On Translating Mathematics

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Abstract: Mathematical texts raise particular dilemmas for the translator. With its arm's-length relation to verbal expression and long-standing "mathematics is written for mathematicians" ethos, mathematics lends itself awkwardly to textually centered analysis. Otherwise sound standards of historical scholarship can backfire when rigidly upheld in a mathematical context. Mathematically inclined historians have had more faith in a purported empathic sixth sense—and there is a case to be made that this is how mathematical authors have generally expected their works to be read—but it is difficult to pin down exact evidentiary standards for this supposed instinct. This essay urges that both of these points of view, for all the tension between them, be kept in the historian's toolbox. It illustrates these considerations with a case study from the Ptolemaic astronomical tradition on computing lunar model parameters from eclipse data.

Mathematics has an uneasy relationship with language. Geometry is intrinsically visual rather than verbal, while algebraic texts often use more symbols than words. One has the sense that mathematics, perhaps more than any other field, is in its essence far removed from immediate textual expression. Written mathematics is an imperfect proxy for mathematical thought itself; it has to be thoroughly digested before it transmutes into the intended idea in the reader's mind. Perhaps this is why mathematicians in every age have stuck to the most formulaic prose and produced voluminous tomes composed solely of combinations of a small set of stock phrases.

This apathetic attitude to the written word, which is arguably nearly universal among mathematicians, makes the classic letter-versus-spirit dilemma in literary translation especially problematic in mathematics. It is one thing to try to strike a balance between literal and essence translation when the author of the original took pride in both. It is quite another to try to decide what

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Isis, volume 109, number 4. 0021-1753/2018/0109-0009$10.00.

to do with literal aspects that the author openly held were merely a pale and incidental shadow of his actual point. One might almost consider mathematical thought a nonverbal language unto itself, so that the textual expression of it is already a translation even in the original—a translation, furthermore, from a nonverbal language into a written one, and hence across a much greater conceptual divide than that between verbal languages to be bridged in any subsequent translation of the written text.

And so the translator and interpreter of historical mathematics has to grapple with a paradox. On the one hand, the text is all we have. It is *the* evidence. As historians, we are obligated, virtually by definition, to treat the text as the truth, the whole truth, and nothing but the truth. But if the above picture is accurate, and even the primary text is in a sense truly a secondhand account, then our professional pride in being sensitive to the subtlest nuances of the text may even be directly counterproductive. From this point of view, perfect textual sensitivity—that methodological cornerstone of our field—may, ironically, amount to the same thing as a widely renounced cardinal sin of our profession: namely, drawing inferences from incidental aspects of a translation without consulting the original with understanding.

Again, modern historians take pride in being sensitive to "actors'" ways of thinking, which is typically taken to mean giving primacy to the text and avoiding any anachronistic terminology or extraneous concepts. But the assumption that perfect textual sensitivity is equivalent to perfect sensitivity to actors' thought is a problematic one—and especially so in mathematics, if there is any truth to what we said above. So we have another paradox on our hands: professional standards of our field designed to ensure certain ends can quite plausibly be construed as directly at odds with those very ends. Being sensitive to the text can mean being sensitive to the author's thought—but can potentially also mean being insensitive to it, insofar as that sensitivity is directed at modes of expression that the author considered unimportant.

This divide tracks disciplinary lines. The scientist seeks to unify and idealize; the humanist thrives on nuance. The scientist is trained never to "multiply causes," the humanist always to "problematize"—the two are virtually opposites. Mathematicians studying history often develop a strong sense of kinship with historical mathematicians and become convinced that they have an empathic sense of those mathematicians' thinking that transcends centuries and superficial differences. Scholars trained in history, on the other hand, view such an approach with great suspicion and instead "take as their working assumption, a kind of null-hypothesis, that there is a discontinuity between mathematical thought of the past and that of the present."[1] The underlying point—that one must try to approach historical sources without preconceived ideas—is no doubt a prudent one. But then again, the *a priori* assumption that the development of mathematical thought is fundamentally discontinuous is itself a preconceived notion. Both approaches—that of the mathematician and that of the historian—have their assumptions and potential pitfalls.

