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Promoting pupils' creative thinking in primary school mathematics: A case study

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A R T I C L E I N F O

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A B S T R A C T

The importance of promoting mathematical creativity in education is increasingly acknowledged. Several strategies have been recommended to foster mathematical creativity such as creating an open atmosphere in the classroom, offering open lessons and possibly enriching mathematics education with ideas from other disciplines and experiences from out-of-school contexts. However, it is not yet clear in what way recommended pedagogical strategies promote pupils' mathematical creativity. Therefore, the purpose of this case study was to gain an in-depth understanding of promoting mathematical creativity in educational practice. To this end, interactions between a teacher and her 22 fourth-grade pupils in three different types of mathematics lessons were investigated. An 'open' in-school mathematics lesson, an 'open' out-of-school mathematicslesson and a regular ('closed') mathematics lesson were video recorded and interactions were transcribed verbatim. Subsequently, dialogic episodes were identified in the video transcripts and were coded on mathematical creative expressions of pupils and strategies used by the teacher. Furthermore, after each lesson the teacher was interviewed regarding her experiences with the given lesson. Interviews were audiotaped, transcribed verbatim and analysed by using constant comparison analyses. Findings indicate that mathematical creativity was only promoted in the two open mathematics lessons. More specifically, mathematical creative expressions were related to longer whole class dialogues in which the teacher created an open atmosphere; she created opportunities for pupils to express their ideas and took these ideas seriously. Although in some episodes of the regular mathematics lesson this open atmosphere was also created, no mathematical creativity occurred.

1. Introduction

Promoting creative thinking is widely regarded as an important goal of present-day primary mathematics education (e.g., Leikin & Pitta-Pantazi, 2013; Leikin & Sriraman, 2017; National Governors Association Center for Best Practices, 2010). To promote mathematical creativity, a supportive pedagogy is considered essential: creating an open atmosphere in the classroom (e.g., Beghetto & Kaufman, 2011), offering open lessons (e.g., Hershkovitz, Peled, & Littler, 2009) and possibly enriching mathematics education with

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ideas from other disciplines and experiences from out-of-school contexts (e.g. Davies et al., 2013). Although national standards seem to provide opportunities in the mathematics curriculum for such a pedagogy, time constraints seem to limit Dutch teachers, as many other teachers around the world, to use such a creative supportive pedagogy (Harris, 2016; Platform Onderwijs 2032, 2016Platform Onderwijs 2032, 2016). Dutch teachers are expected to teach the complete mathematics curriculum, and they typically strongly rely on mathematical textbooks to structure their lessons (Gravemeijer, 2007; Harris, 2016; Platform Onderwijs 2032, 2016Platform Onderwijs 2032, 2016). These textbooks incorporate the entire mathematics curriculum, but do not provide many opportunities for pupils to be creative (Gravemeijer, 2007; Van Zanten & Van den Heuvel-Panhuizen, 2018). Currently, it is unclear which strategies, if at all, teachers use in primary mathematics education to promote mathematical creativity, and how these strategies are related to pupils' emerging mathematical creativity. More insight into these processes can contribute to a theory on the promotion of mathematical creativity in educational practice. This article describes a case study of a teacher and her 4th grade class in Dutch primary education who applied a new pedagogy for promoting mathematical creativity in her classroom. The aim of the case study was to obtain an in-depth understanding of how pupils' mathematical creativity can be promoted in educational practice and, more specifically, to uncover how different strategies may stimulate mathematical creativity in pupils.

1.1. Defining mathematical creativity

Mathematical creativity is commonly defined as the creation of something new and meaningful by breaking away from established mindsets (Haylock, 1987b; Runco & Jaeger, 2012; Sriraman, 2005). In defining mathematical creativity, a distinction is often made between pupil level and professional level creativity (Kaufman & Beghetto, 2009; Sriraman, 2005). Sriraman (2005) has provided a working definition of school-level mathematical creativity in which mathematical creativity is related to problem solving and problem posing. He defined mathematical creativity as:

> (a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination (p. 24).

This definition seems to limit mathematical creativity to the process of mathematical problem posing and problem solving. However, within a classroom context, mathematical creativity might also occur in other ways (e.g., in whole-class dialogues about a mathematical subject); this can be the creation of a new and meaningful mathematical idea or conception, which is not necessarily a solution or a problem. Therefore, we considered the definition of Sriraman (2005) to be incomplete for studying mathematical creativity in educational practice in a broader sense. In this study, we propose an adapted view on mathematical creativity, in which mathematical creativity (also) refers to the cognitive act of combining known concepts in an adequate, but for the pupil new way, thereby increasing or extending the pupil's (correct) understanding of mathematics (cf., Ervynck, 1991; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). For example, a pupil may have a limited understanding of shapes (e.g., a triangle has three angles, a circle is round) and not relate this to his or her understanding of infinity. We consider it an instance of creativity in the subject mathematics, when the pupil combines the two aforementioned concepts into a new and deeper conceptualization of 'shape', in which a circle is conceived as a polygon and thus as similar to the triangle, but with infinitely many sides of infinitely small magnitude. Although this is not a new idea in mathematics, it is new (and useful) for the pupil and deepens his or her mathematical understanding.

