Asymmetric and Symmetric Exchange in a Generalized 2D Rashba Ferromagnet

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Dzyaloshinskii-Moriya interaction (DMI) is investigated in a 2D ferromagnet (FM) with spin-orbit interaction of Rashba type at finite temperatures. The FM is described in the continuum limit by an effective s-d model with arbitrary dependence of spin-orbit coupling (SOC) and kinetic energy of itinerant electrons on the absolute value of momentum. In the limit of weak SOC, we derive a general expression for the DMI constant D from a microscopic analysis of the electronic grand potential. We compare D with the exchange stiffness A and show that, to the leading order in small SOC strength α_R, the conventional relation $D = (4m\alpha_R/\hbar)A$, in general, does not hold beyond the Bychkov-Rashba model. Moreover, in this model, both A and D vanish at zero temperature in the metal regime (i.e., when two spin sub-bands are partly occupied). For nonparabolic bands or nonlinear Rashba coupling, these coefficients are finite and acquire a nontrivial dependence on the chemical potential that demonstrates the possibility to control the size and chirality of magnetic textures by adjusting a gate voltage.

Chiral magnetic structures have attracted a great deal of interest in recent years with the observation of novel exotic magnetic phases such as Skyrmion lattices [1], single Skyrmions [2–4], chiral domain walls [5–7], chiral magnons [7–9], and helimagnets [10]. The source of chiral symmetry breaking, required for the formation of such structures, is the asymmetric exchange interaction that is referred to as Dzyaloshinskii-Moriya interaction (DMI) [2,11–17]. DMI originates from spin-orbit coupling (SOC) in magnetic systems with broken inversion symmetry, e.g., in noncentrosymmetric crystals or at surfaces and interfaces of thin magnetic films. The latter, effectively low-dimensional systems, which are of particular interest for applications, are in the focus of our study.

Recently, both bulk and interfacial DMI have been measured by employing Brillouin light scattering and, indirectly, using spin-polarized electron energy-loss spectroscopy [18–29]. On the other hand, for calculation of DMI in realistic materials, there exist effective computational techniques that provide decent agreement with certain experimental data [30–33]. A comprehensive understanding of the asymmetric exchange in generic systems requires model studies as well.

A widely used strategy for addressing DMI in systems with magnetic order is to utilize an s-d type model approach with noninteracting itinerant electrons mediating magnetic interactions. Within this ideology, the authors of Ref. [34] derived formulas for the asymmetric exchange between two single magnetic ions embedded in a 1D- or 2DEG with Rashba SOC. A decade later, their result was generalized by allowing for finite uniform magnetization [35].

As far as smooth noncollinear magnetic structures are concerned (e. g., domain walls or Skyrmions), it is more convenient to describe a magnet in the continuum limit by sending the lattice spacing to zero in the first place. In this paradigm, Berry phase type expressions for the asymmetric exchange have been recently derived [36] and the relation between DMI and ground-state spin currents has been pointed out [37,38]. Surprisingly, though, the only 2D ferromagnet (FM) model for which DMI has, so far, been calculated in the continuum limit refers to the system of a FM deposited on top of a topological insulator [39–41].

In this Letter, we focus on a less exotic model that captures the effects of both Rashba SOC and the s-d type exchange interaction between localized FM spins and 2DEG. The following Hamiltonian of one conduction electron is considered:

$$\mathcal{H} = \xi(p) + \alpha_R\zeta(p)[\boldsymbol{p} \times \boldsymbol{\sigma}]_z + J_{sd}S\,\boldsymbol{n}(\boldsymbol{r}, t) \cdot \boldsymbol{\sigma}, \quad (1)$$

where $\xi(p)$ and $\zeta(p)$ are arbitrary functions of the absolute value of momentum that parametrize free electron

dispersion (kinetic energy) and momentum dependent Rashba SOC, respectively. The last term stands for the effective s-d exchange interaction with strength J_{sd}. We assume that the system is deep in the FM phase and the temperature is far below the corresponding Curie temperature; hence the localized spins of the absolute value S can be described by the continuous vector field $\boldsymbol{n}(\boldsymbol{r}, t)$ with the constraint $|\boldsymbol{n}| \equiv 1$. We also assume the dynamics of itinerant electrons to be much faster than that of FM spins and treat the field $\boldsymbol{n}$ as time independent. The notation $\boldsymbol{\sigma}$ refers to a vector of Pauli matrices.

