3.1 Mathematics Education in the Netherlands Viewed from Four Perspectives

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3.1.1 The Role of Mathematics and Mathematicians in Mathematics Education in the Netherlands

When mathematics became a compulsory subject in primary and secondary school in the Netherlands at the start of the 19th century, professional mathematicians were not much involved. Furthermore, the Dutch government took a somewhat restrained position on what mathematics was taught and particularly on how it was taught. Decisions regarding the curriculum were considered an internal school affair. In 1917 this policy was formalized in the Dutch Constitution as Freedom of Education. In practice, this meant that changes in the school mathematics curriculum were usually discussed by teacher unions, or special committees of teachers, then approved by the school inspectors and only after that ratified by the government. Professional mathematicians played hardly any formal part in this process. They sometimes participated in secondary education final examinations, by acting as assessors in the oral examinations and checking the grading of students' work on the written examinations, but they were not responsible for these examinations, which were devised by a selected group of teachers and approved by school inspectors. Furthermore, professional mathematicians were scarcely involved in the production of textbooks for secondary and primary mathematics education, which were mainly written by teachers. The government left the production of textbooks to the market. Schools were free to choose those books they liked most. Although there is currently more government involvement in the 'what' of teaching through formulating standards and a series of compulsory tests, the freedom regarding textbooks still exists.

Professional mathematicians' interest in school mathematics began to grow in the first half of the 20th century. For example, in 1924 a mathematics journal was extended by an addendum in which didactical questions could be discussed; this addendum later became the still existing journal *Euclides*. The first issue of this journal paid attention to the argument between the mathematician Dijksterhuis and the physicist Ehrenfest-Afanassjewa. The latter had also studied mathematics in Göttingen with Klein and Hilbert. While Dijksterhuis insisted on keeping to the traditional approach to teaching school geometry, which was based on the formal characteristics of the discipline, Ehrenfest argued for making use of students' intuitive knowledge and starting with concrete activities in three-dimensional space. However, Ehrenfest's ideas and those of her discussion group on teaching mathematics which was to become the Mathematics Working Group in 1936, did not find much response from mathematicians. Even the famous Dutch mathematician Brouwer did not have any affinity with teaching mathematics in school.

Yet, from 1950 onwards, more attention was paid to mathematics education within the community of mathematicians. This was reflected, for example, in the establishment in 1954 of the Dutch Education Commission for Mathematics, an ICMI subgroup, in which mathematics teachers and mathematicians cooperated. Freudenthal became chair of the group shortly after its foundation. Although in those days more mathematicians felt that mathematics teaching needed modernisation, it was Freudenthal in particular who had a genuine and deep interest in the didactics of mathematics. So, it is no wonder that, after World War II, he had also joined Ehrenfest's group.

In 1961, the Dutch government appointed the Commission Modernisation of the Mathematics Curriculum (CMLW), which was a new phenomenon in the long history of government that was rather aloof with respect to curriculum issues. The founding of this new commission, consisting of professional mathematicians and teachers, was a direct consequence of the Royaumont conference. The Dutch government became convinced of the urgency of the modernization of mathematics education. Freudenthal was the most outstanding mathematician in this commission and he was also the only one who was heavily involved in the didactics of mathematics. He convinced the other commission members to focus on the teaching of mathematics instead of on the content of the curriculum and he also moved the attention of the commission to the lower grades of schooling. Later on, in 1971, IOWO (Institute for Development of Mathematics Education) was established with Freudenthal as its first director. The opening of the institute was the beginning of a long period of over forty years in which mathematics teachers and educators worked on the design and research of mathematics education in primary and secondary school and teacher education. The instructional designs and the underlying theory of Realistic Mathematics Education (RME) developed at this institute have changed the Dutch mathematics curriculum and the approach to teaching mathematics, and this happened without any government interference. Characteristic of this approach is that it starts with offering students problems in meaningful situations, from which contexts can gradually evolve into models that can be used to solve a broader scope of problems; through the process of progressive schematisation, students eventually end up understanding mathematics at a more formal level.