1.2. Promoting mathematical creativity

To promote mathematical creativity a pedagogical environment is needed characterized by an open atmosphere in which pupils have the opportunity to develop new mathematical concepts in interaction with others (Colucci-Gray et al., 2017; Kaufman, Beghetto, Baer, & Ivcevic, 2010; Leikin & Dinur, 2007; Sawyer, 2014; Soh, 2000). Teachers are advised to guide pupils by asking them suitable (open) questions and by giving them opportunities to reflect on mathematical ideas (Bostic, 2011; Hong & Milgram, 2008; Levenson, 2011; Shen, 2014). Furthermore, it is considered vital that teachers are open and flexible with regard to pupils' responses (Beghetto & Kaufman, 2011; Davies et al., 2013; Hershkovitz et al., 2009; Leikin & Dinur, 2007), encourage pupils to share (multiple) ideas, and stimulate pupils to collaborate (Kaufman et al., 2010; Soh, 2000; Taggar, 2002). In this way, multiple perspectives can be discussed which could help pupils to step back from fixed cognitive schemas (Bakker, Smit, & Wegerif, 2015). Moreover, it is important that teachers stimulate pupils' intrinsic motivation; teachers should create opportunities for choice and discovery, and should not use extrinsic motivators (Beghetto & Kaufman, 2011, 2014; Davies et al., 2013; Schacter, Thum, & Zifkin, 2006).

In addition to the teachers' strategies to create an open atmosphere in the classroom, it is advised that teachers offer 'open' mathematics lessons to stimulate pupils' mathematical creativity (Beghetto & Kaufman, 2011; Davies et al., 2013; Hershkovitz et al., 2009). A lesson is open if it invites different solutions, methods of solution or are open for interpretation (Silver, 1995). Creating new and useful mathematical concepts requires a certain degree of freedom in the lesson. This means that the lessons should enable pupils to search, explore, make conjectures, hypothesize, examine, refute, adapt strategies, devise plans, conclude, reason and/or justify their conclusions and/or reflect on them (Nadjafikhah et al., 2012). Lessons that do not enable these features and require pupils to solely apply rules and arithmetical procedures (i.e. 'closed' lessons), are expected to restrain pupils' mathematical creativity. Regular mathematical teaching practices and textbooks mainly contain this closed type of learning lessons (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009)

Furthermore, to promote mathematical creativity it may be important that a teacher enriches mathematics education by making connections with subjects other than mathematics or with contexts outside school (Colucci-Gray et al., 2017). More generally, this phenomenon can be characterized as boundary crossing (Akkerman & Bakker, 2011). To create something new and meaningful in mathematics, it is important to break away from established mindsets (e.g. Haylock, 1987a). It could be expected that by crossing disciplinary boundaries it is easier for pupils to break away from established mindsets and to think and act in a mathematically creative way. Integrating different conceptual systems, for example from the disciplines of visual arts and mathematics, could activate pupils to combine familiar concepts in new ways. This principle might also apply to the location of the mathematics lesson. Teaching mathematics in an outdoor environment might activate a different conceptual system, which could stimulate pupils to combine concepts into new, enriched or deeper mathematical understanding or to discover unknown relations between mathematical concepts (Bancroft, Fawcett, & Hay, 2009; Davies et al., 2013).

1.3. The aim of this study

To conclude, several pedagogical strategies seem to be important for promoting mathematical creativity: strategies for creating an open environment, offering open mathematics lessons and enrichment of mathematics education with ideas from other disciplines and out-of-school experiences. It is yet unclear how these strategies can be implemented in the classroom and how they relate to pupils' mathematical creativity as expressed in classroom dialogues (Kazak, Wegerif, & Fujita, 2015). To address these questions, we conducted a case study of a teacher and her class. To be able to distinguish the role of creativity promoting strategies, we asked the teacher to conduct three different mathematics lessons: a regular mathematics lesson following the textbook, an open in-school interdisciplinary lesson enriched with visual arts reception and production, and an open out-of-school interdisciplinary lesson in which artworks in the neighbourhood were discussed. Subsequently, classroom dialogues were recorded, transcribed and analysed to answer the following research questions:

- Which strategies for promoting mathematical creativity in pupils' thinking are used by the teacher in the three lessons and do lessons differ in this regard?
- How are the mathematical creativity promoting strategies of the teacher related to pupils' creativity as expressed in classroom dialogues?

2. Method

A single case study (Creswell, 2007) was conducted in the context of the Mathematics Arts Creativity in Education (MACE) programme.

2.1. Research context

2.1.1. The MACE programme

The MACE programme aims to increase pupils' creative problem-solving skills in geometry and visual arts and to increase pupils' geometrical ability in the upper grades of primary school. To achieve these goals, a teaching sequence for pupils in Grades 4–6 (aged 9–12) was designed in which geometry and visual arts education were integrated. The teaching sequence consisted of nine lessons, which were related to the theme 'space' and 'patterns' and took about 60–90 minutes to complete (Schoevers & Kroesbergen, 2017). To support implementation of the teaching sequence, a professional development (PD) programme for teachers was developed (Keijzer, Oprins, De Moor, & Schoevers, 2018). The PD programme consisted of five sessions (2.5 h each). After each PD session teachers had to teach one or two lessons of the MACE teaching sequence in their own schools. Lessons 2 and 5 of this teaching sequence were observed in the current study. The content of the PD sessions was focused on how to teach an integrated visual arts and mathematics curriculum and how to promote pupils' creative thinking within visual arts and mathematics education. In these sessions exchanging experiences with other teachers, reflection and discussion of the background theory and didactics related to MACE was important. Furthermore, (hands-on) activities were carried out in which the teachers could practice with and experience activities from the teaching sequence. An overview of the MACE programme can be found in Fig. 1.

2.1.2. School and class context

The school that participated in the current study was situated near Amsterdam, the Netherlands. The school and 12 teachers

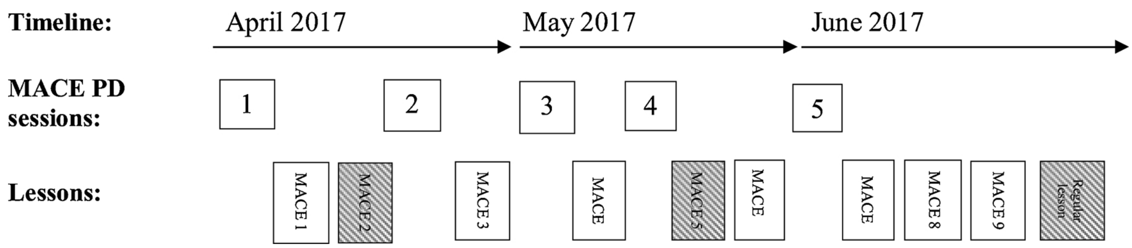

Fig. 1. Timeline of the data collection and overview of the MACE programme. The grey lessons are observed for this study.

participated in the MACE programme because they had the aim to improve their mathematics education. The aims of the MACE programme were in line with the vision of the school.

