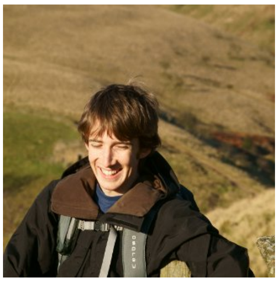
forces us to deal directly and unavoidably with the usual issues on our preconceptions about philosophy, logic and rationality, vs their nature and the shapes that they have taken throughout history, especially in contexts where they don't seem immediately recognisable. This is why the upcoming workshop, bringing together scholars working on philosophy in Britain in the earlier middle ages, is particularly interesting and exciting, especially for a late medievalist like myself. http://www.dcamp.uk/britains-early-philosophers/

The lineup is great and includes talks on Alcuin's logic (Jack Coopey) and on Abbo of Fleury's arithmetic (Clelia Crialesi), which might be of particular interest to our readers.

If you would like to join us, just drop Sara a line (s.l.uckelman@durham.ac.uk).

Graziana Ciola
Durham University

Uncertain Reasoning



I'm writing this month's "What's Hot..." column about a paper that is quite new, and I'm still trying to digest its results. So this column is my attempt to understand what's going on.

Let's start with your standard betting argument for constraints on rational belief. It's irrational to accept a set of bets that guarantee you a sure loss, and depending on what further rational constraints you agree to, this will give you some form of norm for how you ought to set your prices for gambles (probabilism for example). How easy is it to decide whether a particular set of gambles is immune to sure loss? Even if the state space is finite – so the problem is at least decidable – this problem is, in general NP-hard. That is, it is a computationally demanding task to figure out whether a certain set of gambles avoids sure loss. You're essentially searching a really big space of sets of gambles to find out if there are any that (i) you are committed to finding acceptable because of the rationality constraints and (ii) suffer a sure loss. If there are any such, then your set of gambles is incoherent. Is there a less demanding kind of rationality, where you are only expected to be able to perform computationally less demanding search tasks? It turns out that there is. (I'll give you the reference at the end of the next paragraph, because writing it out here would spoil the punchline of this column.) If you only require your agent to search for gambles that satisfy (i) and (ii) in a systematically smaller space of sets of gambles (one for which the search is achievable in polynomial time) then you could be "P-coherent". One can then prove that if you are not also coherent in the stronger sense – that is, if you do not avoid sure loss in the more demanding sense – then there are only a limited number of ways your set of gambles and associated price assignments could be. This is an interesting, if slightly weird result. But the really wild stuff happens when we add one further ingredient.

So far we've been assuming that we're dealing with your standard classical set-up of gambles over a set of states. What if, instead, we considered gambles that result from the action of a self-adjoint operator on a Hilbert space? That is, what if we described events not by sets of states, but by projections onto a subspace of a vector space? What happens is this: for a suitable definition of polynomial-time-searchable space of gambles, you can essentially derive quantum mechanics from just the principle that you ought to be P-coherent! That is, from just that idea that you ought to figure out if you're suitably coherent in a computationally tractable way, plus some assumptions about what gambles you're committed to accepting, you can show that your prices for gambles can exhibit all the weird properties of quantum mechanics (e.g. entanglement). This is a surprising result. You get the theory of quantum mechanics out of just insisting that you ought to be able to figure out if you're coherent in a computationally tractable way. Here's the reference: Benavoli, Facchini and Zaffalon "Computational Complexity and the Nature of Quantum Mechanics" arXiv:1902.04569v1.

As you can probably tell, I'm still digesting this result. I find the result intriguing. I can't pretend I fully understand it yet, but it's a weird and surprising fact. I'm not sure I know what this means about the interpretation of quantum mechanics, or whether it really even tells us anything new (after all, "Quantum Bayesianism" is not new). But the connection to computational complexity is a new twist. It's another interesting and possibly deep connection between computational complexity and quantum mechanics (the more standard connection being the fact that quantum computers can solve some kinds of hard problems much faster than conventional computers). (For more on the topic of computational complexity in a very accessible form, see Aaronson "Why Philosophers Should Care About Computational Complexity" arXiv:1108.1791).

Seamus Bradley
Philosophy, University of Leeds

Mathematical Philosophy

In this column, I'd like to talk about *truthmakers*: those things in the world (states of affairs, actions, events, etc.) that are responsible for the truth or falsity of our claims about the world.—Truthmakers have been around in philosophy for a while, especially in metaphysics. The project of truthmaker metaphysics is to use truthmakers as a guide to metaphysics and ontology. The idea would be, for example, to take statements about numbers, like "there are just seven swans on the lake" or Kant's ubiquitous example of "7 + 5 = 12," to determine their truthmakers, and to let the results of this investigation tell us whether numbers exist, what's their nature, etc. This is roughly the metaphysical project of David Armstrong and others.

But truthmaker metaphysics is not what I think is "hot" in mathematical philosophy. What I'd like to talk about, instead, is a different kind of project involving truthmakers: the project of truthmaker *semantics*. I wish to propose that we mathematical philosophers can help realize the philosophical potential of this project, which has recently been championed by Kit Fine and others (see Fine's overview piece "Truthmaker Semantics," in *A Companion to the Philosophy of Language, Second Edition*, edited by Bob Hale, Crispin Wright, Alexander Miller, Wiley 2017).