The model of Eq. (1) describes a generic FM layer coupled to 2DEG with spin-orbit interaction of Rashba type. One possible realization of such a system is a $LaAlO_3/SrTiO_3$ interface [42]. The model might also be used to describe a $SrRuO_3/SrIrO_3$ interface, which has recently gained considerable attention in the context of the so-called topological Hall effect—the phenomenon intrinsically linked to DMI [43,44].

In the continuum limit, DMI (or the antisymmetric exchange) is recognized as a contribution $\Omega_D[\boldsymbol{n}]$ to the micromagnetic free energy density that is linear with respect to the first spatial derivatives of the vector field $\boldsymbol{n}$. The symmetric exchange, on the other hand, is associated with a contribution $\Omega_A[\boldsymbol{n}]$ that is quadratic with respect to the first spatial derivatives of $\boldsymbol{n}$. The ratio between the two contributions plays a key role in formation of chiral magnetic structures, affecting their stability and size. Relation between $\Omega_D[\boldsymbol{n}]$ and $\Omega_A[\boldsymbol{n}]$ for the model of Eq. (1) is interesting for one more, historical, reason. Standard symmetry analysis shows [17] that in an isotropic 2D FM system, one has

$$\Omega_A[\boldsymbol{n}] = A[(\nabla_x \boldsymbol{n})^2 + (\nabla_y \boldsymbol{n})^2], \qquad (2)$$

where A is the exchange stiffness. For a particular choice, $\xi(p) = p^2/2m$ and $\zeta(p) \equiv 1$ in Eq. (1), which is referred to below as the Bychkov-Rashba model [45], the authors of Ref. [46] argued that, in the limit of weak SOC, the form of Eq. (2) necessarily leads to

$$\Omega_D[\boldsymbol{n}] = D\,\boldsymbol{n} \cdot [(\boldsymbol{e}_z \times \boldsymbol{\nabla}) \times \boldsymbol{n}] \qquad (3)$$

and, moreover, to $D = (4m\alpha_R/\hbar)A$. Unfortunately, the actual calculation of $\Omega_D[\boldsymbol{n}]$ has been performed neither in Ref. [46] nor, to the best of our knowledge, anywhere else even for the particular case of the Bychkov-Rashba model.

Below, we undertake an accurate microscopic treatment of the model of Eq. (1) in the leading order with respect to small α_R and, under rather general assumptions on $\xi(p)$ and $\zeta(p)$ [47], directly derive Eqs. (2) and (3). Furthermore, we report that the exchange stiffness A and the DMI constant D are given by remarkably concise expressions, namely,

$$A = \frac{\Delta_{sd}}{32\pi} \frac{\partial}{\partial \Delta_{sd}} \left(\int_0^\infty dp \, \frac{p[\xi'(p)]^2}{\Delta_{sd}} (f_- - f_+) \right), \qquad (4)$$

$$D = \frac{\alpha_R \Delta_{sd}}{8\pi\hbar} \frac{\partial}{\partial \Delta_{sd}} \left(\int_0^\infty dp \, \frac{p^2 \zeta(p)\xi'(p)}{\Delta_{sd}} (f_- - f_+) \right), \quad (5)$$

where $\Delta_{sd} = |J_{sd}|S$ is half of the exchange splitting, $\xi'(p) = \partial\xi/\partial p$ and $f_\pm = f(\xi(p) \pm \Delta_{sd})$ are expressed via the Fermi-Dirac distribution

$$f(\varepsilon) = (1 + \exp[(\varepsilon - \mu)/T])^{-1}, \qquad (6)$$

with the chemical potential μ and temperature T.

We would like to draw the reader's attention to the fact that the result of Eq. (4) is well-known, though, in a different form [see, e.g., Eq. (70) in Ref. [48]]. It is, however, useful to cast A in the form of Eq. (4) in order to compare the symmetric and asymmetric exchange for several particular choices of $\xi(p)$ and $\zeta(p)$ as we do later in the text.

