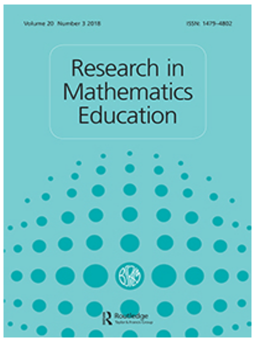

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Tools and taxonomies: a response to Hoyles

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ABSTRACT
In this response paper to Hoyles' contribution "Transforming the mathematical practices of learners and teachers through digital technology" focuses on three points. First, more knowledge is needed on why teaching and learning practices should transform, into what will they transform, and by what or by whom will they be transformed. Second, a suggestion is made for a more specific taxonomy on the didactical functionality of digital tools in mathematics education. Third, a plea is made for a future research agenda that addresses the ways in which activities with digital tools mediate the learning of mathematics in a fruitful way. This includes the interpretation and grading of online student work through intelligent mathematical software, and the notion of embodiment, as to do justice to the bodily experiences in which mathematical experiences are rooted.

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> Tools matter: they stand between the user and the phenomenon to be modelled, and shape activity structures. (Hoyles & Noss, 2003, p. 341)

It was the acknowledgement that tools are not neutral, expressed in the above quotation, that made me follow the work by Hoyles and colleagues on the use of digital tools in mathematics education since a long time. Already in 1996, her seminal book with Noss witnessed a vision on the roles that digital tools can play in opening "windows on mathematical meaning" (Noss & Hoyles, 1996). Since then, this has led to an impressive number of insightful publications that show the need for research to go beyond the naïve optimistic view that tools just "do the job". In line with the work done by Papert (1980), this corpus of work shows how digital technology may support a constructionist approach. Indeed, the current interest in computational thinking and programming might initiate a revival of these ideas.

Hoyles' contribution, entitled "Transforming the mathematical practices of learners and teachers through digital technology" (Hoyles, 2018), is based on her excellent plenary address on the 2008 ICME conference, which I happened to have the chance to attend. Some readers might be surprised to see an article based on a lecture appearing ten years later, though Hoyles does explain in a footnote that "The text is inevitably in part retrospective. It has been updated in relevant parts without the pretence of producing

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an exhaustive review of the research in the intervening years." In the paper, I sometimes seem to feel that the author struggled with the double perspective of issues at stake by that time, and the current state-of-the-art.

Clearly, Hoyles' paper stands in the constructionist tradition and describes the developments in the field from this perspective. The point of departure is that "mathematical knowledge, and its related pedagogy, is inextricably linked to the tools in which the knowledge is expressed" (Hoyles, 2018), and that digital tools may play important roles in shaping mathematical meanings and in transforming the mathematical practices of learners and teachers. Even if I completely agree with this stance, I would like to claim that not enough is yet known about how to exploit this link between mathematics and tool use, and the way in which this transforms practices for the sake of mathematical learning. One can wonder if the integration of digital technology in mathematics teaching is as successful as may be hoped and expected some decades ago. From this somewhat critical position, my response addresses three main points: (1) the transforming practices and its actors; (2) tools and taxonomies; and (3) the future research agenda.

Transforming practices

The title of Hoyle's paper speaks about transforming practices of learners and teachers. In the text, Hoyles highlights the potential power of digital tools to transform the teaching and learning of mathematics. This raises several questions: *why* should teaching and learning practices transform, *into what* will they transform, and *by what* or *by whom* will they be transformed?

As for the *why*-question, the main argument in the paper is the tools' potential to enhance conceptual engagement. If this is in terms of measurable student outcomes, however, it should be acknowledged that the effect sizes reported in experimental studies in the field are significant but only small to moderate; from this perspective, the evidence for benefit is not overwhelming (Drijvers, 2018). Another argument to answer the why-question, which to my surprise is not brought afore in the paper, concerns the students' future practices. There is no doubt that digitalisation highly transforms practices in both professional and private contexts. To prepare for this, education inevitably should integrate digital tools and undergo a transformation of practices. In my view, the need to transform practices is largely inspired by developments outside the educational context.

The question of *into what* might teaching and learning practices transform is not addressed explicitly in the paper. What are mentioned are the shifts in how mathematics is known, and the windows on students' conceptions that digital technologies can open. As for the latter, there is evidence indeed that digital tools may offer opportunities to explore mathematical situations and to develop mathematical conceptions (e.g. Heid, 2018). Still, it is not very clear how to exploit this potential in the mathematics classroom. As for the first point of how mathematics is known, Hoyles is probably thinking of students using the mathematical knowledge that is embedded in digital tools, their practice being that of an informed user. This makes sense, even if much remains unknown about what competences such an informed user should have, and how these can be acquired.

