first.
> **Teacher:** First the cents.

Furthermore, we found that there did not seem to be a difference between the in-school and out-of-school context regarding the occurrence of mathematical creativity. In the out-of-school interdisciplinary lesson 14 dialogic episodes were identified, of which five in the out-of-school context. In both the in-school (6 out of 9 dialogic episodes) and the out-of-school dialogic episodes (4 out of 5 dialogic episodes) mathematical creativity occurred. Furthermore, the same strategies were used by the teacher in these episodes. However, out-of-school dialogues differed from in-school dialogic episodes in that students frequently connected shapes they were not familiar with to well-known concepts from daily life, like ‘this is the shape of a baguette’ and ‘ugly circle’.

4.4. Discussion

The aim of this case study was to obtain a more in-depth understanding of the teaching strategies an elementary school teacher can implement to promote mathematical creativity in the classroom and how these strategies, and particular combinations of them, relate to students' mathematical creativity as expressed in classroom dialogues. Mathematical creativity was defined as students' thinking acts, expressed in classroom dialogues, in which they posed or solved problems in, for them, novel ways, or in which they combined ideas from mathematics and other conceptual domains into, for them, new ideas, thereby expanding or deepening their understanding of mathematics.

The first research question addressed the strategies the teacher in focus, Anne, used to promote creativity in students' mathematical thinking during three mathematics lessons, varying in openness and interdisciplinarity. Teaching strategies identified in the research literature as potentially promoting mathematical creativity guided the analysis of classroom dialogues during these lessons. We found that all strategies for promoting mathematical creativity in students' thinking were used by Anne, but some more often than others. More, and more diverse, strategies were used during the two open interdisciplinary mathematics lessons compared to the regular textbook-based mathematics lesson.

The second research question concerned the relation between mathematical creativity promoting strategies of the teacher and students' creativity as expressed in classroom dialogues. We examined this relation within thematically coherent episodes to ensure that the co-occurrence of particular teaching strategies and expressions of creativity could be meaningfully interpreted. There were several important findings. First, expressions of mathematical creativity only occurred during the two open interdisciplinary lessons, involving both out-of-school and in-school mathematics and arts education. During these lessons, Anne created an open climate and facilitated longer whole-class dialogues, in which she gave students ample opportunities to express their ideas and, through specific pedagogical actions, she clearly signaled to students that she took these ideas seriously. The importance of the use of these strategies for promoting creativity in mathematics education is in line with the literature on promoting mathematical creativity (e.g., Beghetto & Kaufman, 2011). However, a second important finding was that these same teaching strategies were also observed during the regular textbook-based mathematical lesson, but now expressions of mathematical creativity did not occur. This finding illustrates that making some aspects of the lesson open (i.e., giving opportunities for alternative and personal strategies), does not necessary stimulate mathematical creativity as expressed in dialogues.

The reason why more, and more different, creativity promoting strategies were used in the two open interdisciplinary lessons compared to the regular lesson, and why the use of these strategies was not related to mathematically creative expressions in the regular mathematics lesson may be the fact that the learning goals differed, suggesting that creativity promoting teaching strategies only stimulate creativity when this is an explicit learning goal. The regular mathematical lesson, in this regard, had a more specific learning goal than the other two lessons, namely practicing arithmetical strategies versus learning about conceptualizations of shapes, space and patterns. For the two interdisciplinary lessons the main learning goal was to teach new concepts (tool-for-result), but also to use mathematical dialogues in which concepts are questioned and developed (tool-and-result; Bakker et al., 2015; Kazak, Wegerif, & Fujita, 2015). The teacher also experienced this and stated in the interview after the regular mathematics lesson, that she felt less freedom in teaching, because she also "(…) had to teach them things". Another explanation may be that during this regular mathematics lesson, students behave in accordance with particular sociomathematical norms, indicating in an implicit way what is expected and rewarded (Yackel & Cobb, 1996). Students may have considered expressing mathematical creativity as unacceptable according to the sociomathematical norm of the regular lesson. Furthermore, the interdisciplinary character of the lessons may have made it easier for students to relate two different ideas or concepts (i.e. to express mathematical creativity). Thus, the fact that the two interdisciplinary lessons were different (i.e., more open, other multidisciplinary content) from this regular mathematical lesson may be the cause that the teacher used different strategies and that teacher and students interacted in a different way.

Although the findings of this study indicate that creating an open climate in open interdisciplinary lessons is indeed related to students' mathematical creative expressions, it can be questioned whether this enhances students' mathematical learning. One the one hand, it can be argued that most dialogues between teacher and students were rather superficial because the ideas and concepts of students were central, and students did not learn specific mathematical content. On the other hand, it can be argued that students did engage in a mathematical dialogue in which mathematical concepts were questioned and may further developed (tool-and-result; Bakker et al., 2015; Kazak et al., 2015), which could also be considered as mathematical learning. In this respect, however, it seems to be important that the teacher asks (follow-up) questions that make the student reason about mathematical concepts in a deep way. Within the lessons, students may had obtained deeper insights into their mathematical creative ideas or solutions, if the teacher would have asked more eliciting questions that challenged students' thinking.

Another finding in this study was that not all theory-driven strategies for promoting mathematical creativity were related to students' expression of mathematical creativity. We found that in dialogic episodes in the in-school interdisciplinary lesson in which students collaborated, the teacher used different strategies compared to other dialogic episodes: the teacher mainly supported and encouraged creative thinking and stimulated their autonomy. However, these dialogic episodes and strategies were not related to students' expression of mathematical creativity. A cause for this may be the short time of interaction (i.e., average of 7 utterances per episode). Furthermore, in these episodes the aim of teacher was mainly focused on determining what artworks the students hitherto made and on providing help to students who were stuck in the process of creating the artwork. Contrary to the literature (e.g., Beghetto & Kaufman, 2014; Beghetto & Kaufman, 2010), this study may indicate that some strategies in itself are not related to mathematical creative expressions.

4.4.1. Limitations & future research

A limitation of this study is that the findings cannot be extended beyond this single case. Nevertheless, the findings provide a detailed presentation of how creativity promoting strategies relate to students' expressions of mathematical creativity in a classroom dialogue.

This study can be a starting point for future research in which more teachers, classrooms and lessons can be observed. Furthermore, future research could investigate how many and which students in a classroom come up with mathematically creative expressions. Another limitation of this study is that mathematical creativity was coded on students' verbal expressions only. However, mathematical creativity may also be expressed in non-verbal ways, for example, by combining two known concepts (e.g., shape and infinity) in a new way when designing an artwork. Moreover, students may not have verbalized all their thoughts, which limited this study to describing the associations between creativity promoting strategies used by the teacher and students' verbal expressions. However, we believe that this new way of coding mathematical creativity in educational dialogues adds to the literature on mathematical creativity, because it can give more detailed information about how mathematical creativity manifests in a mathematics classroom context and what factors (e.g., expressions of teachers or peers) contribute to this manifestation of mathematical creativity. Nevertheless, future research could also take into account non-verbal expressions of creativity.

4.5. Conclusion and Implications

To conclude, findings of the current study indicate that mathematical creativity was promoted if the teacher created longer whole class dialogues and an open atmosphere in open interdisciplinary lessons with open learning goals. In this study, an open atmosphere was created if the teacher created opportunities for students to express their ideas and if the teacher was open to these ideas. Furthermore, the interdisciplinary content, a less specified learning goal (i.e., learning about conceptualizations instead of strategies), and different sociomathematical norms might have contributed to the presence of mathematical creativity in the open interdisciplinary lesson and the absence of mathematical creativity in the regular mathematics lesson.

For educational practice this finding implies that mathematical creativity may be easier encouraged in open and interdisciplinary lessons. In these lessons the teacher should focus on the use of strategies like, asking activating open questions that invite students to generate multiple answers and ideas, creating longer whole class dialogues in which ideas of students' ideas are central, making sure that students answers are heard and respected and that follow-up question are asked to learn more about students' ideas. Furthermore, our study contributes to the literature on (promoting) mathematical creativity, since we used a new approach in defining and measuring mathematical creativity in elementary school mathematical dialogues. In our study we used a broader definition of mathematical creativity, because, as shown in this study, in elementary school mathematics education mathematical creativity also occurs in other situations than mathematical problem posing and solving.

4.6. Appendices

4.6.1. Appendix A. Example of a coded dialogue (Episode 1, in-school interdisciplinary lesson)

Dialogic episode	Codes
TEACHER: Pattern, who sees a pattern? STUDENT 5: Nick's drawing TEACHER: Nick's drawing on the dartboard STUDENT 6: there are exclamation marks and in between of those things TEACHER: What is the pattern in this? How can you..? If I do not know what a pattern is, and I think 'pattern, pattern', can you explain me what a pattern is? STUDENT 6: all shapes in an order TEACHER: all shapes in... STUDENT 6: an order TEACHER: an order. Ok. That is a pattern. STUDENT 14: Over there, where the drink bottles are TEACHER: Where the drink bottles are? STUDENT 14: A square kind of kitchen TEACHER: A square kind of kitchen? STUDENT 14: Yes, at the kitchen TEACHER: the tiles? STUDENT 14: the kitchen counter TEACHER: the kitchen counter. What is the pattern? STUDENT 14: The pattern is just strange squares TEACHER: Strange squares... Jenny, patterns? STUDENT 5 (Jenny): my shirt TEACHER: your shirt is also a pattern STUDENT 5: yes TEACHER: Tell me. STUDENT 5: White, black, white black, white, black TEACHER: Ok. White, black, white black. Felix? STUDENT 3 (Felix): Also, the gauze on my shoes. TEACHER: The gauze on your shoes. Is that a pattern? Let's see, what is the pattern? STUDENT 3: Sort of diamond shape. TEACHER: A diamond shape. Oh, fantastic. Lisa? STUDENT 10 (Lisa): Evelyn's dress. TEACHER: Evelyn's dress. Evelyn, can you get up, please? STUDENT 10: Also, the one of Hanna. TEACHER: Also, the one of Hanna. Get up Hanna! What patterns do you all have! STUDENTS: *(inaudible, students are not talking at once).* TEACHER: O wow (STUDENT stands up). This is not some pattern. STUDENT: No, because it is not in the same order. TEACHER: A pattern is not in a certain order. STUDENT 10: A pattern is an order, and this is no order, because it is mixed up. TEACHER: Ok, so a pattern... is something different. What is pattern then? STUDENT 10: It is alternately the same. TEACHER: Ok, alternately the same. Ok. STUDENT 5: It does not have to be! It does not have to be alternately, but it is repeatedly the same. TEACHER: Does not have to be the same, you say?	1: Students introduced new and relating concepts to the discussion, namely that a pattern is related to repetition of shapes, repetition of colors, a location and that a pattern should at least have one constant aspect (e.g. color or shape). 2A: The teacher asked students to connect detect patterns in their environment. 2B: the teacher asked activating questions that encourage students to talk, like 'who sees patterns?'. 2D1: Students ideas about patterns are central in this episode. 2D2: the student starts a dialogue about that a pattern does not have to be alternately. The teacher asks more about this idea. Later, this happens again with student 18. 2E1: The teacher implicitly indicate that ideas are personal, and explicitly one time: ''what patterns do you all have''. 2E2: She let the students know that she heard their answers (she repeats them or, ok/super), or answers with interest (Oh wow!). 2F: Follow-up questions are asked to know more about students' ideas, like 'what is the pattern in there'.

STUDENT5 : No, because…
TEACHER: I hear something. Jenny says it does not have to be. (points at another student).
STUDENT15: If you have three colors for example. After that 3 colors emerge again. Then, you have constantly these three colors.
STUDENT 5: or 5 colors.
TEACHER: Wait, one student talking at once. please! Because, otherwise my mind gets chaotic, and I do not have a pattern in my mind.
STUDENT 15: Look, if you have three colors, they come always next to each other. Or five colors, they come always next to each other. So, it does not matter how many colors there are, but they recur.
TEACHER: Ok. J., what do you want to say?
STUDENT 5: Yes, I wanted to say the same, and here is a pattern (the student points to her pencil case).
TEACHER: What pattern is in there?
STUDENT 5: You see purple, blue and pink all the time. And you see four yellow circles all the time.
STUDENT 10 (Eric): But is not always on the same place.
STUDENT 5: That is not necessary.
STUDENT 10: Yes, it is, if it always on the same spot, it is a pattern.
STUDENT 5: In the circle is a circle.
TEACHER: Eric, (student 10) was talking, remember? Ok Eric, you say it is not a pattern.
STUDENT 10: No, because it is not on the same place.
TEACHER: No, because it is not on the same place. L. (student 21).
STUDENT 21: It is not on the same place, but the circles are all different. And, within the circle, there is a pattern.
TEACHER: In the circle there is a pattern. Interesting. Interesting, interesting patterns. What an interesting subject.
STUDENT 18: A pattern is also a shape. For example, you make a flower, in within the flower there is a circle, a yellow flower. And you make the recur. If your patterns mingle, there is no order anymore.
TEACHER: If you mingle, the pattern disappears?
STUDENT 18: Yes, because you have blue, purple, green for example. After that blue has to come, next purple and then green.
TEACHER Ok, should that be, or could that be in another color or in another shape? Is that possible, is it still a pattern or…?
STUDENT 18: It cannot be both different, but you can use another shape or another color
TEACHER: Ok that is possible. Have a look behind you. Something is standing out.
STUDENT 9: Yes, that was my hand teacher.
TEACHER: tell me! Can you tell me about it?
STUDENT 9: in the letters are patterns.
TEACHER: in the letters are patterns? Wow, but if I look at the U. What you just told me Sara, what do you see?
STUDENT 18: triangle, circle, triangle
TEACHER: Ok, and I also see lines. Is the U a pattern or is it not?
STUDENT 18: they are pattern, it is a pattern, this a pattern and that is a pattern.
TEACHER: interesting, patterns! That is where we are talking about. Interesting that you see a lot of pattern is class already!

Appendix B. Overview of the coded episodes

mathematical creativity	2A	2B	2C	2D1	2D2	2E1	2E2	2F	2G1	2G2	2G3	2H	utterances	episodes	lesson
1	1	1	0	1	1	1	1	1	0	0	0	0	77	episode 01	in-school
1	1	1	0	1	0	1	1	1	0	0	0	0	46	episode 03	in-school
1	1	1	1	1	0	1	1	1	1	0	0	0	17	episode 04	in-school
1	1	1	0	1	0	0	1	1	0	0	0	0	41	episode 01	in-school
1	1	1	0	1	1	1	1	1	0	0	0	0	20	episode 02	in-school
1	1	1	0	1	1	1	1	1	1	0	0	0	51	episode 05(outside)	out-of-school
1	1	1	0	1	0	0	1	1	0	0	0	0	25	episode 07(outside)	out-of-school
1	0	0	0	1	0	1	1	1	0	0	0	0	29	episode 14	in-school
1	0	1	1	0	1	1	1	1	0	0	0	0	16	episode 03	in-school
1	0	0	0	0	1	0	1	1	1	0	0	0	12	episode 04	out-of-school
1	0	1	0	1	0	1	0	1	1	1	0	0	8	episode 12	in-school
1	1	1	0	1	1	0	0	1	1	0	0	0	83	episode 06(outside)	out-of-school
1	1	1	0	1	0	1	1	0	1	0	0	0	24	episode 02	in-school
1	1	1	0	1	0	0	1	0	1	0	0	0	16	episode 10	out-of-school
1	0	1	0	1	0	1	1	0	0	0	0	0	10	episode 14	out-of-school
1	0	1	0	1	0	0	0	0	0	1	1	0	11	episode 09	in-school
1	0	0	0	1	1	0	0	0	0	0	0	0	9	episode 09(outside)	out-of-school
0	0	1	0	0	0	0	1	1	0	0	0	0	30	episode 12	out-of-school
0	0	0	0	0	1	1	0	1	0	0	0	0	13	episode 3	regular math
0	0	1	0	1	0	0	1	0	0	1	0	0	3	episode 10	in-school
0	0	1	0	1	0	1	1	0	0	0	0	0	48	episode 4	regular math
0	0	1	0	1	0	1	0	0	0	1	0	0	5	episode 06	in-school
0	0	1	0	1	0	0	0	0	1	1	0	0	6	episode 13	in-school
0	0	1	0	1	0	1	0	0	0	0	0	0	9	episode 5	regular math
0	0	1	1	0	0	1	0	0	0	1	0	1	16	episode 05	in-school
0	1	1	1	0	0	1	0	0	0	1	0	1	3	episode 07	in-school
0	0	1	0	0	0	0	0	0	0	1	0	1	4	episode 11	in-school
0	1	1	1	0	0	1	0	0	0	0	1	0	16	episode 11	out-of-school
0	0	1	0	0	0	0	0	0	0	0	0	0	25	episode 6	regular math
0	0	0	1	0	1	1	0	0	0	0	0	1	8	episode 08	in-school
0	0	0	0	0	0	0	0	0	0	0	0	0	31	episode 08(outside)	out-of-school
0	0	0	0	0	0	0	0	0	0	0	0	0	7	episode 13	out-of-school
0	0	0	0	0	0	0	0	0	0	0	0	0	13	episode 1	regular math
0	0	0	0	0	0	0	0	0	0	0	0	0	13	episode 2	regular math
0	0	0	0	0	0	0	0	0	0	0	0	0	37	episode 7	regular math

Note. 0 means that the strategy is not coded and 1 means it is coded.

4

4.6.3. Appendix C. Example of the coding process of the interview after the out-of-school lesson

Fragment	Open code	Axial code	Axial core code
Anne: yes, but measuring, time and money. Yes, I would like to go to the market with my students. Here you have 10 euros. Buy food for 25 students and prepare it. Researcher: Instead of the regular mathematics lesson?	Ideal_out-of-school lesson	Teaching beliefs' regarding 'open' mathematics education	Teachers' beliefs
Anne: Yes, yes. You need to skip it. They call it program lessons in the mathematical textbook, but it still on a flat surface. In what way do children learn to understand money? Researcher: it is not meaningful. Anne: yes, you know, what can I buy for ten euro's? What can I buy for 100 euro's? They have no idea. Also, the change. And what is more expensive, a bag or a box with more volume? Researcher: yes, a store in the classroom or the checkout of the preschooler. Anne: yes, we are talking about it, but why do you only have those playing-learning corners in preschool? Why is it gone it grade 1 and why do you need to sit behind a desk for learning mathematics and language? I am in favor of a playing-learning corner! Make a geometry corner.	Ideal_inside of school lesson	Teaching beliefs' regarding 'open' mathematics education	Teachers' beliefs

Chapter 5

Enriching mathematics education with visual arts: Effects on elementary school students' ability in geometry and visual arts

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Authors contribution: E.S., and E.K. designed the study. E.S. collected and analyzed the data and wrote the paper. E.K., and P.L. critically reviewed the paper.

Abstract

This study evaluates the effects of the Mathematics, Arts and Creativity in Education (MACE) program on students' ability in geometry and visual arts in the upper grades of elementary school. The program consisted of a lesson series for fourth, fifth, and sixth grade students in which geometry and visual arts were integrated, alongside with a professional development program for teachers. A quasi-experimental study was conducted in which three groups of teachers and their classes were investigated. One group of teachers taught the lesson series and followed a professional development program (n = 36), one group of teachers only taught the lesson series (n = 36), and a comparison group taught a series of traditional geometry lessons from mathematical textbooks (n = 43). A geometrical ability, creativity, and vocabulary test, and a visual arts assignment were used in a pre- and post-measurement to test the effects of the MACE program. Results showed that students who received the MACE lesson series improved more than students who received regular geometry lessons only in geometrical aspects perceived in a visual artwork. Regarding students' understanding and explanation of geometrical phenomena and geometrical creative thinking, all students improved, but no differences between the groups were found, which implies that on these aspects the MACE program was as effective as the comparison group that received a more traditional form of geometry education.

5.1. Introduction

Within elementary school mathematics education, non-routine problem solving is considered important, since it is at the heart of mathematics (Kolovou, 2011). For example, creating a paper model of a 12-sided dice can be considered a geometrical problem for elementary school students. Students cannot simply apply a strategy, but have to recall, use and combine facts, skills, procedures and ideas in a new and meaningful way to solve the problem. This requires creative and flexible thinking (Chapter 4; Warner, Alcock, Coppolo Joseph, & Davis, 2003), which is also considered important in other disciplines in elementary education, such as visual arts in which creativity is a central element (Sawyer, 2014; Stichting Leerplanontwikkeling, 2015). However, in educational practice, most teachers do not provide many opportunities for students to act creatively in mathematics (Gravemeijer, 2007; Kolovou, 2011) and visual arts (Bresler, 1999; Elfland, 1976). A possible explanation is that teachers tend to structure their lessons around mathematical textbooks which as such provide only few of these opportunities (Kolovou, 2011). Furthermore, the curriculum and the targets to be reached may constrain the creative practices that the teachers feel able and willing to engage in (Dobbins, 2009).

The Mathematics, Arts, Creativity in Education (MACE) program was designed to change educational practice in this respect. The program aimed to teach domain-specific and overlapping learning goals of visual arts and geometry and to promote students' creative skills in both disciplines by creating opportunities for students to act creatively in an integrated visual arts and geometry context. To reach the goals of the program, a lesson series was designed for fourth, fifth and sixth grade students, along with a professional development (PD) program for teachers to enhance implementation of the lesson series. This study evaluated the intended effects of the MACE program.

