From sample to population: A hypothetical learning trajectory for informal statistical inference

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This paper presents the results of a teaching experiment to enhance 9th-grade students' understanding of informal statistical inference (ISI). The teaching experiment was conducted to evaluate and revise a hypothetical learning trajectory (HLT) as a step towards an empirically and theoretically based HLT-design for ISI. The challenge was to invite young students, inexperienced with sampling, to making statistical inferences without knowledge of formal probability theory. In this trajectory, the students proceeded from a first experience with sampling physical objects, through an understanding of sampling variation and resampling, to reasoning with sampling distribution. The results of the intervention suggest that young students can informally interpret sample data with corresponding uncertainty. Engaging in concrete sampling, in simulations and in deepening whole-class discussions seem essential parts of this HLT-design.

Keywords: Informal Statistical Inference, Hypothetical Learning Trajectory, TinkerPlots

The need for informal statistical inferences

The use of sampling to draw inferences about a population is at the heart of statistics, and therefore important to learn. Statistical inference includes both a generalization from sample to population, and an estimation of the reliability of this generalization. Recent research investigated if and how this type of statistical reasoning can already be developed informally by young learners (Paparistodemou & Meletiou-Mavrotheris, 2008). This so-called informal statistical inference has different definitions (Wild & Pfannkuch, 1999). It is clearly not about formal statistical procedures such as the testing of hypotheses, but about the ways students use their informal statistical knowledge to support their inference about an unknown population based on observed sample data (Zieffler, Garfield, delMas, & Reading, 2008). Makar and Ruben (2009) identify three key principles of informal statistical inference: *generalize* beyond data; *data as evidence* for these generalizations; and *probabilistic reasoning* about the generalization. Because of the importance of conceptualizing statistical inferences among young students, we wanted to develop it in a prototypical HLT and therefore, we focus on the question: *What are the features of a theoretically and empirically based HLT-design for enhancing 9th-grade students' informal statistical inference?*

The design of the hypothetical learning trajectory

The trajectory as a whole

As educational materials that focus on the development of statistical reasoning for grade 9 in the Netherlands hardly exists, we designed such materials. In this design study, a so-called hypothetical learning trajectory (Simon, 1995) was formulated for students in this grade. Design guidelines were identified through literature review, and the possibilities of educational software were explored. This resulted in the HLT-design in Figure 1. In this paper we focus on the sixth step, in which students investigate what happens while the size of computer simulated samples increases.

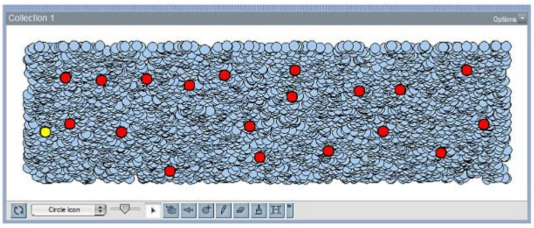
The sixth step: 'What happens if we increase the sample size?'

As preparation for step 6, students will conduct the physical "Black Box with notes" experiment in step 5, in which students are expected to manually draw a sample of 40 from a box containing almost 5,000 notes with data about the length and gender of 14-year-olds. In this preparatory step, students will be confronted with sampling variation.

HLT–design								
Step	1 Conduct physical experiment (Black Box with small and large window)	2 Examine frequency-distribution >100.000 repetitions of physical experiment	3 Simulate sampling distribution of physical experiment *(with ICT)*	4 Test a claim with sampling distribution (Social Media) *(with ICT)*	5 Conduct physical experiment (Black Box with notes)	6 Examine sample size and repeated sampling *(with ICT)*	7 Test a claim with sampling distribution *(with ICT)*	8 Compare groups based on samples *(with ICT)*
Data	Ordinal – nominal level of measurement				Interval – ratio level		All levels of measurement	
Con-cept	Sampling variation		Repeated sampling + Frequency distribution	Sample size + Sampling distribution	Sampling variation	Sample size + repeated sampling	Sampling distribution	
Proba-bilistic reason-ing	In words	In words + argument frequency distribution	In words + argument sampling distribution		In words	In words + argument sample size	In words + argument sampling distribution	

Step 5:
Generalization about the length of 14-year olds based on a sample (size 40) and accompanied by probabilistic reasoning

Step 6:
Generalization about the length of 14 year olds based on simulated samples with TinkerPlots and probabilistic reasoning with an argument on sample size



Step 7:
Estimate the quantity of jam in a jar by sampling and probabilistic reasoning with the sampling distribution

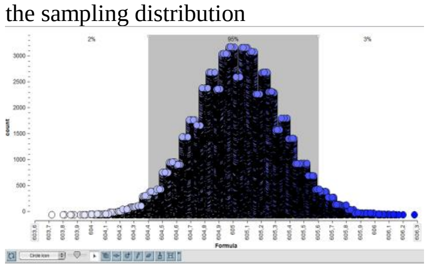


Figure 1: HLT-design

In step 6, the step we are focusing on, students will investigate what happens if the sample size increases. The hypothesis in this step is that students will understand that the characteristics (e.g. the mean) and the distribution of a larger sample usually better resemble the underlying population. To conceptualize this idea, students will use TinkerPlots (Konold & Miller, 2005) to easily and quickly simulate samples of different sizes. Therefore, the dataset from step 5 will be uploaded into TinkerPlots, in which the paper notes are then displayed as data cards. Students will be asked to simulate three small samples (size n=20) and three larger samples (n=200). A learning activity based on growing samples and the use of TinkerPlots is expected to help students to develop aspects of informal inference and argumentative reasoning (Ben-Zvi, 2006). Next, the students will be asked to compare similarities and differences between their simulated sample results, and, during a whole-class session, to the underlying population. Embedding students' findings in a classroom discussion is expected to enhance their statistical reasoning (Bakker, 2004). This activity will prepare for probabilistic reasoning with sampling distribution in step 7, in which sample size plays an important role.

