MAPPING KINDERGARTNERS' NUMBER COMPETENCE

Iliada Elia[(1)], *Marja van den Heuvel-Panhuizen*[(2)]

[(1)] ***Department of Education, University of Cyprus,*** [(2)] ***Freudenthal Institute, Faculty of Science & Faculty of Social and Behavioural Sciences, Utrecht University, The Netherlands***

Abstract

In this study the number competence of kindergartners was investigated. Based on a series of items involving counting, subitizing, additive reasoning and multiplicative reasoning we collected data from a sample of kindergartners in the Netherlands ($N = 334$) and Cyprus ($N = 304$). A confirmatory factor analysis showed that the four-factor structure fit to the empirical data from the Netherlands, and that the competence of the kindergartners in Cyprus reflected a two-factor structure including extended counting and additive reasoning. With respect to this latter, common number component, the Netherlands children outperformed those from Cyprus. In both countries the children who were in the second year of kindergarten did better than those who were in the first year. In the Netherlands, out of the four components, multiplicative reasoning was the most difficult, whereas in Cyprus additive reasoning was more difficult than extended counting.

Key words: differences between countries, kindergartners, number performance, structure of number competence

Introduction

Number is the most fundamental topic of mathematics in primary school (Sarama and Clements, 2008), but children start learning about numbers and develop basic skills and concepts in arithmetic already before they are taught mathematics formally from Grade 1 on (e.g., Gervasoni and Perry, 2015). In fact, the foundation for children's understanding of number is laid in their preschool and kindergarten years when they learn about quantities, numbers, operations, and relations between quantities as a way of modelling their world (Nunes, 2012).

The number concept comprises several components, which children need to develop and link to each other to build a deep understanding of the concept (Sarama and Clements, 2008). Most studies on early number development have examined children's competences by focusing on these number components separately from each other. Taking a more comprehensive approach, this study aims to investigate the structure of young children's number competence with a focus on four major sub-domains of number development: counting, subitizing, additive reasoning and multiplicative reasoning. The nature of this structure in children in two countries is also a concern of the study.

The domain of number

Counting

Counting is considered a key component in the development of the concept of number (e.g., Sarama and Clements, 2008). By using counting in everyday experiences children construct basic knowledge about numbers resulting in being able to find the numerosity of a collection of objects. In order to succeed in this, children have to acquire the ability of oral counting (knowing the sequence of number words), the one-to-one correspondence between the set of objects and the number words, the ability to keep track of the counted objects and the objects that have not been counted and the cardinality principle (that the numerosity of a set of objects is indicated by the last number word of the counting process) (Baroody and Wilkins, 1999; Kilpatrick, Swafford and Findel, 2001).

Subitizing

Another way of determining the numerosity of a collection of items is subitizing. This means that children can recognise instantly the number of small collections (Baroody and Wilkins, 1999). The development of this ability is considered to take place even before children have learned to count objects reliably (Baroody, 2004). Based on the different mechanisms underlying subitizing, a distinction is made between perceptual and conceptual subitizing (Clements, 1999). Perceptual subitizing refers to directly seeing how many objects there are. Conceptual subitizing is quickly figuring out the numerosity of a larger collection of objects by viewing it as being composed of smaller groups of objects.

Additive reasoning

Children's early experiences with counting (Eisenhardt et al., 2014) and subitizing (Clements, 1999) form the basis for additive reasoning (addition and subtraction). This starts already at a young age. Most preschoolers can understand and solve simple additions and subtractions at the age of three, often by using real objects to model the tasks (Kilpatrick et al., 2001) through perceptual counting (see Eisenhardt et al., 2014). Playing with collections of objects supports children's development of the intuitive ideas of adding to/having something more and taking away/having something less (Baroody and Wilkins, 1999). Later, at the age of five or six, children acquire a basic understanding of part-whole relationships (Sophian and McCorgay, 1994), which is a great achievement in the development of the understanding of additive relations and of number sense in general. It means that they understand that any number can be represented as the sum of other numbers (additive composition) (Nunes, 2012), which helps them, for example, to solve missing-addend problems (Sarama and Clements, 2008).