A female teacher, Anna (pseudonym), and her fourth-grade class (9–10-year-olds) participated in this study. Her class consisted of 22 pupils, all from a middle to high socioeconomic background. On average, these pupils had an above average mathematical achievement on standard tests. The teacher used a mathematical textbook for the regular mathematics education, which was inspired by the concept of Realistic Mathematics Education. Other mathematical teaching materials were used to supplement the textbook.

Anna had 18 years of experience as a teacher in the upper grades of primary school. She was very open and enthusiastic about the MACE programme. Her motivation to take part in this programme was to learn how to make geometry more meaningful to her pupils. She believed that, for many pupils, this domain of mathematics was rather difficult to grasp. We consider Anna to be representative for the other eleven teachers in the programme.

2.2. Data collection

Data were collected from April until June 2017. The third author video-recorded the three selected instruction lessons: a regular lesson, an open in-school arts and mathematics lesson, and an open partially out-of-school arts and mathematics lesson. After the first PD session, the out-of-school MACE lesson was conducted. Six weeks later, the in-school MACE lesson was conducted. At that time, the teacher had attended four PD sessions. Three weeks after the in-school MACE lesson, the regular mathematics lesson was conducted. At that time the PD trajectory was already finished. To record the lessons without disturbing the classroom processes the researcher positioned herself in the back of the classroom using a video camera mounted on a tripod. The video-recordings were transcribed verbatim to enable coding of the data on a creativity promoting dialogues.

Furthermore, after each lesson, the teacher was interviewed. The interviews always started with an open question about her experiences with the lesson and took 10–15 min. Subsequently, questions were asked to invite the teacher to elaborate on issues that arose. The three interviews were audio-recorded and also transcribed verbatim. Findings from the interviews were solely used to interpret and corroborate the findings of the analyses of the classroom dialogues.

2.3. Establishing trustworthiness

The main researcher (first author) was involved in the design of the MACE programme. To decrease possible bias and to assure the credibility of the findings, a second coder (i.e. third author) was involved in analysing the data as described in the former paragraph. Similar to the first author, she was trained as an education researcher, but she was not involved in the MACE programme. To assure transparency, a log was kept during the collection and analyses of the data to keep track of the decisions made.

Within this research we tried to act in an ethical way by trying to keep the burden low for the teacher. She participated voluntarily and did not have any problems with us videotaping three lessons and with having a short interview after each lesson. We always discussed whether time and date of the video-observations and interviews were convenient the teacher. We also asked and received permission of the teacher to use her quotes in this article. For the whole MACE project ethical approval was obtained from the ethics committee of the Faculty of Social and Behavioural Sciences of Utrecht University (FETC15-083).

2.4. Data analysis

2.4.1. Analysis of the video transcripts

To examine how pedagogical actions can influence pupils' mathematical creativity, the video transcripts were analysed by the first author. A first step in the analysis was to identify dialogic teaching episodes in which mathematical creativity could occur, to ensure that the co-occurrence of particular teaching strategies and expressions of creativity could be meaningfully interpreted. Episodes were identified based on the definition of Muhonen, Rasku-Puttonen, Pakarinen, Poikkeus, and Lerkkanen (2016):

> A dialogic teaching episode was identified as an extended exchange in which the topic continued essentially unchanged between the teacher and child or between children and which manifested three of the five principles of dialogic teaching described by Alexander (2006): purposefulness (teachers plan and steer classroom talk with specific educational goals in mind), collectiveness (teachers and children address learning tasks together as a small group or as a the whole classroom) and reciprocity (teachers and children listen to each other, share ideas and consider alternative viewpoints; p. 146)

In total, 33 episodes were identified over the three lessons, of which 14 concerned the out-of-school lesson (of which 10 took place in the classroom), 14 the in-school lesson and 7 the regular mathematics lesson. Regarding the out-of-school lesson we noted that a large part of the dialogues that took place outside the school were not considered as dialogic teaching episodes since often no collectiveness or purposefulness was involved (Muhonen et al., 2016). In these cases, the teacher walked around while the pupils were working on their assignment. The teacher occasionally said something to pupils, but this often concerned one pupil or only a very short exchange.

A second step in analysing the video transcripts was to identify dialogic episodes in which pupils expressed mathematical creativity. An operational definition of the code for expressing mathematical creativity is presented in Table 1, based on our conceptual definition in the introduction. For each episode, we coded if pupils expressed mathematical creativity or not, without taking into account how often mathematical creativity was expressed during an episode.

A third step in the analysis was to identify the strategies used by the teacher to promote mathematical creativity. This part of the

Table 1
Operationalization of the codes.