We have checked that the DMI constant of Eq. (5) can also be obtained either by evaluation of ground-state spin currents [37,38] or by using the formalism of Ref. [36]. We have also checked that one may restore both Eqs. (4) and (5) by calculation of spin density of conduction electrons $\boldsymbol{s}$ [49] followed by an integration of the relation $\boldsymbol{n} \times (\delta\Omega/\delta\boldsymbol{n}) = J_{sd}S\,\boldsymbol{n} \times \boldsymbol{s}$, as it was done in Ref. [39] for DMI in the Dirac model. It must also be possible to compute A and D from an effective action [41,50].

Nevertheless, we believe that the most natural and straightforward way to derive Eqs. (4) and (5) is to extract $\Omega_A[\boldsymbol{n}]$ and $\Omega_D[\boldsymbol{n}]$ from the electronic grand potential density Ω. In this approach, there is no need to assume *a priori* the symmetry form of the final result as it is often done in the literature. Using the standard formulation of statistical physics, we express the grand potential density at $\boldsymbol{r} = \boldsymbol{r}_0$ as

$$\Omega = -\frac{T}{2\pi i} \text{Tr} \int d\varepsilon \, g(\varepsilon)[\mathcal{G}^A(\boldsymbol{r}_0, \boldsymbol{r}_0) - \mathcal{G}^R(\boldsymbol{r}_0, \boldsymbol{r}_0)], \quad (7)$$

where $\mathcal{G}^{A(R)} = (\varepsilon \mp i0 - \mathcal{H})^{-1}$ is the advanced (retarded) Green's function for the model of Eq. (1), Tr stands for the matrix trace, and the notation

$$g(\varepsilon) = \ln\left(1 + \exp[(\mu - \varepsilon)/T]\right), \qquad (8)$$

is employed.

Now, let us show how Eq. (7) can be used to obtain the DMI contribution to micromagnetic free energy density. First, one should Taylor expand $\boldsymbol{n}(\boldsymbol{r})$ around $\boldsymbol{n}(\boldsymbol{r}_0)$ and use the result to generate the Dyson series

$$\mathcal{G}(\boldsymbol{r}_0, \boldsymbol{r}_0) = G(\boldsymbol{r}_0 - \boldsymbol{r}_0) + J_{sd}S \int d\boldsymbol{r}' \, G(\boldsymbol{r}_0 - \boldsymbol{r}')$$
$$\times \left[\sum_{\beta\gamma} (\boldsymbol{r}' - \boldsymbol{r}_0)_\beta \nabla_\beta n_\gamma(\boldsymbol{r}_0)\sigma_\gamma \right] G(\boldsymbol{r}' - \boldsymbol{r}_0), \quad (9)$$

where G is the Green's function of a homogeneous system with fixed $\boldsymbol{n}(\boldsymbol{r}) \equiv \boldsymbol{n}(\boldsymbol{r}_0)$. In Eq. (9), we have disregarded all the gradients of $\boldsymbol{n}$ but the first, which is only accounted for in the linear order. The second term in Eq. (9) is precisely the one that determines the asymmetric exchange. Substituting it into Eq. (7), we switch to momentum representation and symmetrize the result to obtain the general formula

$$\Omega_D[\boldsymbol{n}] = \sum_{\beta\gamma} \Omega_{\beta\gamma}^{\mathrm{DMI}} \nabla_\beta n_\gamma, \quad (10)$$

with the DMI tensor defined as

$$\Omega_{\beta\gamma}^{\mathrm{DMI}} = T \frac{J_{sd}S}{2\pi\hbar} \mathrm{Im} \int d\varepsilon \, g(\varepsilon) \int \frac{d^2 p}{(2\pi)^2}$$
$$\times \mathrm{Tr}(G^R \sigma_\gamma G^R v_\beta G^R - G^R v_\beta G^R \sigma_\gamma G^R), \quad (11)$$

where $\boldsymbol{v} = \partial\mathcal{H}/\partial\boldsymbol{p}$ is the velocity operator. Note that we have dropped the argument of $\boldsymbol{n}(\boldsymbol{r}_0)$ in Eq. (10) and further, below.