5.1.1. Integrating mathematics and arts education

The mathematical domain of geometry in education has the aim to teach students to understand and explain geometric phenomena from reality, and to order and organize spatial situations (Jones, 2002; Van den Heuvel-Panhuizen & Buys, 2005), such as to draw a map or to reason about the effect of the height of the sun on the length of the shadow. This also requires students to obtain geometrical vocabulary to explain these phenomena (Buijs, Klep, & Noteboom, 2008). Furthermore, the ability the solve geometrical problems is considered important, since it is central to mathematics (Kolovou, 2011) and can be a way to construct new mathematical knowledge (Levav-Waynberg & Leikin, 2012b). Problem solving requires creative thinking (Silver, 1997): students need to be able to combine known concepts, skills, procedures and ideas from mathematics and other domains in a new way to solve the problem (Chapter 4; Sriraman, 2005), which can contribute to the construction of new knowledge and deeper understanding of geometrical concepts (Levav-Waynberg & Leikin, 2012b; Warner et al., 2003). Based on these core aspects of geometry education, we defined geometrical ability in this study as students' ability to understand and explain geometric phenomena, to describe these phenomena by using geometrical vocabulary, and to creatively solve geometrical problems.

Visual arts education has the aim to teach students to develop their visual-imaginative abilities by using their experiences of reality and by teaching them to visualize these experiences (Braakhuis, Von Piekartz, Vogel, & De Graaf, 2012). The main aspects of visual arts education are visual art production, perception (observing, interpreting and analyzing) and reflection (thinking and speaking about a visual art product during or after visual art production; Haanstra, 2014). The (cyclic) creative process is central in teaching the visual arts curriculum (Sawyer, 2014; Stichting Leerplanontwikkeling, 2015).

Thus, in both visual arts and geometry, creative processes play a central role. One of the key cognitive processes of creativity is to overcome fixation on ideas and to break away from established mindsets. Integrating visual arts and mathematics education could help students to think and act more creatively and flexibly, since it could make it easier to 'break out' their thinking rut (Haylock, 1987b). Currently, visual arts and mathematics are usually taught in separate disciplines in elementary school and to solve a problem in mathematics or visual arts, students may rely on highly familiar domain-specific knowledge and routines. We hypothesize that by integrating mathematics and visual arts education, students will be stimulated to integrate different conceptual systems from the disciplines of visual arts and mathematics, which could activate students to create something new and meaningful (Plucker & Zabelina, 2009).

5.1.2. The MACE pedagogy

The MACE pedagogy comprises the following key features: visual arts perception, open activities, reflection, communication with peers, and a specific role of the teacher. In this section we elaborate on these features and we describe how these features are thought to enhance students' ability in both geometry and visual arts. Other, more formal features of the lesson series (e.g., duration and number of lessons) and of the associated PD program for teachers are discussed in the Method section.

Visual arts perception. Within an integrated pedagogy, visual arts perception plays a role right at the start of the MACE lesson series. Artworks are discussed in a whole class setting by using an interactive whiteboard in relation to the interdisciplinary theme of the lesson (e.g., in a lesson about *perspective*: 'Can you tell me how the photo would have looked like if the photographer had used another point of view?') and with the use of visual thinking strategies (e.g., 'What's happening in this picture?', 'What do you see that makes you say that?', 'What more can we find?'; Housen, 2002). Visual aspects of a piece of art are carefully observed and analyzed because students are asked to back up their ideas and interpretations (Hailey, Miller, & Yenawine, 2015). Furthermore, they have to keep observing visual art, consider the view of others, and discuss many possible interpretations (Hailey et al., 2015).

Educating visual arts perception could change students' visual perception. If students have to carefully observe and analyze artworks, they may become more able to extract shapes and objects from a visual scene which, in turn, can influence their recognition and representation of visual information (Kozbelt, 2001; Tishman, MacGillivray, & Palmer, 1999). Since artworks are discussed in an interdisciplinary context, students may be better able to change their recognition and representation of visual aspects in artworks such as ‘space’, ‘shapes’ and ‘composition’ (Stichting Leerplanontwikkeling, 2018). Furthermore, visual arts perception could improve students’ geometrical reasoning by asking students to imagine how an artwork would look like if particular changes were made (Tishman et al., 1999; Walker, Winner, Hetland, Simmons, & Goldsmith, 2011).

Open activities. Within the MACE program open activities are used in which students have to produce a visual artwork related to a theme at the boundaries of visual arts and geometry (e.g., relating to perspective or symmetry; Roucher & Lovano-Kerr, 1995). Tasks contain a problem students do not immediately know how to solve because there is no predetermined or obvious solution (Kolovou, 2011). Activities are ‘open’ if they invite different solutions and if problem solving methods are open for interpretation (Silver, 1995). For example, in one of the open tasks of the MACE program, students have to make a 3-dimensional representation of a 2-dimensional painting by using free materials like egg boxes and rolls of toilet paper.

By using these types of activities, students can learn to visualize their experiences by exploring materials, meaning, and several visual aspects (Stichting Leerplanontwikkeling, 2018). In addition, students can learn to order and organize spatial situations in which they may need to reason geometrically (e.g., how certain objects need to be structured to construct the 3-dimensional representation of the painting). Furthermore, investigation and manipulation of the materials may expand students’ exploration of the physical environment. This, in turn, can help them to form visuospatial and sensorimotor representations of geometrical structures that are embedded in the environment, which may advance their thinking during geometrical problem solving (Carbonneau, Marley, & Selig, 2013; Núñez, Edwards, & Matos, 1999).

Reflection. Reflection at the end of each lesson is considered very important within this integrated pedagogy. The knowledge and skills acquired during visual arts reception and production may still be rather implicit. It is important that students learn to reflect on their (creative) process of producing an artwork, and to communicate about the thinking processes with classmates and their teacher. By clarifying what was going on and what they have learned, the implicit knowledge and skills acquired can be made explicit; reflection can extend and modify existing knowledge (Chi, De Leeuw, Chiu, & Lavancher, 1994; Clark & Karmiloff-Smith, 1993).

Communication with peers. Within the MACE lessons there is ample space for students to discuss, exchange and communicate ideas with classmates; students can collaborate in the learning activities and situations are created in which they are asked to communicate their ideas. In this way, students can compare their ideas and view ideas from different standpoints, which can enhance creative thinking (Beghetto & Kaufman, 2010; Taggar, 2002). It can also increase students’ learning in geometry, because they have to explain their thinking, they get feedback and encounter other points of view (Jarvis, 2001).

This may enable students to reach a higher level of understanding (Van den Heuvel-Panhuizen & Drijvers, 2014) and encourage (geometrical) language development (Van Lier, 1996).

Specific role of the teacher. The MACE lessons also require a specific role of the teacher. Teachers are advised to act as a facilitator: asking questions to extend students' thinking and reasoning, instead of merely transferring knowledge (Bostic, 2011). In addition, it is important that teachers use academic geometrical vocabulary, for example in reformulating students' thinking. As a result, it is expected that students will improve their geometrical vocabulary (Henrichs & Leseman, 2014). To stimulate creativity, teachers are advised to create an open atmosphere in which students' ideas are central, ask open and activating questions that invite students to generate multiple answers and to be open to these ideas (Chapter 4; Davies et al., 2014).

Although this specific role of the teacher is explained in the MACE teaching manual, also a PD program for teacher was designed to assist teachers in implementing the MACE lesson series in educational practice.

5.1.3. Aim of the study

This study investigated the effects of the integrated MACE approach on students' geometrical ability and perception of visual arts in a quasi-experimental design. The comparison condition consisted of students that received regular (textbook-based) geometry and visual arts education.

It was hypothesized that students in the MACE program would improve more with regard to their geometrical ability than students in the control group. More specifically, students were expected to better explain and understand geometric phenomena than students in the comparison group, to use more geometrical words to explain these phenomena, and to think more creatively in solving geometrical problems. Compared to students who received the regular geometry lessons, students who received the MACE lessons could engage in open, hands-on, and interdisciplinary activities, had more opportunities to express, discuss and exchange ideas with classmates and teachers, and had more time to reflect on the lesson. The presence of these aspects in the MACE lessons, as explicated in the introduction, relative to the absence or reduced presence of these aspects in the regular textbook-based geometry lessons (e.g., Van Grootheest et al., 2011) were the reasons to hypothesize that students in the MACE program would improve more with regard to their geometrical ability than students in the comparison condition.

Furthermore, we hypothesized that students who received the MACE program would observe and describe more geometrical aspects in visual artworks, especially regarding the aspects 'space', 'space suggestion', 'shapes', and 'composition', compared to students in the comparison condition, because students who receive regular visual arts education do not often observe and discuss visual artwork, and especially not with a specific focus on the geometrical aspects (Kruiter, Hoogeveen, Beekhoven, Kieft, & Bomhof, 2016)

5.2. Methods

5.2.1. Participants

In this study, 2909 students in grades 3, 4, 5, and 6 situated in 121 classes and 57 schools participated. 4th, 5th and 6th grade teachers were recruited for the MACE program by sending flyers of the MACE program to 428 regular elementary schools in all regions of the Netherlands. 11.68% of the schools were willing to participate. To evaluate the effect of the MACE lesson series and the MACE PD program, the participating schools were assigned to one of three conditions. Although we planned to randomly assign schools to the three conditions, this was not completely possible. For logistic reasons, schools had to be within a reasonable travelling distance of the PD training locations. Furthermore, some teachers were not available for the MACE PD program. In the first condition, the teachers followed the MACE PD program and taught the MACE lesson series to their students; in the second condition, the teachers taught the lesson series, but without following the PD program; teachers in the third condition taught regular geometry lesson from existing mathematical textbooks (but were offered to follow the MACE program after the study). Since geometry lessons are taught irregularly, spread over the school year, a lesson series was established for this study in which geometry lessons from various Dutch mathematical textbooks were combined to have the same time and intensity of geometry instruction as in the MACE program. In this way, students in the comparison group received the same geometrical content as students in the MACE program, but in a regular way. Geometry lessons from mathematical textbooks were used that were similar in geometrical content, but different from MACE lessons in pedagogy and enrichment with visual arts aspects. In the study, 10 classes and 197 students dropped out, because teachers experienced a too high workload (6.8 % of the total number of students). In Table 1 the three groups are described.

Table 1.

School and class characteristics

	Experimental group 1[a]	Experimental group 2[b]	Comparison group
Number of schools	21	17	18
Number of classes	33	33	45
Number of students	801	811	1100
grade 3	24	10	12
grade 4	140	153	307
grade 5	326	340	458
grade 6	300	297	316
% with a very low SES[c]	3.2%	0.3%	3.8%
% boys	47.8%	47.8%	50.4%

[a] Students participated in the lesson series and their teachers in the PD program. [b] Students participated in the lesson series and teachers did not participate in a PD program. [c] Students had a very low Socioeconomic Status (SES) if elementary school was the highest completed education of at least one of the parents.

5.2.2. The MACE program

The MACE program consisted of a lesson series for fourth, fifth, and sixth grade students in which geometry and visual arts education were integrated, along with a PD program for teachers (Keijzer, Oprins, De Moor, & Schoevers, 2018). In a pilot study with 15 teachers, an initial version of the program was formatively evaluated and adjusted afterwards (Schoevers & Kroesbergen, 2017).

Description of the MACE lessons. The series of MACE lessons consisted of nine lessons which took each 60–90 minutes; five lessons related to the theme space and four to the theme patterns (Meetkunst projectteam, 2018b). Each lesson started with a whole-class introduction, which took 15–25 minutes, in which students had to observe and discuss visual artworks in relation to the topic of the lesson. The introduction was followed by an open activity in which students had to create an artwork and this took 25–30 minutes. During the activities students worked mostly in (small) groups. A lesson ended with whole-class reflection, which took about 10 minutes, in which teachers discussed students' creative process and the artwork they had created, and what students may have learned.

Below we describe the MACE lesson "playing with perspective" in more detail. In Appendix A, a short description of each MACE lesson is given. In the lesson "Playing with perspective" the teacher has to start with an introduction in which teacher and students discuss six artworks in which the artists have explored perspective and viewpoints, such as the artwork 'Another World' by M.C. Escher and photo's in which photographers manipulated perspective. Questions that a teacher could ask during this introduction are stated in the manual, such as 'How did the artist create this effect?', 'Can you tell something about the viewpoint of the artist, and what could be the reason for this?' and 'How would the photo look like when they would have used another point of view?'. After the introduction, students have to make photos in which they create visual illusions by playing with perspective and point of view in groups of 3–4 students in- or outside the school building. After 15–20 minutes, students have to select their two best photos. At the end of the lesson the teacher has to discuss the selected photos of the students and the process of making the photos in a whole-class setting. Questions that the teacher could ask are stated in the manual, such as 'What effect did you want to create?', 'What did you do to create this effect?', 'What perspective did you use?' and 'Where would you stand if we would draw a map?'. Furthermore, the teacher has to ask students to reflect on what they have learned (Schoevers & Kroesbergen, 2017). Examples of two other MACE lessons are described in the case study in Chapter 4 of this dissertation.

Description of the PD program. The PD program for teachers consisted of five sessions (2.5 hours each), which were guided by experts in the field of mathematics and visual arts education (Meetkunst projectteam, 2018c). After each PD session, teachers had to teach one or two activities of the MACE lessons series in their own schools. An overview and timeline of the MACE program can be found in Figure 1. The aim of the PD program was to train teachers how to stimulate students' creative thinking in this integrated visual arts and

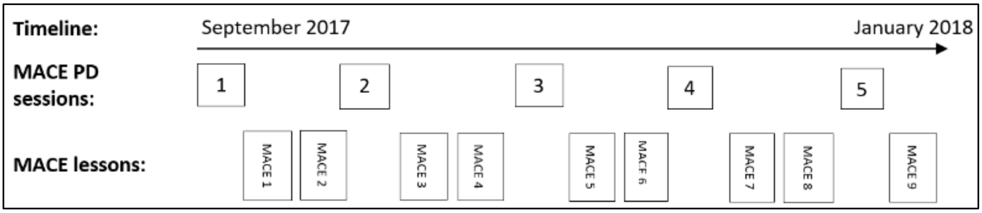


Figure 1. An overview of the MACE program.

mathematics program. Furthermore, the aim was to create a positive attitude of the teachers towards geometry, visual arts and the integration of both. Moreover, it aimed to increase teachers' geometrical knowledge and their pedagogical content knowledge of geometry and visual arts education.

Within the PD program, active learning was considered important. Therefore, interactive methods were used in the sessions. Teachers, for example, had to experience the MACE lessons themselves, watch film fragments of other teachers and had to design a hypothetical learning trajectory. Afterwards, teachers always had to discuss and reflect on these activities. The content of the PD program was related to the classroom practice. Furthermore, reflection on the MACE lessons was important; it could support on-going learning and encourage change.

5.2.3. Regular geometry lesson series

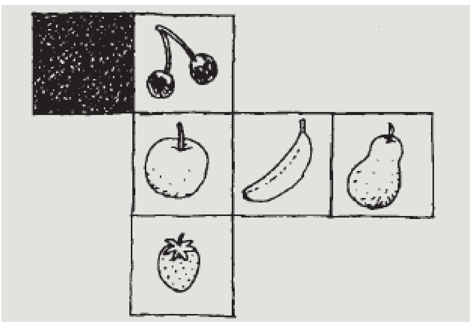
A lesson series was established for the comparison group in which geometry lessons from several widely used Dutch mathematical textbooks were combined and adjusted to have the same time and intensity of geometry instruction as in the MACE lesson series. We considered mathematical textbook lessons as regular geometry lessons, because most teachers, especially in the Netherlands, use mathematical textbooks to teach the geometry curriculum (Hop, 2012). Most used Dutch mathematics textbooks mainly offer closed-ended routine problems (Van Zanten & Van den Heuvel-Panhuizen, 2018). The (type of) problems, duration and lesson structure of these textbook-based geometry lessons were applied and used in the regular geometry lesson series for the comparison condition. The regular geometry lessons series consisted of 7 lessons, of which four related to two- and three dimensionality, block constructions, floor maps and perspective. The other three lessons were related to patterns, symmetry and rotation. The regular geometry lessons took between 30–40 minutes. Each lesson started with a short whole class introduction (5–10 minutes) in which the subject of the lesson was introduced. Afterwards, students independently worked on geometry problems in a workbook (15–20 minutes). These were mainly multiple choice problems. The lesson was completed by a short (5 minutes) whole class reflection on the lesson.

5.2.4. Instruments

Geometric Ability Test. The GAT measured whether students understood and could explain geometrical phenomena. The GAT was developed for this study. The test took between 20–30 minutes and was stopped after 30 minutes. The test consisted of 11 geometry problems. The test started with four closed-ended routine problems that called upon spatial sense and spatial visualization (see the left sample question in Figure 2) and ended with seven art-geometry problems in which students mainly had to reason geometrically in relation to a painting (see the right sample question in Figure 2). As the test was specifically developed for this study, no a priori information on item difficulty and other item characteristics was available. Therefore, within problem type, the order of the problems was randomized and the same for all students to prevent unknown order effects. Two equivalent versions (i.e., A and B) of the GAT were respectively used as a pre-, and post-test.

Scoring. The spatial visualization problems were relatively straightforward with clearly one correct answer. Therefore, one point was given for a correct answer and zero points for an incorrect answer. The geometry problems related to visual arts were more complex and could also yield partially correct solutions. Therefore, two points were given for an answer in which students showed to be able to reason correctly about geometric phenomena (e.g., when students explained why things in front of the painting are bigger than in the back of the painting and, for example, would use the term perspective). One point was given for an answer in which the reasoning was not complete (e.g., 'by painting a line down in a curve'). Zero points were given for answers in which the question was repeated or there was no reasoning about the question involved (e.g., 'because when you paint you can make everything'). The last art-geometry question of the test was considered too difficult for grade 4 and 5; only 9% of the students scored one or two points (Hopkins & Antes, 1978). Therefore, this question was not used in this study. Furthermore, 27.1% of the students were able to finish the GAT in the maximum time of 30 minutes. Therefore, we calculated an average score based on the number of questions the students were able to complete. The maximum score that could be obtained was 1.6. Since items were not in an ascending order of difficulty, this was the most valid way to calculate a total score.

Peter folds the cube. He puts the black side of the cube at the bottom. What fruit is on top of the cube?



The painting is flat, but you can see depth. How is that possible? Explain in words. You can also draw.

Figure 2. Sample questions of the GAT

Reliability and validity of the GAT. Test-retest reliability of version A is acceptable (r = .66) and good for version B (r = .77; Schoevers, 2018). Alternative forms reliability of the GAT is sufficient (r = .80; Schoevers, 2018). Furthermore, form A and B are similar regarding difficulty of the items (Schoevers, 2018). Criterium validity of the GAT, as measured by the correlation between the GAT and general math ability score, was moderate (r = .40 - .42), but sufficient since the mathematical domain geometry covers only a small part of the general mathematical ability test. Tests at the pre- and posttest were scored by four raters. Interrater reliability (IRR) was sufficient to excellent for all items on both the pre- and posttest (κ = .81 – 1.00, ICC = .67 – 1.00). In line with our expectations, the internal consistency of the GAT was not very high for both versions (version A, α = .62; version B, α = .58), because the GAT represents a heterogenous set of skills.

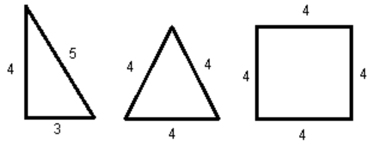
Geometrical vocabulary. Active geometrical vocabulary was scored if students correctly used geometrical words in the open questions of the GAT. A list of geometrical words was composed by using the learning goals of elementary school geometry education (Van den Heuvel-Panhuizen & Buys, 2005) and elementary school teacher education (Van Zanten, Barth, Faarts, Van Gool, & Keijzer, 2009). We distinguished between tier 1 and tier 2 words (Henrichs & Leseman, 2014). Domain-general academic tier 3 words were not included in this study, because they were not of interest. Tier 1 words are geometrical words used in a daily language environment (e.g. ‘turned’ or ‘flat’). Tier 2 words are domain-specific academic geometrical words used in mathematics education (e.g. ‘vertical’, ‘horizontal’, ‘square’ and ‘triangle’). The complete list of words can be found in Appendix B.

5

Scoring. The total number of words, of tier 1 and of tier 2 words appearing in the writings were counted for each question in the GAT. Regarding tier 2 words a token (i.e., total number of tier 2 words used) and type score (i.e., the number of different tier 2 words used) were calculated. Since, these measures were highly correlated (r > .90), we only used the token score. Next, a ratio of the number of tier 2 geometrical words to the total number of words used was calculated for each question of the GAT. This ratio was calculated to adjust for differences in students’ wordiness. A total score for the GAT was calculated by averaging the ratio scores of the different questions. Geometrical vocabulary was scored by four raters. IRR was good in this study with regard to the total word written (ICC > .99), the number of tier 1 words (ICC > .88), and the total number of tier 2 words (ICC > .86) for both the pre- and posttest.

Geometrical Creativity Test (GCT). The domain-specific GCT was used to measure geometrical creative thinking. It was developed for this study and based on the Mathematical Creativity Test used in Chapter 2 of this dissertation. The GCT took between 20 – 30 minutes and was stopped after 30 minutes. It consisted of four geometry questions and one problem posing question, which were open-ended and could have multiple correct answers. Multiple solution and problem posing tasks are commonly used to measure creativity in mathematics (Leikin, Koichu, & Berman, 2009; Leikin, 2009; Silver, 1997). Students were instructed to provide multiple, but distinct solutions, which moreover, had to be original. In the problem posing questions, students were asked to pose mathematical questions based on a photo.

Look carefully at the three given shapes. Which shape does not belong in the same group as the other two? Explain your answer. Are there more than one possible answers? If so, write down as many answers as possible.



Shape _____ does not belong in this group of shapes because……………………………………………………

Figure 3. A sample question of the GCT Version A

Version A was used for the pre-test, and version B was used for the post-test. A sample question of the GCT is depicted in Figure 3.