Core code	Code	Operationalization
1. Pupils' expressed mathematical creativity	1_mathematicscreativity	During the episode, an expression of a pupil was considered as mathematically creative if an idea, solution or problem was expressed in which two or more concepts were integrated, if the idea, solution or problem could be considered new to the pupil, and if at least one of the integrated concepts was related to mathematics. A pupil's mathematical creative utterance could occur in a single utterance or in a short interaction sequence with the teacher or a peer.
2.Teaching for creativity	2 A_ask pupils to connect concepts	During the episode, the teacher asked pupils to connect a mathematical concept to another concept. It was only coded as teaching for creativity when it could be assumed that the pupils used to consider these concepts as unrelated (e.g., The teacher asked pupils to look for shapes in their environment).
	2B_activating questions	During the episode, the teacher asked one or several activating questions that elicited multiple answers, ideas, reasons et cetera (e.g., 'What shapes do you see in your environment?'). The questions started a new (sub)dialogue and were not mere follow-up questions (code 2 F).
	2C_support or encourage creative thinking	During the episode, the teacher provided at least one suggestion that supported pupils' creative thinking (e.g., 'you could try to do X, this may help you') or encouraged pupils to think creatively (e.g., 'try to think out of the box', 'try to think creatively').
	2D_opportunties express ideas	2D1_pupils' ideas central: Ideas of pupils were central in the episode. The teacher aimed to elicit pupils' ideas about a specific subject and did not predominantly instruct or explain, or give his or her own ideas or opinion. 2D2_sub-dialogue allowed: Within the episode, one or more pupils started a sub-dialogue (with the teacher). The teacher allowed this by asking more about what pupil is saying. This sub-dialogue was not directly related to a question asked by the teacher in the main dialogic episode.
	2E_react with respect	2E1_idea personal: Within the episode, the teacher emphasized and valued that pupils' ideas are personal. The teacher implicitly or explicitly stated that there are no wrong ideas, for example by saying 'So in your opinion' or 'everyone can have his own opinion'. 2E2_answers heard: Within the episode, the teacher reacted with respect and interest to all ideas and gave no explicit feedback (e.g. correct/incorrect). She let the pupils know their answers are heard (e.g., by saying 'Ok', 'thank you', or by repeating or elaborating upon the pupil's answer).
	2 F_teacher asks more about pupils' ideas	During the episode, the teacher asked a least one follow-up question about pupils' ideas to learn more about them, to support pupils' thinking or to obtain clarification (e.g., What do you exactly mean by that?').
	2G_pupils share ideas	The teacher encourages pupils to share (multiple) ideas and to let pupils collaborate to hear different points of view. 2G1_pupils react on each other: The teacher stimulates pupils to react on ideas of each other in a dialogic episode. 2G2_pupils collaborate: Within the episode pupils are collaborating. 2G3_exchange ideas: The teacher asks pupils to exchange ideas in dyads or (small) groups.
	2H_autonomy	The teacher stimulates pupils' autonomy in the episode: the teacher stimulates pupils to make their own choices and/or the teacher uses language such as 'you may, 'you can'.

analysis was applied to all observed dialogic episodes to be able to evaluate which strategies used by the teacher co-occurred with expressions of mathematical creativity by the pupils, and which did not. Based on the literature discussed in the introduction about teachers' strategies to promote mathematical creativity, a codebook was established (see Table 1). For each dialogic episode, codes were assigned indicating per strategy if the teacher used that particular strategy to promote mathematical creativity, resulting in multiple codes per episode. Furthermore, the number of utterances per episode was counted. In Appendix A an example is given of how a dialogic episode was coded. Appendix B gives an overview of all codes in all episodes.

As the last step of our analyses, we examined to what degree particular strategies co-occurred, focusing again on all episodes. Furthermore, we calculated the number of episodes in which the strategies from Table 1 were used, broken down by type of lesson and the occurrence of mathematical creativity in pupils' expressions.

As described in the previous section, the video transcripts were double-coded by a second coder (third author) in three steps. First, the initial identification of dialogical episodes across the three instruction lessons was checked. The majority of dialogical episodes were identified in the same way by the two coders, showing high intercoder agreement. However, regarding the in-school mathematics and arts lesson, the second coder identified 3 episodes less and regarding the out-of-school lesson, she identified one additional episode and considered another episode to last longer than did the first coder. In a discussion with both coders, the definition of dialogic episodes of Muhonen et al. (2016) was used to agree on the final number and lengths of the dialogic episodes. Regarding the regular mathematics lesson, both coders identified the same episodes. Second, the coding scheme for assessing pupils' mathematical creativity and teachers' creativity promoting teaching strategies was checked. The first and second coder independently applied the first version of the scheme to 9 selected episodes, taken from all three lessons. Based on a comparison of the results, the coding scheme was slightly adjusted, and ambiguous codes were clarified. Third, after consensus was reached, all episodes were double-coded with the adapted coding scheme. With regard to the codes for mathematical creativity, 100% agreement was reached. With regard to all codes for teaching for creativity, interrater agreement was good (κ = .74; Cicchetti, 1994). Differences were discussed afterwards, and a final code was given.

2.4.2. Analysis of the interview transcripts

To analyse the transcripts of the interviews, a constant comparison method was used to identify underlying themes in the data (Leech & Onwuegbuzie, 2007). The starting point of the analysis was the question: What are Anna's experiences in teaching the three mathematics lessons, and why might these experiences differ? Her experiences were used to corroborate and further explain the findings of the video transcript analyses. With this question in mind, the first interview was read and subsequently coded in an open way by the first and second coder separately. The interview transcripts were divided into meaningful parts and labelled with a descriptive code. The fragments and codes selected by the two coders were mostly similar. Differences were discussed, and a final code was assigned. Similar fragments were labelled with the same code. The same coding process was used for the second interview. With regard to the second interview, the second coder segmented the transcripts in smaller fragments than the first coder. Differences were discussed until consensus was reached on the most appropriate segmentation and corresponding codes. For the third interview, the same procedure was used. Most fragments and codes were similarly identified, and differences were discussed and easily resolved. In the end, 66 open codes emerged from the data. After the open coding, axial coding was applied. The obtained codes were grouped according to similarity and for each grouping, a common theme was identified. Each coder made a mind map of the list of grouped open codes to find the axial codes. Both mind maps were discussed, and six final axial codes emerged. Next, the interviews were coded together by both coders again with axial codes. The codes were found satisfactory and well applicable to the identified fragments. Subsequently, selective coding was applied. Both coders analysed how the axial codes related to each other within each interview and how they related to themes in the scientific literature. It was concluded that there were three main codes under which all axial and open codes could be placed, namely *context*, *teachers' beliefs*, and *teachers' acting*. Axial (core) codes that were related to the research question were used to corroborate the findings of the analyses of the classroom dialogues. In Appendix C an example is given of the coding process.