Until the late 1990s, RME was generally accepted for primary and secondary education. Also university mathematicians were involved in secondary education reform projects such as HEWET and PROFI. However, after 2000 some university mathematicians started to blame RME for the lack of basic mathematical skills of their first-year students. They wanted to return to the way of teaching mathematics that (in their view) was common some forty years ago. For primary school mathematics education, the Ministry of Education and the Netherlands Royal Academy of Sciences (KNAW) appointed a commission to arbitrate this Dutch Math War. The commission's conclusion was that there was no evidence that students' achievements would be better with either RME or the mechanistic back-to-basics approach. This conclusion resulted in RME being less in the firing line, and it became possible again to have a professional discussion among all stakeholders about primary school mathematics education. In the area of secondary education, the Ministry of Education appointed the Commission Future Mathematics Education (cTWO), consisting of mathematicians, mathematics education researchers and mathematics teachers, to revise the mathematics curriculum for upper secondary mathematics education. In addition, Regional Support Centres (Bètasteunpunten) were established to facilitate connections between secondary schools and universities. Also, the Mathematical Society and the mathematics teachers' association (NVvW), set up Platform Mathematics Netherlands, a new organisation for collaboration, which included, among other things, a commission for mathematics education. This commission only covers secondary mathematics education and not the teaching of mathematics in primary school.

Background information about the role of mathematics and mathematicians in mathematics education in the Netherlands can be found in Goffree, Van Hoorn, and Zwaneveld (2000), La Bastide-van Gemert (2015) and Smid (2018), for example.

3.1.2 The Role of Theory in Mathematics Education in the Netherlands

Mathematics education is not just about the process of teaching mathematics, but also encompasses ideas and knowledge about how students learn mathematics, about how mathematics can best be taught and what mathematical content should be taught and why. Finding answers to these questions is the main goal of the scientific discipline that in the Netherlands—in line with the European tradition—is called the didactics of mathematics.

At the beginning of the 19th century when the first textbooks were published in the Netherlands, the prefaces of these textbooks showed the initial efforts towards a theory of mathematics education that contributed to the development of the didactics of mathematics as a scientific discipline. A next step forward came in 1874 when the Dutch schoolteacher Versluys published his book on methods for teaching mathematics and for the scientific treatment of the subject. However, a decisive move towards a theoretical basis of mathematics education in the Netherlands was Freudenthal's unfinished manuscript *Rekendidactiek* (Arithmetic Didactics), written in 1944, but never published. Freudenthal's interest in mathematics education in primary school was triggered during World War II when he was teaching arithmetic to his sons, and observed their learning processes. He also carried out an extensive literature review of the didactics of arithmetic. A further advancement in Freudenthal's thinking about mathematics education occurred at the end of the 1950s when he worked with the Van Hieles and became familiar with the theory of levels. Inspired by this, he developed the very important didactic principle of guided reinvention in which decisions about guidance should be informed by analysing learning processes. For Freudenthal, the 're' in reinvention points to students' learning processes, and the adjective 'guided' to the instructional environment. Viewing learning as guided reinvention means striking a subtle balance between the students' freedom of invention and the power of the teachers' guidance.

Freudenthal's intention of giving mathematics education a scientific basis resulted in the publication of *Weeding and Sowing* in 1978, which he called a preface to a science of mathematics education. In this book, he introduced the didactical phenomenological analysis of mathematics, an approach which was further elaborated in *Didactical Phenomenology of Mathematical Structures*, published in 1983. According to Freudenthal, thorough analysis of mathematical topics is needed in order to show where the student might step into the learning process of mankind. In other words, a didactical phenomenology, rather than a pure epistemology of what constitutes mathematics, is considered to inform us on how to teach mathematics. This phenomenology includes how mathematical 'thought objects' can help organising and structuring phenomena in reality, which phenomena may contribute to the development of particular mathematical concepts, how students can come into contact with these phenomena, how these phenomena beg to be organised by the mathematics intended to be taught, and how students can be brought to higher levels of understanding.