Multiplicative reasoning

The domain of multiplicative reasoning, which includes multiplication and division, is clearly distinct from the domain of additive reasoning (e.g., Clark and Kamii, 1996; Vergnaud, 1983). Previous studies have shown that in the first grades of primary school, before formal instruction on multiplicative reasoning, children can resolve a substantial amount of problems in this domain (Bakker et al., 2014; Mulligan and Mitchelmore, 1997). A study by Carpenter et al. (1993) revealed that even children at the kindergarten level were able to solve various multiplication and division word problems. Research suggests that children's multiplicative knowledge is strongly influenced by the characteristics of the problems offered to them. A study of Bakker et al. (2014) showed that 'equal groups' problems were the easiest problems and that problems with pictures representing the multiplicative situation are easier than problems which are presented without countable objects. The same study also showed that multiplication and division problems were at same difficulty level, which could be accounted to an intuitive understanding of the connection between the two operations (see also Mulligan and Mitchelmore, 1997).

Cultural aspects in the learning of number

Most of the comparative studies between different cultures have concentrated on comparing the mathematics competences between Asian, Oceanian and South American students and students from the western countries. A common finding of these studies, which starts to appear even in the earliest years of children's development, is that, for example, Asian children outperform Western children in the domain of number concepts (Anderson, Anderson and Thauberge, 2008; Starkey and Klein, 2008). A number of factors that were found to account for this difference include "linguistic regularities, parental and teacher mediation styles, different cultural expectations, and how mathematics is practised within different cultural groups, both in and out of school" (Anderson et al., 2008, p. 119). Whether these differences have been found also within Western countries whose cultural traditions may be closer to each other is to our knowledge unknown.

The present study

The aim of the study was to map kindergartners' number competence by identifying its key components and investigating their performance in these key components in two countries. Considering the major subdomains of early number knowledge, we investigated: (a) whether kindergartners' number competence can be distinguished into four factors, namely, counting, subitizing, additive reasoning and multiplicative reasoning, (b) how able kindergartners are in the number competence component each factor stands for and (c) whether the previous issues differ for children in different countries.

Methods

Set up of the study

A survey was carried out in the Netherlands and in Cyprus. The kindergartners' number competence was assessed by administering two booklets, each containing items about counting, subitizing, additive reasoning and multiplicative reasoning.

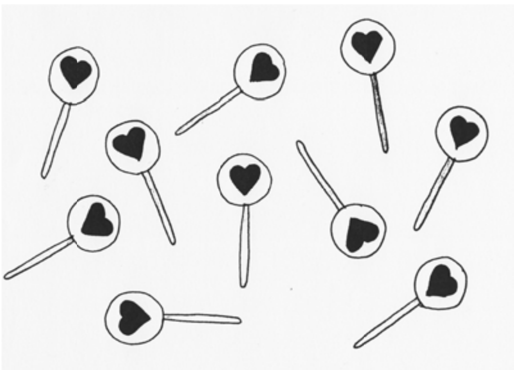
Participants

The participating children in the Netherlands were from kindergarten classes in 18 primary schools situated in the province of Utrecht. Each school participated only with one class containing first (K1) and second year (K2) kindergartners. The total Netherlands sample included in the analysis contained 334 children, 176 girls and 158 boys; 123 K1 children and 211 K2 children. The K1 children had an average age of 4.67 years and the K2 children were on average 5.69 years old. The participating kindergartners in Cyprus were from 10 primary schools situated in the province of Nicosia. Four schools had integrated kindergarten classes containing first-year kindergartners (K1) and second-year kindergartners (K2), while six schools had K1 and K2 children in separate classes. Also different from the Netherlands sample, the schools in Cyprus participated with more than one class. The analysis was based on 23 classes involving 304 children, 163 girls and 141 boys; 86 K1 children and 218 K2 children. The K1 children had an average age of 4.67 years and the K2 children were on average 5.61 years old.