3. Findings

The main purpose of this case study was to gain in-depth insight into how pupils' mathematical creativity can be promoted in educational practice. The results are reported below in three parts. Since the context of this case study is highly relevant to interpret the findings, we first give a short description of the three lessons that were studied. Next, we report the ways in which the pupils expressed mathematical creativity in classroom dialogues. Finally, we describe the strategies the teacher, Anna, used to promote mathematical creativity in pupils' thinking and how they related to pupils' expressions of mathematical creativity.

3.1. Descriptions of the three different settings

Three lessons were observed to study the Anna's strategies in relation to pupils' mathematical creativity. Anne taught these lessons in the order described below.

3.1.1. The out-of-school interdisciplinary lesson 'Space outside the classroom'

The lesson plan of the MACE programme indicated that the teacher had to provide this lesson outside the classroom; this could be a museum or the environment outside the school. Anne chose for the latter. The lesson plan was not prescriptive but provided several suggestions for lessons and questions relating to monuments, statues, buildings, (wall) paintings, open and closed spaces in museums or the environment outside school. Anne started her lesson in the classroom with a discussion about shapes and spaces. Next, the pupils went outside and walked to an artwork in the neighbourhood. During this walk, pupils were encouraged to name the shapes they encountered. Arrived at the artwork, pupils were stimulated to discuss whether the artwork had a front side (note: it was a 3-dimensional round artwork) and whether it took space. After this, pupils had to make a drawing of the artwork from one angle. Next, they had to draw the artwork from another angle of their choice. After this, the class returned to the classroom. Back in the classroom, Anne asked her pupils whether they had seen particular shapes in the artwork (visualized on the interactive whiteboard) or during their walk to the artwork. Furthermore, pupils had to discuss from which angle they drew the artwork using pictures of the artwork as memory support, which were taken beforehand by Anne. Finally, Anne discussed with the class how the artist could have made the artwork and what the pupils had learned from the lesson.

3.1.2. The in-school interdisciplinary lesson: 'what is a pattern?'

With regard to the in-school interdisciplinary lesson, Anne mainly followed the lesson plan. Anne started the lesson with an introduction of the concept patterns. She initiated a classroom discussion inviting the pupils to share where they had seen patterns. After this, Anne further explored the concept of patterns with her class by discussing 3 artworks. Anne used the suggested artworks of the lesson plan. Next, pupils had to shortly think about the following questions: 'Do patterns arise by coincidence?' and 'Does a pattern always stay a pattern?' Simultaneously, pupils had to throw a handful of small blocks on their table. Then, the main lesson started (25 min). Pairs of pupils threw the blocks on a sheet of white paper on the table and then marked the location of the blocks on the paper. They discussed whether the blocks formed a pattern or not, and whether they could make it into a pattern and, if so, how. In the second part of the lesson, pairs of pupils placed the small blocks alternately in a pattern and again marked them on the paper. Next, the pupils needed discussed the same questions again. In addition to the lesson plan, Anne also asked the pairs of pupils to join in groups of four and to combine the patterns they created into a new pattern. After the lesson, Anne reflected on the different products and processes, and discussed pupils' discoveries.

3.1.3. The regular mathematics lesson

Anne taught a lesson from the mathematics textbook *A world in numbers* [*Wereld in Getallen*] (Van Grootheest et al., 2011). The regular mathematics lesson we observed was about arithmetic. The teacher discussed arithmetical problems from pupils' workbooks and asked pupils to solve these in front of the class on the digital whiteboard. Seven arithmetical problems were discussed in a whole-class setting. In-between and after these whole-class discussions, pupils had to do exercises in their mathematical workbooks.

3.2. Mathematical creativity in the classroom dialogues

Mathematical creativity occurred in 10 out of 14 dialogic episodes during the out-of-school interdisciplinary lesson, in 7 out of 14 episodes during the in-school interdisciplinary lesson, and in 0 out of 7 episodes during the regular mathematics lesson. An expression of a pupil was considered as mathematically creative, when they posed or solved problems in, for them, novel ways, or in which they combined ideas from mathematics and other conceptual domains into, for them, new ideas, thereby expanding or deepening their understanding of mathematics. The following sub-dialogue illustrates how a pupil combined the concept of the appearance of shapes with the concept of movement (i.e. jumping):

Teacher: We go to the statue and observe it from different points of view. It is the statue in front of the museum, not behind the museum. M. would like to say something. It might be wise to listen to him.

Pupil M.: When I was there with my dad I saw that the shape changed. When I looked into the water it seems very small and when I looked at it from the front, it was very different.

Teacher: So, the shapes are very different.

Pupil M.: Yes

Teacher: And what caused this? Why do you think that these shapes were different?

Pupil M.: If you jump, you see other shapes. If you stand still you only see the length.

Teacher: Yes, your perception depends on your point of view.

3.3. Promoting mathematical creativity

3.3.1. Used teacher strategies in the three mathematics lessons

In this section, findings are presented about the teaching strategies used by Anne to promote mathematical creativity in the three mathematics lessons and whether the lessons differed in this regard (i.e. research question 1). Table 2 shows that all theoretically-derived teaching strategies were used by Anne, but some more than others, while the distribution of the different strategies differed by type or lesson.

More strategies and more different strategies were used in both open interdisciplinary lessons compared to the regular

Table 2
Overview of the percentages of episodes in which strategies (2 A-2 H) occurred in the three lessons.