Evaluation of Eq. (11) for the present model is performed with the help of the momentum-dependent Green's function

$$G^{R(A)} = \frac{\varepsilon - \xi(p) + \alpha_R \zeta(p)[\boldsymbol{p} \times \boldsymbol{\sigma}]_z + J_{sd}S \boldsymbol{n} \cdot \boldsymbol{\sigma}}{[\varepsilon - \varepsilon^+(\boldsymbol{p}) \pm i0][\varepsilon - \varepsilon^-(\boldsymbol{p}) \pm i0]}, \quad (12)$$

where we introduce the spectral branches $\varepsilon^\pm(\boldsymbol{p}) = \xi(p) \pm \sqrt{(J_{sd}S)^2 + [\alpha_R p \zeta(p)]^2 - 2\alpha_R J_{sd}S p \zeta(p) \sin\theta \sin\phi}$, the angle θ stands for the polar angle of $\boldsymbol{n}$ with respect to the z axis, while ϕ is the angle between the momentum $\boldsymbol{p}$ and the in-plane projection of the vector $\boldsymbol{n}$. We substitute Eq. (12) into Eq. (11), calculate the matrix trace, expand the integrands to the linear order in α_R, and straightforwardly integrate over ϕ. This results in the following form of the DMI tensor:

$$\Omega_{\beta\gamma}^{\mathrm{DMI}} = D \sum_{ij} n_i \epsilon_{ij\gamma} \epsilon_{jz\beta}, \quad (13)$$

where $\epsilon_{q_1 q_2 q_3}$ denotes the three-dimensional Levi-Civita symbol, while

$$D = \frac{\alpha_R \Delta_{sd}^2}{2\pi^2 \hbar} T \int_0^\infty p \, dp \int_{-\infty}^\infty d\varepsilon \, g(\varepsilon)$$
$$\times \mathrm{Im}\left(\frac{2\zeta(p) + p\zeta'(p)}{[\varepsilon - \varepsilon_0^+(\boldsymbol{p}) + i0]^2 [\varepsilon - \varepsilon_0^-(\boldsymbol{p}) + i0]^2} \right), \quad (14)$$

where $\zeta'(p) = \partial\zeta/\partial p$ and $\varepsilon_0^\pm(\boldsymbol{p}) = \xi(p) \pm \Delta_{sd}$. From Eqs. (13) and (14), it is already evident that, up to the linear order in α_R, the asymmetric exchange does, indeed, have the form of Eq. (3) with the DMI constant D which is totally independent of the direction of magnetization.

Integration over ε in Eq. (14) leads to

$$D = \frac{\alpha_R \Delta_{sd}}{8\pi\hbar} T \int_0^\infty dp \frac{g'_- - g'_+}{\Delta_{sd}} \frac{\partial[p^2\zeta(p)]}{\partial p}$$
$$- \frac{\alpha_R \Delta_{sd}}{8\pi\hbar} T \int_0^\infty dp \frac{g_- - g_+}{\Delta_{sd}^2} \frac{\partial[p^2\zeta(p)]}{\partial p}, \quad (15)$$

where $g'_\pm = \partial g_\pm / \partial\Delta_{sd}$ and $g_\pm = g(\xi(p) \pm \Delta_{sd})$ [51]. Eventually, the above two integrals are combined to form a full derivative with respect to Δ_{sd}. Partial integration concludes the derivation of the DMI constant D of Eq. (5) once the identity $\partial g(\varepsilon)/\partial\varepsilon = -f(\varepsilon)/T$ is used.

The symmetric exchange can be treated similarly. In order to derive Eqs. (2) and (4), one should take $\alpha_R = 0$ and extract all terms proportional to $\nabla_\beta n_\gamma \nabla_{\beta'} n_{\gamma'}$ and $\nabla_\beta \nabla_{\beta'} n_\gamma$ in Eq. (7). We relegate the details of the calculation to the Supplemental Material [47].

In the rest of the Letter, we apply the general expressions of Eqs. (4) and (5) to three particular cases. All further analytical results are presented in Table I, and the corresponding plots are given in Figs. 1–3.