Scoring. For the scoring of the GCT we used the scoring scheme of Leikin for creativity in the individual solution space (Leikin, 2009; Levav-Waynberg & Leikin, 2012a). Within this scheme a distinction between fluency, flexibility and originality is made. The scheme was further elaborated in Leikin (2009), Levav-Waynberg and Leikin (2012a), and Leikin, Koichu and Berman (2009). Fluency was calculated by adding the number of correct answers for each question. With the use of the scheme, each solution of a student was scored regarding flexibility and originality. Next, a final score per solution was calculated as the product of $Flexibility_i$ x $Originality_i$. Afterwards, a creativity score per question was concluded as: $Fluency_j$ x (∑ ($Flexibility_i$ x $Originality_i$)). Last, the creativity scores of all questions were added into a total creativity score.

Reliability and validity of the GCT. With regard to the test-retest and alternative forms reliability we used the strategy of calculating a creativity score as described by Kattou, Kontoyianni, Pitta-Pantazi and Christou (2013). We used the fluency score as indicator of creativity. Test-retest reliability of versions A (r = .84) and B (r = .89) are good. Alternative forms reliability of the GAT is sufficient (r = .68; Schoevers, 2018). Furthermore, versions A and B are similar regarding difficulty of the items. In this study, the GCT was scored by seven raters. IRR was sufficient to excellent for all scores per solution on both the pre- and posttest (ICC = .72 – .99). In line with our expectations, the internal consistency of the GCT was not very high in this study for both versions (version A, α = .68; version B, α = .55), because geometry comprises a heterogenous set of knowledge and skills.

Visual Arts Assignment (VAA). In the VAA students have to write down as much as they can about a painting (see Figure 4) in answering the following questions: 'What is going on in this painting?', 'What do you see that makes you say that?', and 'What more can you find?' (Housen, 2002). The same task was used as a pre- and posttest.

Figure 4. Emmanuel De Witte – Interior with a woman at the virginal (Museum Boijmans van Beuningen, Rotterdam, the Netherlands).

Scoring. A scoring scheme for the VAA was created for this study based on literature about visual arts perception in education (See Table 2; KPC-Groep, 2000; Stichting Leerplanontwikkeling, 2015; Van Onna & Jacocobse, 2008). The VVA was scored on four aspects that were related to geometry, namely 'space', 'space suggestion', 'shape' and 'composition'. We scored how often each aspect occurred in the written text of the student. Furthermore, the number of words written was counted. Next, each score on each aspect was divided by the number of words written and multiplied by 100, to take the talkativeness of students into account. The pre- and posttest was scored by five raters. Interrater reliability of most aspects was good to excellent (ICC = .76 – 99).

5.2.5. Procedure

Data were collected in the fall of 2017 by the first author and twelve research assistants with a bachelor or master's degree in (special) education. Before the start of the MACE program, in one session, the pre-tests were administered to the whole class by a research assistant who read aloud the test instructions. Post-tests were administered after the MACE program in the same way as the pre-tests. Furthermore, teachers had to provide information about students' age, gender, mathematical ability (based on the national mathematical ability tests), and the educational level of both students' parents, based on information from school records. Pre- and posttests were coded afterwards by the same research assistants. Research assistants received extensive training and had to reach sufficient interrater reliability with the master coder before they were allowed to administer the pre- and posttests in the classroom, to conduct observations in the classroom, and to code part of the tests. Passive informed consent of the parents was obtained before the start of the study. This form of consent was

Table 2.
Scoring aspects of the VVA

Scoring aspect	Operationalization
a) space	The aspect of ‘space’: The student expresses whether something is in front/back/in/ between/under etc., how the space is ordered, spatial constructions, filling up space (closed), encompassing space (open). *Example. I see a lady cleaning in the back room*
b) space suggestion	Space suggestion: big-small; mirroring; depth etc. *Example: I see depth in the painting; it is a big house; I see 3D in the painting; I see that through the mirror.*
c) shape	Shapes (2dimensional and 3 dimensional); shape characteristics (point, angle, etc.); lines. *Example: I see a squared mirror*
d) composition	Composition – how the whole painting is built up in pieces. *For example, ordering by shape, color or texture; rhythm, repetition of shapes; motives for decoration; pattern; ordering; balance and meaning; symmetric-asymmetric, horizontal-vertical). Example: I see a pattern on the tiles; the painting is light in front and dark in the back; There are three rooms in the house*

approved by the Ethical Committee of the Faculty of Social and Behavioural Sciences of Utrecht University (FETC15-083). 0.8% of the students did not have consent for this study.

5.2.6. Analyses

To take the nested structure of the data into account, multilevel analyses with three levels were conducted in HLM6 (Hox, Moerbeek, & Van de Schoot, 2018). The first level represented the repeated measurements (pre- and posttest), as nested within individuals. The second level represented the students, and the third level the classes. It was not necessary to include the school level as a fourth level, since there was only little variance in the different pretests located at this level (1–5 % of the variance). For each outcome variable, first a base model was created with time as a predictor. In a second model, student-level covariates were added to control for spuriousness: grade, gender (Frost, Hyde, & Fennema, 1994; Mann, 2006), SES (e.g., Crane, 1996), and general mathematical ability. It was expected that students' general mathematical ability would not have an effect on students' performance on the VAA and student's use of geometrical vocabulary, and therefore, this variable was not used as a covariate in the multilevel models related to these measures. Furthermore, it was expected that students with a low SES would score lower on the GAT, GCT and VAA (Crane, 1996). Therefore, we controlled for low SES by using two dummy variables: students with a low and students with a very low SES. Students had a very low SES if elementary school was the highest completed education of at least one of the parents. Since only a small percentage of students had a very low SES, we also included students with a low SES: vocational education was the highest completed education of both parents. In a third model, third level predictors were added as dummy variables: lesson series condition, PD program condition. Furthermore, two class-level covariates were added that were expected to influence the results of the effects of the MACE program: the number of MACE lessons given by the teacher and the number of MACE PD sessions followed by the teacher. In the fourth model, the random slope of time was added to investigate whether students' growth on an outcome variable differed per class. In the fifth model dummy variables of condition (whether or not students had received MACE lessons and whether or not the teachers had participated in the MACE PD program) were added as predictors of the slope of time to investigate whether differences between classes could be explained by conditions.

Before the multilevel analyses were conducted, data were screened and prepared and a missing value analysis was conducted. On the pre-test between 6.6 – 9.2 % of the data were missing. On the post-test between 9.7 – 10.6% of the data were missing. Data were missing because some students were ill or for other reasons not present in the class during the test administration. Furthermore, assumptions for multilevel analysis were checked (Hox et al., 2018). The main assumption is a sufficient sample size. With the use of SPA-ML (Moerbeek, 2015), we calculated the required sample size. With a desired power of .80 and an assumed effect size of .30 in a three-level model, this study required a sample of at least 52 schools, 110 classes and 2496 students. These criteria were met well. Furthermore, the assumptions of linearity and absence of outliers were checked, by inspecting scatterplots at the different levels. The assumption of linearity was met for all variables, but for some variables outliers were detected. One outlier was detected for the tier 2 academic words used in the GAT on the student level. This outlier was deleted, because the score was unrealistic. In addition, the assumption of normally distributed residuals at all levels was tested. This assumption was mildly violated for the residuals at the first and second level of the GCT. However, this was not considered a problem since the maximum likelihood estimator is robust against this violation with a large sample size (Hox et al., 2018). Normality of residuals was more seriously violated for the geometry words used in the GAT, and for the aspects on the VAA. Therefore, robust standard errors were used and reported (Hox et al., 2018).

5.3. Results

5.3.1. Descriptive statistics

In Tables 3 to 6 the means and standard deviations of students' pre- and posttest scores on the GAT, geometrical vocabulary, GCT and VAA are described. In Table 7 Spearman correlations between all measures on the pre-test are presented. The correlations, for example, indicated that the measures of geometrical ability—GAT, GCT and daily geometrical words—are significantly positively related. However, students' use of academic geometrical words was negatively related to the GAT, GCT and daily geometrical words.

Table 3.

Descriptive statistics of students' scores on the GAT

	Experimental group 1	Experimental group 2	Comparison group
	M (*SD*)	*M* (*SD*)	*M* (*SD*)
GAT Pretest	.54 (.30)	.60 (.28)	.52 (.29)
GAT Posttest	.66 (.29)	.73 (.28)	.61 (.28)

Table 4.

Descriptive statistics of ratio scores of geometrical vocabulary

	Experimental group 1		Experimental group 2		Comparison group	
	Pre-test *M* (*SD*)	Post-test *M* (*SD*)	Pre-test *M* (*SD*)	Post-test *M* (*SD*)	Pre-test *M* (*SD*)	Post-test *M* (*SD*)
Tier 1[a] words in the GAT	.11 (.06)	.12 (.06)	.11 (.06)	.12 (.06)	.11 (.07)	.12 (.07)
Tier 2[b] words in the GAT	.06 (.09)	.06 (.09)	.06 (.08)	.05 (.07)	.07 (.10)	.05 (.07)
Number of words written down	40.94 (26.71)	51.09 (31.02)	46.62 (27.48)	51.70 (27.56)	41.23 (23.39)	42.74 (24.54)

[a]Daily geometrical words, [b]Academic geometrical words

Table 5.

Descriptive statistics of the GCT

	Experimental group 1	Experimental group 2	Comparison group
	M (*SD*)	*M* (*SD*)	*M* (*SD*)
GCT Pretest	900.42 (915.66)	1032.85 (1040.36)	865.58 (919.15)
GCT Posttest	912.41 (991.04)	1151.43 (1197.60)	910.06 (1022.64)

Table 6.

Descriptive statistics of the VAA

Aspect	Experimental group 1		Experimental group 2		Comparison group	
	Pre-test *M* (*SD*)	Post-test *M* (*SD*)	Pre-test *M* (*SD*)	Post-test *M* (*SD*)	Pre-test *M* (*SD*)	Post-test *M* (*SD*)
a) space	3.10 (3.66)	4.65 (4.07)	3.53 (3.66)	5.71 (4.47)	3.32 (3.45)	4.00 (4.02)
b) space suggestion	0.77 (1.58)	1.49 (3.61)	0.97 (1.79)	1.29 (2.21)	0.86 (1.60)	0.76 (1.60)
c) shape	0.10 (0.54)	0.19 (0.96)	0.10 (0.60)	0.17 (0.72)	0.20 (1.35)	0.14 (0.66)
d) composition	0.11 (0.50)	0.51 (1.42)	0.06 (0.36)	0.56 (1.33)	0.10 (0.50)	0.14 (0.57)

Table 7.

Spearman correlations between the measures on the pre-test

	1	2	3	4	5	6	7
1. GAT	-						
2. GCT	.28**	-					
3. Daily geometrical words	.24**	.09**	-				
4. Academic geometrical words	-.14**	-.06**	-.18**	-			
5. VAA ‘space’	.20**	.16**	.06**	-.03	-		
6. VAA ‘space suggestion’	.13**	.11**	.04	.01	.09**	-	
7.VAA ‘shape’	.01	.03	.01	-.01	.07**	.05*	-
8.VAA ‘composition’	.08**	.05*	.02	.00	.06**	.03	.07*

** Significant at $p < .001$ (2-tailed), *Significant at $p < .05$ (2-tailed)

5.3.2. Results of the multilevel analyses

Geometrical ability. With regard to the GAT, analyses showed that there was a linear relation between time and geometrical ability $t(3831) = 18.71$, $p < .001$. On average, students' scores on the GAT increased between the pre- and posttest. Furthermore, we found that the relation between time and GAT differed per class, $\chi^2(88) = 343.50$, $p < .001$. These differences, however, could not be explained by the MACE lessons series ($t(90) = 1.61$, $p = .11$), nor by the MACE PD program ($t(90) = -.29$, $p = .77$). In Table 8, only the results of the final model are presented. The results of all other models are included in Appendix C.

With regard to geometrical vocabulary, the analyses showed that there was a significant linear relationship between time and students' use of daily (tier 1: $t(4083) = 4.74$, p $< .001$) and academic geometrical words (tier 2: $t(4083) = -4.11$, $p < .001$) in the GAT. The proportion of daily geometrical words increased between the GAT pre- and posttest, but decreased for academic geometrical words. Next, we found that the rate of improvement differed between classes (tier 1; $\chi^2(92) = 263.58$, $p < .001$; tier 2; $\chi^2(92) = 134.52$, $p < .01$); students in some classes improved more regarding their use of daily and academic geometrical words in the GAT than students in other classes. With regard to the daily geometrical words (tier1), the difference could be explained by the participation in the MACE lesson series ($t(94) = 2.60$, $p < .05$), but not by participation in the MACE PD program ($t(94) = -1.42$, $p = .16$). When a class had received the MACE lesson series, the positive relation between time and daily geometrical words (tier 1) used in the GAT, became stronger. With regard to academic geometrical words (tier 2), the difference between classes could not be explained by the MACE lesson series ($t(94) = 0.36$, $p = .72$), nor by participation of the teacher in the MACE PD program ($t(94) = 0.99$, $p = .33$). See Table 8 for the final models of the geometrical words used in the GAT.

Furthermore, analyses showed a linear relationship between time and geometrical creativity ($t(3977) = 3.04$, $p < .01$). On average, students' scores on the GCT increased between the pre- and posttest. Furthermore, we found that the rate of improvement on the GCT varied between classes, $\chi^2(90) = 238.94$, $p < .001$. This variation, however, could not be explained by the MACE lesson series ($t(91) = 0.74$, $p = .46$), nor by the MACE PD program ($t(91) = -1.28$, $p = .20$; see final model in Table 8).

Visual arts perception. Regarding visual arts perception, we found a linear relationship between time and the aspects 'space' as used in the VAA ($t(4206) = 7.80$, $p < .001$), 'space suggestion'($t(4206) = 2.37$, $p < .05$), and 'composition' ($t(4206) = 6.19$, $p < .001$). Students, on average, described more of these aspects in the VAA posttest compared to the VAA pretest. The rate of improvement on the aspects 'space' ($\chi^2(94) = 260.57$, $p < .001$), 'space suggestion'($\chi^2(94) = 232.81$, $p < .001$) and 'composition' ($\chi^2(94) = 300.95$, $p < .001$) differed between classes; students in some classes improved more in their use of these aspects in describing visual artworks than students in other classes. These differences between classes could be explained by the MACE lesson series (space, $t(94) = 3.66$, $p < .01$; space suggestion, $t(94) = 2.14$, $p < .05$; composition, $t(94) = 4.88$, $p < .001$). However, the MACE PD program did

Table 8.

Multilevel results (final models) regarding geometrical ability

	Understanding and explanation of geometric phenomena (GAT)	Daily geometrical words used in the GAT (tier1)[a]	Academic geometrical words used in the GAT (tier 2)[a]	Geometrical creativity (GCT)
Fixed part	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)
Intercept	0.37 (.02)**	0.11 (.00)**	0.08 (.01)**	444.12 (66.40)**
Time	0.09 (.02)**	0.01 (.00)	-0.02 (.00)**	65.84 (55.65)
Grade 3 (dummy)[b]	-	-	-	24.29 (148.03)
Grade 5 (dummy)[b]	0.10 (.02)**	0.01 (.00)*	0.00 (.00)	299.18 (64.08)**
Grade 6 (dummy)[b]	0.21 (.02)**	0.01 (.00)**	-0.01 (.00)*	555.55 (66.91)**
Gender[c]	0.10 (.01)**	-0.01 (.00)**	-0.01 (.00)**	269.84 (34.22)**
SES low[d] (dummy)	-0.08 (.03)**	-0.01 (.01)*	0.01 (.01)	-126.79 (107.48)
SES very low[d] (dummy)	-0.15 (.03)**	-0.02 (.01)**	0.02 (.01)	-143.69 (106.72)
General math ability	0.12 (.01)**	-	-	223.12 (18.99)**
Lesson series condition (dummy)[e]	-0.12 (.11)	-0.02 (.01)	-0.01 (.01)	343.96 (382.42)
PD program condition (dummy)[f]	-0.11 (.08)	0.02 (.01)	0.02 (.02)	142.65 (289.21)
Number of MACE lessons given	0.02 (.01)	0.00 (.00)	0.00 (.00)	-35.62 (44.38)
Number of PD sessions followed	0.01 (.02)	-0.00 (.00)	-0.00 (.00)	-38.73 (65.30)
*Time*Lesson series*	0.04 (.03)	0.02 (.01)*	0.00 (.01)	68.19 (91.77)
*Time*PD program*	-0.01 (.03)	-0.01 (.01)	0.01 (.01)	-139.02 (108.23)
Random part				
σ_e^2	0.029	0.00320	0.00572	549285.85
σ_{u0}^2	0.024	0.00049	0.00089	282663.42
σ_{v0}^2	0.007	0.00016	0.00029	69448.79
σ_{v1}	0.008	0.00048	0.00027	82455.98

*Significant at p <.05, **Significant at p <.01, [a]Robust standard errors are used, [b]Grade 4 is used as a reference group, [c](Male=0, Female=1), [d](medium or high SES =0, very(low) SES=1), [e](received no MACE lessons=0, received MACE lessons=1), [f](received no MACE PD sessions=0, received MACE PD sessions =1)

Table 9.

Multilevel results (final models) regarding the aspect used in the VAA

	Aspect 'space'[a]	Aspect 'space suggestion'[a]	Aspect 'shape'[a]	Aspect 'Composition'[a]
Fixed part	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)	Unstandardized coefficient (s.e.)
Intercept	2.85 (.24)**	0.61 (.09)**	0.18 (.05)**	0.03 (.03)
Time	0.67 (.23)**	-0.09 (.09)	0.01 (.03)	0.04 (.03)
Grade 3 (dummy)[b]	-1.61 (.67)*	-0.23 (.26)	-0.11 (.06)	-0.01 (.06)
Grade 5 (dummy)[b]	0.10 (.22)	0.22 (.09)*	-0.04 (.04)	0.05 (.04)
Grade 6 (dummy)[b]	0.80 (.22)**	0.25 (.09)*	-0.05 (.04)	0.10 (.04)*
Gender[c]	0.56 (.14)**	0.15 (.06)*	0.03 (.03)	0.03 (.03)
SES low (dummy)[d]	-0.80 (.30)*	-0.08 (.13)	-0.03 (.06)	0.03 (.07)
SES very low (dummy)[d]	-0.92 (.37)**	-0.05 (.13)	-0.06 (.05)	-0.09 (.05)
Lesson series condition (dummy)[e]	-1.17 (.73)	-0.27 (.36)	-0.04 (.13)	-0.03 (.08)**
PD program condition (dummy)[f]	-0.63 (.74)	-0.27 (.22)	0.06 (.12)	-0.10 (.07)
Number of lessons given	0.14 (.09)	0.05 (.04)	0.00 (.02)	0.03 (.02)**
Number of PD sessions followed	0.02 (.18)	-0.01 (.05)	-0.01 (.03)	0.03 (.01)*
*Time*Lesson series*	1.48 (.40)**	0.42 (.20)*		0.44 (.09)**
*Time * PD program*	-0.43 (.42)	0.27 (.24)		-0.03 (.13)
Random part				
σ_e^2	10.92	2.78	0.77	0.59
σ_{u0}^2	2.58	0.29	0.00	0.03
σ_{v0}^2	0.79	0.03	0.02	0.00
σ_{v1}	1.32	0.30	0.01	0.08

*Significant at p <.05, **Significant at p <.01, [a]Robust standard errors are used, [b]Grade 4 is used as a reference group, [c](Male=0, Female=1), [d](medium or high SES =0, very(low) SES=1), [e](received no MACE lessons=0, received MACE lessons=1), [f](received no MACE PD sessions=0, received MACE PD sessions =1)

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not show such an effect (space, $t(94) = -1.02$, $p = .31$; space suggestion, $t(94) = 1.13$, $p = .26$; composition, $t(94) = -0.30$, $p = .76$).

Regarding the aspect of 'shape' used in the VAA, no linear relationship with time was found ($t(4206) = 0.39$, $p = .70$), indicating that students, on average, did not describe this aspect more frequently in the VAA posttest compared to the VAA pretest. This relation was the same for all classes ($\chi^2(94) = 94.96$, $p = .45$), indicating no effect of the conditions. The final multilevel models regarding the aspects used in the VAA are presented in Table 9. The results of the other models can be found in Appendix C.

5.3. Discussion

The MACE program aimed to teach the (overlapping) curriculum goals of visual arts and the mathematical domain of geometry, and to promote students' creative thinking skills in both disciplines, by creating opportunities for students to act creatively in an integrated visual arts and geometry context. The program was evaluated in a quasi-experimental study in which students were assigned to three conditions: (1) students who received the MACE lesson series from their teachers who received a PD program, (2) students who received the MACE lesson series, and (3) students who received regular geometry lessons. Students' growth between pre- and post-measurements of geometrical ability and visual arts perception were examined to test the effect of the conditions.

Students' ability to understand and explain geometrical phenomena improved in all conditions. However, contrary to our expectations, no differences between conditions were found. A possible explanation is that the lessons of the comparison condition were actually also an intervention that improved regular geometry education. Teachers indicated that the lessons they taught were similar to regular geometry lessons, but that they enabled more interaction between students than they were used to. As a consequence, students in the comparison condition had more than in regular lessons opportunities to explain their thinking, to receive feedback and to encounter other points of view, which could have enhanced their geometrical thinking (Beghetto & Kaufman, 2010; Taggar, 2002). Furthermore, more interaction between students could also evoke reflection, which might have enabled students to reach the same level of understanding as students who participated in the MACE program (Van den Heuvel-Panhuizen & Drijvers, 2014). Furthermore, in the comparison condition, the lessons were offered in a sequence, while usually the 4 to 6 geometry lessons are spread over the school year. Interestingly, our analyses showed that students in some classes significantly improved more on their ability to understand and explain geometrical phenomena than students in other classes. Differences between classes could not be explained by the type of lessons they received. This result seems to imply that for students' ability to understand and explain geometrical phenomena, the content and structure of the lesson material is of less importance than other factors. Plausible factors could be the implementation of the lesson and the quality of the teacher. For example, despite the teaching materials and its related teaching approach, some teachers may stimulate more communication between students than other teachers, or may be better able to ask open questions that can extend students thinking, reasoning and understanding than other teachers (Bostic, 2011). Future research should investigate these possible factors.