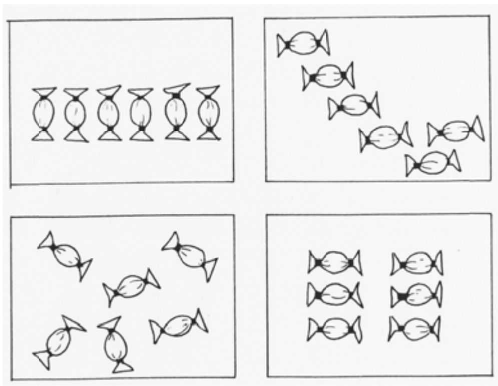
Items

To measure early number competence a series of pictorial paper-and-pencil items was developed (see Fig.1 for examples of items), including three items that refer to counting, three items that are about subitizing, three items that include additive reasoning and five items that refer to multiplicative reasoning.

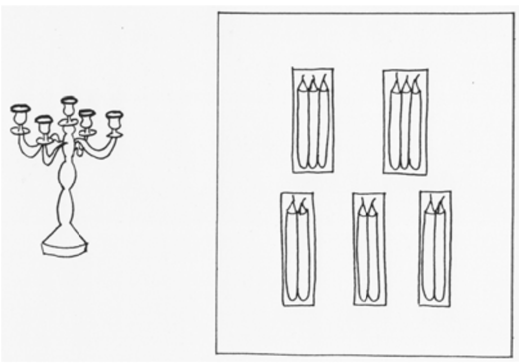
Lollipops: Mommy buys 5 lollipops. Put a circle around 5 lollipops.



Sweets: Underline the picture on which you can tell the fastest that there are 6 sweets.



Candle holder: Underline the boxes of candles you need for this candleholder.



Mittens: Underline the amount of mittens these children need in total.

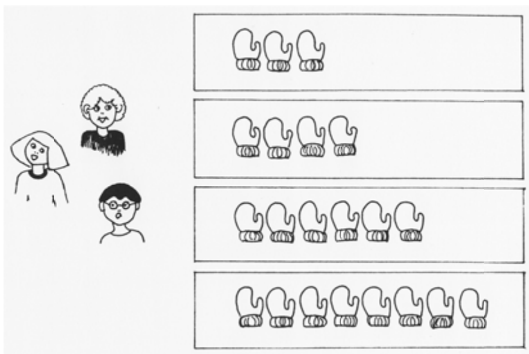


Fig. 1: Examples of items: *Lollipop* (counting), *sweets* (subitizing), *candle holder* (additive reasoning), *mittens* (multiplicative reasoning)

Every item covers one page and contains an illustration depicting a situation and/or a number of small illustrations that represent the possible answers. After a test item was read aloud to them, the children had to answer by underlining or putting a circle around the picture or pictures that represent the correct answer. Correct responses were coded as 1, and incorrect ones as 0.

Results

Components of early number competence

Confirmatory factor analysis (CFA) was applied, using MPLUS (Muthén and Muthén, 2010) to investigate whether different items in the topic of number can form different factors which reflect different types of competence (early number competence components) in the two samples. In order to evaluate model fit, three fit indices were computed (Marcoulides and Schumacker, 1996): the chi-square to its degree of freedom ratio (χ^2/df should be less than 2); the comparative fit index (CFI should be higher than .9); and the root mean-square error of approximation (RMSEA should be close to or lower than .08).