Although mathematics plays a central role in these analyses, Freudenthal rejected the idea of taking the structure of mathematics or the expert knowledge of mathematicians as his point of departure. The goal was making mathematics accessible and understandable for students by taking their learning processes seriously. Freudenthal viewed working on the design of education and experience with educational practice as necessary requirements for making theory development possible. His work at IOWO, which was founded in 1971, and particularly his collaboration with the Wiskobas group around Treffers, and later with the Wiskivon group for secondary education, which both did a great deal of work with students and teachers in schools, was therefore crucial for Freudenthal's thinking.

At the same time, however, Freudenthal's involvement was important for IOWO as well. In addition to promoting mathematisation and mathematics as a human activity that is connected to daily life or an imagined reality, and emphasizing that students and even young children can generate a large amount of mathematical thinking—yet in an informal context-connected way—Freudenthal's participation was essential for another reason too. Being an authority in the field of mathematics as a discipline, Freudenthal legitimised the work done at IOWO from the perspective of mathematics.

Although the activities at IOWO, due to its focus on designing education, could be characterised as engineering work rather than as research, IOWO produced 'valuable splinters' which could be counted as research output. Freudenthal saw this approach as paradigmatic for how theory development must take place: from designing educational practice to theory. The theory that evolved from this work at IOWO was later called Realistic Mathematics Education (RME) and was initially described by Treffers in 1978, and published in 1987 in his book *Three Dimensions*. The principles of this domain-specific instruction theory have been reformulated over the years, including by Treffers himself, but are presently still seen as leading for RME.

The first principle, the activity principle, follows from the interpretation of mathematics as a human activity, and implies that students are treated as active participants in the learning process. The reality principle arises from considering mathematics as based in reality and developing from it through horizontal mathematisation, and entails that mathematics teaching provides students with meaningful problems that can be mathematised. The level principle highlights the idea that students pass through several stages of understanding, from informal, context-connected to formal mathematics. In this process, didactical models serve a bridging function and vertical mathematisation is stimulated. This level principle is also reflected in the procedure of progressive schematization. The intertwinement principle states that the mathematics curriculum is not split into isolated strands, but that, following the mathematisation of reality, the focus is on the connection and coherence of mathematical structures and concepts. The interactivity principle signifies the social-cognitive aspect of learning mathematics, and entails that students are offered opportunities to share their thinking with others in order to develop ideas for improving their strategies and to deepen their understanding through reflection. The guidance principle refers to organising education in such a way that guided reinvention is possible, through a proactive role of the teacher and educational programs based on coherent long-term teaching-learning trajectories that contain scenarios which have the potential to work as a lever to effect shifts in students' understanding. Several local instruction theories focusing on specific mathematical topics have been developed which align with these general principles of RME.

RME is not a fixed and finished theory of mathematics education and is still in development. Over the years the successors of IOWO—OW&OC (Research of Mathematics Education & Education Computer Centre), the Freudenthal Institute and the lately established Freudenthal Group[1]—have made different emphases. As a result of collaboration with researchers in other countries, RME has also been influenced by theories from abroad such as social constructivist approaches which contributed to the interaction principle of RME and provided RME with a lens for investigating classroom discourse. More recently, elicited by the use of new technology in mathematics education, approaches inspired by instrumentation theory have connected with RME to achieve a better understanding of how tool use and concept development are related. Finally, a further new avenue is the revitalisation of the activity principle of RME through the incorporation of embodied cognition and perception-action theories in which the focus is also on how students' concept development and deep learning can be understood and fostered.

However, when working on the further development of RME through integrating it with other theories, it is still important that mathematics should maintain its central place. Developing mathematics education and investigating learning and teaching processes should always be grounded in mathe-didactical analyses which unpack mathematics in didactic terms and take into account phenomenological, genetic-epistemological, and historical-cultural perspectives.