Also, students' geometrical creative thinking improved in all conditions, but did not differ between the conditions. This result was not in line with our expectations. We expected that implicitly and explicitly stimulating students to act creatively and think divergently (Sawyer, 2014), especially in integrated visual arts and geometry lessons, would lead to more improvement in students' geometrical creativity compared to students who did not receive such stimulation. However, all students improved regarding geometrical creativity, which could be due to natural growth of students. Although we found in a qualitative case study that students expressed more mathematically creative ideas and solutions in classroom dialogues during the MACE lessons than they did during a regular mathematics lesson (Chapter 4), one MACE lesson per week may not be enough to bring large improvement of students' geometrical creativity.

Regarding students' use of geometrical vocabulary to explain geometric phenomena, we found a partial effect of the MACE program. Students who participated in the MACE program increased in the use of daily geometrical words proportional to the total number of words in their written explanations in the geometrical ability test more than students who received regular geometrical lessons. However, the increase of students' daily geometrical words was not a goal of the MACE program. Instead, increase of the use of academic geometrical words was a goal, which however was not influenced by the MACE program. In fact, students in all conditions used less academic geometrical words, controlled for the number of words in the written explanations at the post-test. Furthermore, no effect was found for the MACE PD program. This was also not in line with our expectations. Since the importance and stimulation of, especially, academic geometrical words was emphasized in the manual of the lesson series and in the PD program, we expected that students' use of academic geometrical words would increase. One explanation for these results is that students used more words at the posttest to explain their answers compared to the pretest, but did not use more academic geometrical words. As a result, the proportion of academic geometrical words used decreased.

In contrast to the previous results, we did find that students' perception changed regarding geometrical aspects in visual arts. Students who received the MACE lesson series showed more improvement in describing part of the geometrical aspects addressed in the lesson series (i.e., space, space suggestion and composition, but not regarding the aspect of shape) in a visual artwork compared to students in the comparison group. This result is in line with our expectations. Since students had to observe and analyze visual artworks mainly with regard to the aspects of space and patterns in every MACE lesson, they changed their recognition of visual information in visual artworks in this respect (Kozbelt, 2001; Tishman et al., 1999). The aspect of shapes played a smaller role in the lesson series, explaining why no effects were found with regard to this aspect. Participation of the teacher in the MACE PD program did not affect students' perception of the spatial aspects of visual art. A possible explanation for this result is that all teachers indicated that the teaching manual of the MACE lesson series was elaborate and clear. For example, many sample questions were stated in the manual that a teacher could use during visual art perception that were related to these spatial aspects of visual art. Since the teaching manual was elaborate in this regard, the PD program probably had not much added value and did not affect students' perception of spatial aspects of visual art.

5.3.1. Limitations and future research

It is important to take the limitations of the present study into account. A first limitation of this study is that we were not able to randomly assign the teachers and students to the different conditions. A second limitation is our measure of geometrical creativity. Although our measure of geometrical creativity, a multiple solution task, is commonly used in the field (Leikin, 2009), it may not have been sensitive enough to do full justice to the multidimensional construct of geometrical creativity. Our measure does provide information on the cognitive aspect of geometrical creativity, but other quantitative and qualitative measures could be used in future to measure other dimensions of creativity, such as how creativity was verbally or non-verbally expressed by students in the classroom. A third limitation is that no classroom observations were conducted in the comparison group. If these would have been conducted, classroom factors could have been investigated that could possibly explain the differences between classes in students' growth in understanding and explaining of geometric phenomena. Furthermore, another comparison group could have been used. Our current comparison group also received a lesson series with regular geometry lessons. The lessons differed in several aspects from the MACE lessons. However, we were not able to investigate differences with regard to real educational practice, in which geometry lessons are usually taught spread over the whole school year. Future research, could take this into account.

5.3.2. Conclusion

The MACE program had the aim to teach the (overlapping) curriculum goals of visual arts and the mathematical domain of geometry, and to promote students' creative skills in both disciplines by creating opportunities for students to act creatively in an integrated visual arts and geometry context. Where in regular education geometrical concepts are directly and time-efficiently taught, students that received the MACE lesson series reached the same level of geometrical understanding with open lessons in which they were able to express themselves creatively. Although students who received the MACE lesson series did not improve more on geometrical creativity and academic geometrical vocabulary than students who followed regular education, students did improve more in their ability to perceive geometrical aspects in visual arts and their use of daily geometrical words to describe geometric phenomena.

For educational practice, the results of this study imply that teachers could use the integrated MACE lesson series instead of regular geometry lessons, in order to teach the geometry curriculum and visual art perception. Teaching in this integrated way may save time for teachers who experience a lot of time pressure to teach the curricula. If the integrated

MACE approach and pedagogy are also applied to other mathematics lessons during the week, the integrated MACE approach may also be more effective for students' creative skills. However, more research is necessary. This study, is to our knowledge, the first study that evaluated the effectivity of integrated visual arts and geometry education with a clear theoretical framework and research design. The study and the theoretical framework can be a valuable contribution to research on interdisciplinary arts and mathematics education.

5.4. Appendices

5.4.1. Appendix A. Short description of the MACE lesson series

MACE Lesson	Description
1. Catching space in the classroom'	In this lesson the concept of 'space' is explored. Students and teacher discuss their perception of space and how visual artists used space in their artworks. In the activity, students explore how they can enclose space with one sheet of A4 paper.
2. From art to space: a scale model	Visual art is often an interpretation of (3-dimensional) reality. To visualize reality, it is often scaled down and made 2 dimensional. Students and teacher discuss this theme in relation to visual artworks. Afterwards, students have to create a 3-dimensional representation of a 2-dimensional painting.
3. Space outside the classroom	In this lesson students explore the concept of space in an out-of-school context (e.g., museum, neighborhood of the school or statue near the school).
4. From space to flat	In relation to paintings, teachers and students discuss how visual artists suggest spatial depth by painting a flat surface (e.g. walls/floors/ceilings). In the activity students have to draw the corner of their classroom on an angle-folded A4.
5. Playing with perspective	During the introduction visual artworks are discussed in which artists played with perspective and proportions and created optical illusions. Afterwards, students made photo's in groups of four, in which they create an optical illusion by playing with perceptive.
6. What is a pattern?	Students explore the concept of patterns in examples of daily life (pattern in a day, in music, stories, numbers, artworks, decorations). Next, students will explore if and how they can make a pattern on randomly fallen blocks.
7. Laying tiles	The subject of this lesson is about characteristics of (regular) patterns. Different ways of repetition, composition and symmetry in visual artworks are discussed. In the activity, students will explore, describe and compare different patterns with simple basic tiles.
8. Mirror, Mirror, what do I see?	In this lesson, students explore (mirror) symmetry and balance in diverse situations and artworks. In the activity, students have to make a new pattern with the use of two mirrors and an image of a colored decagon. Next, they have to do make a new artwork with the use of two mirrors and an image of a painting.
9. Spatial patterns	Students start with the perception of 3-dimensional artworks in relation to patterns. In the activity, students have to create a pattern on a cuboid. Afterwards, students have to discuss if they can predict the pattern of the cuboid of their peers.

5.4.2. Appendix B. List with geometrical words

Tier 1: Daily geometrical words		
(in) between	full	repetition/iteration/recurrence
about	height/up/ on top/above	reverse/upside down/ inverse
above/ upper/ upper side	high	right
around	inside	round
back (ground)/ back side	large/ wide/	same as/ as many
behind	left	shape/ figure/
below/ down/ beneath/ underneath/ low / lower	level /elevation/ altitude	side
big/large/wide	tiny/ small	similar
boundary	long/tall/length	space
brief/short/	many/much / a lot/ plenty	straight
by	middle/ in the middle of/amidst	therein/in/on/ into/
direction	More	thick(ness)/fat
distance	move/shift/	thin/narrow
empty/hollow	near/beside/at/next to/ at the side of	through
far away/distant	nearby/close by	to/in front of/toward/front side/ahead/fore ground
few/little/less	oblique/slanting/sloping	towards
fill	oppositional/inversely	under/underneath/below/beneath
fit in	outside	upon
flat	over/across/behind/	width
for	place/position/locate/location	within
Tier 2: Academic geometrical words		
Easy 2D shapes (e.g. square/circle/triangle)	Parallel	Regularity/order
Difficult 2D shape (e.g. equilateral triangle, isosceles triangle, parallelogram).	Angle	Edge Map/plan
2dimensional (2D)	Horizontal	Spatial/spacious
3D shapes	Volume	Scale
3dimensional (3D)	Line of sight	Line symmetry
Cardboard building scheme	Line segment	Mirroring
Upper/side/front view	perpendicular	Point of view
Composition	Scale model	Symmetry
Diagonal	Area	Paper model
Depth	Optical illusion	Vanishing point
Rotation	Pattern	Proportion/ratio
Equilibrium	Perspective	Vertical

5.4.3.Appendix C. Complete multilevel results

Table 1

Multilevel results regarding geometrical ability

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 covariates	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.53 (.01)**	0.37 (.02)**	0.37 (.02)**	0.37 (.02)**	0.37 (.02)**
Time	0.11 (.01)**	0.11 (.01)**	0.11 (.01)**	0.11 (.01)**	0.09 (.02)**
Grade 5 (dummy)		0.10 (.02)**	0.10 (.02)**	0.10 (.01)**	0.10 (.02)**
Grade 6 (dummy)		0.20 (.02)	0.20 (.02)	0.21 (.02)	0.21 (.02)**
Gender		0.10 (.01)**	0.10 (.01)**	0.10 (.01)**	0.10 (.01)**
SES low (dummy)		-0.08 (.03)**	-0.08 (.03)**	-0.08 (.03)**	-0.08 (.03)**
SES very low (dummy)		-0.15 (.03)**	-0.15 (.03)**	-0.15 (.03)**	-0.15 (.03)**
General mathematical ability		0.12 (.01)**	0.12 (.01)**	0.12 (.01)**	0.12 (.01)**
Lesson series condition (dummy)			-0.10 (.11)	-0.11 (.11)	-0.12 (.11)
PD program condition (dummy)			-0.11 (.08)	-0.11 (.08)	-0.11 (.08)
Number of lessons given			0.02 (.01)	0.02 (.01)	0.02 (.01)
Number of PD sessions followed			0.01 (.02)	0.01 (.02)	0.01 (.02)
*Time*Lesson series*					0.04 (.03)
*Time * PD program*					-0.01 (.03)
Random part					
σ_e^2	0.033 (39.76%)	0.033	0.033	0.029	0.029
σ_{u0}^2	0.036 (43.37%)	0.022	0.022	0.024	0.024
σ_{v0}^2	0.013 (16.87%)	0.007	0.007	0.008	0.007
σ_{v1}				0.008	0.008
Deviance	230.45	-432.26	-441.42	-563.11	-566.11

* Significant at $p<.05$, **Significant at $p<.01$

Table 2

Multilevel results regarding the use of daily geometrical words in the GAT (tier1)

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.11 (.00)**	0.11 (.00)**	0.11 (.00)**	0.11 (.00)**	0.11 (.00)**
Time	0.01 (.00)**	0.01 (.00)**	0.01 (.00)**	0.01 (.00)**	0.01 (.00)
Grade 5 (dummy)		0.01 (.00)*	0.01 (.00)*	0.01 (.00)*	0.01 (.00)*
Grade 6 (dummy)		0.01 (.00)**	0.01 (.0)**	0.01 (.00)**	0.01 (.00)**
Gender		-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**
SES low (dummy)		-0.01 (.01)*	-0.01 (.01)*	-0.01 (.01)*	-0.01 (.01)*
SES very low (dummy)		-0.02 (.01)**	-0.02 (.01)**	-0.02 (.01)**	-0.02 (.01)**
Lesson series condition (dummy)			-0.01 (.01)	-0.01 (.01)	-0.02 (.01)
PD program condition (dummy)			0.01 (.01)	0.01 (.01)	0.02 (.01)
Number of lessons given			0.00 (.00)	0.00 (.00)	0.00 (.00)
Number of PD sessions followed			-0.00 (.00)	-0.00 (.00)	-0.00 (.00)
*Time*Lesson series*					0.02 (.01)*
*Time * PD program*					-0.01 (.01)
Random part					
σ_e^2	0.00348 (87.9%)	0.00349	0.00349	0.00320	0.00320
σ_{u0}^2	0.00038 (9.6%)	0.00034	0.00034	0.00049	0.00049
σ_{v0}^2	0.00010 (2.5%)	0.00008	0.00007	0.00017	0.00016
σ_{v1}				0.00055	0.00048
Deviance	-11058.79	-11098.28	-11102.63	-11173.40	-11181.05

Note. Robust standard errors are used, * Significant at $p<.05$, **Significant at $p<.01$

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Table 3

Multilevel results regarding academic geometrical words used in the GAT (tier 2)

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.06 (.00)**	0.07 (.01)**	0.08 (.00)**	0.07 (.00)	0.08 (.01)**
Time	-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**	-0.02 (.00)**
Grade 5 (dummy)		-0.00 (.00)	-0.00 (.00)	0.00 (.00)	0.00 (.00)
Grade 6 (dummy)		-0.01 (.00)*	-0.01 (.00)	-0.01 (.00)*	-0.01 (.00)*
Gender		-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**	-0.01 (.00)**
SES low (dummy)		0.01 (.01)	0.01 (.01)	0.01 (.01)	0.01 (.01)
SES very low (dummy)		0.01 (.01)	0.01 (.01)	0.02 (.01)	0.02 (.01)
Lesson series condition (dummy)			-0.00 (.02)	-0.00 (.01)	-0.01 (.01)
PD program condition (dummy)			0.02 (.02)	0.02 (.02)	0.02 (.02)
Number of lessons given			0.00 (.00)	0.00 (.00)	0.00 (.00)
Number of PD sessions followed			-0.01 (.00)	-0.00 (.00)	-0.00 (.00)
*Time*Lesson series*					0.00 (.01)
*Time * PD program*					0.01 (.01)
Random part					
σ_e^2	0.00586 (85.80%)	0.00585	0.00585	0.00572	0.00572
σ_{u0}^2	0.00087 (12.74%)	0.00083	0.00082	0.00088	0.00089
σ_{v0}^2	0.00010 (1.46%)	0.00008	0.00008	0.00029	0.00029
σ_{v1}				0.00028	0.00027
Deviance	-8821.32	-8858.75	-8861.91	-8880.16	-8881.63

Note. Robust standard errors are used, * Significant at p <.05, **Significant at p <.01

Table 4

Multilevel results regarding (geometrical) creativity (GCT)

Model	Model 1: Intercept-only model	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	897.80 (42.32)**	454.71 (60.56)**	436.59 (68.46)**	443.40 (65.92)**	444.12 (66.40)**
Time	75.40 (24.83)**	71.58 (24.80)**	71.63 (24.81)**	68.39 (38.95)	65.84 (55.65)
Grade 3 (dummy)		33.17 (148.28)	29.33 (149.13)	26.55 (148.03)	24.29 (148.03)
Grade 5 (dummy)		300.31 (65.14)**	300.02 (65.36)**	299.28 (64.10)**	299.18 (64.08)**
Grade 6 (dummy)		578.38 (68.02)**	578.38 (68.02)**	555.84 (66.93)**	555.55 (66.91)**
Gender		266.88 (34.28)**	266.97 (34.28)**	269.84 (34.22)**	269.84 (34.22)**
SES low (dummy)		-132.44 (107.52)	-128.09 (107.69)	-126.50 (107.48)	-126.79 (107.48)
SES very low (dummy)		-158.18 (106.96)	-152.86 (107.10)	-143.93 (106.73)	-143.69 (106.72)
General mathematical ability		222.58 (19.05)**	222.11 (19.06)**	223.24 (18.99)**	223.12 (18.99)**
Lesson series condition (dummy)			182.25 (401.67)	354.85 (382.25)	343.96 (382.42)
PD program condition (dummy)			-136.28 (300.43)	111.24 (288.31)	142.65 (289.21)
Number of lessons given			-12.78 (46.63)	-35.25 (44.40)	-35.62 (44.38)
Number of PD sessions followed			13.43 (68.11)	-38.33 (65.33)	-38.73 (65.30)
*Time*Lesson series*					68.19 (91.77)
*Time * PD program*					-139.02 (108.23)
Random part					
σ_e^2	589766.09 (56.66%)	590849.99	590918.06	549371.84	549285.85
σ_{u0}^2	326948.62 (31.41%)	262624.36	262510.83	282683.92	282663.42
σ_{v0}^2	124191.37 (11.93%)	860090.20	84855.01	69624.69	69448.79
σ_{v1}				84072.83	82455.98
Deviance	65824.88	65587.91	65586.84	65522.05	65520.41

* Significant at *p* <.05, **Significant at *p* <.01

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Table 5

Multilevel results regarding aspect 'space' used in the VAA

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	3.22 (.13)**	2.70 (.23)**	2.51 (.25)**	2.58 (.25)**	2.85 (.24)**
Time	1.38 (.18)**	1.38 (.18)**	1.38 (.18)**	1.35 (.17)**	0.67 (.23)**
Grade 3 (dummy)		-1.77 (.60)**	-1.69 (.68)*	-1.61 (.67)*	-1.61 (.67)*
Grade 5 (dummy)		0.12 (.23)	0.10 (.22)	0.10 (.22)	0.10 (.22)
Grade 6 (dummy)		0.79 (.23)**	0.79 (.22)**	0.80 (.22)**	0.80 (.22)**
Gender		0.57 (.14)**	0.56 (.14)**	0.56 (.14)**	0.56 (.14)**
SES low (dummy)		-0.92 (.29)**	-0.85 (.30)**	-0.81 (.30)**	-0.80 (.30)*
SES very low (dummy)		-0.96 (.38)*	-0.91 (.37)*	-0.92 (.37)*	-0.92 (.37)**
Lesson series condition (dummy)			-0.52 (.71)	-0.61 (.71)	-1.17 (.73)
PD program condition (dummy)			-0.85 (.75)	-0.81 (.75)	-0.63 (.74)
Number of lessons given			0.14 (.09)	0.14 (.09)	0.14 (.09)
Number of PD sessions followed			0.02 (.19)	0.02 (.19)	0.02 (.18)
*Time*Lesson series*					1.48 (.40)**
*Time * PD program*					-0.43 (.42)
Random part					
σ_e^2	11.78 (18.18%)	11.79	11.79	10.92	10.92
σ_{u0}^2	2.31 (15.32%)	2.15	2.14	2.58	2.58
σ_{v0}^2	0.98 (6.50%)	0.76	0.65	0.85	0.79
σ_{v1}				1.75	1.32
Deviance	23147.54	23092.24	23081.58	23014.13	22997.99

Note. Robust standard errors are used, * Significant at p <.05, **Significant at p <.01

Table 6

Multilevel results regarding 'Space suggestion' used in the VAA

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.87 (.04)**	0.60 (.09)**	.49 (.09)*	0.52 (.09)*	0.61 (.09)**
Time	0.18 (.08)*	0.18 (.08)*	.18 (.08)*	0.20 (.08)*	-0.09 (.09)
Grade 3 (dummy)		-0.25 (.23)	-0.23 (.28)	-0.23 (.25)	-0.23 (.26)
Grade 5 (dummy)		0.25 (.10)*	0.22 (.10)*	0.22 (.10)*	0.22 (.09)*
Grade 6 (dummy)		0.27 (.10)**	0.25 (.09) **	0.25 (.09)**	0.25 (.09)**
Gender		0.15 (.06)**	0.15 (.06)**	0.15 (.06)**	0.15 (.06)**
SES low (dummy)		-0.15 (.13)	-0.09 (.13)	-0.08 (.13)	-0.08 (.13)
SES very low (dummy)		-0.08 (.14)	-0.05 (.14)	-0.05 (.13)	-0.05 (.13)
Lesson series condition (dummy)			-0.41 (.38)	-0.15 (.36)	-0.27 (.36)
PD program condition (dummy)			-0.08 (.25)	-0.17 (.22)	-0.27 (.22)
Number of lessons given			0.09 (.05)	0.05 (.05)	0.05 (.04)
Number of PD sessions followed			-0.02 (.05)	-0.01 (.05)	-0.01 (.05)
*Time*Lesson series*					0.42 (.20)*
*Time * PD program*					0.27 (.24)
Random part					
σ_e^2	2.95 (90.77%)	2.95	2.95	2.78	2.78
σ_{u0}^2	0.22 (6.77%)	0.21	0.21	0.29	0.29
σ_{v0}^2	0.08 (2.46%)	0.06	0.04	0.04	0.03
σ_{v1}				0.38	0.30
Deviance	16851.12	16830.80	16812.50	16759.51	16746.60

Note. Robust standard errors are used, * Significant at p <.05, **Significant at p <.01