The different "null hypotheses" of the mathematician and the historian lead to opposite strategies for delineating the meaning of terms in historical texts. The mathematician tries to find the smallest natural category that fits the evidence. The assumption that, whatever the meaning is, it must be something that we can construe in ways natural to us is a leap of faith, but one that the mathematician takes to be true until proven otherwise. The historian instead admits only what can be strictly proven from explicit textual evidence, regardless of whether the total conception so formed seems unnatural or artificially restricted. Thus the mathematician potentially overesti-

[1] Michael N. Fried, "Ways of Relating to the Mathematics of the Past," *Journal of Humanistic Mathematics*, 2018, 8(1):3–23, on p. 16.

mates similarities between ancient texts and modern conceptions, while the historian potentially underestimates them.[2]

This tension formed the core of a notorious feud as the center of gravity of history of mathematics as an academic discipline made a rocky move from mathematics departments to the halls of the humanities in the second half of the twentieth century.[3] With increased specialization came considerable advances in the field and in some respects greatly improved professional standards. It was natural for the new generation to frame this transition as one of unequivocal progress. But it would be much too simplistic to mistake the legitimate advances of the new generation for a clear-cut methodological victory of good over evil. Indeed, that would be an especially ironic fallacy, since it parallels in the domain of historiography that presentist triumphalism in the interpretation of the history of science that the new generation of humanistic historians perceived as one of the most deplorable flaws of older historical scholarship.

One outcome of these disputes is that altering mathematical notation when translating or discussing a text is nowadays treated with the utmost suspicion. Rendering ancient texts using more modern algebraic shorthands in particular has been widely condemned. But it is not entirely clear that such practices are necessarily more pernicious than typographical modernizations that are always made. Classical Greek was written as a continuous string of letters without spaces, punctuation, or distinguished letter case. Arabic words are written without any characters for vowels. No one thinks a translation should retain these characteristics. One could argue that the use of altered mathematical notation in a translation is sometimes equally innocent, not to mention equally helpful in eliminating major obstacles to readability. The aversion to using modern notation is based largely on the assumption that differences in form of expression reflect ways of thinking that differ in functionally significant ways. Although this is sometimes the case, we should not close our minds to the possibility that sometimes it is not, just as the absence of spacing in Greek manuscripts does not imply that the ancient Greeks inhabited a conceptual universe ignorant of the notion of words as distinct units.

Editions and translations with full philological precision are of course crucial for some purposes, but this should not be conflated with the notion that they are also the best way to capture faithfully the mathematical thought expressed in historical treatises. To what extent differences in textual surface form represent conceptual chasms in the underlying mathematical thought is itself an empirical question. And for all their legitimate critiques of older generations' readiness to translate ancient texts into modern notation, the efforts of the more recent and more textually

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[2] See the discussion that follows for an example. Another illustrative case is one analyzed in depth in Reviel Netz, *The Transformation of Mathematics in the Early Mediterranean World: From Problems to Equations* (Cambridge: Cambridge Univ. Press, 2004), pp. 98–104; and Netz, "Archimedes Transformed: The Case of a Result Stating a Maximum for a Cubic Equation," *Archive for History of Exact Sciences*, 1999, 54:1–47, esp. pp. 2–9. This concerns the meaning of "epi," a word Archimedes uses in expressions of the form "[area] epi [line]." As Netz notes, there are ample and compelling reasons (both mathematical and linguistic) to see this as a multiplication that is algebraic rather than geometrical in character. And indeed Heiberg's 1913 Latin translation simply rendered "epi" by a multiplication symbol, *. Yet, Netz objects, the corpus only has expressions of the form "area epi line," and not "line epi area." This makes it possible to question whether "epi" was thought of as a commutative operator. Hence, Netz argues, textual evidence alone is not enough to infer that "epi" is fully equivalent to multiplication in all respects. This illustrates the very high bar for explicit textual evidence set by modern scholars before they are willing to accept identifications of concepts.