The idea is to use truthmakers in order to elucidate semantic concepts, like content, aboutness, or truth. In this way, truthmaker semantics differs from truthmaker metaphysics, which is primarily interested in metaphysical questions, like what exists or what's the nature of things.

Truthmaker semantics is an alternative to standard possible-worlds semantics. This difference comes out clearly when we think about how the two semantics think of semantic content. In possible-worlds semantics, the content of a proposition is typically understood in terms of the possible worlds where the proposition is true. In truthmaker semantics, in contrast, the idea is to model the semantic content of a proposition in terms of its truthmakers.

Truthmaker semantics has cut-and-dry applications in metaphysics, and this is also where we can clearly see the advantage it provides over the possible-worlds approach. In his more programmatic piece "Hyperintensional Metaphysics," Daniel Nolan proclaims: "The twenty-first century is seeing a hyperintensional revolution" (*Philosophical Studies* 171(1): 149–160). By this he means that metaphysicians have come to realize that *hyperintensional concepts*, i.e. concepts which can differ among necessary equivalents, are central to many important metaphysical questions.

The concept of *metaphysical grounding*, which is typically glossed as the relation of one truth holding *in virtue of others*, provides an illustrative example. Intuitively, the propositions that $7 + 5 = 12$ and that $e^{i\pi} + 1 = 0$ have different grounds—the one holds solely in virtue of facts about the natural numbers while the other holds, at least partly, in virtue of facts about the rational and irrational numbers. But the two statements are necessarily equivalent! They are both, after all, necessary truths. This means that grounding is a hyperintensional concept: two necessarily equivalent propositions can have different grounds.

But from this it follows that there is simply no way of giving a semantics of ground purely in terms of possible-worlds. The propositions $7 + 5 = 12$ and $e^{i\pi} + 1 = 0$ are true in exactly the same possible-worlds (all of them!), meaning their semantic content in possible-worlds semantics is the same. Consequently, in possible-worlds semantics, we cannot account for their different grounds.

This is, in a sense, old news. We know that possible-worlds semantics is intensional and thus obviously has issues with hyperintensional concepts. But before the start of the "hyperintensional revolution," this was largely seen as inconsequential for most of philosophy, especially metaphysics, since hyperintensionality was viewed as a primarily epistemological phenomenon. Only in recent years, the significance of hyperintensional phenomena has begun to be appreciated in other subfields of philosophy, like in metaphysics.

And that's where truthmakers enter the picture again. In his "Guide to Ground," which in 2012 was included in the *Philosopher's Annual* (Volume XXXII), Kit Fine argues that we can give a truthmaker semantics for metaphysical ground: the idea is (roughly) that we can understand grounding as a special kind of truthmaker preservation. In this way, truthmaker semantics can do something that possible-world semantics can't: account for the hyperintensional concept of metaphysical ground.

It's perhaps a bit of a grandiose claim, but I'd like to suggest that truthmaker semantics has the potential to play the same role for the hyperintensional revolution that possible-worlds semantics has played for what Nolan calls the "intensional revolution" of the last century. By this, Nolan means the rise of modal/intensional distinctions and the use of the possible-worlds framework that analytic philosophy experienced in the second half of the twentieth century. The formal work on possible-worlds semantics carried out by Barcan-Marcus, Carnap, Hintikka, Kripke, and others functioned as a catalyst for this development. My proposal is that truthmaker semantics can play a similarly catalyzing role for the hyperintensional revolution.

Part of the appeal of the possible-worlds framework derives from its versatility: it has found fruitful applications in metaphysics, epistemology, philosophy of language, ethics, and elsewhere in philosophy. Well, neither are the applications of truthmaker semantics limited to metaphysics. To give just one example of the many interesting applications that have surfaced in recent years, consider Stephan Krämer's paper "A Hyperintensional Criterion of Irrelevance" (*Synthese* 194(8): 2917-2930), in which Krämer uses truthmaker semantics to tackle hyperintensional issues in Bayesian confirmation theory. Now this is where things get interesting for us mathematical philosophers: there are many other applications to explore and work to be done.

My hope is that we truthmaker semanticists will help put the hyperintensional revolution on solid philosophical footing by providing robust mathematical results. But perhaps another hyperintensional semantics, like impossible-worlds semantics, will prove to be more fruitful, or perhaps the hyperintensional revolution will fail altogether. There's only one way to find out: Let's get to work!

PS: If you got interested in truthmaker semantics, consider joining our summer school in Hamburg this year: https://hamburgersommerkurs.wordpress.com/.

Johannes Korbmacher
Munich Centre for Mathematical Philosophy

Events

April

SHE: Seminar on Historical Epistemology, University of Milan, 2 April.

LoE: Workshop on Levels of Explanation, University of Birmingham, 3 April.

ResLog: Reasoning, Argumentation and Logic in Natural Language: Experiments and Models, Ruhr University Bochum, 3–5 April.

Formal Methods and Science in Philosophy III, Dubrovnik, Croatia: 11–13 April,

.MA: Conference on Mathematical Ability, Utrecht University, 17 April.

H-OE: Higher-Order Evidence, University of Southampton, 25 April.