The results of the CFA are presented in Fig. 2. On the left the structural equation model is shown with the latent variables of the number competence components and their indicators for the Netherlands sample. We evaluated the construct validity of this model by examining whether the 14 items loaded adequately on each of the four number competence factors described above: counting, subitizing, additive reasoning and multiplicative reasoning. The CFA showed that this model reflected the empirical data quite well, as the descriptive-fit measures indicated support for the hypothesised model (χ^2/df = 1.10, CFI = .99 and RMSEA = .02). This means that students' early number competence in the Netherlands can be distinguished into four factors: counting, subitizing, additive reasoning and multiplicative reasoning. All factor loadings were statistically significant and most of them were rather large; the total range is from .36 to .86. The interrelations between the factors were significant and considerably strong, ranging from .72 to .95. In a general sense, this indicates that children in the Netherlands who were efficient in one number subdomain were quite competent in another subdomain and vice versa.

To evaluate the construct validity of this model in the Cyprus sample, CFA was used. The results of the analysis showed that the correlations between some latent variables (factors) were greater than 1 indicating that the four-factor structure did not make sense for the empirical data on number competence of young students in Cyprus. Another model with a smaller number of factors had to be explored. The model that best fitted the Cyprus data (CFI = .95, χ^2/df = 1.16, RMSEA = .02) was the one presented in the right part of Fig. 2, which includes two factors.

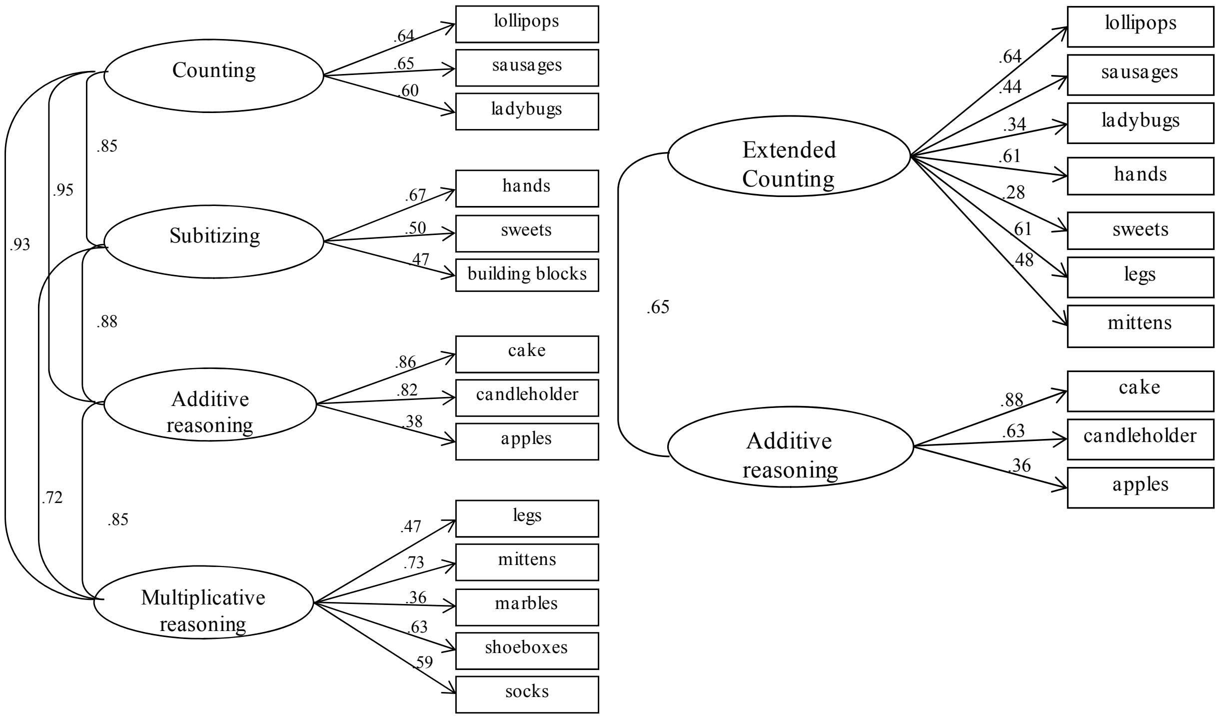


Fig. 2: Structural equation model for early number competence components in the Netherlands (left) and Cyprus (right)