[1]In 2012 the Freudenthal Institute was split into two. The research and design work in early childhood, primary education, special education, and intermediate vocational education were moved to the Faculty of Social and Behavioural Sciences (FSW) and carried out by the *Freudenthal Group*. The research and design work in secondary education have remained part of the *Freudenthal Institute* in the Faculty of Science.

Background information about the role of theory in mathematics education in the Netherlands can, for example, be found in De Lange (1987), Freudenthal (1991), Treffers (1987a), and Van den Heuvel-Panhuizen and Drijvers (2014).

3.1.3 The Role of Design in Mathematics Education in the Netherlands

Making things work, looking for pragmatic solutions and being creative and innovative are typical features of Dutch culture and they occupy an important place in Dutch society. This emphasis on design can also be recognized in mathematics education and can be considered the most significant characteristic of the Dutch didactic tradition in the past half century.

The reform movement in mathematics education that started in the Netherlands at the end of the 1960s was all about designing 'new' education, which in those days meant working on an alternative to the mechanistic approach to teaching mathematics that was prevalent in the Netherlands at the time. This approach, which still has some followers today, is characterised by teaching mathematics at a formal level from the outset, an atomized and step-by-step way of teaching in which the teacher demonstrates how problems must to be solved, and the scant attention paid to the application of mathematics. At the same time that the need arose for an alternative for this mechanistic approach, two new approaches from abroad appeared: the empiricist trend in which students were set free to discover a great deal by themselves and were stimulated to carry out investigations, and the structuralistic trend propagated by the New Math movement in which the mathematics to be taught was directly derived from mathematics as a discipline. However, neither of these new approaches was well received in the Netherlands.

Therefore, at the end of the 1960s the Wiskobas group started to think about another way to improve teaching mathematics in primary school. From 1971 on this took place at the newly-established IOWO, which some time later was extended to include the Wiskivon group that had been formed to design a new approach to teaching mathematics in secondary education. All staff members of these two groups, except Freudenthal, had experience in school practice either as a mathematics teacher or as a mathematics teacher educator. This meant that their work was very practice-oriented. The theory development which resulted in RME was considered a derivative of this practical work and would later serve as a guide for further design activities. Because of the focus on the practice of teaching, it is not surprising that Freudenthal often stated that IOWO was not a research institute, and IOWO staff members did not regard themselves as researchers, but as producers of instruction, as engineers in the educational field. As implied by the latter term, this work was not done in isolation, but carried out with teachers and students in classrooms. Moreover, there was a strong collaboration with mathematics teacher educators, counsellors at teacher advisory centres, and textbook authors with whom the materials were discussed and who also contributed to their development. In this way, the implementation of the reform happened more or less naturally without specific government interference. By having these strong networks of people and institutions involved in mathematics education, new ideas for teaching mathematics could immediately be used in pre-service teacher education, in-service courses, and above all in textbooks. Of all the possible change agents, textbooks have played a key role in the reform of mathematics education in the Netherlands. For primary school mathematics, the same is also true for the mathematics education infrastructure that evolved from these networks. For secondary mathematics education a teachers' association (NVvW) had already been founded in 1925 and for primary school mathematics the infrastructure came into being later. In 1981 Panama was set up, which has come to involve a collaboration of institutions for pre-service and in-service mathematics teacher education, and in 1982 NVORWO was established as an association for primary school mathematics. The main purpose of the infrastructure as a whole was, and still is, to inform the mathematics education community in the Netherlands through national mathematics education conferences, professional journals, in-service courses and websites and to support national mathematical events for students.