To begin with, we return to the Bychkov-Rashba (BR) model characterized by $\xi(p) = p^2/2m$ and $\zeta(p) \equiv 1$. As can be immediately seen from Eqs. (4) and (5), the relation $D_{\mathrm{BR}} = (4m\alpha_R/\hbar)A_{\mathrm{BR}}$, indeed, holds, and the prediction of Ref. [46] is validated. Furthermore, in the limit of zero temperature, one finds from Eq. (5) that

$$D_{\mathrm{BR}} = \frac{\Delta_{sd} m \alpha_R}{8\pi\hbar} \begin{cases} 1 - (\mu/\Delta_{sd})^2, & |\mu| < \Delta_{sd} \\ 0, & \mu > \Delta_{sd} \end{cases}. \quad (16)$$

Thus, if SOC is weak, both A and D are finite in the Bychkov-Rashba model at $T = 0$ only in the half-metal regime $|\mu| < \Delta_{sd}$.
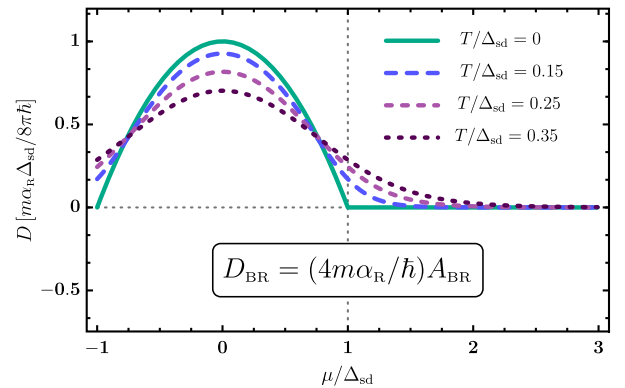

FIG. 1. Dzyaloshinskii-Moriya interaction constant D in the Bychkov-Rashba model as a function of the chemical potential μ at different temperatures T. Both μ and T are normalized by half of the exchange splitting $\Delta_{sd} = |J_{sd}|S$.
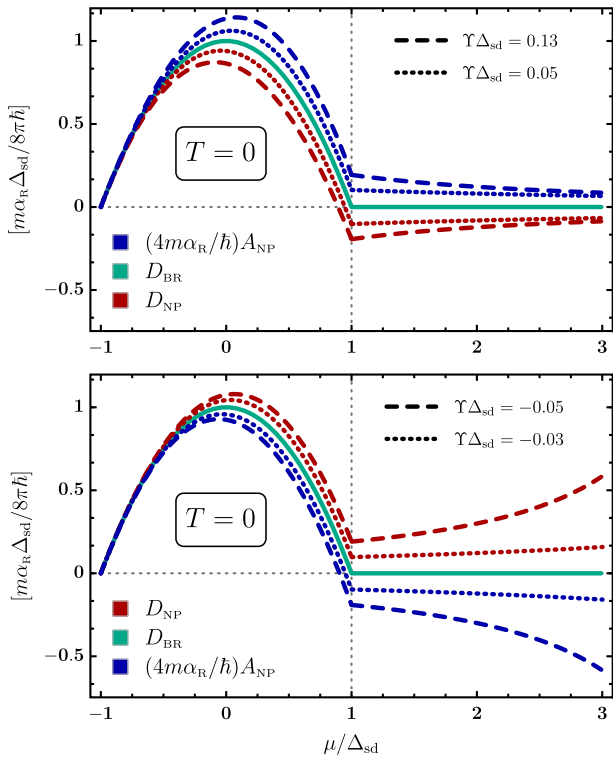
FIG. 2. Dzyaloshinskii-Moriya interaction constant D and "normalized" exchange stiffness $(4m\alpha_R/\hbar)A$ as functions of the chemical potential μ at zero temperature for different values of nonparabolicity coefficient Υ. Both μ and $1/\Upsilon$ are normalized by half of the exchange splitting $\Delta_{sd} = |J_{sd}|S$.

In fact, DMI in this model vanishes identically in the metal regime $\mu > \Delta_{sd}$ irrespective of the SOC strength. At larger α_R, the asymmetric exchange ceases to have the simple symmetry of Eq. (3) in the form of Lifshitz invariants. However, contributions from the two Fermi surfaces still cancel each other within each component of the DMI tensor Ω^{DMI}, no matter what the SOC strength is [52–54]. A nonperturbative in SOC study of DMI in the model of Eq. (1) will be presented elsewhere.