[3] Sabetai Unguru, "On the Need to Rewrite the History of Greek Mathematics," *Arch. Hist. Exact Sci.*, 1975, *15*:67–114, sparked extensive debate. See, e.g., Jean Christianidis, *Classics in the History of Greek Mathematics* (Boston: Springer, 2004); Martina Schneider, "Contextualizing Unguru's 1975 Attack on the Historiography of Ancient Greek Mathematics," in *Historiography of Mathematics in the Nineteenth and Twentieth Centuries*, ed. Volker R. Remmert, Schneider, and Henrik Kragh Sørensen (Basel: Birkhäuser, 2016), pp. 245–267; and Viktor Blåsjö, "In Defence of Geometrical Algebra," *Arch. Hist. Exact Sci.*, 2016, *70*:325–359.

oriented generation of historians have by and large necessitated little revision of the established mathematical understanding drawn from the textually liberal editions of previous generations.[4] The potential danger of invalidly projecting modern conceptions onto historical works by the use of anachronistic notation and terminology, which has often been highlighted, is indeed real. But so are the potential dangers of an overly textual approach. While the former may erase historical differences, the latter runs the risk of perceiving conceptual differences where there are none.

The mathematician's and the historian's points of view are complementary, and a healthy balance between them is advisable. Neither should be taken as inherently superior as a matter of general methodology; rather, each is a tool that can offer its own particular illumination in any given case.

It would furthermore be regrettable, in our opinion, if celebration of textual esoterica, and zero tolerance for moderate readability adaptations, made the reading of historical mathematical and scientific texts a fortified protectorate for specialized scholars only. Let us not give up on trying to reach a wider audience, including modern scientists, teachers, students, and the educated public in general. By striving to make translations as accessible as possible, one can hope to stimulate recognition of and wonder about the past among a wider readership than just fellow professional historians of science.[5]

We shall illustrate the above points using an example from the corpus of mathematical astronomy, which was based on the *Almagest* of Ptolemy (ca. 150 C.E.) and continued through the end of the Islamic astronomical tradition. This corpus is important for the history of mathematics, because in late antiquity and the medieval period astronomy was the main application of advanced mathematics—and also the main reason for studying geometry. Although significant advances were made in the Islamic tradition, the corpus as a whole is quite homogeneous.

Our problem is the computation of a simple form of the orbit of the moon from observations of three lunar eclipses close in time. The problem was treated by Ptolemy in the *Almagest* and then multiple times by Islamic astronomers. We will discuss Ptolemy's computation for three Babylonian eclipses observed in March 721, March 720, and September 720 B.C.E. and also the analogous computation by the Ottoman astronomer Taqi al-Din ibn Ma'ruf for three eclipses in October 1576, April 1577, and September 1577.[6] The computations follow essentially the same method, which was already known to the Greek astronomer Hipparchus (ca. 150 B.C.E.), if not before.

In the simple form of the lunar orbit, the moon moves with constant speed on a small circle, called the "epicycle," whose center also revolves with constant (but different) speed on a larger circle, the "deferent," around the center of the Earth.[7] One is required to compute the numerical ratio between the radii of the big and the small circles.

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[4] For example, the view that ancient quasi-algebraic reasoning in geometric terms is effectively functionally equivalent to modern algebra in key respects was central in older interpretations of Greek mathematics and has strong internalist support. And although it was aggressively challenged by Unguru in "On the Need to Rewrite the History of Greek Mathematics," it is still accepted even among some textually sensitive contemporary scholars. See Nathan Sidoli, "Uses of Construction in Problems and Theorems in Euclid's *Elements* I–VI," *Arch. Hist. Exact Sci.*, 2018, 72:353–402, esp. p. 376; Jens Høyrup, "What Is Geometric Algebra, and What Has It Been in Historiography?" *AIMS Mathematics*, 2017, 2:128–160; and Netz, *Transformation of Mathematics in the Early Mediterranean World* (cit. n. 2), pp. 54–55.

[5] See the inaugural lecture of our predecessor Henk Bos, "Recognition and Wonder," in *Lectures in the History of Mathematics* (Providence, R.I.: American Mathematical Society, 1997), pp. 1–21.

[6] Taqi al-Din's autograph manuscript, MS Istanbul Kandilli 208, is currently under investigation by Huseyin Sen, a Ph.D. student of Jan P. Hogendijk.

[7] For details see Olaf Pedersen, *A Survey of the "Almagest"* (Odense: Odense Universitetsforlag, 1962), pp. 167–182.