The educational designs that have been produced over the years by IOWO and its respective successors, are multifaceted, ranging from tasks containing opportunities for mathematisation and paradigmatic contexts that evolve into level-shifting didactical models, to tasks for mathematics days and competitions for students, to elaborate teaching sequences for particular mathematical domains. Among other things, the design work in primary school mathematics led to helpful contexts such as the pizza context in which students could produce fractions by themselves through fair sharing activities, and the bus context in which students were encouraged to reason about passengers entering and exiting and so invent their own symbolic notations of what happens at a bus stop. The design work for primary school also resulted in some very powerful didactical models that can be found in most current textbooks in the Netherlands, such as the empty number line, the arithmetic rack, the percentage bar and the ratio table. With respect to upper secondary education, new programs were developed in the 1980s and 1990s for Mathematics A (preparing students for studies in the social sciences) and Mathematics B (preparing students for studies in the natural sciences). Additionally, at the turn of the century new RME-based modules on calculus and geometry were developed for Mathematics B in the upper grades of pre-university secondary education. A prominent design project that was carried out with the University of Wisconsin involved the development of a complete textbook series *Mathematics in Context* for Grade 5–8 of the U.S. middle school. This project began in the mid-1990s and ran for some ten years.

Another long-term design project was the TAL project that started in 1997. Its aim was the development of longitudinal conceptual teaching-learning trajectories that describe the pathway that students largely follow in mathematics from Kindergarten to Grade 6. The decision to work on such trajectories was innovative at that time. The basis for this TAL project was the so-called Proeve, a first version of a national curriculum for primary school mathematics that led to the official enactment of the first description of the core goals for mathematics at the end of primary school at the beginning of the 1990s. The teaching-learning trajectories were designed to describe how these core goals could be reached, thus providing teachers, textbook authors and test developers with an insight into the continuous learning line of learning mathematics, so contributing to making the curriculum more coherent.

The advent of computer-based technology in schools again brought new demands and challenges for design. In addition to exploring opportunities for computer-assisted instruction, much effort was also put into rethinking the subject of mathematics within the context of the virtual world and exploring how students could benefit from the dynamic and interactive qualities of the new technology. This led not only to the development of the so-called Digital Mathematics Environment in which teachers can adapt and design instructional material for their students including the use of mathematical tools and feedback, but also resulted in a seemingly inexhaustible flow of applets and mini-games for primary and secondary education that are freely available online.

Background information about the role of design in mathematics education in the Netherlands can, for example, be found in Bakker (2004), Doorman (2005), Drijvers (2003), Gravemeijer (1994), National Center for Research in Mathematical Sciences Education & Freudenthal Institute (1997–1998), Streefland (1993), and Van den Heuvel-Panhuizen (1996, 2003).

3.1.4 The Role of Empirical Research in Mathematics Education in the Netherlands

Research in the Netherlands into the learning and teaching of mathematics since the first half of the 20th century has always been empirical in one way or another. Initially this research was undertaken mostly by psychologists and pedagogues with an interest in mathematics, but later on it was also done by mathematics teachers. A prominent example of such research was the didactical experiment carried out by Van Hiele-Geldof in the 1950s on teaching geometry in the first year of secondary school. Her thesis about this experiment contained a very careful description of how she developed the teaching sequence that brought students from visually supported thinking to abstract thinking. She also recorded precise protocols of what happened in the classroom, which were then thoroughly analysed. Starting with what she called a psychological-didactical analysis of the mathematical content was part of her research method. In fact, Van Hiele-Geldof's work, greatly admired by Freudenthal, contained many important ingredients of the research into the learning and teaching of mathematics that was done in the Netherlands from then on.

Freudenthal's empirical didactical research began with observing his own children as he was teaching them arithmetic during World War II. It is noteworthy that he warned at first against overestimating the value of these observations, stating that he would like to do observations with a more diverse sample and on a larger scale. Later, he apparently changed his opinion. In the 1970s he emphasized the strengths of qualitative small case studies. He even called on research in the natural sciences ("One Foucault pendulum sufficed to prove the rotation of the earth") to prove the power of observing a student's learning process. For Freudenthal, one good observation was worth more than hundreds of tests or interviews. The reason for this preference was that observing learning processes led to the discovery of discontinuities in learning, which Freudenthal regarded as being of great significance for understanding how students learn mathematics. Merely comparing scores of a large sample of students collected at different measuring points would imply that only an average learning process, in which the discontinuities have been extinguished and all essential details have disappeared, can be analysed.

