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Conformal symmetry and the cosmological constant problem*

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We argue that when a theory of gravity and matter is endowed with (classical) conformal symmetry, the fine tuning required to obtain the cosmological constant at its observed value can be significantly reduced. Once tuned, the cosmological constant is stable under a change of the scale at which it is measured.

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1. 't Hooft's Technical Naturalness

In 1979 't Hooft[1] proposed an explanation to why a physical parameter may be small. This *technical naturalness hypothesis* states that:

> *A physical parameter* $\alpha(\mu)$ [...] *is allowed to be very small only if the replacement* $\alpha(\mu) \to 0$ *would increase the symmetry of the system.*

't Hooft then observes that the smallness of the parameter is protected in the sense that — due to the enhanced symmetry — quantum corrections will necessarily be proportional to $\alpha(\mu)$ — and thus will not affect the smallness of the parameter, explaining the term "technical".

In this paper, we combine the technical naturalness hypothesis with conformal symmetry to argue that a classically conformally invariant theory of gravity, matter

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and interactions provides a natural framework in which the cosmological constant (CC) can be small. In what follows we argue that, once a small CC is generated through radiative breaking of conformal symmetry,[2–4] it is protected from growing large (cf. Ref. 5).

2. Conformal Symmetry

Local Weyl symmetry (in the literature often referred to as conformal symmetry) is an internal symmetry that — in addition to diffemorphism invariance — naturally survives a breaking of local conformal symmetry which we assume to be realized at very high energies/short distances. Because spacetime torsion changes lengths of parallelly transported tensors, torsion tensor is the natural candidate which imbues Weyl symmetry in the gravitational sector.[6,a] More precisely, only the torsion trace part of torsion tensor — defined by $\mathcal{T}_\alpha \equiv (2/3)T^{\mu}_{\ \mu\alpha}$ — transforms under Weyl transformations. That is, if the metric tensor transforms as

$$g_{\mu\nu}(x) \to \mathrm{e}^{2\theta(x)} g_{\mu\nu}(x), \tag{1}$$

then

$$T^{\alpha}_{\ \mu\nu} \to T^{\alpha}_{\ \mu\nu} + \delta^{\alpha}_{\ [\mu}\partial_{\nu]}\theta \ \ \Rightarrow \ \ \mathcal{T} \to \mathcal{T} + d\theta. \tag{2}$$

These transformations then imply that classical gravity in vacuum is conformal. It is straightforward to extend the symmetry (1)–(2) to the matter sector.[6] The coupling of gravity to matter can then be made conformal by adding a dilaton field (whose condensate determines the value of the Newton "constant"). Finally, quantum effects break conformal symmetry[3,7] and in the remainder of the paper, we discuss in which way these breakings affect the observed CC.

3. Gravitational Hierarchy Problem

The *cosmological constant problem* (CCP) is by far the most severe hierarchy problem of physics, and up to date no convincing solution has been proposed that is accepted by most physicists. Assuming the observed CC is given by dark energy, then (in dimensionless units): $(8\pi G_N)\Lambda \sim 10^{-122}$. The CCP can be stated as follows[8] (for reviews see also Refs. 9–11):

(1) Why is the CC so small (when measured in natural units)?
(2) Why is it becoming important right now (when we are observing), i.e. why is the energy density in CC so close to the energy density in matter fields, $\rho_\Lambda \equiv \Lambda/(8\pi G_N) \sim \rho_m$?
(3) If CC is different from zero, what sets its magnitude and what stabilizes it against running with the energy scale?

[a]Very much like the longitudinal component of the vector field in an Abelian gauge theory, the longitudinal component of the torsion trace vector contains the compensating scalar that implements Weyl symmetry to Einstein's vacuum equations.

To elaborate on *Problem* 1, note that quantum vacuum fluctuations contribute to ρ_Λ as $\sim k_{\rm UV}^4$, where $k_{\rm UV}$ is an ultraviolet momentum cutoff scale. Given that the natural cutoff of quantum gravity is the Planck scale (the scale at which gravity becomes strongly interacting), $m_P = 1/\sqrt{G_N}$, the first problem can be rephrased as: *Why is* $\Lambda/m_P^2 \ll 1$? In other words, why vacuum fluctuations do not (significantly) contribute to Λ? As regards *Problem* 3, we note that, if one can identify the symmetry which is realized when CC vanishes, then this symmetry protects CC from running fast with scale.

