Transp Porous Med (2018) 122:521–526
https://doi.org/10.1007/s11242-017-0928-7

Reply to the Comments on "Bridging Effective Stress and Soil Water Retention Equations in Deforming Unsaturated Porous Media: A Thermodynamic Approach"—by Nasser Khalili and Arman Khoshghalb

Jacques M. Huyghe[1,2] · **Ehsan Nikooee**[3] ·
S. Majid Hassanizadeh[4]

Received: 30 August 2017 / Accepted: 1 September 2017 / Published online: 16 March 2018

The authors of the paper sincerely thank Nasser Khalili and Arman Khoshghalb for their valuable comments. Khalili and Khoshghalb state that we present a semi-empirical relationship for the determination of the effective stress parameter for unsaturated soils based on thermodynamic considerations. We fully agree with this statement. However, the purpose of our paper is much broader than presenting a semi-empirical relationship for the effective stress parameter. The purpose of the paper is to demonstrate that from a given void ratio-dependent retention formula a corresponding formula for the effective stress parameter can be derived and vice versa.

As an example, only as an example and not as the main purpose of our work, the authors choose to derive a relationship for the effective stress parameter, χ, based on the Brooks–Corey formula that had been generalized by Gallipoli (2012) to include the effect of deformation. In order to provide a verification of the derived relationship, we calibrated the model with the experimental effective stress parameters at a reference net stress and extracted the air entry value ($p_{c,ae}$) of a given soil, the slope of the soil water retention curve (SWRC) in the log–log plane, hereafter denoted by λ_p, and a fitting parameter ξ. These three parameters were then employed to determine the variation of the effective stress parameter with suction at different net stress levels, and we compared our predictions with data from the literature on the effective stress parameter of the same soil. These predictions were made with no tuning parameters for matching the effective stress curve to the values of effective stress parameter

---

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s11242-017-0928-7) contains supplementary material, which is available to authorized users.

✉ Jacques M. Huyghe
jacques.huyghe@ul.ie

[1] Bernal Institute, University of Limerick, Limerick, Ireland

[2] Energy Technology Section Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

[3] Civil and Environmental Engineering Department, School of Engineering, Shiraz University, Shiraz, Iran

[4] Earth Sciences Department, Utrecht University, Utrecht, The Netherlands

at different net stress levels. In fact, the model parameters were already quantified from the equation of the retention curves for deforming media.

Regarding model parameters, one may state that ξ is a fitting parameter. However, reviewing the theoretical framework, one can notice that ξ should in fact be obtained from the slope of the air entry–void ratio curves ($p_{\mathrm{c,ae}} - \mathrm{e}$) in the log–log plane. In the absence of adequate and precise information on ξ, in our predictions, we found it by calibrating the model with the values of effective stress parameter at a reference net stress. Then, with the fixed values for ξ, $\mathrm{p}_{c,ae}$ and λ_p, predictions for effective stress parameter at higher net stress values were obtained. It is noteworthy that as an alternative, first approximations for ξ can be made by taking it equal to the fractal dimension of particle size distribution (Russell 2014). However, the preciseness of such estimation for ξ based on the soil particle size distribution will depend on the applicability of the (mono-) fractal assumption for the considered soil.

Two shortcomings are listed by Khalili and Khoshghalb. One is that the air entry value ($\mathrm{p}_{c,ae}$) is not determined properly; the other is that we have related the state dependency of the SWRC to net stress rather than to the void ratio. We fully acknowledge these points. However, as it will be shown hereafter, the comments raised by Khalili and Khoshghalb do not result in a change in the theoretical framework presented in the original paper. They are mainly concerned about the verification part of the paper which could be performed more properly. This has been addressed in what follows, and predictions have been refined and revised accordingly.

1 SWRCS Measured at Different Net Stress Levels Versus Void Ratio-Dependent SWRCS

In fact, in order to evaluate the theoretical framework and the relationship we proposed for the effective stress parameter, we needed custom-designed experiments and a complete set of information of the variation on the effective stress parameter with suction, values of void ratio at failure, void ratio-dependent soil water retention curves. Altogether, these are hardly available in the current literature. Therefore, to verify the resulting formula, we employed data in the literature reporting the variation of the effective stress with suction and the water retention data, both determined at different net stress levels. We note that, as correctly stated by Khalili and Khoshghalb, this can be at the cost of discrepancies observed in the predictions. On the other hand, we would like to point that the theoretical framework which has been presented in the paper is based on the variation of the air entry value with void ratio as its input and not the net stress. Since void ratio reflects the state of the soil better, as stated by Khalili and Khoshghalb. Further research and better designed experiments are, hence, encouraged to look into the adequacy and preciseness of the proposed equation and its comprehensive comparison with other available formula where complete information on void ratio-dependent retention behaviour of soil and void ratio at failure is gathered and utilized.