Code	Lesson 1 $N_{episodes}$ = 14	Lesson 2 $N_{episodes}$ = 14	Lesson 3 $N_{episodes}$ = 7
2A_Ask pupils to connect concepts	50%	36%	0%
2B_activating questions	71%	86%	43%
2C_ support or encourage creative thinking	14%	29%	0%
2D1_Pupil ideas central	57%	71%	29%
2D2_Sub dialogue allowed	43%	14%	14%
2E1_idea personal	36%	71%	43%
2E2_answers heard	64%	43%	14%
2F_aks more about pupils' ideas	57%	36%	14%
2G1_pupils react on each other	29%	29%	0%
2G2_pupils collaborate	0%	57%	0%
2G3_exchange ideas	7%	7%	0%
2H_autonomy	0%	29%	0%

Note. Lesson 1 is the out-of-school interdisciplinary lesson, lesson 2 is the in-school interdisciplinary lesson and lesson 3 is the regular mathematics lesson.

Table 3
Overview of the percentage of episodes in which strategies (code 2A–2H) occurred divided in episodes with and without the occurrence of mathematical creativity and the three lessons.

Code	Episodes with mathematical creativity			Episodes without mathematical creativity		
	Lesson 1 $N_{episodes} = 10$	Lesson 2 $N_{episodes} = 7$	Lesson 3 $N_{episodes} = 0$	Lesson 1 $N_{episodes} = 4$	Lesson 2 $N_{episodes} = 7$	Lesson 3 $N_{episodes} = 7$
2A_Ask pupils to connect concepts	60%	57%	–	25%	14%	0%
2B_open questions	80%	86%	–	50%	86%	43%
2C_support or encourage creative thinking	10 %	14%	–	25%	43%	0%
2D1_Pupil ideas central	80%	100%	–	0%	43%	29%
2D2_Sub dialogue allowed	60%	14%	–	0%	14%	14%
2E1_idea personal	40%	86%	–	25%	57%	43%
2E2_answers heard	80%	71%	–	25%	14%	14%
2 F_aks more about pupils' ideas	70%	71%	–	25%	0%	14%
2G1_pupils react on each other	40%	43%	–	0%	14%	0%
2G2_ pupils collaborate	0 %	29%	–	0%	86%	0%
2G3_exchange ideas	0 %	14%	–	25%	0%	0%
2H_autonomy	0 %	0 %	–	0%	57%	0%

Note. Lesson 1 is the out-of-school interdisciplinary lesson, lesson 2 is the in-school interdisciplinary lesson and lesson 3 is the regular mathematics lesson.

mathematics lesson. For example, in the regular mathematics lesson the teacher never asked pupils to connect two different mathematical concepts (code 2 A).

The strategy of supporting and encouraging pupil's creative thinking (code 2C), expressed in questions or suggestions such as 'try to think out of the box', was rather infrequent across all episodes and lessons. This strategy mainly occurred in episodes in which pupils were collaborating, for example when they were working in pairs on an assignment (code 2G2) and was always used simultaneously with Anne stimulating pupils' autonomy (code 2 H). Anne used these strategies especially when a pair or group of pupils were stuck in their creative process. Since only the in-school lesson contained such cooperative assignments, this can explain why these strategies were mainly used during this lesson.

3.3.2. Used teacher strategies and pupils' expressions of mathematical creativity

In this section, findings are presented about how teaching strategies relate to pupils' expressions of mathematical creativity. In Table 3 the overview of strategies used by Anne during the three lessons is broken down by episodes where pupils expressed mathematical creativity in educational dialogues and episodes where they did not. The findings show an association between the teaching strategies used and the occurrence of mathematical creativity. Some strategies were clearly more often used in episodes in which pupils expressed mathematical creativity than in episodes in which no mathematical creativity was observed. This holds in particular for the strategies 2A, 2D1, 2E2, 2F, and 2G1. Other strategies, such as supporting and encouraging creative thinking (code 2C) and pupils' autonomy (code 2H), mainly occurred in episodes in which pupils did not express mathematical creativity. For the remaining strategies, for example whether Anne allowed a sub-dialogue (code 2E1), the pattern of use and the relation with the occurrence of mathematical creativity was more diffuse.

A deeper analysis of the dialogic episodes suggested that pupils' expressions of mathematical creativity occurred more often if Anne combined activating open questions (code 2B) with questions eliciting pupil's ideas about a specific subject (code 2D1), provided indications that pupils' answers are heard and respected (code 2E2) and/or asked follow-up questions to learn more about pupils' ideas (code 2 F). If 3 out of 4 of these strategies occurred together, it was related to pupils' expressions of mathematical creativity. In this study, these aspects together seem to create an open atmosphere in which the teacher created opportunities for pupils to express their ideas and she was open to these ideas. Anne asked activating open questions, like 'who sees a pattern', 'who else sees a pattern?' and 'what more do we see?', that stimulated pupils to express their ideas. Several times she repeated these kinds of questions and she gave several pupils a turn to express their ideas. She often repeated or recast the ideas of pupils and sometimes added follow-up questions while contingently building upon pupils' preceding responses. The excerpt below of a dialogic episode, observed during the out-of-school interdisciplinary lesson, may illustrate this.

Pupil K.: A half O

Teacher: A half O, a half O (*teacher makes hand gestures of half O).* Okay. What more do you see? What shapes do you see? J., what shapes do you see?

Pupil J.: a changing shape

Teacher: Oh, a changing shape. Oh, interesting. Shapes can change. Change in what?

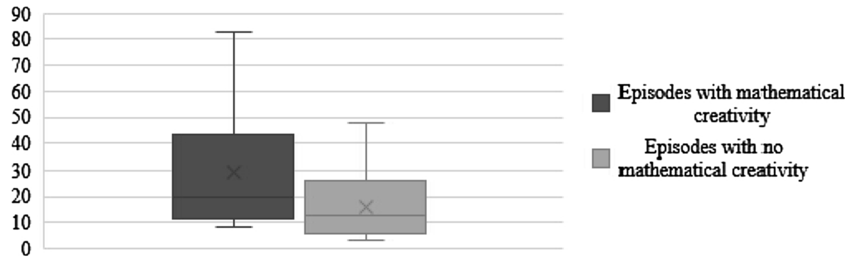

Fig. 2. Boxplots of the numbers of utterances in the dialogic episodes with (N = 17) and without (N = 18) the occurrence of mathematical creativity.