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Table 7

Multilevel results regarding the aspect 'shape' used in the VAA

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.15 (.02)**	0.17 (.05)**	0.19 (.05)**	0.18 (.05)**
Time	0.01 (.03)	0.01 (.03)	0.01 (.03)	0.01 (.03)
Grade 3 (dummy)		-0.12 (.06)*	-0.11 (.06)	-0.11 (.06)
Grade 5 (dummy)		-0.04 (.04)	-0.04 (.04)	-0.04 (.04)
Grade 6 (dummy)		-0.06 (.03)	-0.05 (.04)	-0.05 (.04)
Gender		0.03 (.03)	0.03 (.03)	0.03 (.03)
SES low (dummy)		-0.02 (.05)	-0.03 (.06)	-0.03 (.06)
SES very low (dummy)		-0.05 (.05)	-0.06 (.05)	-0.06 (.05)
Lesson series condition (dummy)			-0.07 (.13)	-0.04 (.13)
PD program condition (dummy)			0.03 (.11)	0.06 (.12)
Number of lessons given			0.00 (.01)	0.00 (.02)
Number of PD sessions followed			-0.00 (.02)	-0.01 (.03)
*Time*Lesson series*				
*Time * PD program*				
Random part				
σ_e^2	0.78 (98.73%)	0.78	0.78	0.77
σ_{u0}^2	0.00 (0%)	0.00	0.00	0.00
σ_{v0}^2	0.01 (1.27%)	0.01	0.01	0.02
σ_{v1}				0.01
Deviance	10936.13	10928.67	10926.99	10922.15

Note. Robust standard errors are used, *Significant at p <.05, **Significant at p <.01

Table 8

Multilevel results regarding aspect 'composition' used in the VAA

Model	Model 1: Intercept-only model (with time)	Model 2: Add Level 2 predictors	Model 3: Add level 3 predictors	Model 4: Random Slope of time	Model 5: Cross-level interactions
Fixed part	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)	*B* (s.e.)
Intercept	0.09 (.01)**	-0.01 (.04)	-0.06 (.11)	-0.01 (.03)	0.03 (.03)
Time	0.27 (.01)**	0.27 (.04)**	0.27 (.04)**	0.27 (.04)**	0.04 (.03)
Grade 3 (dummy)		-0.05 (.07)	-0.07 (.08)	-0.02 (.06)	-0.01 (.06)
Grade 5 (dummy)		0.07 (.05)	0.04 (.04)	0.05 (.04)	0.05 (.04)
Grade 6 (dummy)		0.10 (.05)*	0.07 (.04)*	0.10 (.04)*	0.10 (.04)*
Gender		0.03 (.03)	0.03 (.03)	0.03 (.03)	0.03 (.03)
SES low (dummy)		-0.01 (.07)	-0.01 (.07)	0.03 (.07)	0.03 (.07)
SES very low (dummy)		-0.11 (.05)*	-0.11 (.05)*	-0.09 (.05)*	-0.09 (.05)
Lesson series condition (dummy)			-0.23 (.14)	-0.24 (.08)**	-0.03 (.08)**
PD program condition (dummy)			-0.21 (.12)	-0.10 (.07)	-0.10 (.07)
Number of lessons given			0.05 (.02)*	0.03 (.01)**	0.03 (.02)**
Number of PD sessions followed			0.05 (.03)	0.03 (.02)	0.03 (.01)*
*Time*Lesson series*					0.44 (.09)**
*Time * PD program*					-.03 (.13)
Random part					
σ_e^2	0.66 (95.94%)	0.66	0.66	0.60	0.59
σ_{u0}^2	0.01 (0.73%)	0.00	0.00	0.03	0.03
σ_{v0}^2	0.02 (3.34%)	0.02	0.01	0.00	0.00
σ_{v1}				0.13	0.08
Deviance	10314.71	10306.24	10276.82	10118.94	10093.72

Note. Robust standard errors are used, * Significant at $p < .05$, **Significant at $p < .01$

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Chapter 6

General Discussion

This dissertation responds to the question of how we can accommodate and nurture creativity in education, and contributes to the recent societal attention for creativity in Dutch education (Curriculum.nu, 2018; Platform Onderwijs 2032, 2016). This dissertation specifically focuses on the promotion of creativity in the discipline of mathematics in the upper grades of elementary school. The studies reported in the preceding chapters examine this topic from different perspectives. In this final chapter, the main findings from these studies are summarized, integrated and discussed. Subsequently, this final chapter discusses directions for further research and implications for educational practice.

6.1. Overview of the main findings

The first two studies in this dissertation addressed the relation between creativity and mathematics in elementary schools. *Chapter 2* presented a study that investigated the relations between domain-specific mathematical creativity, domain-general creativity and mathematical ability in fourth-grade students. These relations were studied by testing two competing models, using structural equation modeling: (1) mathematical ability and domain-general creativity both predict mathematical creativity, and (2) mathematical creativity predicts mathematical ability, and mathematical creativity in turn is predicted by domain-general creativity, which indirectly relates to mathematical ability. Multiple measures of domain-general creativity were used to estimate the latent construct of general creativity. The results indicated that domain-general creativity should not be seen as a unitary construct but as consisting of at least two different constructs, which we identified in this study as 'generating ideas', and 'exploring and digging deeper into ideas'. Furthermore, we found support for the first model: both mathematical ability and domain-general creativity predicted mathematical creativity. Although this was a cross-sectional study and no conclusions can be drawn about the causal directions of the relations, the results suggest that both domain-specific mathematical knowledge and skills and domain-general creative processes are necessary to be creative in mathematics. Based on these results, a recommendation for education could be that for the promotion of creativity in mathematics education it is important to focus on both aspects. Further research is needed to explore how teachers can promote creativity in educational practice. It can be conjectured that a teacher could, for example, explicitly promote creativity with tasks, such as a mathematical multiple solution task, that seem to require both aspects.

Chapter 3 reports on a study that further explored the relation between creativity and mathematics with a study that investigated the relations of students' domain-general creativity with their performance on three types of mathematical problems. The aim was to determine whether the relation between students' domain-general creativity and performance differed between closed routine geometrical problems, geometrical multiple solution problems and non-routine visual-art geometry problems. The Test of Creative Thinking-Drawing Production (TCT-DP; Urban & Jellen, 1996) was used to obtain a measure of students' domain-general creativity. Results showed that students' domain-general creativity was positively associated with their performance on three types of geometry problems. However, students' creativity was significantly stronger associated with performance on multiple solution problems than with performance on routine and non-routine visual arts-geometry problems. As several covariates were included to control for spuriousness, we may cautiously conclude that the remaining effects of creativity reflected true involvement of creative thinking processes in geometrical problem solving, especially in the more open multiple solution problems. Providing students with appropriate problems in mathematics education is an important way to promote creative thinking. Multiple solution problems, due to their open character, are likely to serve this goal well and should be prominently included in mathematical textbooks. The results of this study also suggest that multiple solution tasks could be valuable measurement instruments for research on the promotion of creativity in mathematics education, as the results indicate that creative processes are involved in students' performance on multiple solution tasks. This study confirms earlier research which has shown that such a task can be employed to obtain an indication of students' mathematical creativity (e.g., Leikin, 2009).

The last two chapters presented studies that were conducted in relation to the Mathematics Arts Creativity in Education (MACE) [Meetkunst] program (Meetkunst Projectteam, 2018a). The MACE program was developed to support elementary schools in the Netherlands to meet partly overlapping learning goals and objectives of the disciplines visual arts and geometry, and to promote students' creative skills in both disciplines. A lesson series for Grades 4 to 6 (ages 9 to 12 years) was designed in which students could engage in open activities and classroom discussions in an integrated visual arts and geometry teaching context. Furthermore, a professional development program for teachers was designed to enhance implementation of the lesson series.

Chapter 4 presented a case study of a fourth-grade teacher and her class. This study provided in-depth insights into how recommended creativity promoting pedagogical strategies were implemented in the classroom, how they related to different types of lessons and how they related to students' mathematical creativity as expressed in classroom dialogues. In this study we proposed an adapted view on mathematical creativity, since the most often used definition proposed by Sriraman (2005) limits mathematical creativity to the process of mathematical problem posing and problem solving. We considered his definition incomplete for studying mathematical creativity in educational practice in a broader sense. In our adapted view, mathematical creativity (also) refers to the cognitive act of combining known concepts in an adequate, but for the student new way, thereby increasing or extending the student's (correct) understanding of mathematics. Interactions between a teacher and her 22 fourth-grade students in three different types of mathematical lessons were investigated: an 'open' in-school MACE lesson, an 'open' out of school MACE lesson and a regular 'closed' mathematics lesson. The study showed that the teacher employed all previously identified strategies for promoting mathematical creativity in students' thinking, but some more frequently than others. More diverse strategies were used during the two open interdisciplinary mathematics lessons of the MACE program compared to the regular textbook-based mathematics lesson. The findings of this study indicated that mathematical creativity was promoted when the teacher initiated longer whole class dialogues and created an open atmosphere in open interdisciplinary lessons with open learning goals. In this study, an open atmosphere was created when the teacher gave students ample opportunities to express their ideas and, through specific pedagogical actions, clearly signalled to students that she took these ideas seriously. Furthermore, providing interdisciplinary content with a less specific or narrow learning goal (i.e., learning about conceptualizations instead of strategies), and the fact that these lessons presented a different sociomathematical norm than in regular lessons (i.e. less focused on single correct outcomes), might also have contributed to the presence of mathematical creativity in the open interdisciplinary lesson and the absence of mathematical creativity in the regular mathematics lesson.

The study reported in *Chapter 5* investigated the effects of the integrated MACE approach on students' geometrical ability (i.e., geometrical understanding, creativity and vocabulary) and perception of visual arts in a quasi-experimental design. Three groups of teachers and their classes were investigated. One group of teachers taught the lessons series and followed a professional development program, one group of teachers only taught the lesson series, and the teachers of the comparison group taught a series of regular geometry lessons from widely used mathematical textbooks. The results of the MACE evaluation study showed that students who received the MACE lesson series improved more than students who received regular geometry lessons only in geometrical aspects perceived in a visual artwork. The MACE program was as effective as regular geometry lessons with regard to geometrical ability. Whereas in regular education geometrical concepts are directly and time-efficiently taught, students who received the MACE teaching sequence reached the same level of geometrical understanding with open lessons in which they were enabled to express themselves creatively. Although the findings of the qualitative case study, reported in Chapter 4, indicated that students expressed more mathematically creative ideas in the MACE lessons compared to a regular mathematics lesson, the results of the effect study indicated there was no effect of the MACE program on students' geometrical creativity measured with a geometrical multiple solution task. Students who participated in the MACE program did not improve more regarding geometrical creativity than students who followed regular geometry lessons.

6.2. Conclusion and discussion

This dissertation offers several recommendations for the promotion of creativity in elementary school education in general and in mathematics education specifically, building on the findings of the empirical studies presented in this dissertation. In what follows, I will first elucidate two recommendations for the promotion of creativity in education in general. Subsequently, I will describe how the promotion of creativity can be specifically facilitated within the discipline of mathematics.

6.2.1. Promoting creativity in education in general

First, I argue that it is important to introduce activities and assessment tools that can enhance creativity within and across academic disciplines in elementary schools because it can strengthen students' domain-general and domain-specific creativity. A pertinent issue in this field is whether creativity is predominantly domain-general (e.g., Plucker, 1999), domain-specific (e.g., Baer, 2012) or a mix of both (e.g., Jeon, Moon, & French, 2011). The studies in this dissertation demonstrated that creativity is neither completely domain-general nor domain-specific: both domain-general and domain-specific factors are involved in creative performance (Chapter 2; Chapter 3). These results imply that the promotion of creativity should be embedded within one or more academic disciplines, and not be taught by a general creativity training, such as a divergent thinking training (De Souza Fleith, Renzulli, & Westberg, 2002; Renzulli, 1986). Furthermore, in order to improve students' creativity, it is recommended to promote creativity in multiple academic disciplines. As the study in Chapter 5 suggests: having only one creativity promoting mathematics lesson per week might not be enough to substantially improve students' creativity in mathematics. Stimulating students' domain-specific creativity in multiple disciplines may also enhance their domain-general creativity. A recent study showed that similar general creative processes are involved in creative performances within different disciplines (Willemsen, Kroesbergen, & Schoevers, 2019). If creativity is promoted in multiple academic disciplines in elementary school, the likelihood increases that students' general creative skills will improve and, consequently, students may improve their creativity within an academic discipline. Furthermore, promoting creativity in multiple disciplines in education may promote transfer of general creative (cognitive) skills, as research suggests that extensive practice in a variety of contexts is beneficial for transfer (Salomon, & Perkins, 1989). It is considered desirable that students are able to transfer their creative skills to other situations and context, such as daily life or, possibly, in future jobs.

The second recommendation of this dissertation accentuates the importance of crossing boundaries (Akkerman & Bakker, 2011a; 2011b) in order to promote creativity. Creativity may be promoted if disciplinary boundaries in education are crossed, for example when a discipline is enriched with another educational discipline or with a context outside school. "A boundary can be perceived as a socio-cultural difference leading to discontinuity in action or interaction" (Akkerman & Bakker, 2011a, p.133), and can be a resource for learning and for the exploitation of creative processes (Akkerman & Bakker, 2011b). Crossing boundaries in education, which happens in interdisciplinary education, may support students to break away from established mindsets and to act creatively: integrating different conceptual systems could activate students to combine familiar concepts in new and meaningful ways. Although this dissertation did not directly investigate, nor present strong empirical support for this conviction, some initial support was provided in Chapter 4 of this dissertation. In the in-depth case study of a MACE lesson, it was observed that students who were encouraged to name (mathematical) shapes they had encountered during a walk in the neighborhood of the school, integrated different conceptual systems by connecting shapes they were not familiar with to well-known concepts from daily life. One student, for example, said: 'this is the shape of a baguette' (Chapter 4). Another recent study also found support for the idea that conceptual systems from different disciplines can activate students to be creative and showed that knowledge and skills from different disciplines (mathematics and literacy) were related to creative performances in mathematics (Willemsen et al., 2019). Although this dissertation provided some initial support for this new approach to promote creativity in education, more research is needed on the relation between boundary crossing and creativity.

6.2.2. Promoting creativity in mathematics education

Of all creativity promoting strategies described in the literature (e.g., Beghetto & Kaufman, 2010; Davies et al., 2014), I argue that two main pedagogical strategies are central for nurturing creativity in mathematics education. These two main strategies are in line with former research (e.g., Beghetto & Kaufman, 2010; Davies et al., 2014; Sternberg, 2007). My recommendations follow the suggestions of Sternberg (2007) who argued that for promoting creativity, students need opportunities to act creatively and that students' creativity should be encouraged and valued (Sternberg, 2007). More specifically I argue that (1) teachers should offer students 'open' opportunities with less-specific learning goals and (2) encourage students' creativity by creating an open atmosphere in the classroom and by clearly emphasizing that creative responses are valued. Although these strategies are important for fostering creativity in mathematics education, they appear to be general and not specifically related to the domain of mathematics. Therefore, I believe that these recommendations may also be applicable for other educational disciplines.

First, it is important to offer students 'open' opportunities with less-specific learning goals, which is in alignment with creativity literature in mathematics education (e.g., Leikin, 2009), and in education in general (e.g., Davies et al., 2013). Opportunities, such as lessons, tasks, activities or problems, are 'open' if they invite different solutions, methods of solution or are open for interpretation (Silver, 1995). These opportunities can be manifested in different ways: for example in the form of open problem-solving tasks, such as a multiple solution task (Chapter 2; Chapter 3), or in the form of an open whole-class mathematical dialogue (Chapter 4). This dissertation shows that openness is an incitement for students to create novel and meaningful mathematical concepts and solutions that can extend or deepen their understanding of mathematics. Open opportunities in education require also less-specific learning goals (Chapter 4): with open opportunities it is not predetermined what students will learn exactly. On the surface this may appear problematic, because mathematical learning objectives need to be accomplished. Indeed, an open approach might be less conducive for learning specific mathematical learning goals, such as learning a strategy to add fractures. However, less-specific learning goals, such as learning about the structure and coherence of (whole) numbers, fractures and proportions, or solving (open) problems (Buijs, Klep, & Noteboom, 2008) could also be taught by using open opportunities. In this way, mathematical concepts can be developed, but the precise nature of it cannot be determined in advance (Askew, 2007). Askew (2007) illustrates this well:

> “Rather than learning in classrooms being built up in this pre-determined way, I want to suggest that maybe it is more like ants constructing ant-hills: Ants don’t (at least we assume) start out with a blue-print of the ant-hill that will be constructed. The ant-hill emerges through their joint activity. What emerges is recognizably an ant-hill (and not an eagle’s nest) although the precise structure is not determined until completion (if such a state ever exists). Ants are not ‘applying’ a method in order to construct ant-hills, they are simply practicing their method” (p. 39).

Secondly, this dissertation illustrates that next to offering open opportunities to students, it is important to encourage and value students’ creativity within these opportunities. This can be accomplished by creating an open climate in the classroom and by clearly emphasizing that creative responses are valued. An open climate seems to be necessary to nurture creativity and can be created if a teacher combines activating open questions with questions eliciting students’ ideas about a specific subject, provides indications that students’ answers are heard and respected and/or asked follow-up questions to learn more about students’ ideas’ (Chapter 4). Furthermore, this dissertation illustrates that it is important that teachers clearly emphasize that students’ creative responses are valued. Results of the studies in this dissertation showed that creativity was mainly promoted if the stimulation of creativity was a specific goal and if students seemed to know that expressing mathematical creativity was the norm. For example, in a multiple solution task, students were explicitly asked to create multiple novel and different solutions (Chapter 2; Chapter 3). Also, in the interdisciplinary MACE lessons, students knew that creative expressions were expected and valued, since this was an aim of the program (Chapter 4; Chapter 5). Furthermore, by making clear for students when to be creative may not only be important to foster students’ creativity, but may, in general, be valuable for students to know as part of their metacognitive skills, and may be important for the transfer of creativity (Perkins & Salomon, 1992). Kaufman and Beghetto (2013) illustrated this nicely:

> ‘‘We do not want a pilot trying a new water landing technique during a typical commercial flight. (...) However, if a commercial flight somehow runs into trouble over water and requires a novel maneuver to safely land the plane, we want that pilot to pull out all the creative stops’’ (p.159).

Kaufman and Beghetto (2013) refer to knowing when (not) to be creative, as creative metacognition. A teacher could support students’ creative metacognition, for example, by helping students to recognize the context that are more (and less) conducive to creative expression (Kaufman & Beghetto, 2013).

To conclude, this dissertation gave new and empirically grounded answers to the question how we can accommodate and nurture creativity in elementary school education in general, and in mathematics education specifically. The reported research showed that it is important to promote creativity within multiple and integrated disciplines. To nurture creativity within (interdisciplinary) mathematics education, this dissertation showed that it is important that open opportunities with less-specific learning goals are offered to students, and that the teacher encourages students' creativity by creating an open atmosphere in the classroom and by clearly emphasizing that creative responses are valued. Although, we studied the promotion of students' creativity within the discipline of mathematics, findings might also apply to other disciplines in education.

Hindering factors. Next to the insights into how creativity can be fostered in (mathematics) education, this dissertation also showed several factors that may hinder the implementation of the two mentioned creativity promoting strategies in elementary school mathematics education.

A first hindering factor may be that not all (Dutch) teachers are able to encourage both mathematical learning and creativity in education but may be focused on either one of these. The case study (Chapter 4) illustrates this: the teacher was able to create an open atmosphere in the classroom which promoted mathematically creative expressions, but dialogues stayed rather superficial and could have been improved if the teacher would have asked more eliciting questions that challenged students' thinking and reasoning about mathematical concepts in a deep way. Conversely, one could also imagine that other teachers may focus more on teaching the mathematical content but were not able to create a creativity promoting atmosphere. Consequently, professional development (PD) seems to be necessary. Although, the MACE program provided a PD program, the results of the effect study indicated that the PD program did not have an effect on the teachers' skills to promote students' mathematical learning and creativity. Although the relation between the PD program and teachers' instruction behavior was not directly examined, the fact that no effect was found on students' geometrical abilities suggest that the program did not affect teachers' skills (Chapter 5). The reason why the MACE PD program was apparently not effective could be that it had too many foci: the promotion of creativity, geometry education, visual arts education and the integration of both. However, all these foci were deemed necessary, because teachers' (pedagogical) content knowledge of both geometry and visual arts was expected to be not sufficient (Keijzer, Oprins, De Moor, & Schoevers, 2018) to enable them to integrate both domains and to promote creativity in this interdisciplinary context. Therefore, I believe that future professional development should focus only on training more specialized mathematics teachers in nurturing creativity in mathematics education, because it is expected that they will have enough (pedagogical) content knowledge of mathematics. Consequently, professional development could focus solely on promoting creativity within mathematics education and may be more effective.

Second, the currently prominent role of textbooks in elementary education, in particular in the Netherlands, might hinder the promotion of creativity in mathematics education. Results of this dissertation indicated that it is important that open opportunities with less specific or narrow learning goals need to be created within mathematics education, for example by using multiple solution problems and sustained open whole class mathematical dialogues. However, teachers often follow strictly the guidelines of the mathematical textbooks to teach the mathematics curriculum (Gravemeijer, 2007; Hop, 2012; Meelissen et al., 2012; Van Zanten & Van den Heuvel-Panhuizen, 2018), while mathematical textbooks offer only very limited opportunities for working with open and multiple solution problems (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009; Meelissen et al., 2012; Thijs, Fisser, & Hoeven, 2014; Van Zanten & Van den Heuvel-Panhuizen, 2018). Therefore, it is important that either mathematical textbook designers include more open mathematics lessons, or that teachers replace mathematics textbooks lessons with more open mathematics lessons. This may be achieved by emphasizing the importance of creativity in the mathematics curriculum, which consequently may lead to attention for creativity in standardized testing and in educational inspections and vice versa.