The design work that was carried out at the IOWO from 1971 onwards, with the aim of creating materials and teaching methods for the reform of Dutch mathematics education, was also highly informed by empirical research. In agreement with Freudenthal, the emphasis was on small-scale qualitative studies carried out in schools. Based on didactical-phenomenological analyses of the mathematical domains and making use of knowledge of students' learning processes and the classroom context, learning situations were initially designed using thought experiments. These were followed by actual experiments with students and teachers, and the reactions of both students and teachers were observed. In this way, IOWO staff members ran school experiments in which the mathematics to be taught and the teaching methods were continuously adapted based on experiences in classrooms and feedback and input from the teachers. In this development process, design, try-out, evaluation and adaptation followed each other in short, quick cycles. Reflections on what happened in the classrooms focused not only on whether or not a learning process had taken place, but also on what impeded or facilitated its occurrence. These reflections and the accompanying intensive deliberations among IOWO staff members about the designed learning situations provided important theoretical insights which evolved into the RME theory of mathematics education. In their turn, these theoretical insights led further designs. In other words, theory development and the development of education were strongly interwoven.

This type of research, initially called developmental research but later given the internationally more common name design research, was the backbone of RME-based research activities. Over the years, the method of design research was developed further. Data collection and analysis procedures which would contribute to the evidential value of the findings of design research were added. The theoretical grounding was also elaborated, through prior mathe-didactical analyses and through including findings and approaches from the education and learning sciences.

In addition to design research, depending on what specific research questions have to be answered, various other empirical research methods are used, such as quasi-experiments (including pretest-posttest intervention designs and micro-genetic designs), surveys (including questioning teachers about their classroom practice and beliefs about mathematics education and carrying out expert consultations), document studies (including textbook and software analyses and study on the history of mathematics education) and review studies and meta-analyses. Also, outside the circle of RME-affiliated didacticians, there is a large group of researchers in the Netherlands consisting of psychologists, orthopedagogues, and cognitive neuroscientists who focus particularly on investigating how specific student characteristics influence students' learning of mathematics. In this way, they complement the research done by didacticians. Similarly, research by educationalists who, among other things, investigate school organisation and classroom climate also provide relevant knowledge for all involved in mathematics education.

Relevant empirical data to direct the development of mathematics education were also acquired through the PPON studies that, from 1987 on, have been carried out every five years by Cito, the national institute for educational measurement. It is important that these studies gave an overview of changes over time in the mathematics achievements of Dutch primary school students and of the effect of the use of particular textbooks. Finally, PISA and TIMSS provide the international perspective on achievement data for both primary and secondary school students.

Background information about the role of empirical research in mathematics education in the Netherlands can, for example, be found in Freudenthal (1977), Goffree (2002), Van den Brink and Streefland (1979), and Van der Velden (2000).

3.2 Students' Own Productions and Own Constructions—Adri Treffers' Contributions to Realistic Mathematics Education

Marc van Zanten[2]

3.2.1 Introduction

The development of Realistic Mathematics Education (RME) started with the setup of the Wiskobas project in 1968. Wiskobas is an acronym for 'Wiskunde op de basisschool', meaning mathematics in primary school. Treffers was one of the leading persons within Wiskobas from the beginning onwards. He can be considered as one of the founding fathers of RME. In an interview held on the occasion of the ICME13 conference, Treffers (Fig. 3.1) stated that, in his view, the active input of students in the teaching and learning process is the basis for good mathematics education. This was one of the issues that came to the fore in the interview, together with other ideas Treffers published on over the years, which are mentioned also in the text. In the interview, Treffers highlighted the issues that are most important to him, and looked back on his earliest sources of inspiration for his work on mathematics education. He explained how certain people had had a major influence on his vision of mathematics education. Interestingly, they turned out to be very special people not belonging to the

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