We shall use this problem to discuss some of the complexities of translating a mathematical text. We note, first of all, that the problem cannot be solved by text alone; rather, one also needs to draw a rather complicated geometrical figure. See Figure 1, which is Ptolemy's figure.[8] Ptolemy moved the three positions of the epicycle into one, on which A, B, and Γ represent the positions of the moon on the epicycle during the first, second, and third eclipses. Point K is the center of the epicycle and Δ is the center of the Earth. (Ptolemy does not draw the deferent circle in the figure, in order not to confuse matters unnecessarily.)

From the observations of the three Babylonian eclipses, Ptolemy first derives the following data: arc AB = 53 degrees 35 minutes, arc AΓ = 96 degrees 51 minutes; from point Δ, point B is seen left of A, and Γ between A and B, as in the figure; angle BΔA = 3 degrees 24 minutes, angle AΔΓ = 2 degrees 47 minutes. From this information Ptolemy needs to compute the ratio KB : KΔ and the angle BΔK.

In Ptolemy's account, the figure is essential for conveying these relationships, especially those pertaining to relative positions. Already, therefore, the relationship between the text and the mathematical argument is strained. Ptolemy clearly makes no pretense of providing all essential information explicitly in linear prose; instead, he seems to have considerable faith that the reader will be able to make holistic sense of the underlying idea by piecing together visual and textual information in ways that are not spelled out. And while the reader is expected to rely on the figure in some regards, it is to be understood that in others it is only schematic. Notably, the point Δ is drawn unrealistically close to the epicycle—a severe distortion that is clearly inconsistent with the numerical values involved.

We turn now to some issues regarding the use of modern symbolism in translating the Greek. In the course of his derivation, Ptolemy says: "tèn de BAΓ, hèn kekinètai apo tès deuteras ekleipseoos epi tèn tritèn, morioon ousan PN KF aphairein tès mesès moiras ō ΛZ."[9] This reads, in literal translation: "whereas (arc) BAΓ, which is its increment in motion between the second eclipse and the third, is of those parts PN KF, and produces a decrement from the mean of ō ΛZ parts."[10]

The capitals are transcriptions of the Greek alphabetical numbers with meaning A=1, B=2, Γ=3, Δ=4, E=5, F=6, Z=7, . . . , I=10, K=20, Λ=30, M=40, N=50, . . . , P=100, . . . , and ō=0. The alphabetic numbers were used for the values of angles and also to label the points in the geometrical figure.[11] The Islamic astronomers adapted the Greek alphabetic numbering system in the form of the Arabic alphabetic abjad system, which became the norm in astronomical texts. Even Taqi al-Din calls the labels *alif*, *bā*, and so forth in his geometrical figures "numbers" (*ruqūm*).

G. J. Toomer—quite rightly, in our view—decided to render the numbers in the geometrical figures as letters A, B, C, . . . , and the values of angles as numbers in the Hindu-Arabic system, in order to make the translation intelligible to the general modern reader. Thus his translation reads: "while arc BAG, which is its increment in motion between the second and third eclipses, is 150;26 (degrees), and produces a decrement of 0;37 degrees from the mean motion."[12]

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[8] The figure is adapted from G. J. Toomer, *Ptolemy's "Almagest"* (1984; Princeton, N.J.: Princeton Univ. Press, 1998) (hereafter cited as **Toomer, *Ptolemy's "Almagest"***), Fig. 4.4, p. 193; and Claudius Ptolemaeus, *Opera quae exstant omnia*, Vol. 1: *Syntaxis Mathematica*, ed. J. L. Heiberg, 2 pts. (Leipzig, 1888, 1893) (hereafter cited as **Heiberg, *Syntaxis***), p. 305. We have added point K from Ptolemy's subsequent figure (Toomer, *Ptolemy's "Almagest,"* Fig. 4.6, p. 196; and Heiberg, *Syntaxis*, p. 311).

[9] Heiberg, *Syntaxis*, p. 306, ll. 1–3.

[10] Adapted from Toomer, *Ptolemy's "Almagest,"* p. 193, ll. 6–8.