6.3. Future directions for research

Creativity is a multidimensional construct that refers to the act of creating novel and meaningful ideas, solutions and products within a particular (social) context (Plucker, Beghetto, & Dow, 2004). Since it is difficult to grasp this complex construct, more research is needed to explore this construct. Measures of (mathematical) creativity in this dissertation were mainly limited to paper-and-pencil tests, such as a mathematical creativity test or the test of creative thinking-drawing production. Although, these measures are commonly used, I would recommend future studies to study creativity in (mathematics) education in other, various and novel ways. For example, I recommend researchers to use multiple different (mathematical) creativity tests. With the tests in the study reported in Chapter 2, we found that two general creative processes were related to students' performance on multiple solution tasks. However, future research might indicate that more and different types of creative processes are involved if other creativity measures are used. Likewise, I also recommend researchers to measure creativity in mathematics in other and new ways, such as our approach in the study in Chapter 4, in which we coded students' expressions of mathematical creativity in classroom dialogues. For elementary school students, it might be difficult to verbalize their (creative) ideas in a written paper-based test. Future research could also investigate non-verbal ways of mathematically creative expressions, such as in the design of an artwork. Studying creativity in various and novel ways could bring the creativity literature new and more comprehensive insights into how creativity emerges in education, which, in turn, is useful for studies investigating how creativity can be fostered in education.

A limitation of this dissertation is that I studied the relation between creativity and mathematics mainly with the use of cross-sectional studies and with the use of several covariates such as intelligence. However, in order to disentangle whether and how the constructs of creativity and intelligence differ, and how both relate to mathematical performance and mathematical learning, longitudinal studies are needed. Longitudinal and experimental studies with strong randomized designs are particularly needed to obtain more insight into the causality of the relations. This type of studies could bring new insights to the field on the role creativity might play in mathematical performance and learning.

Furthermore, more research is needed on the promotion of creativity in mathematics and other disciplines in elementary education. Although this dissertation showed that two main strategies were important for the promotion of creativity in mathematics education, these results were based on studies in the mathematical domain of geometry. More empirical research in various mathematical domains is needed to confirm these results. Furthermore, these strategies may be applied to other disciplines as well, but more research on nurturing creativity in other disciplines than mathematics is necessary. In this way, more knowledge can be gained on the domain-specificity of the strategies involved in the promotion of creativity in education. Future studies should investigate the relation between pedagogical strategies and creativity with more teachers, classrooms and lessons in multiple disciplines. Furthermore, research could investigate whether creativity promoting strategies used by the teacher could play an explanatory role in the success or failure of programs such as MACE.

6.4. Implications for educational practice

This dissertation has generated a number of implications that could be of interest to policy makers, curriculum and educational textbooks designers, teacher educators, teachers and others who are interested in elementary school education. In this section, I reflect on these implications.

A possible first implication of the research reported in this dissertation is that the importance of creativity in education needs to be reflected in the curricula, (mathematical) textbooks, (standardized) tests, and monitoring system of the national Inspectorate of Education. If the importance of creativity is reflected in these ways, educational practices might change, and the creativity can be structurally accommodated and nurtured in schools.

A second possible implication is that it is important that more ‘open opportunities’ are structurally provided to all students within and across different disciplines in education. These ‘open opportunities’ can be open problems, open educational dialogues, or open activities. In addition, it is important that these opportunities are offered to all students, and not only to the more advanced students, as in current practice (Van Zanten & Van den Heuvel-Panhuizen, 2018). For example, in mathematics education, teachers could replace at least once a week mathematical textbook lessons with more open mathematics lessons. Another example is to include more open problems in mathematical textbooks, such as multiple solution tasks. Moreover, during these opportunities, teacher need to encourage students’ creativity and clearly emphasize that creative responses are valued. Teachers can encourage students’ creativity by using a combination of strategies: the use of activating open questions and questions eliciting students’ ideas about a specific subject, providing indications that students’ answers are heard and respected and/or asking follow-up questions to learn more about students’ ideas.

Third, it is important that (pre-service) elementary teachers receive effective professional development to promote creativity in (mathematics) education. This could, for example, be professional development in which teachers learn to encourage both mathematical learning and creativity simultaneously. Another, maybe even more effective approach for professional development could be to train elementary school teachers into teachers with a specialization in one or two specific subjects, such as mathematics, or mathematics and science. It is expected that more specialized teachers are better able to promote creativity within the classroom, since they have more specialized (pedagogical) content knowledge. They may have more expertise to recognize, create and use domain-specific learning opportunities to encourage students' mathematical learning and creativity.

References

Askew, M. (2007). Scaffolding revisited: From tool for result to tool- and-result. In J. H Woo, H. C. Lew, K. S. P, Park, & D.Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (pp. 33–40). Seoul, Korea. Retrieved from https://files.eric.ed.gov/fulltext/ED499419.pdf

Akkerman, S. F., & Bakker, A. (2011a). Boundary crossing and boundary objects. *Review of Educational Research*, *81*, 132–169. doi:10.3102/0034654311404435

Akkerman, S. F., & Bakker, A. (2011b). Learning at the boundary: An introduction. International *Journal of Educational Research, 50,* 1–5. doi:10.1016/j.ijer.2011.04.002

Alloway, T. P., & Alloway, R. G. (2010). Investigatin the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology, 106,* 20-29. doi: 10.1016/j.jecp.2009.11.003

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education: introduction and review. *ZDM - International Journal on Mathematics Education*, *47*, 1047–1065. doi:10.1007/s11858-015-0738-8

Baer, J. (1996). The effects of task-specific divergent-thinking training. *Journal of Creative Behavior, 30*, 183-187. doi: 10.1002/j.2162-6057.1996.tb00767.x

Baer, J. (2012). Domain specificity and the limits of creativity theory. *Journal of Creative Behavior*, *46*, 16–29. doi:10.1002/jocb.002

Baer, J., & Garett, T. (2010). Teaching for creativity in an era of content standards and accountability. In R. A. Beghetto & J. C. Kaufman (Eds.), *Nurturing creativity in the classroom* (pp. 6–23). New York, USA: Cambridge University Press.

Baer, J., & Kaufman, J. C. (2005). Bridging generality and specificity: The amusement park theoretical (apt) model of creativity. *Roeper Review*, *27*, 158–163. doi:10.1080/02783190509554310

Baer, J., & Kaufman, J. C. (2008). Gender differences in creativity. *The Journal of Creative Behavior, 42,* 75-105. doi: 10.1002/j.2162-6057.2008.tb01289.x

Baddeley, A. D. (2003). Working memory: Looking back and looking forward. *Nature Reviews Neuroscience, 4*, 829–839. doi:10.1038/nrn1201

Bahar, A. K., & Maker, C. J. (2011). Exploring the relationship between mathematical creativity and mathematical achievement. *Asia-Pacific Journal of Gifted and Talented Education, 3*, 33-48. Retrieved from http://apfgifted.org/Tpl/default/Public/Journal/2011/Mathematical_Creativity_and_Achievement.pdf

Bancroft, S., Fawcett, M., & Hay, P. (2009). *Researching children researching the world: 5 × 5 × 5 = creativity.* Stoke-on-trent, England: Trenthem Books.

Baran, G., Erdogan, S., & Çakmak, A. (2011). A study on the relationship between six-year-old children's creativity and mathematical ability. *International Education Studies, 4,* 105–111. doi: 10.5539/ies.v4n1p105

Beaton, D. E., Bombardier, C., Guillemin, F., & Bos Ferraz, M. (2000). Guidelines for the process of cross-cultural adaptation of self-report measures. *Spine, 25,* 3186 – 3191. doi:10.1097/00007632-200012150-00014

Beghetto, R. A. (2007). Does creativity have a place in classroom discussions ? Prospective teachers' response preferences. *Thinking Skills and Creativity*, *2*, 1–9. doi:10.1016/j.tsc.2006.09.002

Beghetto, R. A., & Kaufman, J. C. (2010). *Nurturing creativity in the classroom*. New York, NY, USA: Cambridge University Press.

Beghetto, R. A., & Kaufman, J. C. (2011). Teaching for creativity with disciplined improvisation. In R. K. Sawyer (Ed.), *Structure and improvisation in creative teaching* (pp. 94–109). New York, NY, USA: Cambridge University Press.

Beghetto, R. a., & Kaufman, J. C. (2014). Classroom contexts for creativity. *High Ability Studies, 25*, 53–69. doi:10.1080/13598139.2014.905247

Benedek, M., Jauk, E., Sommer, M., Arendasy, M., & Neubauer, A. C. (2014). Intelligence, creativity and cognitive control: The common and differential involvement of executive functions in intelligence and creativity. *Intelligence, 46,* 73–83. doi:10.1016/j.intell.2014.05.007

Bokhove, C., & Jones, K. (2018). Stimulating mathematical creativity through constraints in problem-solving. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the Scope of Research on Mathematical Problem Solving* (pp. 301–319). Cham, Switzerland: Springer.

+

Bolden, D. S., Harries, T. V, & Newton, D. P. (2010). Pre-service elementary teachers ' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, *73*, 143–157. doi:10.1007/s10649-009-9207-z

Bollen, K. A. (1987). Outliers and improper solutions: A confirmatory factor analysis example. *Sociological Methods and Research ,15,* 375–384. doi:10.1177/0049124187015004002

Bostic, J. D. (2011). *The effects of teaching mathematics through problem-solving contexts on sixth-grade students' problem-solving performance and representation use* (Doctoral dissertation*)*. Retrieved from http://ufdc.ufl.edu/UFE0043164/00001.

Braakhuis, G., Von Piekartz, R., Vogel, H., & De Graaf, T. (2012). *Kennisbasis Beeldend onderwijs op de Pabo [Knowledge base visual arts education at primary school teacher education]*. Retrieved from https://10voordeleraar.nl/kennisbases/publicaties

Bresler, L. (1999). The hybridization and homogenization of school art: Institutional contexts for elementary art specialists. *Visual Arts Research*, *25*, 25–37. Retrieved from https://www.jstor.org/stable/20715983?seq=1#page_scan_tab_contents

Brinkman, W., Miedema, E., & Schreuder, C. (2017). *Kunst=Taal en Rekenen. Drie jaar Boijmans Taal- en Rekenprogramma [Art=Language and Mathematics. Three years of Boijmans Language and Mathematics programme].* Assen/Rotterdam: Van Gorcum/Museum Boijmans Van Beuningen.

Buijs, K., Klep, J., & Noteboom, A. (2008). *TULE - Rekenen/wiskunde [TULE – Arithmetic/Mathematics]*. Enschede: SLO.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*, 380–400. doi:10.1037/a0031084

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*, 45–75. doi:10.1007/s10649-005-0808-x

Castillo-Vergara, M., Barrios Galleguillos, N., Jofré Cuello, L., Alvarez-Marin, A., & Acuña-Opazo, C. (2018). Does socioeconomic status influence student creativity? *Thinking Skills and Creativity*, *29*, 142–152. doi:10.1016/j.tsc.2018.07.005

Charlton, C., Rasbash, J., Browne, W. J., Healy, M., & Cameron, B. (2017). *MLwiN Version 3.02.* Bristol, England: Centre for Multilevel Modelling, University of Bristol.

Chi, M. T. H., De Leeuw, N., Chiu, M.-H., & Lavancher, C. (1994). Eliciting self-sxplanations improves understanding. *Cognitive Science*, *18*, 439–477. doi: 10.1016/0364-0213(94)90016-7

Cicchetti, D. V. (1994). Guidelines, criteria, and rules of thumb for evaluating normed and standardized assessment instruments in psychology. *Psychological assessment, 6,* 284–290. doi:10.1037//1040-3590.6.4.284

Clark, A., & Karmiloff-Smith, A. (1993). The cognizer's innards: A psychological and philosophical perspective on the development of thought. *Mind & Language*, *8*, 487–519. doi:10.1111/j.1468-0017.1993.tb00299.x

Clotfelter, C. T., Ladd, H. F., & Vigdor, J. L. (2007). Teacher credentials and student achievement: longitudinal analysis with student fixed effects. *Economics of Education Review*, *26*, 673–682. doi:10.1016/j.econedurev.2007.10.002

Colucci-Gray, L., Burnard, P., Cooke, C., Davies, R., Gray, D., & Trowsdale, J. (2017). *Reviewing the potential and challenges of developing STEAM education through creative pedagogies for 21st learning: how can school curricula be broadened towards a more responsive, dynamic, and inclusive form of education?* Retrieved from https://www.bera.ac.uk/project/bera-research-commissions/reviewing-the-potential-and-challenges-of-developing-steam-education-2

Craft, A. (2005). *Creativity in schools: Tensions and dilemma's*. New York, USA: Routledge

Crane, J. (1996). Effects of home environment, SES and maternal test scores on mathematics cchievement. *The Journal of Educational Research*, *89*, 305–314. doi:10.1080/00220671.1996.9941332

Creswell, J. W. (2007). *Qualitative Inquiry and Research Design: Choosing Among Five Approaches (2nd ed.).* Thousand Oaks, CA, USA: Sage Publications. doi:10.1016/j.aenj.2008.02.005

Cropley, A. J. (2010). Defining and measuring creativity: Are creativity tests worth using? *Roeper Review, 23,* 72-79. doi:10.1080/02783190009554069

Cropley, A. J. (2012). Creativity and education: An Australian perspective. *The International Journal of Creativity and Problem Solving*, *22*, 9–25.

Csikszentmihalyi, M. (2006). Foreword: Developing creativity. In N. Jackson, M. Oliver, M. Shaw, & J. Wisdom (Eds.), *Developing Creativity in Higher Education: An Imaginative Curriculum*. London, England: Routledge.

Curriculum.nu (2018). *Revised vision of arithmetic-mathematics education (October). [Bijgestelde visie rekenen-wiskunde (oktober)]*. Retrieved from https://curriculum.nu/wp-content/uploads/2018/10/181015-Bijgestelde-visie-Rekenen-en-wiskunde.pdf

Darling-Hammond, L. (2000). Teacher quality and student achievement. *Education Policy Analyses Archives, 8,* 1–44. Retrieved from https://epaa.asu.edu/ojs/article/download/392/515

Davies, D., Jindal-Snape, D., Collier, C., Digby, R., Hay, P., & Howe, A. (2013). Creative learning environments in education—A systematic literature review. *Thinking Skills and Creativity*, *8*, 80–91. doi:10.1016/j.tsc.2012.07.004

Davies, D., Jindal-Snape, D., Digby, R., Howe, A., Collier, C., & Hay, P. (2014). The roles and development needs of teachers to promote creativity: A systematic review of literature. *Teaching and Teacher Education*, *41*, 34–41. doi:10.1016/j.tate.2014.03.003

De Souza Fleith, D., Renzulli, J.S., & Westberg, K. L. (2002). Effects of a creativity training program on divergent thinking abilities and self-concept in monolingual and bilingual classrooms. *Creativity Research Journal, 14*, 373-386. doi:10.1207/S15326934CRJ1434_8

Dobbins, K. (2009). Teacher creativity within the current education system: A case study of the perceptions of elementary teachers. *Education 3–13*, *37*, 95–104. doi:10.1080/03004270802012632

Elfland, A. (1976). The school art style: A functional analysis. *Studies in Art Education*, *17*, 37–44. doi: 10.2307/1319979

Elia, I., Van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM - International Journal on Mathematics Education, 41*, 605–618. doi:10.1007/s11858-009-0184-6

Ervynck, G. (1991). Mathematical creativity. In D. O. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 42–53). Dordrecht, the Netherlands: Kluwer Academic Publishers. doi:10.1007/0-306-47203-1_3

European Parliament and the Council. (2006). *Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning.* Retrieved from http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2006:394:0010:0018:en:PDF

Frost, L. A., Hyde, J. S., & Fennema, E. (1994). Gender, mathematics performance, and attitudes and affect: A meta-analytic synthesis. *International Journal of Educational Research*, *21*, 373–385. doi:10.1016/S0883-0355(06)80026-1

Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences*, *23*, 114–122. doi:10.1016/j.lindif.2012.09.012

Glăveanu, V. P. (2013). Rewriting the language of creativity: The five A's framework. *Review of General Psychology*, *17*, 69–81. doi:10.1037/a0029528

Glăveanu, V. P. (2014). Revisiting the "art bias" in lay conceptions of creativity. *Creativity Research Journal*, *26*, 11–20. doi:10.1080/10400419.2014.873656

Goldstein, H. (2011). *Multilevel statistical models*. Chichester, UK: John Wiley & Sons.

Gravemeijer, K. P. E. (2007). Reken-wiskundeonderwijs anno 2007 - tussen oude waarden en nieuwe uitdagingen [Arithmetic-mathematics education in the year of 2007 – between old values and new challenages]. *Panama-Post:Reken-Wiskundeonderwijs: Onderzoek, ontwikkeling, praktijk*, *26*, 3–10.

Gravemeijer, K. P. E., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R., & Munk, F. (2007). *Measurement and geometry in upper primary school*. Rotterdam, the Netherlands: Sense Publishers.

Guilford, J. P. (1967). *The nature of human intelligence*. New York, USA: McGraw-Hill.

Haanstra, F. (2014). Nationale leerplannen en leerplankaders voor de kunstvakken. *Cultuur + Educatie*, *14*, 8–25. Retrieved from https://www.lkca.nl

Hailey, D., Miller, A., & Yenawine, P. (2015). Understanding visual literacy: The Visual Thinking Strategies approach. In D. M. Baylen & A. D'Alba (Eds.), *Essentials of Teaching and Integrating Visual and Media literacy* (pp. 49-73). London, UK: Springer.

Halmos, P. R. (1980). The heart of mathematics. *American Mathematical Monthly*, *87*, 519–524.

Harris, A. (2016). *Creativity and education*. London, UK: Palgrave macmillan. doi:10.1007/978-94-6209-052-1

+

Haylock, D. W. (1987a). A framework for assessing mathematical creativity in schoolchildren. *Educational Studies in Mathematics*, *18*, 59–74. doi:10.1007/BF00367914
Haylock, D. W. (1987b). Mathematical creativity in schoolchildren. *The Journal of Creative Behavior, 21,* 48–59. doi:10.1002/j.2162-6057.1987.tb00452.x
Henrichs, L. F., & Leseman, P. P. M. (2014). Early science instruction and academic language development can go hand in hand. The promising effects of a low-intensity teacher-focused intervention. *International Journal of Science Education*, *36*, 2978–2995. doi:10.1080/09500693.2014.948944
Hershkovitz, S., Peled, I., & Littler, G. (2009). Mathematical creativity and giftedness in elementary school: Task and teacher promoting creativity for all. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in Mathematics and the Education of Gifted Students* (pp. 255–269). Rotterdam, the Netherlands: Sense Publishers.
Hong, E., & Milgram, R. M. (2008). *Preventing talent loss.* New York, USA: Routledge.
Hong, E., & Milgram, R. M. (2010). Creative thinking ability: Domain generality and specificity. *Creativity Research Journal, 22,* 272-287. doi:10.1080/10400419.2010.503535
Hop, M. (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5 [Balance of mathematics education halfway through elementary school. Periodic assessment 5]*. Arnhem, the Netherlands: Cito.
Hopkins, C. D., & Antes, R. L. (1978). *Classroom measurement and evaluation*. Itasca, USA: Peacock.
Housen, A. C. (2002). Aesthetic thought, critical thinking and transfer. *Arts and Learning Research*, *18*, 99–132. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.467.3752&rep=rep1&type=pdf
Hox, J. J. (2010). *Multilevel analysis. Techniques and applications.* New York, NY, USA: Routledge
Hox, J. J., Moerbeek, M., & Van de Schoot, R. (2018). *Multilevel analysis: Techniques and applications (third edition).* New York, NY, USA: Routledge.
Huang, P., Peng, S., Chen, H., & Tseng, L. (2017). The relative influences of domain knowledge and domain-general divergent thinking on scientific creativity and mathematical creativity. *Thinking Skills and Creativity*, *25*, 1–9. doi:10.1016/j.tsc.2017.06.001
Hwang, D. J., Lee, K. S., & Seo, J. J. (2005). Relationship between divergent thinking in mathematical and non-mathematical situations-based on the TTCT; Figural A and the MCPSAT-. *Journal of Gifted/Talented Education, 15*, 59-76. Retrieved from http://ocean.kisti.re.kr/downfile/volume/ksg/OJHHBM/2005/v15n2/OJHHBM_2005_v15n2_59.pdf
IBM Corporation (2013). *IBM SPSS Statistics for Windows. Version 22.0.* Armonk, NY, USA: IBM Corporation
Isaksen, S. G., Dorval, K. B., & Treffinger, D. J. (2011). *Creative approaches to problem solving: A Framework for innovation and change (Third edition)*. Thousand Oaks, CA, USA: Sage publications.
Janssen, J., Scheltens, F., & Kraemer, J. (2007). *Rekenen-wiskunde. Handleiding. [Arithmetic-Mathematics. Manual].* Arnhem, the Netherlands: Cito.
Janssen, J., Verhelst, N., Engelen, R., & Scheltens, F. (2010). *Wetenschappelijke verantwoording van de toetsen LOVS Rekenen-Wiskunde voor groep 3 tot en met 8 [Scientific justification of the tests LOVS Arithmetic-Mathematics for grade 1-6].* Arnhem, the Netherlands: Cito.
Jarvis, D. H. (2001). *Learning between the lines: A syncretistic experiment in mathematics and visual arts education* (Unpublished master's thesis). Retrieved from https://www.researchgate.net/publication/308347776_Learning_Between_the_Lines_A_Syncretistic_Experiment_in_Mathematics_and_Visual_Arts_Education
Jeon, K.-N., Moon, S. M., & French, B. (2011). Differential effects of divergent thinking, domain knowledge, and interest on creative performance in art and math. *Creativity Research Journal*, *23*, 60–71. doi:10.1080/10400419.2011.545750
Jones, K. (2002). Learning of geometry. In L. Haggarty (Ed.), *Aspects of Teaching Secondary Mathematics: perspectives on practice* (pp. 121–139). London, UK: RoutledgeFalmer.
Kattou, M., & Christou, C. (2013). Investigating the effect of general creativity, mathematical knowledge and intelligence on mathematical creativity. *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education.* Retrieved from http://www.researchgate.net/publication/261286953