Pupil J.: Maybe in space. It may grow as well. It can get slightly increasingly longer. An infinite shape!

Teacher: An infinite shape, does it exist? An infinite shape? *(B. raises his hand)*. B.?

Pupil B.: Yes, a circle never ends, it always continues. It continues, so it is infinite.

Teacher: oh, that is interesting.

Pupil R.: We also have a rectangle.

Teacher: So, it has been said that we have a circle that is infinite, we have a rectangle. What more do you see more?

Another finding is that mathematical creativity mainly occurred in longer whole-class interactions and at the beginning and at the end of the lessons (see Fig. 2 and Appendix A). In episodes where the teacher interacted with small groups and which were often short, hardly any mathematical creativity occurred. This may be related to the fact that the teacher used different strategies in these episodes. Often these dialogues were focused on what the group was thinking, doing or making in relation to their artworks (product).

3.3.3. Used teachers' strategies and pupils' expressions of mathematical creativity in the three mathematics lessons

In addition to the strategies used by the teacher, the type of mathematics lesson also seemed to play a role in the occurrence of pupils' expressions of mathematical creativity. We found that pupils' expressions of mathematical creativity did not occur in the regular mathematics lesson. Although this particular combination of strategies (i.e. codes 2B, 2D1, 2E2, 2F) was related to pupils' mathematical creative expressions in the two open interdisciplinary lessons, it was not in the regular mathematics lesson. This is visible in the following excerpt from the regular mathematics lesson in which the teacher asks pupils a question that invites them to express their own strategies in adding numbers, while listening with interest to what the pupils put forth. Although the teacher did not ask any follow-up questions (code 2F), an open atmosphere was created. However, the pupils express no mathematical creativity as defined in the current study: pupils only came up with different ways to add instead of constructing new concepts in dialogue.

Teacher: 90 cents! Okay, so I have to add €43.80 and 90 cents. How much is that in total?

Pupil L, €44,70

Teacher: Okay, €44.70. Who says I do it in a different way? F.?

Pupil F.: First add 25 and 55.

Teacher: Okay, so you first add the cents. This one, this one and this one.

Pupil F.: No, not yet the 90.

Teacher: Oh, not yet the 90. So, this one and this one.

Pupil F.: and then you calculate. That is 80. And then you take 17 plus 23.

Teacher: Okay than you take 17 and 23.

Pupil F.: Than 10 plus 20. Than you have 30 and then 7 and 3 and then you get 40.

Teacher: Okay, so plus 40.

Pupil F.: And then I add 90 and 80.

Teacher: So, you add 90.

Pupil F.: Than you have €1,70. And then you do 3 plus the 1 euro and then 4 euro plus 40 and then 70 cents.

Teacher: So, this one plus this one, then I have €4.70. And then you add this to 40. Okay. Yes.

Pupil T.: I added the cents first.

Teacher: First the cents

Furthermore, we found that there did not seem to be a difference between the in-school and out-of-school context regarding the occurrence of mathematical creativity. In the out-of-school interdisciplinary lesson 14 dialogic episodes were identified, of which five in the out-of-school context. In both the in-school (6 out of 9 dialogic episodes) and the out-of-school dialogic episodes (4 out of 5 dialogic episodes) mathematical creativity occurred. Furthermore, the same strategies were used by the teacher in these episodes. However, out-of-school dialogues differed from in-school dialogic episodes in that pupils frequently connected shapes they were not familiar with to well-known concepts from daily life, like 'this is the shape of a baguette' and 'ugly circle'.

4. Discussion

The aim of this case study was to obtain a more in-depth understanding of the teaching strategies a primary school teacher can implement to promote mathematical creativity in the classroom and how these strategies, and particular combinations of them, relate to pupils' mathematical creativity as expressed in classroom dialogues. Mathematical creativity was defined as pupils' thinking acts, expressed in classroom dialogues, in which they posed or solved problems in, for them, novel ways, or in which they combined ideas from mathematics and other conceptual domains into, for them, new ideas, thereby expanding or deepening their understanding of mathematics.

The first research question addressed the strategies the teacher in focus, Anne, used to promote creativity in pupils' mathematical thinking during three mathematics lessons, varying in openness and interdisciplinarity. Teaching strategies identified in the research literature as potentially promoting mathematical creativity guided the analysis of classroom dialogues during these lessons. We found that all strategies for promoting mathematical creativity in pupils' thinking were used by Anne, but some more often than others. More, and more diverse, strategies were used during the two open interdisciplinary mathematics lessons compared to the regular textbook-based mathematics lesson.

The second research question concerned the relation between mathematical creativity promoting strategies of the teacher and pupils' creativity as expressed in classroom dialogues. We examined this relation within thematically coherent episodes to ensure that the co-occurrence of particular teaching strategies and expressions of creativity could be meaningfully interpreted. There were several important findings. First, expressions of mathematical creativity only occurred during the two open interdisciplinary lessons, involving both out-of-school and in-school mathematics and arts education. During these lessons, Anne created an open climate and facilitated longer whole-class dialogues, in which she gave pupils ample opportunities to express their ideas and, through specific pedagogical actions, she clearly signalled to pupils that she took these ideas seriously. The importance of the use of these strategies for promoting creativity in mathematics education is in line with the literature on promoting mathematical creativity (e.g., Beghetto & Kaufman, 2011). However, a second important finding was that these same teaching strategies were also observed during the regular textbook-based mathematical lesson, but now expressions of mathematical creativity did not occur. This finding illustrates that making some aspects of the lesson open (i.e., giving opportunities for alternative and personal strategies), does not necessary stimulate mathematical creativity as expressed in dialogues.