[11] One may think of "first point," "second point," etc., but this comparison cannot be taken too far. The letters are used in ascending numerical value, so L usually follows K, but there are no labels KA, KB (for 21, 22). F=6 represents the obsolete Greek letter digamma.

[12] Toomer, *Ptolemy's "Almagest,"* p. 193, ll. 6–8.

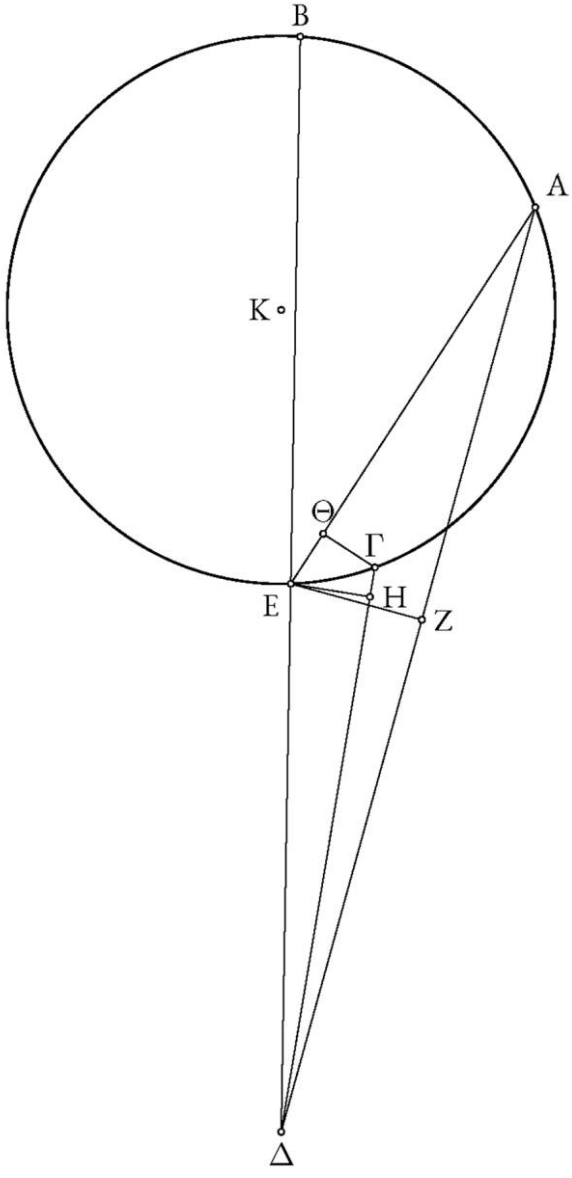

Figure 1. Determination of lunar parameters in Book IV of Ptolemy's *Almagest.*

The distinction between these two types of numbers or letters is clearly central to the reasoning. But once again Ptolemy does not appear particularly concerned that the text on its own should directly represent his thought process. Instead, he evidently trusts that the reader will impose mathematical understanding that is not textually explicit. (Interestingly, Taqi al-Din, in his autograph manuscript, distinguishes between labels in geometrical figures and numbers by using black ink and overbars for the former and red ink for the latter.)

These observations as to the relation between text and mathematics, especially the importance of implicit mathematical assumptions and conventions, become more contentious as Ptolemy's account starts to involve "algebraic" reasoning. In the course of his argument, Ptolemy defines Λ and M as points of intersection of ΔK and the epicycle, and he says: "palin de, epei kai to hypo ΛΔ kai ΔM meta tou apo tès KM poiei to apo ΔK tetragoonon."[13] This literally translates as: "Again, since also the under ΛΔ and ΔM with the of KM is equal in power to the square of ΔK." Here it is understood that "the under ΛΔ and ΔM" means the rectangle whose sides are equal to ΛΔ and ΔM and that "the of KM" is the square with side KM. Toomer translates more freely: "Furthermore, since $LD.DM + KM^2 = DK^2$."[14]