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM - International Journal on Mathematics Education*, *45*, 167–181. doi:10.1007/s11858-012-0467-1
Kaufman, J. C., & Beghetto, R. A. (2009). Beyond big and little: The four C model of creativity. *Review of General Psychology*, *13*, 1–12. doi:10.1037/a0013688
Kaufman, J. C., & Beghetto, R. A. (2013). In praise of Clark Kent: Creative metacognition and the importance of teaching kids when (not) to be creative. *Roeper Review*, *35*, 155–165. doi:10.1080/02783193.2013.799413
Kaufman, J. C., Beghetto, R. A., Baer, J., & Ivcevic, Z. (2010). Creativity polymathy: What Benjamin Franklin can teach your kindergartener. *Learning and Individual Differences*, *20*, 380–387. doi:10.1016/j.lindif.2009.10.001
Kazak, S., Wegerif, R., & Fujita, T. (2015). The importance of dialogic processes to conceptual development in mathematics. *Educational Studies in Mathematics, 90,* 105–120. doi:10.1007/s10649-015-9618-y
Keijzer, R., Oprins, B., De Moor, K., & Schoevers, E. M. (2018). Integrating visual art, geometry and creativity for primary school teachers: A pd trajectory. In M. Friman (Ed.), *EAPRIL 2017 proceedings* (pp. 52–65). Hämeenlinna, Finland.
Kim, K. H. (2005). Can only intelligent people be creative? A meta-analysis. *Journal of Advanced Academics, 16*, 57–66. doi:10.4219/jsge-2005-473
Kim, K. H. (2006). Can we trust creativity tests? A review of the Torrance Test of Creative Thinking (TTCT). *Creativity Research Journal, 18,* 3-14. doi:10.1207/s15326934crj1801_2
Kim, K. H. (2011). The creativity crisis: The decrease in creative thinking scores on the Torrance test of creative thinking. *Creativity Research Journal, 23,* 285-295. doi:10.1080/10400419.2011.627805
Kline, P. (1999). *The handbook of psychological testing (2nd ed.).* London, UK: Routledge
Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in elementary school mathematics textbooks – a needle in a haystack. *Mediterreanean Journal for Research in Mathematics Education*, *8*, 29–66.
Kolovou, A. (2011). *Mathematical problem solving in primary school* (Doctoral Dissertation). Retrieved from https://dspace.library.uu.nl/handle/1874/205718
Kozbelt, A. (2001). Artists as experts in visual cognition. *Visual Cognition*, *8*, 705–723.
KPC-Groep. (2000). *Kunstbeschouwen. Culturele en kunstzinnige vorming 2 [Visual art reception. Cultural and artistic education 2].* Den Bosch, the Netherlands.
Kroesbergen, E. H. & Schoevers, E. M. (2017). Creativity as predictor of mathematical abilities in fourth graders in addition to number sense and working memory. *Journal of Numerical Cognition, 3,* 417–440. doi:10.5964/jnc.v3i2.63
Kroesbergen, E. H., Schoevers, E. M., Jonker, V., Keijzer, R., & Wijers, M. (2019). Meetkunst in de basisschool: de effecten van een programma waarin meetkunde en kunstonderwijs zijn gecombineerd. [Geometry and art in primary school: the effects of a program in which geometry and arts education are integrated]. *Cultuur + Educatie*, *18,* 50-67.
Kroesbergen, E. H., & Van Dijk, M. (2015). Working memory and number sense as predictors of mathematical (dis-)ability. *Zeitschrift Für Psychologie*, *223*, 102–109. doi:10.1027/2151-2604/a000208
Kruiter, J., Hoogeveen, K., Beekhoven, S., Kieft, M., & Bomhof, M. (2016). *Rapport Monitor. Cultuuronderwijs in het primair onderwijs en programma Cultuureducatie met kwaliteit [Report Monitor. Culture education in primary school and program culture education with quality].* Utrecht, the Netherlands: Sardes/Oberon.
Kwon, O. N., Park, J. H. J. S., & Park, J. H. J. S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review*, *7*, 51–61. doi:10.1007/BF03036784
Laidra, K., Pullmann, H., & Allik, J. (2007). Personality and intelligence as predictors of academic achievement: A cross-sectional study from elementary to secondary school. *Personality and Individual Differences, 42,* 441-451. doi:10.1016/j.paid.2006.08.001
Leech, N. L., & Onwuegbuzie, A. J. (2007). An array of qualitative data analysis tools: A call for data analysis triangulation. *School Psychology Quarterly, 22,* 557–584. Doi:10.1037/1045-3830.22.4.557

+

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the fifth conference of the European Society for Research in Mathematics Education—CERME-5* (pp. 2330–2339). Retrieved from http://www.mathematik.uni-dortmund.de/~erme/CERME5b/WG14.pdf#page=112

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman and B. Koichu (Eds.). *Creativity in Mathematics and the Education of Gifted Students* (pp. 129 – 145). Rotterdam, the Netherlands: Sense Publishers.

Leikin, R. (2018). Openness and constraints associated with creativity-directed activities in mathematics for all students. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the Scope of Research on Mathematical Problem Solving* (pp. 387–397). Cham, Switserland: Springer.

Leikin, R., & Dinur, S. (2007). Teacher flexibility in mathematical discussion. *The Journal of Mathematical Behavior*, *26*, 328–347. doi:10.1016/j.jmathb.2007.08.001

Leikin, R., Koichu, B., & Berman, A. (2009). *Mathematical giftedness as a quality of problem-solving acts*. (R. Leikin, A. Berman, & B. Koichu, Eds.), *Creativity in mathematics and the education of gifted students*. Dordrecht, the Netherlands: Sense Publishers.

Leikin, R., & Pitta-Pantazi, D. (2013). Creativity and mathematics education: The state of the art. *ZDM - International Journal on Mathematics Education, 45*, 159-166. doi:10.1007/s11858-012-0459-1

Leikin, R., & Sriraman, B. (2017). *Creativity and Giftednes. Interdisciplinary perspectives from mathematics and beyond.* Basel, Switserland: Springer International Publishing. doi:10.1007/978-3-319-38840-3

Lester, F. K. (2013). Thoughts about research on mathematical problem- solving instruction. *The Mathematics Enthusiast*, *10*, 245–278. Retrieved from https://scholarworks.umt.edu/cgi/viewcontent.cgi?article=1267&context=tme

Levav-Waynberg, A., & Leikin, R. (2012a). The role of multiple solution tasks in developing knowledge and creativity in geometry. *Journal of Mathematical Behavior*, *31*, 73–90. doi:10.1016/j.jmathb.2011.11.001

Levav-Waynberg, A., & Leikin, R. (2012b). Using multiple solution tasks for the evaluation of students' problem-solving performance in geometry. *Canadian Journal of Science, Mathematics and Technology Education*, *12*, 311–333. doi:10.1080/14926156.2012.732191

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *Journal of Creative Behavior*, *45*, 215–234. doi:10.1002/j.2162-6057.2011.tb01428.x

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education*. Cham, Switserland: Springer International Publishing.

Livne, N. L. & Milgram, R. M. (2006). Academic versus creative abilities in mathematics: Two components of the same construct? *Creativity Research Journal, 18,* 199-212. doi:10.1207/s15326934crj1802_6

Mann, E. L. (2005). *Mathematical creativity and school mathematics: indicators of mathematical creativity in middle school students* (Doctoral dissertation). Retrieved from http://www.gifted.uconn.edu/siegle/Dissertations/Eric%20Mann.pdf

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, *30*, 236–260. doi:10.4219/jeg-2006-264

Meelissen, M. R. M., Netten, A., Drent, M., Punter, R. A., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs [PIRLS and TIMSS 2011. Trends in achievement in reading, mathematics and science]*. Nijmegen/Enschede: Radboud Universiteit / Universiteit Twente.

Meetkunst Projectteam (2018a). *Meetkunst: Eindverslag van het onderzoek 'Meetkunde uit de kunst in de klas' [MACE project: Final report]*. Retrieved from elwier.nl/meetkunst

Meetkunst Projectteam (2018b). *Meetkunst. Lessen over ruimte en patronen op het grensvlak van meetkunde en beeldende kunst [MACE project. Lessons about space and patterns on the boundary of geometry and visual arts]*. Retrieved from elwier.nl/meetkunst

Meetkunst projectteam (2018c). *Meetkunst. Nascholing over ruimte en patronen op het grensvlak van meetkunde en beeldende kunst [MACE project. Professional development about space and patterns on the boundary of geometry and visual arts]*. Retrieved from elwier.nl/meetkunst

Mihajlović, A., & Dejić, M. (2015). Using open-ended problems and problem posing activities in elementary mathematics classroom. In F. M. Singer, F. Toader, & C. Voica (Eds.), *Proceedings of the 9th International MCG Conference* (pp. 34–41). Sinaia, Romania. Retrieved from https://studylib.net/doc/18739454/mcg-9-conference-proceedings---the-9th-international-conf...

Moerbeek, M. (2015). *SPA-ML. A software package for power analysis of trials with multilevel data.* Department of Methodology and Statistics, Faculty of Social and Behavioural Sciences, Utrecht University. Retrieved from tinyurl.com/spaml

Muhonen, H., Rasku-Puttonen, H., Pakarinen, E., Poikkeus, A. M., & Lerkkanen, M. K. (2016). Scaffolding through dialogic teaching in early school classrooms. *Teaching and Teacher Education, 55,* 143–154. doi:10.1016/j.tate.2016.01.007

Mumford, M. D., Baughman, W. A., Maher, M. A., Costanza, D. P., & Supinski, E. P. (1997). Process-based measures of creative problem-solving skills: IV. Category combination. *Creativity Research Journal, 10*, 59–71. doi:10.1207/s15326934crj1001_7

Muthén, L.K., & Muthén, B.O. (2012). *Mplus user's guide. Seventh edition.* Los Angeles, CA, USA: Muthén & Muthén.

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: Some definitions and characteristics. *Procedia - Social and Behavioral Sciences, 31*, 285–291. doi:10.1016/j.sbspro.2011.12.056

National Governors Association Center for Best Practices (2010). *Common Core State Standards Mathematics.* Washington D.C., USA: National Governors Association Center for Best Practices.

Neisser, U., Boodoo, G., Bouchard, T. J., Boykin, A. W., Brody, N., Ceci, S. J. , . . . Urbina, S. (1996). Intelligence: Knowns & unknowns. *American Psychologist, 51*, 77–101. doi:10.1037/0003-066X.51.2.77

Núñez, R., Edwards, L., & Matos, J. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics*, *39,* 45–65. doi: 10.1023/A:1003759711966

OECD (2016). *PISA 2015 Results (Volume I): Excellence and Equity in Education*. Paris, France: OECD Publishing. doi:10.1787/9789264266490-en

OECD (2018). *The future of education and skills: Education 2030.* Retrieved from https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf

Onderwijsraad. (2013). *Een smalle kijk op onderwijskwaliteit [A narrow view on educational quality].* Retrieved from https://www.onderwijsraad.nl/publicaties/2013/een-smalle-kijk-op-onderwijskwaliteit/volledig/item7043

Pehkonen, E. (1997). The state-of-art in mathematical creativity. *ZDM - International Journal on Mathematics Education*, *29*, 63–67. doi:10.1007/s11858-997-0001-z

Platform Onderwijs 2032. (2016). *Ons onderwijs 2032. Eindadvies [Our education 2032. Final advice].* Retrieved from https://www.rijksoverheid.nl/documenten/rapporten/2016/01/23/eindadvies-platform-onderwijs2032-ons-onderwijs2032

Plucker, J. A. (1999). Reanalyses of student responses to creativity checklists: Evidence of content generality. *Journal of Creative Behavior, 33*, 126–137. doi:10.1002/j.2162-6057.1999.tb01042.x

Plucker, J. A. (2004). Generalization of creativity across domains: Examination of the method effect hypothesis. *Journal of Creative Behavior*, *38*, 1–12. doi:10.1002/j.2162-6057.2004.tb01228.x

Plucker, J. A., Beghetto, R. A., & Dow, G. T. (2004). Why isn't creativiy more important to educational psychologists? Potentials pitfalls, and future directions in creativity research. *Educational Psychologist*, *39*, 111–133. doi:10.1207/s15326985ep3902

Plucker, J. A., & Zabelina, D. (2009). Creativity and interdisciplinarity: One creativity or many creativities? *ZDM - International Journal on Mathematics Education*, *41*, 5–11. doi:10.1007/s11858-008-0155-3

Preckel, F., Goetz, T., Pekrun, R., & Kleine, M. (2008). Gender differences in gifted and average-ability students comparing girls' and boys' achievement, self-concept, interest, and motivation in mathematics. *Gifted Child Quarterly, 52,* 146-159. doi: 10.1177/0016986208315834

Puccio, G. J. (2017). From the dawn of humanity to the 21st Century: Creativity as an enduring survival skill. *Journal of Creative Behavior*, *51*, 330–334. doi:10.1002/jocb.203

Raven, J. C. (1998). *Manual Section 3 Standard Progressive Matrices.* Oxford, UK: Oxford Psychologists Press Ltd

+

Reilly, D., Neumann, D. L., & Andrews, G. (2015). Sex differences in mathematics and science achievement: A meta-analysis of national assessment of educational progress assessments. *Journal of Educational Psychology, 107,* 645–662. doi:10.1037/edu0000012

Renzulli, J. S. (1986). *New directions in creativity.* Mansfield Center, CT, USA: Creative Learning Press.

Roucher, N., & Lovano-Kerr, J. (1995). Can arts maintain integrity in interdisciplinary learning. *Arts Education Policy Review*, *96*, 20–25. doi:10.1080/10632913.1995.9934554

Runco, M. A., & Jaeger, G. J. (2012). The standard definition of creativity. *Creativity Research Journal, 24,* 92–96. doi:10.1080/10400419.2012.650092

Runco, M. A., & Pritzker, S. R. (2011). *Encyclopedia of creativity. 2nd edition*. London, UK: Academic Press.

Sak, U. & Maker, C. J. (2006). Developmental variation in children's creative mathematical thinking as a function of schooling, age and knowledge. *Creativity Research Journal, 18,* 279–291. doi:10.1207/s15326934crj1803_5

Sawyer, R. K. (2014). *Explaining creativity*. New York, NY, USA: Oxford University Press.

Sawyer, R. K. (2014). How to transform schools to foster creativity. *Teacher College Record*, *118*. Retrieved from http://keithsawyer.com/PDFs/TCR.pdf

Schacter, J., Thum, Y., & Zifkin, D. (2006). How much does creative thinking enhance elementary school students achievement. *Journal of Creative Behavior, 40,* 47–72. doi:10.1002/j.2162-6057.2006.tb01266.x

Schoenfeld, A. H. (1983). The wild, wild, wild, wild, wild, world of problem solving (A review of sorts). *Learning of Mathematics*, *3*, 40–47. doi:10.2307/40247835

Schoenfeld, A. H. (2013). Reflections on problem solving theory and practice. *The Mathematics Enthusiast*, *10*, 9–34. Retrieved from http://scholarworks.umt.edu/tme%5Cnhttp://scholarworks.umt.edu/tme/vol10/iss1/3%5Cnhttp://www.math.umt.edu/TMME/vol10no1and2/1-Schoenfeld_pp9_34.pdf

Schoevers, E. M. (2018). *Reliability and validity of the Geometrical Creativity Test (GCT) and the Geometrical Ability Test (GAT)*. Unpublished manuscript, Utrecht University, Utrecht, the Netherlands.

Schoevers, E. M., & Kroesbergen, E. H. (2017). Enhancing creative problem solving in an integrated visual art and geometry program: A pilot study. In D. Pitta-Pantazi (Ed.) *The 10th Mathematical Creativity and Giftedness International Conference - proceedings* (pp. 27–32). Nicosia: Department of Education, University of Cyprus.

Schreuder, C. (2013). *Projectplan Boijmans Taal- en Rekenprogramma [Project plan Boijmans Language and Mathematics Programme].* Rotterdam, the Netherlands: Museum Boijmans Van Beuningen.

Scott, G., Leritz, L. E., & Mumford, M. D. (2009). The effectivess of creativity training: A quanitative review. *Creativity Research Journal, 16,* 361–388. doi:10.1080/10400410409534549

Shen, Y. (2014). *Elementary school teachers' interpretation and promotion of creativity in the learning of mathematics: A grounded theory study* (doctoral dissertation). Retrieved from https://digitalcommons.unl.edu/cgi/viewcontent.cgi?referer=&httpsredir=1&article=1204&context=cehsdiss

Silver, E. A. (1995). The nature and use of open problems in mathematics education. *ZDM - International Journal on Mathematics Education*, *27*, 67–72. Retrieved from https://www.researchgate.net/publication/234698606_The_Nature_and_Use_of_Open_Problems_in_Mathematics_Education_Mathematical_and_Pedagogical_Perspectives

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM - International Journal on Mathematics Education*, *29*, 75–80. doi:10.1007/s11858-997-0003-x

Silvia, P. J. (2008). Creativity and intelligence revisited: a latent variable analysis of Wallach and Kogan (1965). *Creativity Research Journal, 20,* 34–39. doi:10.1080/10400410701841807

Sirin, S. R. (2005). Socioeconomic status and academic achievement: A meta-analytic review of research. *Review of Educational Research, 75*, 417–453. doi:10.3102/00346543075003417

Sloper, D.S. (2015). A-priori sample size calculator for structural equation models [Software]. Retrieved from http://www.danielsoper.com/statcalc

Soh, K.-C. (2000). Indexing creativity fostering teacher behavior: A preliminary validation study. *Journal of Creative Behavior*, *34*, 118–134. doi:10.1002/j.2162-6057.2000.tb01205.x

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Journal of Secondary Gifted Education*, *17*, 20–36. doi:10.4219/jsge-2005-389

Stein, M., Remillard, J. T., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second Handbook of research on mathematics teaching and learning* (pp. 319–369). Charlotte, USA: Information Age Publishing.
Stein, M., & Smith, M. (2010). The influence of curriculum on students' learning. In B. J. Reys, R. E. Reys, & R. Rubenstein (Eds.), *Mathematics Curriculum. Issues, trends, and future directions* (pp. 351–362). Reston: National Council of Teachers of Mathematics.
Sternberg, R. (2007). Creativity as a habit. In A.-G. Tan (Ed.), *Creativity. A handbook for teachers* (pp. 3–25). Singapore: World Scientific Publishing. doi:10.1142/6211
Sternberg, R. J. (2015). Teaching for creativity: The sounds of silence. *Psychology of Aesthetics, Creativity, and the Arts*, *9*, 115–117. doi:10.1037/aca0000007
Sternberg, R. J. & Lubart, T. I. (1999) The concept of creativity: Prospects and paradigms. In R.J. Sternberg (ed.) *Handbook of Creativity* (pp. 3–16). London, UK: Cambridge University Press.
Stichting Leerplanontwikkeling (2015). *Nieuw elan voor kunstzinnige oriëntatie in het primair onderwijs. Een praktische handreiking voor leraren [new diligence for arts education in primary school].* Enschede, the Netherlands: SLO.
Stichting Leerplanontwikkeling (2018). *Leerplankader kunstzinnige oriëntatie.* Retrieved from http://kunstzinnigeorientatie.slo.nl/
Tabachnick, B. G. & Fidell, L. S. (2013). *Using Multivariate Statistics. Sixth edition.* New Jersey, NJ, USA: Pearson.
Taggar, S. (2002). Individual creativity and group ability to utilize individual creative resources: A multilevel model. *The Academy of Management Journal*, *45*, 315–330. doi:10.2307/3069349
Thijs, A., Fisser, P., & Hoeven, M. van der. (2014). *21eeeuwse vaardigheden in het curriculum van het funderend onderwijs [21st century skills in the curriculum of established education].* Enschede, the Netherlands: Stichting Leerplan Ontwikkeling (SLO). Retrieved from http://downloads.slo.nl/Repository/21e-eeuwse-vaardigheden-in-het-curriculum-van-het-funderend-onderwijs.pdf
Tishman, S., MacGillivray, D., & Palmer, P. (1999). *Investigating the educational impact and potential of the Museum of Modern Art's visual thinking curriculum*. Cambridge, MA, USA: Harvard University.
Torrance, E. P. (2008). *Torrance Test of Creative Thinking.* Benseville, IL, USA: Scholastic Testing Service.
Torrance, E. P., & Safter, H. T. (1999). *Making the creative leap beyond.* Buffalo, NY, USA: Creative Education Foundation Press.
Treffinger, D. J., Young, G. C., Selby, E. C., & Shepardson, C. (2002). *Assessing creativity: A guide for educators.* Storrs, CT, USA: University of Connecticut, The national research center on the gifted and talented. Retrieved from http://www.gifted.uconn.edu/nrcgt/reports/rm02170/rm02170.pdf
Urban, K. K. (2005). Assessing creativity: The Test for Creative Thinking - Drawing Production (TCT-DP). *International Education Journal*, *6*, 272–280. Retrieved from https://files.eric.ed.gov/fulltext/EJ854980.pdf
Urban, K. K., & Jellen, H. G. (1996). *Test for Creative Thinking - Drawing Production (TCT-DP).* Lisse, the Netherlands: Swets & Zeitlinger.
Van de Weijer-Bergsma, E., Kroesbergen, E.H., Prast, E., & Van Luit, J.E.H. (2014). *Dutch percentile norms for the Raven Standard Progressive Matrices.* Internal report. Utrecht, The Netherlands: Utrecht University.
Van de Weijer-Bergsma, E., Kroesbergen, E. H., Prast, E. J., & Van Luit, J. E. H. (2015). Validity and reliability of an online visual–spatial working memory task for self-reliant administration in school-aged children. *Behavior Research Methods*, *47*, 708–719. doi:10.3758/s13428-014-0469-8
Van den Heuvel-Panhuizen, M., & Buys, K. (2005). *Young children learn measurement and geometry. A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school.* Rotterdam, the Netherlands: Sense Publishers.
Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521–525). Dordrecht, the Netherlands: Springer. doi:1007/978-94-007-4978-8
Van Grootheest, L., Huitema, S., Erich, L., Van Hijum, R., Nillesen, C., Osinga, H., … Van de Wetering, M. (2011). De wereld in getallen [The world in numbers]. Den Bosch, the Netherlands: Malmberg.