2 Determination of Air Entry Value and Revised Predictions of the Effective Stress Parameter

The air entry value is an essential input in our method. In our paper and for weathered granite, we used regression, and the values of $\mathrm{p}_{c,ae}$ and λ_p were obtained by fitting a Brooks–Corey (B.C.) type equation to the retention data of Lee et al. (2005). For other soil samples, the values reported in the literature by Ajdari et al. (2012) were used.

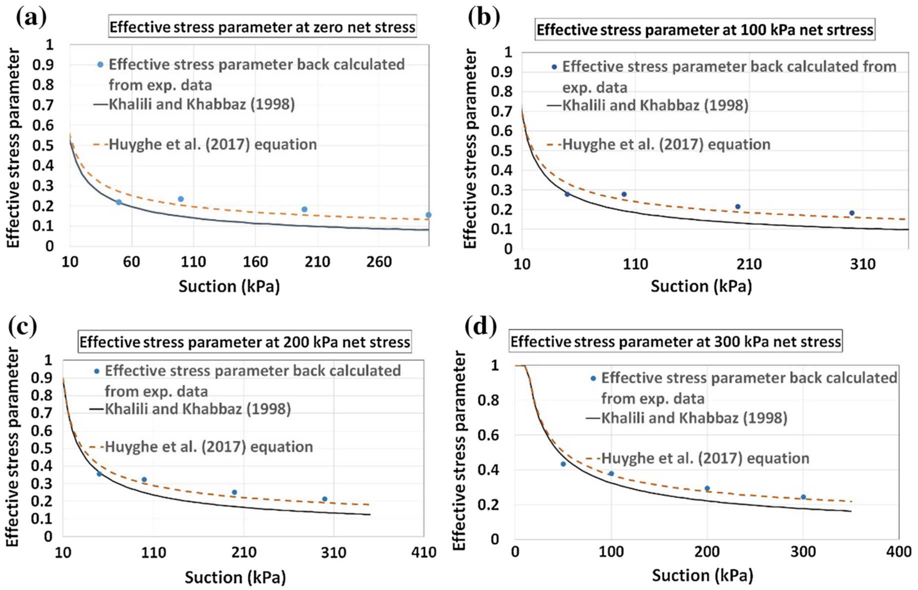

Fig. 1 **a** Revised calibration for effective stress parameter at zero net stress; **b**, **c** and **d** revised predictions for effective stress parameter at higher net stress levels (weathered granite)

We fully acknowledge the point raised by Khalili and Khoshghalb. Hence, given the fact that the estimation of air entry value based on fitting an equation to retention data may not always result in the precise determination of air entry value, we have revised our calculations and predictions based on the comments of Khalili and Khoshghalb have been made.

It should be noted that in our original formulation and the following predictions, the residual degree of saturation in Brooks–Corey equation was assumed to be zero (where B.C. reduces to the model of Campbell (1974). Hereafter, we would like to present the updated predictions based on the comments of Khalili and Khoshghalb.

3 Predictions of Effective Stress Parameter for Weathered Granite

The values of air entry parameter at different net stress levels suggested by Khalili and Khoshghalb (interpreted from original SWRCs of Lee et al. (2005)) are now used for revised predictions, namely 3.1, 5.1, 8 and 13 kPa for net stress values of 0, 100, 200 and 300 kPa, respectively. Based on the air entry value of 3.1 kPa, from SWRC obtained at zero net stress, and by fitting Campbell model, the slope of retention curve, λ_p, was determined to be 0.3742. From the calibration of the model to the effective stress parameters at net stress zero, ξ parameter was found to be 1.199. The new predictions for higher net stress values show a reasonable agreement with the experimental data (Fig. 1).

4 Predictions of Effective Stress Parameter for Kidston Tailing

The values of air entry parameter as suggested by Khalili and Khoshghalb (based on shear strength data) are now used for revised predictions, namely 15, 32 and 60 kPa for net stress