Some historians of Greek mathematics have insisted that translations should be made very literally, in order not to read modern concepts into the ancient texts.[15] In their view, it is misleading to render the rectangle contained by LD and DM as LD*DM, because in the orthodox Euclidean view LD and DM are line segments, not numbers, and therefore cannot be multiplied as numbers. However, immediately after the quoted passage Ptolemy shows that if KM is 60 "parts," then the square of ΔK is 476,300 + 5/60 + 32/3,600 parts, from which he concludes that ΔK = 690 + 8/60 + 42/3,600 parts, which is a rounded form of this irrational number. (We have written the numbers in modern form, sparing the reader the details about the Greek alphabetic number system.) In other words, he routinely computed numerical approximations, and he never even called attention to the fact that most of these segments are irrational. Blurring the distinction between line segments and numbers is precisely the alleged danger in using notation such as the multiplication dot and the exponent 2 for the square. But the case for upholding this distinction is greatly undermined when Ptolemy himself switches freely between the geometrical and arithmetical viewpoints. Taqi al-Din operates similarly; like Ptolemy, he has no qualms about equating segments to numbers.[16]

This illustrates our point about the mathematical versus the humanistic-historical ways of getting at the meaning of mathematical texts. Had Ptolemy given the general "algebraic" description only, the extent to which he thought of this in numerical or geometrical terms would be open to interpretation. And since the language is geometrical, the historian's approach would require us to play it safe and say that we have no basis for inferring that Ptolemy's notion went beyond a geometrical context. The mathematician's view, on the other hand, would be that this is surely algebra and that the geometrical language is simply a textual artifact that is conceptually incidental. This is inferred on the basis of the mathematician's intuitive empathic understanding of colleagues past and present. Accordingly, translations using algebraic symbolism have been commonplace among scholars with such inclinations. On the basis of the "algebraic" expression alone, the matter could not be definitively settled. Fortunately, in this case we also have the numerical part of the argument, showing that the mathematician's sense was indeed correct. The historian's view has to be updated in light of this additional information, and when this is done the two interpretations converge.

Ptolemy explained his solution for two concrete cases—namely, the three Babylonian eclipses and three lunar eclipses that he observed in his own time. In both cases he arrived at the result KB : KΔ ≈ (5+1/4):60, and he concluded that the lunar epicycle had not changed over time. Yet his intention was to explain not only the concrete computations but also the general method to be used for any three lunar eclipses. He therefore offers a "generally applicable description," "in order to make the sequence of the proof readily transferable for computation of this kind."[17] That is to say, Ptolemy wants to convey how to find BK : KΔ in terms of arbitrary input arcs AB, BΓ and

[13] Heiberg, *Syntaxis*, p. 312, ll. 4–6.
[14] Toomer, *Ptolemy's "Almagest,"* p. 197, l. 5.
[15] This line of argument started with E. J. Dijksterhuis; see, e.g., Dijksterhuis, *Archimedes* (1938; Princeton, N.J.: Princeton Univ. Press, 1987), p. 51.
[16] See MS Istanbul Kandilli 208, 43a:15–17.
[17] Toomer, *Ptolemy's "Almagest,"* pp. 193–194; and Heiberg, *Syntaxis*, p. 306.

arbitrary input angles ABΔ, BΔΓ. Even in modern terms, the general formula or expression for BK : KΔ in terms of these givens is very complicated.[18] Using the mathematical tools of antiquity and the Islamic Middle Ages, it was impossible to express the general computation in this way. All one could do was give the computation for concrete numbers and hope that the reader would implicitly extract the general procedure from the text. Of course, this also involves drawing the geometrical figure in the correct way, putting A, B, and Γ near their correct positions. Islamic astronomers were able to do this, as is clear from Taqi al-Din and many others. Thus the "deep" knowledge that was only implicit in the texts and the geometrical figures was nevertheless transmitted in this case.

Once again we see that Ptolemy places considerable faith in the reader to perceive mathematical ideas that go well beyond what is textually explicit. One might say that Ptolemy expects the reader to take the treatise as a starting point for probing thought that will give rise in the reader's mind to the intended message, the text itself being merely a shadow of the idea it is meant to convey. In short, mathematical authors expect mathematical readers. This speaks in favor of not dismissing as historiographically naive the mathematician's point of view in the interpretation and, consequently, the translation of historical texts.

[18] See Pedersen, *Survey of the "Almagest"* (cit. n. 7), pp. 174–176.