Finally, there is the issue of *by what or by whom* would these practices be transformed. The tools themselves do have a certain agency, but in my opinion cannot be the real actors in the play. Rather, teachers and educational designers are in the leading role. Experiences with the large-scale introduction of interactive whiteboards in the UK, however, show that new technologies may lead to traditional teacher-centred teaching practices: "the mere introduction of such technologies is insufficient to promote greater interactivity in the classroom, and indeed, that use may have had detrimental effects" (Rudd, 2007, p. 2). If teachers, as the main actors, prefer to use new technology for old teaching styles, then the transformational potential highlighted by Hoyles is unlikely to be exploited.

Tools and taxonomies

The backbone of Hoyles' paper is formed by the identification and description of six categories of digital tools "that support an agenda for research into transformational change in mathematics teaching and learning". Of course, it can be very helpful to organise an eclectic research field through a categorisation, a classification or a (non-hierarchical) taxonomy. The fact that the categories are not complete nor mutually exclusive does not necessarily hinder their functionality. In spite of the many interesting points that are made and relevant projects that are mentioned by Hoyles, however, I do have some difficulties with the proposed taxonomy for several reasons.

My first problem lies in the confusion between tools and tool use. Hoyles speaks about categories of tool use, but the categories themselves are called tools and indeed seem to focus on tools rather than on their use. This is an important point, as one of the main research outcomes in our field is that tools are not always used in the ways which, and for the purposes that, the designers had in mind. A well-known example concerns students using a graphing calculator for applying a linear regression procedure to a set of two points to find an equation of the line through two given points. Clearly, the technique is used without any understanding of linear regression and I expect this type of use was not foreseen by the designers. A second, more hilarious, example of a mismatch between tool designers' and teachers' intentions on the one hand, and students' use on the other, concerns students' fanciful solutions for using a barometer to find the height of a skyscraper.[1] Indeed, the appropriation of a mathematical tool for a specific purpose requires the co-emergence of technical and conceptual knowledge, the subtle development of which is the heart of instrumental genesis, described in Hoyles' paper. The difference between tools and tool use becomes even more relevant now that many digital tools for mathematics offer a whole range of mathematical options from sometimes quite different domains: tools are in fact collections of smaller tools. The software *Geogebra*, to mention just one example, originated as a dynamic geometry tool, but nowadays comes with a spreadsheet, a statistics module and a symbolic manipulation part.

My second difficulty with Hoyles' categorisation is that the categories are so different. Take cars as an example; I could live very well with a taxonomy of passenger cars as being black, white or red. The fact that pink cars, or red cars with white roofs are neglected for the sake of conciseness, is not a problem; they are rare and not important for a global overview. If needed, I might add a category *different colour*. What would be worrying, however, is to have categories like red cars, cars mainly used for weekend outings, electric cars, and cars that the mechanic finds easy to repair. In the six categories presented by Hoyles, there is an incompatibility that is not very helpful in organising the field. In addition, I am puzzled by some of the categories. For example, the first one is labelled dynamic and graphical tools, but non-graphical tools can be very dynamic as well, and graphical tools can be quite static. As a second example of this difficulty with Hoyles' categorisation, the outsourcing category seems to be very encompassing, if not all-inclusive, as using a digital tool always implies outsourcing work. The description of the outsourcing category, however, stresses tools that embed, and may even hide, mathematical knowledge, which is a relevant but different aspect; tool used for outsourcing can be transparent, whereas embedded tools usually are not. Finally, the example in Figure 1 of Hoyles' paper, on using a digital tool for geometric drawings, would, in my opinion, fit better in the outsourcing category, rather than in the new representational infrastructure category, as access is given to no really new representation.

This brings me to my third concern. It is not so clear what are the underlying principles that guide this categorisation, so that it would be traceable for the reader. In other words, it is not clear how the six categories emerged, and what exactly is the goal of the taxonomy. Thinking of such a taxonomy for tool use in mathematics education, I would prefer to distinguish two different goals, each with a corresponding dimension: the mathematical functionality of the tool, and its didactical functionality. A categorisation of the *mathematical functionality* of a tool would not be so hard to make and would be close to a taxonomy of mathematics itself. Tools can be used to carry out algebraic work, graphing tasks, statistical analyses, calculus procedures, and geometric jobs. As noted above, some tools may serve more than one mathematical functionality, but in such a case an integration of sub-tools could be considered; that is just a matter of granularity, and the type of task determines which functionality may be used.

For the *didactical functionality* taxonomy, Figure 1 shows a very simple model described elsewhere (Arcavi, Drijvers, & Stacey, 2017; Drijvers, Boon, & Van Reeuwijk, 2011). Compared to the mathematical dimension, the didactical functionality is not so much on the tool features themselves, but more closely related to the task and the way in which the tool is used in the teaching and learning process. On the one hand, this distinguishes the didactical function to "do the mathematics" for the user, which resonates with the outsourcing category mentioned by Hoyles. This didactical function is not targeting the heart of the activity itself, but is to relieve the user's mind so that energy can be saved for the core matter, which is not outsourced. On the other hand, this identifies two types of didactical functionality that focus on the learning more directly: tool use for practicing mathematical skills, and tool use for the development of mathematical concepts. For practicing mathematical skills, digital tools have much to offer, such as variation