Van Lier, L. (1996). *Interaction in the language curriculum: awareness, autonomy and authenticity*. London, UK: Longman.
Van Onna, J., & Jacobse, A. (2008). *Laat maar zien. Een didactische handleiding voor beeldend onderwijs [Show me. A didactical manual for visual arts education]*. Houten, the Netherlands: Wolters-Noodhoff.
Van Zanten, M., Barth, F., Faarts, J., Van Gool, A., & Keijzer, R. (2009). *Kennisbasis rekenen-wiskunde voor de pabo*. Retrieved from https://10voordeleraar.nl/documents/kennisbases_pabo/kb_rekenen_wiskunde_pabo.pdf
Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch elementary school mathematics textbooks. *ZDM - International Journal on Mathematics Education*, *50*, 827–838. doi:10.1007/s11858-018-0973-x
Volman, M. (2006). Het 'nieuwe leren': oplossing of nieuw probleem? ['New learning': solution or a new problem?]. *Pedagogiek, 26,* 14-25.
Walker, C. M., Winner, E., Hetland, L., Simmons, S., & Goldsmith, L. (2011). Visual thinking: Art students have an advantage in geometric reasoning. *Creative Education*, *2*, 22–26. doi:0.4236/ce.2011.21004
Warner, L. B., Alcock, L. J., Coppolo, J. J., & Davis, G. E. (2003). How does flexible mathematical thinking contribute to the growth of understanding? *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education Held Jointly with the 25th Conference of PME-NA*, *4*, 371–378. Retrieved from https://eric.ed.gov/?id=ED500860
Weisberg, R. W. (1999). Creativity and knowledge: A challenge to theories. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 226–250). Cambridge, UK: Cambridge University Press.
Willemsen, R., Kroesbergen, E. H., & Schoevers, E. M. (2019). The structure of creativity in primary education: An empirical confirmation of the amusement park theory. *Journal of Creative Behavior*. Advance online publication. doi:10.1002/jocb.411
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458–477.

Nederlandse Samenvatting (Summary in Dutch)

Het bevorderen van creativiteit staat hoog op de onderwijsagenda. De groeiende aandacht voor creativiteit in het onderwijs lijkt onder andere gerelateerd te zijn aan de grote ontwikkelingen in de samenleving. We leven in een snel veranderde samenleving met snelle technologische ontwikkelingen en een snel groeiende hoeveel informatie. Dit heeft tot gevolg dat er in onderwijsdebatten wordt gediscussieerd over de competenties die nodig zijn om leerlingen voor te bereiden op deze snel veranderende samenleving. Creativiteit wordt genoemd als één van deze competenties en is daarom belangrijk om te bevorderen in het onderwijs. Echter, meer expertise is nodig hoe dit kan worden gedaan in het onderwijs, met name in een discipline als rekenen-wiskunde. Dit proefschrift had daarom tot doel om meer inzicht te krijgen in hoe creativiteit van (bovenbouw) leerlingen kan worden bevorderd in het reken-wiskunde onderwijs op de basisschool. Dit is in vier studies op verschillende manieren onderzocht.

Om meer inzicht te krijgen in hoe creativiteit kan worden bevorderd in het reken-wiskundeonderwijs, is het wenselijk om te weten hoe creativiteit en wiskunde aan elkaar zijn gerelateerd: is creativiteit domein-specifiek of domein-algemeen? Dit is belangrijk omdat het ook verband houdt met hoe creativiteit van leerlingen kan worden gestimuleerd: kan dit alleen binnen één domein, zoals rekenen-wiskunde, of kan het worden gestimuleerd door een algemene creativiteitstraining? In *Hoofdstuk 2* zijn daarom de relaties tussen domein-specifieke wiskundige creativiteit, domein-algemene creativiteit en reken-wiskunde prestaties onderzocht. Door middel van *structural equation modelling* zijn deze relaties onderzocht. Twee tegenstrijdige modellen werden getest: 1) reken-wiskundeprestaties en domein-algemene creativiteit voorspellen samen wiskundige creativiteit, 2) domein-algemene creativiteit voorspelt wiskundige creativiteit, dat vervolgens reken-wiskundeprestaties voorspelt. Verschillende creativiteitsmetingen zijn gebruikt om het latente construct domein-algemene creativiteit te meten. Resultaten laten zien dat domein-algemene creativiteit niet één construct is, maar bestaat uit minstens twee verschillende constructen, die we in dit onderzoek identificeerden als 'genereren van ideeën' en 'exploreren en dieper ingaan op ideeën'. Verder vonden we onderbouwing voor het eerste model waarin reken-wiskunde prestaties en domein-algemene creativiteit samen de wiskundige creativiteit voorspelden. Alhoewel dit een cross-sectioneel onderzoek is en er geen conclusies kunnen worden getrokken over de causale verbanden van de relaties, suggereren de resultaten dat zowel domein-specifieke wiskundige kennis en vaardigheden en domein-algemene creatieve processen nodig zijn om creatief te zijn in rekenen-wiskunde. Op basis van deze resultaten wordt aanbevolen om bij het bevorderen van creativiteit in het reken-wiskundeonderwijs te focussen op beide aspecten. Verder onderzoek is nodig om te bestuderen hoe leerkrachten dit kunnen doen in de onderwijspraktijk.

In *hoofdstuk 3* zijn de relaties tussen domein-algemene creativiteit van leerlingen en hun prestaties op drie typen meetkundeproblemen onderzocht. Het doel was om te onderzoeken of de relatie tussen domein-algemene creativiteit en meetkundeprestaties van leerlingen verschilden tussen drie soorten meetkundeproblemen: (1) een probleem waarbij een routinematige oplossing werd gevraagd en waarbij één antwoord goed was (gesloten routine probleem), (2) een meetkundeprobleem waarbij leerlingen meerdere oplossingen moesten geven (meerdere-oplossingen-probleem) en (3) een probleem waarbij leerlingen meetkundig moesten redeneren over een kunstwerk (non-routine kunst-meetkundeprobleem). Resultaten lieten een positief verband zien tussen de creativiteit van leerlingen en hun prestaties op alle typen meetkundeproblemen. Echter, de creativiteit van leerlingen was sterker gerelateerd aan prestaties op een meerdere-oplossingenprobleem dan aan hun prestaties op de routine en non-routine kunst-meetkundeproblemen. Aangezien er in de analyses verschillende covariaten zijn gebruikt, kunnen we voorzichtig concluderen dat de effecten van creativiteit de ware betrokkenheid van creatieve denkprocessen in het oplossen van meetkundige problemen weerspiegelen. Creatieve denkprocessen lijken dus het meest betrokken te zijn bij het oplossen van open meerdere-oplossingenproblemen. Het is belangrijk dat leerlingen de juiste problemen krijgen aangeboden in het reken-wiskundeonderwijs om creativiteit te bevorderen. Meerdere-oplossingenproblemen lijken, door hun open karakter, dit goed te kunnen doen. In deze studie wordt dan ook aanbevolen om dit type probleem op te nemen in reken-wiskundeboeken. De resultaten van dit onderzoek laten ook zien dat een meerdere-oplossingenprobleem een waardevol instrument kan zijn om wiskundige creativiteit van leerlingen te meten. Dit onderzoek bevestigd daarin eerder onderzoek.

Hoofdstuk 4 en 5 beschrijven studies die zijn gedaan in relatie tot het meetkunstproject. Het meetkunstproject had tot doel om basisscholen in Nederland te ondersteunen bij het behalen van leerdoelen van zowel reken-wiskunde en beeldende kunst en bij het bevorderen van creativiteit van leerlingen in beide disciplines. Om dit doel te bereiken is er een lessenserie van 9 lessen ontwikkeld voor leerlingen uit groep 6 – 8. In deze lessenserie stonden open activiteiten en klassendiscussies in een geïntegreerde beeldende kunst- en meetkundeleercontext centraal. Daarnaast is er een nascholing voor leerkrachten ontwikkeld om leerkrachten de ondersteunen bij de implementatie van deze meetkunstlessen.

Hoofdstuk 4 beschrijft een casusstudie over een leerkracht van groep 6 en haar leerlingen. We wilden met deze casusstudie meer inzicht verkrijgen in hoe aanbevolen pedagogische strategieën om creativiteit te bevorderen zijn geïmplementeerd in de klas en hoe die strategieën relateren aan verschillende typen reken-wiskundelessen. Daarnaast onderzochten we hoe die strategieën van leerkrachten relateren aan de wiskundige creativiteit van leerlingen in klassendialogen. Verder hebben we in deze studie een vernieuwde definitie van wiskundige creativiteit voorgesteld. De veelgebruikte definitie van Sriraman (2005) limiteert wiskundige creativiteit tot het proces van probleemoplossen en het zelf bedenken van wiskundige problemen. Wij vonden deze definitie echter incompleet om wiskundige creativiteit in de onderwijspraktijk in de brede zin te onderzoeken. Onze herziene definitie van wiskundige creativiteit refereert daarom naar het combineren van bekende concepten in een adequate, maar voor de leerling nieuwe manier, waarbij het wiskundig begrip van leerlingen wordt vergroot. In deze studie onderzochten wij interacties tussen de leerkracht en haar leerlingen in drie verschillende reken-wiskunde lessen: een ‘open’ meetkunstles op school, een ‘open’ meetkunstles buiten de school en een reguliere ‘gesloten’ reken-wiskundeles. De studie laat zien dat de leerkracht alle geïdentificeerde strategieën gebruikt voor het bevorderen van wiskundige creativiteit, maar sommige strategieën meer frequent dan andere. Meer diverse strategieën zijn gebruikt tijdens de twee open interdisciplinaire reken-wiskundelessen vergeleken met de reguliere methode-gebonden reken-wiskundeles. De resultaten van dit onderzoek laten zien dat wiskundige creativiteit bevorderd werd als de leerkracht een langere klassendialoog initieerde en een open sfeer creëerde in de open interdisciplinaire lessen met open leerdoelen. Een open sfeer werd gecreëerd als de leerkracht de leerlingen ruime kansen gaf om hun ideeën te uiten en duidelijk aan leerlingen liet merken dat ze hun ideeën serieus nam. Verder lijken er een aantal factoren te zijn die er aan hebben bijgedragen dat wiskundige creativiteit aanwezig was in de open interdisciplinaire lessen en afwezig was in de reguliere reken-wiskunde lessen: de interdisciplinaire lesinhoud, een minder specifiek leerdoel (bijv. leren over conceptualisaties in plaats van strategieën), en het feit dat in deze lessen een andere socio-wiskundige norm aanwezig was dan in de reguliere les (een norm die minder gericht was op een correcte oplossing van een wiskundig probleem).

De studie in *hoofdstuk 5* onderzocht de effecten van het meetkunstprogramma op het meetkundig vermogen van leerlingen (i.e. meetkundig begrip, meetkundige creativiteit en meetkundige vocabulaire) en de perceptie van leerlingen op beeldende kunst in een quasi-experimenteel design. Drie groepen leerkrachten en hun klassen zijn onderzocht. Een groep leerkrachten gaf de meetkunst lessenserie en participeerde in het nascholingsprogramma, een groep leerkrachten gaf alleen de lessenserie, en de vergelijkingsgroep gaf een serie reguliere meetkundelessen afkomstig uit veelgebruikte reken-wiskundemethoden. Het resultaat van het meetkunstonderzoek laat zien dat leerlingen die de lessenserie hebben gevolgd meer meetkundige aspecten zijn gaan herkennen in beeldende kunstwerken dan leerlingen die de reguliere meetkundelessen hebben gehad. Verder was het meetkunstprogramma even effectief als reguliere meetkundelessen met betrekking tot het verbeteren van het meetkundig vermogen van leerlingen. Leerlingen uit het meetkunstprogramma bereikten dus een zelfde meetkundig begripsniveau met open lessen waarin ze zich creatief konden uiten als leerlingen die de reguliere lessen volgden waarin de meetkundeconcepten direct en tijdsefficiënt werden aangeleerd. Alhoewel de resultaten van de kwalitatieve casusstudie uit hoofdstuk 4 lieten zien dat leerlingen meer wiskundige creatieve ideeën uitten in de meetkunstlessen vergeleken bij de reguliere reken-wiskunde les, geeft het resultaat van dit onderzoek aan dat er geen effect was van het meetkunst programma op de meetkundige creativiteit van leerlingen gemeten met een meerdere oplossingen taak. Leerlingen die deelnamen aan het meetkunstprogramma zijn niet meer vooruit gegaan in wiskundige creativiteit dan leerlingen die de reguliere lessen hebben gevolgd.

In de algemene discussie in *hoofdstuk 6* wordt geconcludeerd dat dit proefschrift nieuwe en empirische gefundeerde antwoorden geeft op de vraag hoe we creativiteit in het

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onderwijs in het algemeen, en specifiek het reken-wiskundeonderwijs, een plek kunnen geven en dit kunnen bevorderen. Onderzoek in dit proefschrift laat zien dat het belangrijk is om creativiteit te bevorderen in meerdere en geïntegreerde disciplines. Tevens laat het zien dat creativiteit van leerlingen in (interdisciplinair) reken-wiskundeonderwijs kan worden bevorderd door in een les enigszins ruim gedefinieerde leerdoelen op te stellen en door leerlingen open problemen, activiteiten of lessen aan te bieden. Daarbij is het belangrijk dat leerkrachten een open sfeer creëren in de klas en expliciet aangeven dat creatieve ideeën en antwoorden van leerlingen gewaardeerd worden. Uit het onderzoek komt echter ook een aantal factoren naar voren dat de implementatie van deze strategieën kan belemmeren. Zo lijken niet alle leerkrachten in staat om tegelijkertijd de creativiteit van leerlingen te bevorderen en het wiskundig begrip van leerlingen te verbeteren. Daarnaast zou de rol van (reken-wiskunde)methoden in het basisonderwijs het bevorderen van creativiteit kunnen belemmeren. Reken-wiskundemethoden worden veel gebruikt door leerkrachten, maar er zitten maar weinig open problemen of opdrachten in de methoden die creativiteit van leerlingen kunnen bevorderen. Daarnaast vinden leerkrachten het vaak moeilijk om van de methode af te wijken om bijvoorbeeld een andere en open les te geven die creativiteit van leerlingen wel kan bevorderen.

Als laatst worden er naar aanleiding van het onderzoek in dit proefschrift een aantal aanbevelingen gedaan voor de onderwijspraktijk. De eerste aanbeveling is dat het belang van creativiteit in het onderwijs weerspiegeld moet worden in de curricula, methodeboeken, testen en onderwijsinspecties. Als het belang op verschillende manieren wordt weerspiegeld, kan dit een manier zijn om creativiteit structureel een plek te geven in scholen. Een tweede aanbeveling is dat we meer open mogelijkheden moeten bieden aan leerlingen in het basisonderwijs. Dit kunnen open problemen zijn, open onderwijsdialogen of open onderwijsactiviteiten. Hierbij is het van belang dat niet alleen de hoog presterende leerlingen dit krijgen aangeboden. Verder is het belangrijk dat er bij deze open mogelijkheden een open sfeer wordt gecreëerd in de klas en dat expliciet wordt aangegeven dat creatieve ideeën en antwoorden van leerlingen gewaardeerd worden. Leerkrachten kunnen de creativiteit van leerlingen bevorderen door een combinatie van strategieën toe te passen: het gebruik van activerende open vragen, het gebruik van vragen die ideeën van leerlingen uitlokken over een bepaald onderwerp, het respecteren van antwoorden van leerlingen, leerlingen laten weten dat hun antwoorden zijn gehoord en het stellen van vragen om zo meer over de ideeën van leerlingen te weten te komen. Daarnaast is het van belang dat basisschoolleerkrachten effectieve nascholing krijgen om dit soort strategieën te leren toepassen in de onderwijspraktijk om daarmee zowel het wiskundig vermogen als de wiskundige creativiteit van leerlingen te bevorderen.

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About the author

Eveline Schoevers was born on 5 December 1990 in Woerden, the Netherlands. After completing high school (Athenaeum) in 2009 at 'Minkema College' in Woerden, she began her Bachelor Pedagogical Sciences at Utrecht University. In 2011, Eveline spent a semester abroad and followed courses on educational sciences at Middlesex University in London. After obtaining her Bachelor's Degree in 2012, Eveline took a gab year and worked as an educational assistant in special education, as a research assistant at Utrecht University, as a volunteer at 'Platform Voorleesexpress' in the Netherlands and at a community project in Masaka, Uganda, and travelled across Eastern Africa. During the gab year, Eveline realized she was mainly interested in education and conducting research. Hence, in September she began the two-year research master program 'Educational Sciences: Learning in Interaction' at Utrecht University. During this Master's program Eveline worked as a research assistant and completed an internship with the dyscalculia expertise center (*in Dutch: Ambulatorium*), which rendered her a clinical degree as an educational psychologist. After her graduation, in September 2015, Eveline started her doctoral research at the department of Education & Pedagogy at Utrecht University under supervision of Prof. dr. Paul Leseman and Prof. dr. Evelyn Kroesbergen. Eveline participated in 'the Mathematics, Arts, Creativity in Education Project' *(in Dutch: Meetkunst Project)* in collaboration with Museum Boijmans van Beuningen, University of Applied Sciences Rotterdam and iPabo, the Freudenthal Institute, and two elementary schools in Rotterdam. The project focused on creative problem solving in elementary mathematics and visual arts education, and was funded by the Netherlands Initiative for Educational Research (NRO). During her PhD trajectory, Eveline was involved in the PhD council of the Faculty of Social and Behavioural Sciences, co-organized three international conferences on creativity research in Utrecht, and supervised several student theses. Moreover, she frequently presented her work during national and international conferences and gave workshops for Dutch teachers.

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Publications

International peer-reviewed publications

Kroesbergen, E. H. & Schoevers, E. M. (2017). Creativity as predictor of mathematical abilities in fourth graders in addition to number sense and working memory. *Journal of Numerical Cognition, 3*, 417 – 440. doi:10.5964/jnc.v3i2.63

Schoevers, E. M., Kroesbergen, E. H. & Kattou, M. (2018). Mathematical creativity: A combination of domain-general creative and domain-specific mathematical skills. *Journal of Creative Behavior*. Advance online publication. doi:10.1002/jocb.361

Schoevers, E. M., Leseman, P. P. M., & Kroesbergen, E. H. (in press). Enriching mathematics education with visual arts: Effects on elementary school students' ability in geometry and visual arts. *International Journal of Science and Mathematics Education.*

Schoevers, E. M., Leseman, P. P. M., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking Skills and Creativity, 31,* 323 – 334. doi:10.1016/j.tsc.2019.02.003

Willemsen, R., Schoevers, E.M., & Kroesbergen, E.H. (2019). The Structure of Creativity in Primary Education: An Empirical Confirmation of the Amusement Park Theory. *Journal of Creative Behavior.* Advance online publication. doi:10.1002/jocb.411

Conference proceedings

Schoevers, E.M. & Kroesbergen, E.H. (2017). Enhancing creative problem solving in an integrated visual art and geometry program: A pilot study. In Demetra Pitta-Pantazi (Eds.), *The 10th Mathematical Creativity and Giftedness International Conference - proceedings* (pp. 27 – 32). Nicosia, Cyprus: Department of Education, University of Cyprus.

Keijzer, R., Oprins, B., De Moor, & Schoevers, E.M. (2017). Integrating visual art, geometry and creativity for primary school teachers: a pd trajectory. *EAPRIL 2017 proceedings* (pp. 52 – 65). Hämeenlinna, Finland: EAPRIL

National scholarly publications

Kroesbergen, E. H., Schoevers, E.M., Jonker, V., Wijers, M. & Keijzer R. (2019). Meetkunst in de basisschool: De effecten van combinatie van meetkunde en kunstonderwijs. *Cultuur + Educatie. 51,* 50 – 67.

Meetkunst projectteam (2018). *Meetkunst: Eindverslag van het onderzoek 'Meetkunst uit de kunst in de klas'. .* Retrieved from elwier.nl/meetkunst

Schoevers, E.M., Kroesbergen, E.H., Keijzer, R. & Jonker, V. (2016). Meetkunde, kunst en creativiteit - Een lessenserie meetkunde ook voor zwakke rekenaars. *Volgens Bartjens*, *36,* 26 – 28.

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Wijers, M.M., **Schoevers, E.M.,** Jonker, V.H. & Keijzer, R. (2016). Ruimte vangen - Meetkunde en kunst. *Volgens Bartjens*, *36,* 8 – 11.

Manuscripts under review

Schoevers, E.M., Kroesbergen, E.H., Moerbeek, M., & Leseman, P.P.M. (under review). The relation between creativity and elementary school students' performance on different types of geometrical problems.