4. Conformal Symmetry and Cosmological Constant

If a theory of gravity, matter and interactions is classically conformal, still quantum effects can violate the classical symmetry. The couplings constants can start running in such a way that the potential develops a new minimum, away from the origin of the field space thus introducing an energy scale and breaking conformal symmetry.[4] If the couplings are small at some fiducial large energy scale μ_*, their running will typically be slow, allowing for a large hierarchy between the UV scale and the scale of symmetry breaking. The latter can be estimated as the scale at which a given coupling turns negative, allowing for a minimum to form. From the perturbative treatment of the running, we obtain an estimate on that scale, $\mu \sim \mu_* \exp(-1/\lambda_*)$, where λ_* denotes the relevant coupling at the scale μ_* and we have dropped factors of $\mathcal{O}(1)$ in the exponent. Assuming that the vacuum expectation value of the scalar field is of the order of the scale μ, we obtain a rough estimate

$$\rho_\Lambda \sim -v^4 \sim -\mu^4 \sim -\mu_*^4 \exp\left(-\frac{4}{\lambda_*}\right). \tag{3}$$

In light of the above and with the right choice of the couplings at μ_* ($\lambda_* \sim 10^{-2}$), one could, in principle, get a CC as small as the observed one (though negative). In practice, however, nature has chosen to break conformal symmetry in the matter sector at the electroweak scale, $\mu \sim 10^2\,\mathrm{GeV}$, at which the Higgs field acquires an expectation value of $\langle h\rangle \equiv v \simeq 246\,\mathrm{GeV}$, which is responsible for the mass generation of all standard model particles (except perhaps of the neutrinos).[b] This then sets the natural energy density scale for the CC,

$$\rho_\Lambda^{\rm EW} \sim -v^4 \sim -10^8\,\mathrm{GeV}^4. \tag{4}$$

The contribution (4) must be negative, since in the absence of Higgs condensate $\rho_\Lambda^{\rm EW}$ must vanish.[c]

It seems that we have a *no-go* theorem: The contribution from matter field condensates is necessarily large and negative, while the observed CC is small and

[b] In the case of conformally symmetric theory the BEH mechanism of the Standard Model is replaced by the Coleman–Weinberg mechanism.[4]

[c] Another negative contribution is generated by the chiral condensate of mesons generated as the chiral symmetry of QCD gets broken by the chiral anomaly but — when compared with (4) — that contribution can be neglected since it is of the order -10^{-4} GeV.

positive. In order to overcome this impasse, we ought to dig deeper into the model and understand how gravity contributes to the CC. To get a clearer picture, let us consider the following simple conformal model of gravity consisting of the metric field $g_{\mu\nu}$, the torsion field $T^{\alpha}_{\ \rho\sigma}$, the dilaton ϕ and matter fields ψ_i. The action is given by,[6]

$$S[\phi, g_{\mu\nu}, T^{\alpha}_{\ \rho\sigma}, \psi_i] = \int \sqrt{-g} d^4x \left\{ \frac{\alpha}{2}\phi^2 R + \frac{\beta}{2} R^2 - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{\lambda}{4}\phi^4 \right\} + S_m[\psi_i, g_{\mu\nu}, T^{\alpha}_{\ \rho\sigma}], \tag{5}$$

where $R = R[g_{\mu\nu}, T^{\alpha}_{\ \rho\sigma}]$ is the Ricci curvature scalar, $g = \det[g_{\mu\nu}]$, $\nabla_\mu = \partial_\mu + \mathcal{T}_\mu$ is the conformal covariant derivative ($\mathcal{T}_\alpha \equiv (2/3) T^{\mu}_{\ \mu\alpha}$) and S_m denotes the matter action.

This action can be discerning for inflation,[12] but it can be also used to get an insight on how the gravitational sector contributes to the CC. The action (5) contains three scalars: ϕ, R (scalaron) and the longitudinal component of torsion trace,

$$\mathcal{T}^L_\mu = \partial_\mu \theta(x). \tag{6}$$

In the absence of scalar condensates, the theory (5) is at its conformal fixed point, and the vacuum energy must vanish. In order to understand what happens away from the conformal point, it is instructive to replace R in (5) by a scalar field Φ. This can be done by exacting $R \to \Phi$ and by adding a lagrange multiplier term to the lagrangian in (5), $\Delta\mathcal{L} = \frac{1}{2}\omega^2(R - \Phi)$. Varying the resulting action with respect to Φ then gives, $\Phi = -[\alpha\phi^2 - \omega^2]/(2\beta)$. Inserting this into (5) yields an (on-shell) equivalent action for the gravitational sector,

$$S_g[\phi, g_{\mu\nu}, T^{\alpha}_{\ \rho\sigma}] = \int \sqrt{-g} d^4x \left\{ \frac{\omega^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{8\beta}(\alpha\phi^2 - \omega^2)^2 - \frac{\lambda}{4}\phi^4 \right\}. \tag{7}$$

