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One loop graviton corrections to dynamical photons in de Sitter

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Abstract

We employ a recent, general gauge computation of the one loop graviton contribution to the vacuum polarization on de Sitter to solve for one loop corrections to the photon mode function. The vacuum polarization takes the form of a gauge independent, spin 2 contribution and a gauge dependent, spin 0 contribution. We show that the leading secular corrections derive entirely from the spin 2 contribution.

Keywords: quantum field theory on curved space, gauge dependence, observables in cosmology, de Sitter space, quantum gravity, effective field equations

1. Introduction

Serious study of quantum field theory during inflation leaves one with a poignant appreciation for the genius of the physicists who laid the foundations of flat space quantum field theory during the middle of the last century. Among other things, they settled on the S-matrix as the fundamental observable [1, 2]. They also showed how to carefully define this quantity [3, 4] so that it is independent of the choice of local field variable [5, 6] and consequently, independent of the choice of gauge [7].

These are powerful results whose utility can be seen in many ways. One example is inferring quantum gravitational corrections to the Coulomb potential of a charged particle. Naively

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one might find this by computing the quantum gravitational contribution to the vacuum polarization $i[^{\mu}\Pi^{\nu}](x;x')$ and then using this to quantum correct Maxwell's equations,

$$\partial_{\mu}\Big[\sqrt{-g}\; g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}(x)\Big] + \int d^4x'[^{\mu}\Pi^{\nu}](x;x')A_{\nu}(x') = J^{\mu}(x). \tag{1}$$

However, the vacuum polarization is highly dependent on the general coordinate gauge in flat space background. For example, if one defines the quantum metric as $g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$, with $\kappa^2 \equiv 16\pi G$, then the vacuum polarization in the 1-parameter family of exact covariant gauges $\eta^{\rho\sigma}\partial_{\rho}h_{\sigma\nu} = \frac{b}{2}\partial_{\nu}\eta^{\rho\sigma}h_{\rho\sigma}$ is [8, 9],

$$i[^{\mu}\Pi^{\nu}](x;x') = \frac{\kappa^2}{384\pi^4}\left(\frac{2b-1}{b-2}\right)^2\Big[\eta^{\mu\nu}\partial'\cdot\partial - \partial'^{\mu}\partial^{\nu}\Big]\partial^2\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right], \tag{2}$$

where $\Delta x^2(x;x') \equiv \eta_{\mu\nu}(x-x')^{\mu}(x-x')^{\nu}$. One can nonetheless derive gauge independent results for the graviton correction to the Coulomb potential by computing the scattering amplitude for two charged, massive particles and then solving the inverse scattering problem to reconstruct the potential [10].

The problem for inflationary cosmology is that we presently have no analogue of the S-matrix which has been shown to be gauge independent. *This is one of the crucial problems of perturbative quantum gravity* [11]. The primordial power spectra [12, 13] represent the first-ever potentially observable quantum gravitational effects [14–16], and the scalar power spectrum has actually been resolved to three significant figures [17]. Any quantity which can be measured must correspond to some operator, but it is not at all clear what this operator might be. It is certainly not an S-matrix element. At tree order both spectra seem to be 2-point correlators of different components of the graviton field in the gauge of Salopek, Bond and Bardeen [18]. However, it has been shown that loop corrections to these correlators possess infrared divergences [19] which mean that the naive correlators cannot represent what is being measured. Extending the naive correlators to make them gauge invariant is known to extinguish the infrared divergence [20, 21], but at the cost of introducing a new class of ultraviolet composite operator divergences which no one understands how to renormalize [22]. It would be very much better if there were some way of avoiding gauge dependence while still continuing to work with non-coincident one-particle-irreducible functions such as the vacuum polarization.

The vacuum polarization on an inflationary background—hereafter taken to be de Sitter—cannot be less gauge dependent than its flat space limit (2). However, it has been conjectured that there might be no gauge dependence in the leading secular effects of solutions to the effective field equations such as (1) [22]. If this conjecture proves correct then the procedure for making predictions in cosmological quantum field theory would be to discard the gauge-dependent, non-growing contributions just as one derives the flat space S-matrix by dropping any part of a Green's function which lacks poles on each external leg.

Secular effects in electromagnetism plus general relativity were first noted when one uses the simplest version of the graviton propagator [23, 24] to compute the one loop vacuum polarization [25]. In this gauge the Coulomb potential of a co-moving observer was found to grow with time [26]. A similar growth occurs in the electric field strength of plane wave photons [27].