Method

The teaching experiment comprised a ten 45-minute lesson series and was piloted in one class with twenty students, which was taught by the first author. The sixth step was carried out in lesson 5 and included one lesson. The collected data were video-data from classroom discussion, students' worksheets and teacher notes. The data analysis consisted of verifying whether the designed hypotheses actually occurred. To this end, for each step of the design, several detailed and measurable hypotheses were formulated and translated into hypothesized visible student behavior.

Results

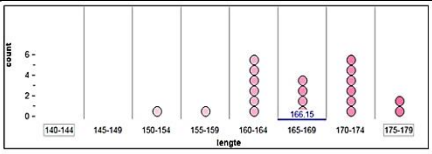
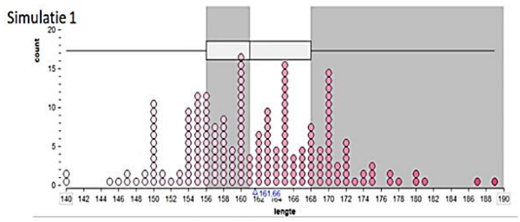
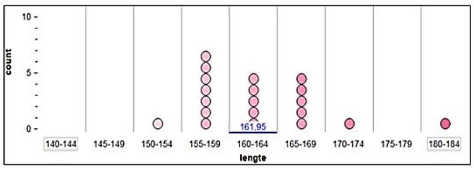
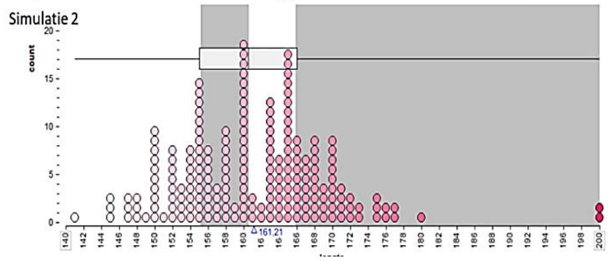
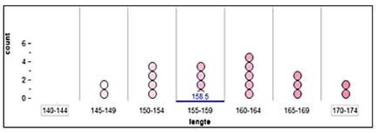
Example of students' findings when simulating repeated samples from a large dataset			
Small sample (n=20)		Large sample (n=200)	
Mean	Distribution	Mean	Distribution
166,15		161,66	
161,95		161,21	
158,5			
Example of students' probabilistic reasoning			
"We think the average length of students is between 157-167. We don't know exactly because it is only a sample of 20 and you compare this with the population of almost 5,000 students."		*"We now think that the average is between 160-162 cm."* *"The actual population is likely to be more similar to this distribution as the sample size is larger. This gives more information."*	

Figure 2: Students' findings with simulating repeated samples with different sizes (worksheet)

In this step, students investigated what would happen if the sample size increased. After simulating repeated samples with different sample sizes (n=20 and n=200), all students indicated that the mean for a larger sample was more stable. For example a student wrote: *"Here (large sample) the averages* *are more similar".* An overview of the students' findings is displayed in Figure 2. During the classroom discussion, students mentioned that their means ranged between 157-166 for a size of 20 and between 159-162 for a sample size of 200. For example, a student said: "*Here (small sample) our lowest measurement was 157.2 and here (large sample) it is 159.7".* Comparison with the actual population (population mean 160.7) confirmed their expectation that a larger sample size would better reflect to the distribution and characteristics of the underlying population. Although the students were only briefly introduced to TinkerPlots, the teacher notes show that the tool enabled students to compare and explore samples of different sizes in a quick and easy way.

Conclusion

In step 6, the students investigated what would happen if the sample size of the physical experiment increased. The hypothesis was that students would understand that the characteristics and the distribution of a larger sample better resemble the underlying population. The intervention data show that students, based on their findings with the simulated samples, became aware of the effect of sample size. Both in student's work and during the whole-class discussion, they indicated that larger samples more closely resemble the underlying population. Key components in this step seem: (1) the strong correlation between the physical experiment in step 5 and the simulations in step 6; (2) the comparison between the simulated sample results, and; (3) the comparison with the underlying population. These key components will be elaborated on in further research in which specific attention will also be paid on instrumental genesis (Gueudet & Trouche, 2009) .

References

Bakker, A. (2004). *Design research in statistics education. On symbolizing and computer tools; A study in statistics education*. Utrecht: Universiteit Utrecht.

Ben-Zvi, D. (2006). *Scaffolding students' informal inference and argumentation*. Paper presented at the ICOTS 7: International Conference on Teaching Statistics 2006, Salvador, Brazil.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Konold, C. and Miller, C. D. (2005). *TinkerPlots: Dynamic Data Explorations* [software, Version 1.0]. Emeryville, CA: Key Curriculum Press.

Makar, K., & Rubin, A. (2009). A framework for thinking about informal statistical inference. *Statistics Education Research Journal, 8*(1), 82-105.

Paparistodemou, E., & Meletiou-Mavrotheris, M. (2008). Developing young students' informal inference skills in data analysis. *Statistics Education Research Journal, 7*(2), 83-106.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223-265.

Zieffler, A., Garfield J., delMas, R., Reading, C. (2008). A framework to support research on informal inferential reasonings, *Statistics Education Research Journal, 7*(2), 40-58.