This (Einstein frame) action is (classically) conformal only if the Lagrange multiplier field transforms as, $\omega \to \Omega^{-1}\omega$ and it reduces to the usual general relativity coupled to a real scalar in a gauge in which ω is gauge fixed to the (reduced) Planck mass,

$$\omega = M_{\mathrm{P}} \ \text{(gauge fixing)}, \tag{8}$$

with $M_P \equiv 1/\sqrt{8\pi G_N}$ (any choice for ω is in principle allowed). Since M_P is the only scale in the problem, it has no absolute meaning, i.e. conformal symmetry of (7) teaches us that the choice (8) is *physically equivalent* to any other nonvanishing (local) scale $\omega'(x) = \Omega^{-1}(x) M_P$. The two remaining scalars, θ and ϕ, are the physical (scalar) degrees of freedom of the theory. Since θ stems from Weyl symmetry, θ retains a flat direction, i.e. it exhibits a (global) shift symmetry, $\theta(x) \to \theta(x) + \theta_0$ and thus cannot contribute to the CC. On the other hand, ϕ exhibits a nontrivial potential. A simple calculation shows that, when $\beta/\alpha > 0$, ϕ is tachyonic and condenses to, $\phi_0^2 = \alpha\omega^2/(\alpha^2 + 2\lambda\beta)$, at which the mass and potential energy are given by

$$m(\phi_0)^2 = \frac{\alpha}{\beta}\omega^2, \quad V(\phi_0) = \frac{\lambda}{4(\alpha^2 + 2\lambda\beta)}\omega^4. \tag{9}$$

Let us pause to try to understand the result (9). As a consequence of conformal symmetry breaking, the gravitational sector produces a positive CC whose size in natural (dimensionless) units is given by

$$\frac{V(\phi_0)}{\omega^4} = \frac{\lambda}{4(\alpha^2 + 2\lambda\beta)}. \tag{10}$$

For this to compensate the negative CC generated in the matter sector, one ought to fine tune (10) to be $\sim 10^{-65}$, such that when (10) is added to the matter contribution (4), one obtains the observed CC, $\Lambda/\omega^4 \sim 10^{-122}$. We emphasize that, once the CC is tuned to the observed value, 't Hooft's technical naturalness ensures that quantum corrections (both from gravitational and matter fields) will not affect it. This can be made more precise as follows. Let us assume that $V_{\rm eff} \sim 10^{-122}\omega^4$ represents the total contribution to the CC. Then, the RG improved $V_{\rm eff}$ must obey the Callan–Symanzik equation,

$$\mu\frac{d}{d\mu}V_{\rm eff}(\phi, \text{all other fields}) = \mu\frac{d}{d\mu}V_{\rm eff}(0, \text{all other fields} \to \text{fixed point}), \tag{11}$$

where the second term constitutes $V_{\rm eff}$ with all fields set to their respective conformal fixed point (at which all scalars vanish), which vanishes for the conformal theory under study.[d] Equation (11) tells us then that $V_{\rm eff}$ (and therefore also Λ) does not change if the scale μ changes. This means that, while the precise value of the fields and couplings can depend on μ, the value of the effective potential at its minimum cannot.[e]

To conclude, let us recall that the classical conformal symmetry alleviates the hierarchy problem associated with the mass of the Higgs boson present in the Standard Model.[15] In this way imposing conformal symmetry on physical theories can elucidate the most notorious hierarchy problems in physics — the CC problem and the gauge hierarchy problem.

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[d] If the potential is nonzero at the origin of the field space, one has to cancel the zero-point energy order by order to obtain a homogenous RG equation for the effective potential.[13]

[e] In practical applications $V_{\rm eff}$ is always approximated by its truncated version at a finite order in loop expansion. Such a truncated effective potential contains some residual dependence on μ,[14] which is however suppressed by a suitable power of $\hbar$.

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