Next, it is instructive to see how the deviations from parabolic dispersion, a common property of, e. g., narrow gap semiconductors and quantum wells [55–57], affect A and D and the relation between them. To model non-parabolicity (NP) we use $\xi(p) = (p^2/2m)(1 + \Upsilon p^2/2m)$ and $\zeta(p) \equiv 1$ with the parameter Υ quantifying the deviation from the parabolic band. We shall assume that $\xi(p)$ is an increasing function even for negative values of Υ; i.e., our choice of $\xi(p)$ is understood as an approximation at small values of p. Temperature is set to zero.

We find, in this case, that the DMI constant and the exchange stiffness remain finite for all values of μ. Moreover, the NP corrections to D and $(4m\alpha_R/\hbar)A$ are of different signs, but have equal magnitudes,

$$D_{\text{NP}} - D_{\text{BR}} = -(4m\alpha_R/\hbar)(A_{\text{NP}} - A_{\text{BR}}), \qquad (17)$$

independently of the sign of Υ (see Fig. 2 and Table I). This leads, in the metal regime, to a particularly unexpected relation

$$D_{\text{NP}} = -(4m\alpha_R/\hbar)A_{\text{NP}}, \qquad \mu > \Delta_{sd}, \qquad (18)$$

[cf. the relation $D_{\text{BR}} = (4m\alpha_R/\hbar)A_{\text{BR}}$ for the Bychkov-Rashba model].

For $\Upsilon < 0$, the exchange stiffness becomes negative in the metal regime, which may eventually make the FM phase unstable. Of course, within our study, we do not consider direct contributions to magnetic exchange that may remain sufficiently large to be overcome by negative A_{NP}. Nevertheless, the reduction of the direct exchange in nonparabolic FM layers may have a serious impact on the size of noncollinear magnetic textures. In a particular case of a single Skyrmion, a simple estimate of its size is $\propto A/D$ [58]. We note that, for $\Upsilon < 0$, the DMI constant is enhanced; hence, the deviations from parabolicity may reduce the size of magnetic Skyrmions leading to miniaturization of Skyrmion-based technology. In general, nontrivial dependence of A and D on the chemical potential shown in Fig. 2 clearly demonstrates the possibility to control the size of Skyrmions by means of a gate voltage.

Finally, motivated by theoretical [59], computational [60], and experimental [61] demonstrations of generally nonlinear (NL) dependence of Rashba SOC on momentum, we model the effect of the latter on the asymmetric exchange. Since Rashba spin splitting is usually reported [59–61] to either saturate or decrease with increasing p, we use $\zeta(p) = 1/(1 + \lambda p^2/2m)$ with positive parameter λ and $\xi(p) = p^2/2m$. At zero temperature, we then find a finite DMI constant D_{NL} for any value of the chemical potential (see Fig. 3 and Table I). Moreover, D_{NL} exhibits a sign change around $\mu = \Delta_{sd}$. This demonstrates that a gate voltage can also be used to manipulate chirality of magnetic order in 2D FM.
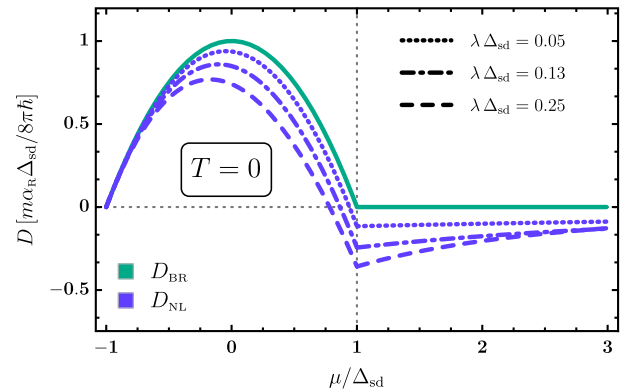

FIG. 3. Dzyaloshinskii-Moriya interaction constant D as a function of the chemical potential μ at zero temperature for different values of nonlinearity coefficient λ. Both μ and $1/\lambda$ are normalized by half of the exchange splitting $\Delta_{sd} = |J_{sd}|S$.

TABLE I.   Analytical results for D and $(4m\alpha_R/\hbar)A$ for particular choices of $\xi(p)$ and $\zeta(p)$. Results that correspond to the Bychkov-Rashba model are shown both with full temperature dependence (upper row) and, for clarity, at zero temperature (second row). Notation $\text{Li}_2(\cdot)$ stands for the dilogarithm, which is the polylogarithm of the order 2. Sign of Υ can be taken arbitrary, whereas λ is assumed positive. For $\Upsilon < 0$ the expressions shown are valid only as long as $\mu < (4|\Upsilon|)^{-1} - \Delta_{sd}$. Here, signs of Υ, u, and U coincide, while w and W are positive.