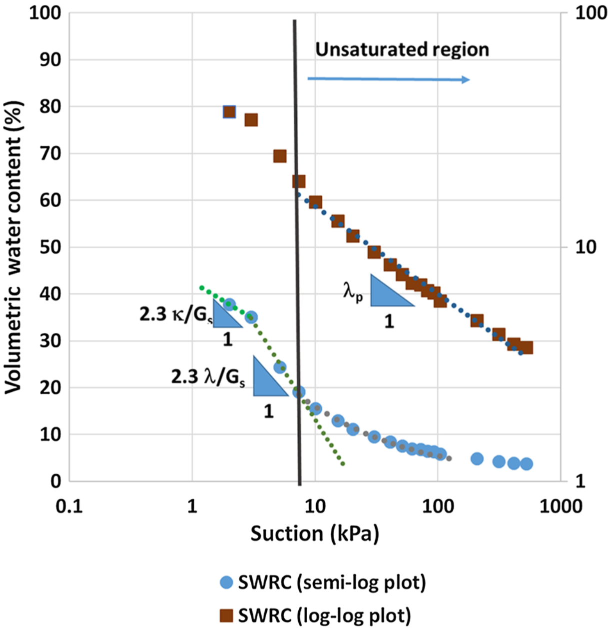

Fig. 2 Determination of air entry value for Kidston tailing (G_S: specific gravity of solid grains, λ_P: pore size distribution index of the soil, λ: slope of normal compression line, κ: slope of swelling line; the vertical line demonstrates the approximate location of air entry value)

levels of 30, 125 and 250 kPa, respectively. Rassam and Williams (1999) presented retention data of zero net stress. Based on the graphical method of Pasha et al. (2015), the location of air entry value at zero net stress is examined, as shown in Fig. 2. It is found to be at [8, 10 kPa]. Then, a regression technique (the "Trust-region" algorithm in curve fitting toolbox of MATLAB) was employed to refine the value of air entry and to find the slope of retention curve. The values of air entry, $p_{c,ae}$, and the slope of retention curve, λ_P, were found to be 9 kPa and 0.3934, respectively. Then, based on the air entry value of 15 kPa (corresponding to the net stress level of 30 kPa), and by calibrating the model with the effective stress parameters associated with net stress level of 30 kPa, ξ parameter was found to be 0.9188. The revised predictions for higher net stress levels (125, 250 kPa) are presented in Fig. 3, where a good agreement is observed with the experimental data.

5 Predictions of Effective Stress Parameter for Talybont Clay

The values of air entry parameter, as suggested by Khalili and Khoshghalb (based on shear strength data of Bishop and Blight (1963)), are now used for revised predictions, namely 12 and 25 kPa for net stress values of 0 and 207 kPa, respectively. Based on the soil grain size distribution of Talybont clay presented in Blight (1961), sand, silt and clay fractions were found to be 83.8, 9.6 and 6.6%, respectively. The soil is, therefore, classified as loamy sand based on USDA soil classification. Therefore, a value of 0.553 for λ_P (recommended by Rawls et al. (1982) for loamy sands) was considered for our modelling. From the calibration of the model to the effective stress parameters at zero net stress level, ξ parameter was

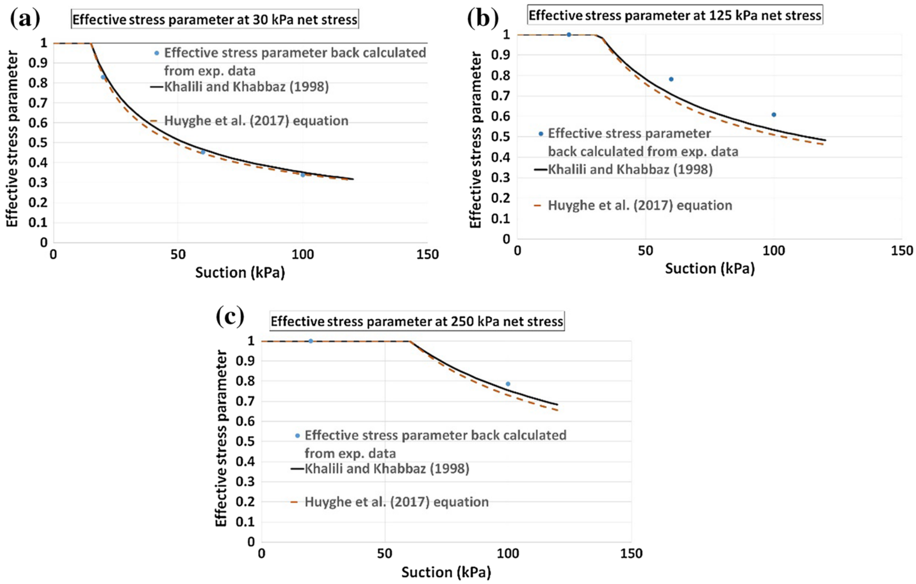

Fig. 3 **a** Revised calibration for the effective stress parameter at 30 kPa net stress; **b** and **c** revised predictions for effective stress parameter at higher net stress levels (Kidston tailing)