This paper is the second step of checking the conjecture of secular gauge independence. In the first step [9] we computed the one loop graviton contribution to the vacuum polarization using the graviton propagator [28] in the de Sitter analogue of the same 1-parameter family of covariant, exact gauges which gave (2). In this work our result for $i[^{\mu}\Pi^{\nu}](x;x')$ is used to solve (1) for the one loop correction to plane wave photons.

Section 2 reviews our result for the vacuum polarization and summarizes notation. Because the graviton propagator in our gauge consists of a transverse-traceless, spin two part and a spin zero part, it is natural to treat each separately; section 3 works out the spin two contribution and section 4 gives the spin zero contribution. Our conclusions comprise section 5.

Before concluding this section we should place the current work in the context of low energy effective field theory [29]. Electromagnetism plus general relativity is not perturbatively renormalizable [30, 31], and that does render it an incomplete theory. However, not being able to compute *everything* is not the same as being unable to compute *anything*. As long as quantum corrections are small one can absorb divergences order-by-order using local counterterms in the sense of BPHZ (Bogoliubov and Parsiuk [32], Hepp [33], and Zimmermann [34, 35]). The result will be ambiguous up to the finite parts of these counterterms, but the contributions from loops of massless particles are finite even before renormalization, and are characterized by a nonlocal dependence on spacetime which can never be absorbed into a local counterterm. Further, these finite loop corrections dominate at large distances and times, so they are the most important part of any loop correction (at large separations) as long as the finite parts of counterterms are not chosen to be unreasonably large.

The literature on this subject is vast and dates back to the 1937 resolution of the infrared problem of quantum electrodynamics by Bloch and Nordsieck [36], long before it was understood how to control ultraviolet divergences in that theory. We *still* do not understand how to control ultraviolet divergences in quantum general relativity, but this did not prevent Weinberg from achieving a similar resolution of that theory's infrared problem on flat space background in 1965 [37]. It is important to understand that these sorts of results are unchanged by whatever turns out to be the ultraviolet completion of the theory. For example, Feinberg and Sucher used the nonrenormalizable Fermi theory in 1968 [38] to compute the long range force due to the exchange of massless neutrinos, and that result is identical in the fully renormalizable Standard Model [39]. Similar examples exist when string theory provides the ultraviolet completion [40]. More recently Donoghue has exploited the same technology to compute the one graviton loop correction to the Newtonian potential [41, 42]. Though there are many important complications from working on de Sitter background, the validity of our analysis relies on the very same theoretical framework of low energy effective field theory.

2. Notation

The purpose of this section is to summarize notation and carry out a preliminary general analysis. We begin by reviewing the de Sitter background, then we reduce the effective field equation (1) to a relation for the one loop correction to the photon mode function. The section closes after presenting our results [9] for the structure functions.

2.1. Background geometry

We use spatially flat conformal coordinates in de Sitter space with Hubble constant H. The invariant element is,

$$ds^2 = a^2[-d\eta^2 + d\vec{x}\cdot d\vec{x}], \qquad a(\eta) \equiv -\frac{1}{H\eta}. \tag{3}$$

Note that the conformal time η lies in the range $-\infty < \eta < 0$, while each of the spatial coordinates runs from $-\infty$ to $+\infty$. We shall many times need to refer to functions of two coordinates, x^μ and x'^μ. In this case an unprimed scale factor is $a \equiv a(\eta) = -\frac{1}{H\eta}$, while the primed scale factor is $a' \equiv a(\eta') = -\frac{1}{H\eta'}$.

Our results for the structure functions depend extensively on the de Sitter invariant bi-scalar function $y(x;x')$, whose definition is,

$$y(x;x') \equiv aa'H^2[\|\vec{x}-\vec{x}'\|^2 - (\eta-\eta')^2] \equiv aa'H^2\Delta x^2. \tag{4}$$

Quantum field theory propagators on de Sitter depend upon a slight modification of $y(x;x')$ which includes an infinitesimal imaginary part to specify the appropriate boundary conditions. The two versions we require are,

$$y_{++}(x;x') \equiv aa'H^2[\|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'| - \mathrm{i}\epsilon)^2], \tag{5}$$

$$y_{+-}(x;x') \equiv aa'H^2[\|\vec{x}-\vec{x}'\|^2 - (\eta-\eta' + \mathrm{i}\epsilon)^2]. \tag{6}$$

Note that $y_{++}(x;x')$ and $y_{+-}(x;x')$ agree for $\eta < \eta'$, whereas they are complex conjugates for $\eta > \eta'$.