The reason why more, and more different, creativity promoting strategies were used in the two open interdisciplinary lessons compared to the regular lesson, and why the use of these strategies was not related to mathematically creative expressions in the regular mathematics lesson may be the fact that the learning goals differed, suggesting that creativity promoting teaching strategies only stimulate creativity when this is an explicit learning goal. The regular mathematical lesson, in this regard, had a more specific learning goal than the other two lessons, namely practicing arithmetical strategies versus learning about conceptualizations of shapes, space and patterns. For the two interdisciplinary lessons the main learning goal was to teach new concepts (tool-for-result), but also to use mathematical dialogues in which concepts are questioned and developed (tool-and-result; Bakker et al., 2015; Kazak et al., 2015). The teacher also experienced this and stated in the interview after the regular mathematics lesson, that she felt less freedom in teaching, because she also "(…) had to teach them things". Another explanation may be that during this regular mathematics lesson, pupils behave in accordance with particular sociomathematical norms, indicating in an implicit way what is expected and rewarded (Yackel & Cobb, 1996). Pupils may have considered expressing mathematical creativity as unacceptable according to the sociomathematical norm of the regular lesson. Furthermore, the interdisciplinary character of the lessons may have made it easier for pupils to relate two different ideas or concepts (i.e. to express mathematical creativity). Thus, the fact that the two interdisciplinary lessons were different (i.e. more open, other multidisciplinary content) from this regular mathematical lesson may be the cause that the teacher used different strategies and that teacher and pupils interacted in a different way.

Although the findings of this study indicate that creating an open climate in open interdisciplinary lessons is indeed related to pupils' mathematical creative expressions, it can be questioned whether this enhances pupils' mathematical learning. One the one hand, it can be argued that most dialogues between teacher and pupils were rather superficial because the ideas and concepts of pupils were central, and pupils did not learn specific mathematical content. On the other hand, it can be argued that pupils did engage in a mathematical dialogue in which mathematical concepts were questioned and may developed (tool-and-result; Bakker et al., 2015; Kazak et al., 2015), which could also be considered as mathematical learning. In this respect, however, it seems to be important that the teacher asks (follow-up) questions that make the pupil reason about mathematical concepts in a deep way. Within the lessons, pupils may had obtained deeper insights into their mathematical creative ideas or solutions, if the teacher would have asked more eliciting questions that challenged pupils' thinking.

Another finding in this study was that not all theory-driven strategies for promoting mathematical creativity were related to pupils' expression of mathematical creativity. We found that in dialogic episodes in the in-school interdisciplinary lesson in which pupils collaborated, the teacher used different strategies compared to other dialogic episodes: the teacher mainly supported and encouraged creative thinking and stimulated their autonomy. However, these dialogic episodes and strategies were not related to pupils' expression of mathematical creativity. A cause for this may be the short time of interaction (i.e. average of 7 utterances per episode). Furthermore, in these episodes the aim of teacher was mainly focused on determining what artworks the pupils hitherto made and on providing help to pupils who were stuck in the process of creating the artwork. Contrary to the literature (e.g., Beghetto & Kaufman, 2014, 2010), this study may indicate that some strategies in itself are not related to mathematical creative expressions.

4.1. Limitations & future research

A limitation of this study is that the findings cannot be extended beyond this single case. Nevertheless, the findings provide a detailed presentation of how creativity promoting strategies relate to pupils' expressions of mathematical creativity in a classroom dialogue. This study can be a starting point for future research in which more teachers, classrooms and lessons can be observed. Furthermore, future research could investigate how many and which pupils in a classroom come up with mathematical creative expressions. Another limitation of this study is that mathematical creativity was coded on pupils' verbal expressions only. However, mathematical creativity may also be expressed in non-verbal ways, for example, by combining two known concepts (e.g., shape and infinity) in a new way when designing an artwork. Moreover, pupils may not have verbalized all their thoughts, which limited this study to describing the associations between creativity promoting strategies used by the teacher and pupils verbal expressions. However, we believe that this new way of coding mathematical creativity in educational dialogues adds to the literature on mathematical creativity, because it can give more detailed information about how mathematical creativity manifests in a mathematics classroom context and what factors (e.g. expressions of teachers or peers) contribute to this manifestation of mathematical creativity. Nevertheless, future research could also take into account non-verbal expressions of creativity.

5. Conclusion and implications

To conclude, findings of the current study indicate that mathematical creativity was promoted if the teacher created longer whole class dialogues and an open atmosphere in open interdisciplinary lessons with open learning goals. In this study, an open atmosphere was created if the teacher created opportunities for pupils to express their ideas and if the teacher was open to these ideas. Furthermore, the interdisciplinary content, a less specified learning goal (i.e. learning about conceptualizations instead of strategies), and different sociomathematical norms might have contributed to the presence of mathematical creativity in the open interdisciplinary lesson and the absence of mathematical creativity in the regular mathematics lesson.

For educational practice this finding implies that mathematical creativity may be easier encouraged in open and interdisciplinary lessons. In these lessons the teacher should focus on the use of strategies like, asking activating open questions that invite pupils to generate multiple answers and ideas, creating longer whole class dialogues in which ideas of pupils' ideas are central, making sure that pupils answers are heard and respected and that follow-up question are asked to learn more about pupils' ideas. Furthermore, our study contributes to the literature on (promoting) mathematical creativity, since we used a new approach in defining and measuring mathematical creativity in primary school mathematical dialogues. In our study we used a broader definition of mathematical creativity, because, as shown in this study, in primary school mathematics education mathematical creativity also occurs in other situations than mathematical problem posing and solving.

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Supplementary data

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