Cases			D and $(4m\alpha_R/\hbar)A$ in units $m\alpha_R\Delta_{sd}/8\pi\hbar$	
			$|\mu| < \Delta_{sd}$	$\mu > \Delta_{sd}$
$\xi(p) = p^2/2m,\ \zeta(p) \equiv 1$		$\begin{matrix}D\\(4m\alpha_R/\hbar)A\end{matrix}$	$1 - (\mu/\Delta_{sd})^2 - (\pi^2/3)(T/\Delta_{sd})^2 + S(\mu/\Delta_{sd}, T/\Delta_{sd})$	
$\xi(p) = p^2/2m,\ \zeta(p) \equiv 1$	$T = 0$	$\begin{matrix}D\\(4m\alpha_R/\hbar)A\end{matrix}$	$1 - (\mu/\Delta_{sd})^2$	0
$\xi(p) = (p^2/2m)(1 + \Upsilon p^2/2m),\ \zeta(p) \equiv 1$		$\begin{matrix}D\\(4m\alpha_R/\hbar)A\end{matrix}$	$\begin{matrix}1 - (\mu/\Delta_{sd})^2 - u(\mu/\Delta_{sd}, \Upsilon\Delta_{sd})\\1 - (\mu/\Delta_{sd})^2 + u(\mu/\Delta_{sd}, \Upsilon\Delta_{sd})\end{matrix}$	$\begin{matrix}-U(\mu/\Delta_{sd}, \Upsilon\Delta_{sd})\\U(\mu/\Delta_{sd}, \Upsilon\Delta_{sd})\end{matrix}$
$\xi(p) = p^2/2m,\ \zeta(p) = 1/(1 + \lambda p^2/2m)$		D	$1 - (\mu/\Delta_{sd})^2 - w(\mu/\Delta_{sd}, \lambda\Delta_{sd})$	$-W(\mu/\Delta_{sd}, \lambda\Delta_{sd})$
Notation	$s(a,b) = 2b\{\ln(1 + \exp[-(a+1)/b]) - b\text{Li}_2(-\exp[-(a+1)/b])\},$ $u(a,b) = (q-1)^2[24b - (q-1)(3q+1)]/48b^2,\ q = \sqrt{1 + 4(a+1)b},$ $w(a,b) = \{[2b(r-1) - r(r-3)](r-1) - 2r\ln r\}/rb^2,\ r = 1 + (a+1)b,$			$S(a,b) = s(a,b) + s(-a,b)$ $U(a,b) = u(a,b) - u(-a,-b)$ $W(a,b) = w(a,b) - w(-a,-b)$

Tuning of DMI has, so far, been realized by different approaches to interface engineering [62,63]. The ambition to manipulate the stability parameter, size, and density of Skyrmions was very recently achieved as well, by means of similar methods [64]. Based on our findings, we argue that a gate voltage variation may add yet another important and flexible tool for controlling chiral magnetic domains, paving the way towards novel material design.

To conclude, we considered the asymmetric exchange in generalized 2D Rashba FM. In the weak SOC limit, we established the full form of the corresponding contribution to micromagnetic free energy density and derived a general formula for the DMI constant. We showed that, to the leading order in small α_R in the Bychkov-Rashba model, a linear relation between the exchange stiffness A and the DMI constant D, indeed, holds, while at zero temperature, both vanish once the two spin sub-bands are partly occupied. At the same time, deviations from the Bychkov-Rashba model prevent this cancellation. There is no general linear dependence between A and D. In particular, the relation $D = (4m\alpha_R/\hbar)A$ for the Bychkov-Rashba model is replaced at zero temperature by the relation $D = -(4m\alpha_R/\hbar)A$ in the metal regime of the same model if nonparabolicity of the kinetic term is taken into account. For nonparabolic bands or nonlinear Rashba coupling, both A and D acquire a nontrivial dependence on the chemical potential that demonstrates the possibility of controlling the size and chirality of magnetic textures by adjusting a gate voltage.

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