2.2. The effective mode equation

It turns out that de Sitter invariance, even when it is present, complicates rather than simplifies representations of the tensor structure of the vacuum polarization [43]. We therefore employed the simple, but noncovariant, representation which was introduced to represent the vacuum polarization from scalar quantum electrodynamics [44],

$$\mathrm{i}[{}^\mu\Pi^\nu](x;x') = \left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)\partial_\rho\partial'_\sigma F(x;x') + \left(\overline{\eta}^{\mu\nu}\overline{\eta}^{\rho\sigma} - \overline{\eta}^{\mu\sigma}\overline{\eta}^{\nu\rho}\right)\partial_\rho\partial'_\sigma G(x;x'), \tag{7}$$

where $\overline{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^\mu_0\delta^\nu_0$ is the purely spatial part of the Minkowski metric. The transformation to a de Sitter covariant representation has been worked out [45] and could be employed if desired.

Substituting (7) and $g_{\mu\nu} = a^2\eta_{\mu\nu}$ into the effective Maxwell equation (1), and then performing some partial integrations, gives an equation in terms of the field strength tensor,

$$\partial_\nu F^{\nu\mu}(x) + \partial_\nu \int \mathrm{d}^4x' \{\mathrm{i}F(x;x')F^{\nu\mu}(x') + \mathrm{i}G(x;x')\overline{F}^{\nu\mu}(x')\} = J^\mu(x). \tag{8}$$

(Here and henceforth we raise and lower indices with the Minkowski metric so $F^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\nu\sigma}F_{\rho\sigma}$ and $\overline{F}^{\mu\nu} \equiv \overline{\eta}^{\mu\rho}\overline{\eta}^{\nu\sigma}F_{\rho\sigma}$.) By setting $J^\mu(x) = 0$ we see that the $\mu = 0$ component of (8) is obeyed by a solution of the form,

$$A_0(x) = 0, \qquad A_i(x) = u(\eta,k)\epsilon_i(\vec{k})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}, \qquad k_i\epsilon_i(\vec{k}) = 0. \tag{9}$$

Substituting (9) into (8) and factoring out both the polarization vector and the spatial plane wave factor gives rise to the effective mode equation,

$$(\partial_0^2 + k^2)u(\eta,k) = -\partial_0 \int \mathrm{d}^4x'\, \mathrm{i}F(x;x')\partial'_0 u(\eta',k)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\Delta\vec{x}} - k^2 \int \mathrm{d}^4x' \Big[\mathrm{i}F(x;x') + \mathrm{i}G(x;x')\Big] u(\eta',k)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\Delta\vec{x}}, \tag{10}$$

where $\Delta\vec{x} \equiv \vec{x} - \vec{x}'$.

Relation (10) is valid to all orders. However, the structure functions $F(x;x)$ and $G(x;x')$ are only known at order κ^2. We therefore expand the mode function in powers of κ^2 as,

$$u(\eta,k) = u_{(0)}(\eta,k) + u_{(1)}(\eta,k) + \mathcal{O}(\kappa^4), \tag{11}$$

with $u_{(n)}(\eta,k) \propto \kappa^{2n-2}$. Then we segregate to first order,

$$(\partial_0^2 + k^2)u_{(1)}(\eta,k) = -\partial_0 \int d^4x'\, iF^{(1)}(x;x')\partial_0' u_{(0)}(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}} - k^2 \int d^4x' \Big[iF^{(1)}(x;x') + iG^{(1)}(x;x')\Big] u_{(0)}(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}}, \tag{12}$$

where the tree order mode function is the usual plane wave,

$$u_{(0)}(\eta,k) = \frac{e^{-ik\eta}}{\sqrt{2k}}. \tag{13}$$

The sort of secular correction we seek is $u_{(1)}(\eta,k) \sim \ln(a)/a$, which means the right hand side of (12) must grow like a. Any slower growth does not contribute to the leading secular effect.

2.3. Structure functions

In an earlier work [9] we applied a general gauge propagator [28] to evaluate the one loop graviton contribution to the vacuum polarization. The computation was made with Einstein + Maxwell using dimensional regularization. Of course Einstein + Maxwell is not perturbatively renormalizable [30, 31] but its divergences can still be absorbed into local higher derivative counterterms, according to the technique of Bogoliubov, Parasiuk [32], Hepp [33] and Zimmermann [34, 35]. Our one loop computation required three such counterterms and their finite parts can be regarded as parameterizing our ignorance of the ultraviolet completion of gravity + electromagnetism in the standard sense of effective field theory [41, 42]. Reliable results are still derivable at late times because the counterterms show no secular increase. Focussing on the late time regime is also necessary because we have not perturbatively correctioned the initial state from free vacuum [46].