In this model the items *cake, candleholder* and *apples*, were found to load adequately to one factor which stands for additive reasoning, as it is the case for the Netherlands sample. The items *lollipops, sausages, ladybugs, hands sweets, legs* and *mittens* were found to load adequately on the other factor. Although these items were initially used to measure three different number competences, namely, counting, subitizing and multiplicative reasoning, they can all be solved with the use of counting. Therefore, we considered the second factor in the Cyprus model to stand for counting. To distinguish this factor from the factor that stands for counting in the model of the Netherlands we named it "extended counting". This means that students' early number competence in Cyprus can be distinguished into two factors: extended counting and additive reasoning. As it is shown in the right part of Fig. 2, most factor loadings were rather high; the total range was from .28 to .88. It is to be noted that four items, that is, *marbles, shoe boxes, socks* and *building-blocks* are not included in the model because although we expected these items to be regressed on the factor "extended counting" their loadings on the particular factor were not statistically significant. This indicated that children's observed performance on these items was not related to the latent factor of "extended counting" and as a result they were eliminated from the model (Brown, 2006). The interrelation between the two factors, corresponding to students' competence in counting and additive reasoning, was significant and considerably strong (.65). This indicates that students in Cyprus who were competent in counting were efficient also in additive reasoning and vice versa.

Kindergartners' performance in each number competence component

Tab. 1 shows that for the total sample of kindergartners in the Netherlands, performance in counting ($M = .52$), additive reasoning ($M = .51$) and subitizing ($M = .55$) was significantly higher than performance in multiplicative reasoning ($M = .35$). These differences were found to be significant (due to limited space we left out detailed statistical information). Also, the children in the Netherlands appeared to perform better in subitizing than in additive reasoning. This difference was found to be marginally significant. For the common factor between the structural models of the two countries, that is, additive reasoning, kindergartners in the Netherlands performed significantly better than the kindergartners in Cyprus.

Similar to the results in the total sample of the Netherlands, we also found for the two kindergarten years that children performed significantly better in counting, subitizing and additive reasoning than in multiplicative reasoning. K1 children in the Netherlands demonstrated signifantly higher performance in subitizing than in counting and additive reasoning, which was not the case for the K2 children. For Cyprus, the results in the total sample was similar to those found for the two kindergarten years, namely that children performed significantly better in extended counting than in additive reasoning. Moreover, in both kindergarten years the Netherlands kindergartners outperformed the kindergartners in Cyprus in additive reasoning. When we compared the scores in the two kindergarten years for the various number competence components, we found that in both countries the K2 children significantly outperformed the K1 children in all components.

Component	NL				Component	Cyprus			
	M			M_{K2}-M_{K1}		*M*			M_{K2}-M_{K1}
	K1+K2	K1	K2			K1+K2	K1	K2	
Counting	.52	.34	.63	.29*	Ext counting	.49	.36	.54	.18*
Add reasoning	.51	.35	.59	.24*	Add reasoning	.38	.26	.43	.17*
Subitizing	.55	.45	.60	.15*					
Mult reasoning	.35	.23	.42	.19*					
N	334	123	211			304	86	218	

Tab. 1: Mean score for each early number competence component for the whole sample and each kindergarten year in the Netherlands and in Cyprus; differences in mean scores between kindergarten years in both countries (* $p < .01$)