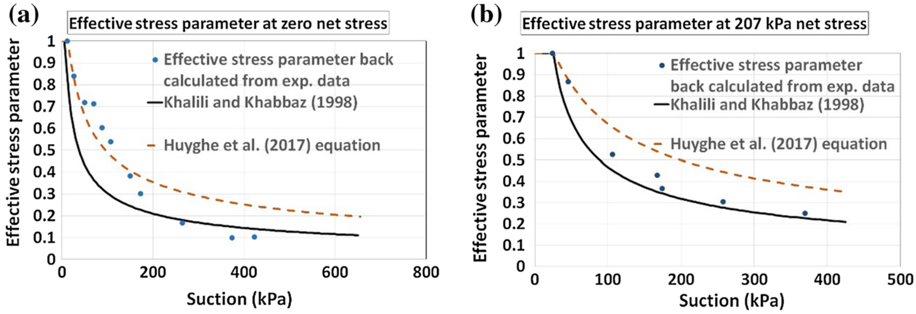

Fig. 4 **a** Revised calibration for effective stress parameter at zero net stress; **b** revised predictions for effective stress parameter at 207 kPa net stress level (Talybont clay)

found to be 1.577. The new predictions for higher net stress values are presented in Fig. 4. The predictions made by Khalili and Khabbaz (1998) and Huyghe et al. (2017) both cover the general trend of the experimental data, but they are not in full agreement. Part of this discrepancy can root in the possible difference between the state of the soil at failure (i.e. its void ratio) and associated parameters (retention parameters, etc.) with those considered in our modelling.

6 Supplementary Material

In the online supplementary material, a flowchart has been presented (Figure 1.S.) for practical purposes. It shows the procedure for estimating the effective stress parameter using our proposed equation.

Acknowledgements The authors would like to thank Dr. Arman Khoshghalb (University of New South Wales), Professor Nasser Khalili (University of New South Wales), Professor David Toll (University of Durham), Dr. David Rassam (CSIRO Land and Water) and Professor Martinus Th. (Rien) van Genuchten (Universidade Federal do Rio de Janeiro) for their very constructive comments and discussions during the preparation of this reply. E.N. and S.M.H. would like to thank European Research Council for the support they have received under the ERC Grant Agreement No. 341225.

Appendix: Notes of correction in the original paper

1. In Huyghe et al. (2017), (Eq. (4) therein), a typo is present (i.e. Ω should be -Ω)
2. We have recalculated the experimental values of χ based on the constructive comments by Khalili and Khoshghalb on the back calculation of effective stress parameter from experimental data of Lee et al. (2005), Rassam and Williams (1999) and Bishop and Blight (1963). We had used a shear box model in Huyghe et al. (2017) to interpret and back calculate the experimental values for χ, which is improper for triaxial data. Thanks to the note of Khalili and Khoshghalb, it is corrected in this reply.

References

Ajdari, M., Habibagahi, G., Ghahramani, A.: Predicting effective stress parameter of unsaturated soils using neural networks. Comput. Geotech. **40**, 89–96 (2012)

Bishop, A.W., Blight, G.E.: Some aspects of effective stress in saturated and partly saturated soils. Geotechnique **13**(3), 177–197 (1963)

Blight, G.E.: Strength and consolidation characteristics of compacted soils. Doctoral dissertation, Imperial College London, bibliographical data available online at: http://ethos.bl.uk/OrderDetails.do?uin=uk.bl.ethos.599565 (1961)

Campbell, G.S.: A simple method for determining unsaturated conductivity from moisture retention data. Soil Sci. **117**, 311–314 (1974)

Gallipoli, D.: A hysteretic soil–water retention model accounting for cyclic variations of suction and void ratio. Geotechnique **62**(7), 605–616 (2012)

Huyghe, J.M., Nikooee, E., Hassanizadeh, S.M.: Bridging effective stress and soil water retention equations in deforming unsaturated porous media: a thermodynamic approach. Transp. Porous Media **117**(3), 349–365 (2017)

Khalili, N., Khabbaz, M.: A unique relationship for the determination of the shear strength of unsaturated soils. Geotechnique **48**, 1–7 (1998)

Lee, I., Sung, S.G., Cho, G.: Effect of stress state on the unsaturated shear strength of a weathered granite. Can. Geotech. J. **42**, 624–631 (2005)

Pasha, A.Y., Khoshghalb, A., Khalili, N.: Pitfalls in interpretation of gravimetric water content-based soil–water characteristic curve for deformable porous media. Int. J. Geomech. **16**(6), D4015004 (2015)

Rassam, D., Williams, D.: A relationship describing the shear strength of unsaturated soils. Can. Geotech. J. **36**, 363–368 (1999)

Rawls, W.J., Brakensiek, D.L., Saxtonn, K.E.: Estimation of soil water properties. Trans. ASAE **25**(5), 1316–1320 (1982)

Russell, A.R.: How water retention in fractal soils depends on particle and pore sizes, shapes, volumes and surface areas. Géotechnique **64**(5), 379 (2014)