Our graviton propagator consists of a transverse-traceless, spin two term and a spin zero term on which all the gauge dependence resides [28]. Only a single graviton propagator enters the vacuum polarization at one loop so it makes sense to report results for the spin two and spin zero contributions separately. The spin two contribution to $F(x;x')$ was found to be,

$$F_2^{(1)}(x;x') = \frac{85\kappa^2H^2}{72\pi^2}\ln(a)i\delta^4(x-x') - \frac{\kappa^2H^2}{16\pi^4}\left[\ln\left(\frac{aa'}{4}\right) + \frac{1}{3} - 2\gamma\right]\nabla^2\left(\frac{1}{\Delta x^2}\right) + \frac{5\kappa^2H^2}{144\pi^4}\partial^2\left(\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right) - \frac{5\kappa^2H^6(aa')^2}{144\pi^4}\left\{\frac{\mathcal{L}(y)}{2} + \frac{2(2-y)\ln(\frac{y}{4})}{4y-y^2} + \frac{2}{y}\right\}, \tag{14}$$

where we define the function $\mathcal{L}(y)$ as,

$$\mathcal{L}(y) \equiv \mathrm{Li}_2\left(\frac{y}{4}\right) + \ln\left(1-\frac{y}{4}\right)\ln\left(\frac{y}{4}\right) - \frac{1}{2}\ln^2\left(\frac{y}{4}\right). \tag{15}$$

Here $\mathrm{Li}_2(z)$ is the dilogarithm function,

$$\mathrm{Li}_2(z) \equiv -\int_0^z \mathrm{d}t\, \frac{\ln(1-t)}{t} = \sum_{n=1}^{\infty} \frac{z^n}{n^2}. \tag{16}$$

The spin two contribution to $G(x;x')$ is,

$$G_2^{(1)}(x;x') = -\frac{5\kappa^2 H^2}{4\pi^2}\ln(a)\mathrm{i}\delta^4(x-x') + \frac{\kappa^2 H^2}{24\pi^4}\Big[\ln\Big(\frac{aa'}{4}\Big) + \frac{1}{3} - 2\gamma\Big]\nabla^2\Big(\frac{1}{\Delta x^2}\Big) + \frac{\kappa^2 H^4 aa'}{96\pi^4}(\partial_0^2 + \nabla^2)\ln(H^2\Delta x^2) + \frac{5\kappa^2 H^6 (aa')^2}{72\pi^4}\Big\{\frac{(1-y)\mathcal{L}(y)}{4} + \frac{(y-3)\ln(\frac{y}{4})}{4-y}\Big\}. \tag{17}$$

Next, the spin zero contributions to $F(x;x')$ and $G(x;x')$ are[1],

$$F_0^{(1)}(x;x') = \frac{\beta^2\kappa^2 H^2}{4}\Bigg\{\frac{\ln(a)}{48\pi^2 aa'}\frac{\partial^2}{H^2}\mathrm{i}\delta^4(x-x') - \frac{(\beta-5)}{72\pi^2}\ln(a)\mathrm{i}\delta^4(x-x') - \frac{1}{48\pi^2 a}\frac{\partial_0}{H}\mathrm{i}\delta^4(x-x') + \frac{1}{384\pi^4}\frac{\partial^4}{aa'}\Bigg[\frac{\ln\big(\frac{H^2}{4}\Delta x^2\big)}{H^2\Delta x^2}\Bigg] + \frac{(\beta-5)}{576\pi^4}\partial^2\Bigg[\frac{\ln\big(\frac{H^2}{4}\Delta x^2\big)}{\Delta x^2}\Bigg] - \frac{H^4(aa')^2}{6\pi^4}\mathcal{N}_F(y)\Bigg\}, \tag{18}$$

$$G_0^{(1)}(x;x') = \frac{\beta^2\kappa^2 H^2}{4}\Bigg\{\frac{[1-\ln(a)]}{24\pi^2}\mathrm{i}\delta^4(x-x') - \frac{\partial^2}{192\pi^4}\Bigg[\frac{\ln\big(\frac{H^2}{4}\Delta x^2\big)}{\Delta x^2}\Bigg] + \frac{H^4(aa')^2}{12\pi^4}\mathcal{N}_G(y)\Bigg\}. \tag{19}$$

Here $\mathcal{N}_F(y)$ and $\mathcal{N}_G(y)$ are complicated functions which are defined in appendix A.