Discussion and conclusion

This study provides evidence for the multidimensional structure of kindergartners' number competence. The investigated four-factor structure including counting, subitizing, additive reasoning and multiplicative reasoning indeed reflected the number competence, but only in the children from the Netherlands. For the children in Cyprus, a two-factor structure, including extended counting and additive reasoning was more adequate to capture their number competence. A possible reason for this finding could be that the children's profile might be influenced by the Cyprus kindergarten's mathematics curriculum and teaching practices, which emphasize counting and additive reasoning and give less attention to subitizing and multiplicative reasoning. To solve items referring to these latter competences, Cyprus children may have applied counting strategies instead, which were quite familiar to them and which could be applied because the items included countable objects. Another finding was that for additive reasoning, the component the two samples had in common, the Netherlands kindergartners outperformed the children from Cyprus. In sum, our study revealed differences in children from two countries in key components of early number competence. However, this conclusion should be taken with prudence, because our sample was small, and was not representative for the countries' population. A further limitation of our study was that our collection of items did not cover the full domain of number and operations. Further research is necessary to cancel out these limitations and also to identify more in-depth the sources of differences in number competence of kindergartners in different countries.

Acknowledgements

This study was made possible by the Netherlands Organization for Scientific Research (NWO-MaGW/PROO: Project 411-04-072), Utrecht University, the University of Cyprus and the teachers in the Netherlands and Cyprus who participated in the study.

References

Anderson, A., Anderson, J., & Thauberge, C. (2008). Mathematics learning and teaching in the early years. In Saracho, O. N. & Spodek, B., *Contemporary perspectives on mathematics in early childhood education* (pp. 95-132). Charlotte, NC: Information Age Publishing.

Bakker, M., Van den Heuvel-Panhuizen, M., & Robitzsch, A. (2014). First-graders' knowledge of multiplicative reasoning before formal instruction in this domain. *Contemporary Educational Psychology,* 39, 59-73.

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic sills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48-65). Reston, VA: National Council of Teachers of Mathematics.

Baroody, A. J. (2004). The developmental bases for early childhood number and operations standards. In Clements, D. H. & Sarama, J. (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 173- 219). Mahwah, NJ: Lawrence Erlbaum Associates.

Brown, T. (2006). *Confirmatory factor analysis for applied research*. New York: Guilford Press.

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education,* 24, 428-441.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1–5. *Journal for Research in Mathematics Education,* 27, 41-51.

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics,* 5, 400-405.

Eisenhardt, S., Fisher, H.M., Thomas, J., Schack, O. E., Tassell, J., & Yoder, M. (2014). Is it counting or is it adding? *Teaching Children Mathematics*, 20(8), 498-507.

Gervasoni, A., & Perry, B. (2015). Children's mathematical knowledge prior to starting school and implications for transition. In Perry, B., MacDonald, A., & Gervasoni, A. (Eds.), *Mathematics and transition to school: International perspectives* (pp. 47-64). Singapore: Springer.

Kilpatrick, J., Swafford, J., & Findel, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Marcoulides, G. A., & Schumacker, R. E. (1996). *Advanced structural equation modelling: Issues and techniques*. NJ: Lawrence Erlbaum Associates.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education,* 28, 309-330.

Muthén, L.K., & Muthén, B.O. (1998-2010). *Mplus user's guide* (6th ed.). Los Angeles, CA: Muthén & Muthén.

Nunes, T. (2012). *Number, quantities and relations: understanding mathematical reasoning in primary school*. Regular Lecture presented at 12th International Congress on Mathematical Education, Seoul, Korea, 8-15 July.

Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. In Saracho, O. N., & Spodek, B., *Contemporary perspectives on mathematics in early childhood education* (pp. 67-94). Charlotte, NC: Information Age Publishing.

Sophian, C., & McCorgay, P. (1994). Part-whole knowledge and early arithmetic problem-solving. *Cognition and Instruction,* 12, 3-33.

Starkey, P., & Klein, A. (2008). Sociocultural influences on young children's mathematical knowledge. In Saracho, O. N., & Spodek, B., *Contemporary perspectives on mathematics in early childhood education* (pp. 253-276). Charlotte, NC: Information Age Publishing.

Vergnaud, G. (1983). Multiplicative structures. In Lesh, R. & Landau, M. (Eds.), *Acquisition of mathematics concepts and processes* (pp. 127-174). New York: Academic Press.