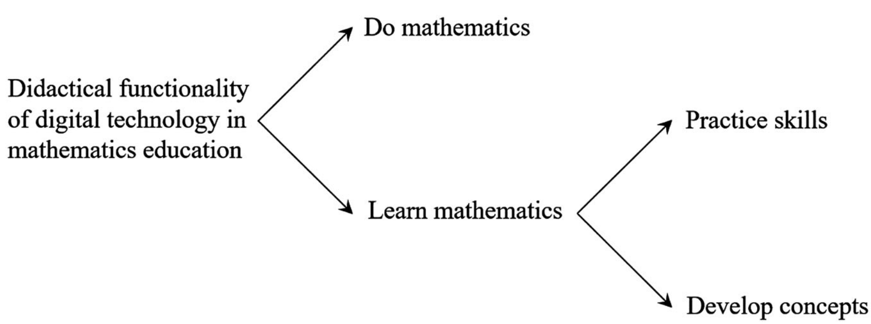

Figure 1. Three didactical functionalities of digital technology in mathematics education.

and randomisation of tasks, automated and intelligent feedback, and a personal environment in which one can safely make mistakes and learn from them. Tool use for concept development, finally, is probably the most challenging didactical functionality. Much of the work done by Hoyles, for example with microworlds, might best fit into this category. It is the category that may raise much enthusiasm among educators and researchers, but that also may be the hardest type of didactical functionality to exploit.

It goes without saying that this didactical functionality taxonomy, again, is not complete and that categories are not mutually exclusive; the developing concepts didactical functionality, for example, is likely, in many cases, to include some sort of outsourcing for doing mathematics. Once more, this is not a big issue, as its main goal is to help teachers and educators to become aware of the types of functionalities that might be exploited while integrating digital tools in mathematics education.

The future research agenda

As a future research agenda, Hoyles recommends attention to (1) embedding digital tools to support epistemological transformation through design research, (2) supporting teachers and teaching in using digital technologies, and (3) managing methodological complexity. These are relevant points, even if I found it hard to see the exact relationship with the previous categorisation. Also, I was surprised to see the research topic related to the methodology of design research in the first point. Not that research theme and research methods are independent, but prescribing a method for a research agenda sounds strange to me. The stress on methods also appears in point (3), whereas I would rather be interested in managing the theoretical complexity. I remember engaging in the challenging but fruitful exercise to confront and compare the notions of webbing (Hoyles, Noss, & Kent, 2004; Noss & Hoyles, 1996) and instrumental orchestration (Trouche & Drijvers, 2014). It is these types of research activities that should be prioritised on research agendas.

My research agenda would include some different perspectives. First, in spite of the optimism that is reflected in the engaging projects that Hoyles described, I feel that some "old" research issues are still waiting to be addressed. As a main point, more needs to be known about the way in which activities with digital tools mediate the learning in a fruitful way, so that such potential can fully exploited (Monaghan, Trouche, & Borwein, 2016). This point, very present in the ICMI study group (Hoyles & Lagrange, 2010), was also expressed in earlier work:

> How might the activity of construction mediate the ways in which learners come to develop techniques and how might this constructive dimension influence the relationship between technical and conceptual fluency? (Hoyles & Noss, 2003, p. 340)

Still, knowledge on this core point is limited, and, as a consequence, clear guidelines for teachers are lacking, which impacts on point (2) of Hoyles' agenda.

In addition, I would like to add two relevant topics to the wider research agenda that seem to be absent in Hoyles' priorities. The first one concerns assessment, both formative and summative, and, in particular, the ways in which online student work can be interpreted, commented upon and eventually graded through intelligent mathematical software. As an example, I refer to so-called domain reasoners that include the rules of a particular mathematical topic, as well as the "buggy rules" that students might use, and that provide very sophisticated feedback based on these interpretations (Heeren & Jeuring, 2014). Or combinations of this type of "intelligence" with learning curve analysis so as to provide students with an integrated view of their knowledge in a certain domain (Tacoma, Sosnovsky, Boon, Jeuring, & Drijvers, 2018).

My second addition to the agenda would refer to notions of embodiment. There can a danger, while using digital technology, that the bodily experiences in which mathematical experiences are rooted might get neglected. As this clearly also holds for traditional teaching resources such as text books and paper, perhaps technology can be used to overcome this limitation. Recent developments in this field suggest promising relationships between the use of digital tools, gesture, and embodiment (e.g. see Duijzer, Shayan, Bakker, van der Schaaf, & Abrahamson, 2017), but much is to be explored in more detail in this field.

This being said, I do believe Hoyles' retrospective outline of the field, in combination with the future research agenda, provides an excellent starting point for important progress in this exciting field of research, with the hope that this indeed can lead to more student engagement and success in mathematics education.

Note

1. https://en.wikipedia.org/wiki/Barometer_question.

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No potential conflict of interest was reported by the author.

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