3. Spin two contribution

The purpose of this section is to work out the leading secular contribution to the source integrals on the right hand side of (12) from the spin two structure functions. We begin by converting the in-out structure functions, (14) and (17), to Schwinger–Keldysh form. This leads to table 1 of seven temporal and eight spatial terms. The next step is substituting each term into the effective mode equation (12) and performing the angular integrations. The total contribution from terms 1–3 are obvious at this stage, however, some analysis is required before the leading secular contribution can be extracted from terms 4–7 and 8.

[1] The result (18) differs slightly from formula (202) of [9] (the second term on the first line of equation (202) ought to be multiplied by $[-\ln(a)]$ and the sign of the first term on the third line of equation (202) of [9] ought to be switched). The result for $G_0^{(1)}$ given in (19) agrees with equation (203) of Ref. [9].

Table 1. Different terms in the temporal and spatial parts of the Schwinger–Keldysh structure functions. To save space we have defined $\Theta \equiv \theta(\Delta\eta - r)$, and extracted a common factor of $\frac{\kappa^2 H^2}{72\pi^3}$ from each term.

k	Terms from $-\mathrm{i}F(x;x')$	Terms from $-\mathrm{i}[F(x;x') + G(x;x')]$
1	$85\pi \ln(a)\delta^4(x-x')$	$-5\pi \ln(a)\delta^4(x-x')$
2	$9[\ln(\frac{1}{4}aa') + \frac{1}{3} - 2\gamma]\nabla^2[\frac{1}{2r}\delta(\Delta\eta - r)]$	$3[\ln(\frac{1}{4}aa') + \frac{1}{3} - 2\gamma]\nabla^2[\frac{1}{2r}\delta(\Delta\eta - r)]$
3	$\frac{5}{4}\partial^4\{\Theta[\ln[H^2(\Delta\eta^2 - r^2)] - 1]\}$	$\frac{5}{4}\partial^4\{\Theta[\ln[H^2(\Delta\eta^2 - r^2)] - 1]\}$
4	$-\frac{5}{2}(H^2aa')^2\Theta\ln[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}]$	$\frac{5}{2}(H^2aa')^2\Theta(\Delta\eta^2 - r^2)\ln[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}]$
5	$5H^2aa'[\ln(\frac{1}{4}aa') + 2]\frac{1}{2r}\delta(\Delta\eta - r)$	$5H^2aa'[\ln(\frac{1}{4}aa') + 2]\frac{1}{2r}\delta(\Delta\eta - r)$
6	$\frac{5H^2aa'\Theta}{(\eta+\eta')^2 - r^2}$	$\frac{15H^2aa'\Theta}{(\eta+\eta')^2 - r^2} - 10(H^2aa')^2\Theta$
7	$-\frac{5}{4}H^2aa'\partial^2\{\Theta[\ln[H^2(\Delta\eta^2 - r^2)] - 1]\}$	$-\frac{5}{4}H^2aa'\partial^2\{\Theta[\ln[H^2(\Delta\eta^2 - r^2)] - 1]\}$
8	0	$\frac{3}{2}H^2aa'(\partial_0^2 + \nabla^2)\Theta$

3.1. Schwinger–Keldysh structure functions

We employ the Schwinger–Keldysh formalism [47–51] to obtain effective field equations which are both real and causal [52–54]. Expressions (14) and (17) give the in-out structure functions. The procedure for converting them to Schwinger–Keldysh form is simple [55]:

- Derive the $++$ structure functions by replacing each factor of the de Sitter length function $y(x;x')$ by $y_{++}(x;x)$ as defined in expression (5); Derive the $+-$ structure function by dropping the delta function terms, adding an overall minus sign, and replacing $y(x;x')$ by $y_{+-}(x;x')$ as defined in expression (6); and Adding the $++$ and $+-$ structure functions.

When $\eta < \eta'$ the $y_{++}(x;x')$ and $y_{+-}(x;x')$ agree so the $++$ and $+-$ structure functions cancel. For infinitesimal ϵ they also cancel whenever $r \equiv \|\vec{x} - \vec{x}'\| > \Delta\eta \equiv \eta - \eta'$. Hence the Schwinger–Keldysh structure function vanishes unless the point x'^{μ} lies on or within the past light-cone of x^{μ}. Because $y_{++}(x;x')$ and $y_{+-}(x;x')$ are complex conjugates in this region, the sum of i times the two structure functions is real.

Our results for the Schwinger–Keldshy structure functions are reported in table 1. As an example, consider the contribution to $-\mathrm{i}F_2^{(1)}(x;x')$ from the prepenultimate term of expression (14),

$$(\text{Term } 4) \longrightarrow \frac{\mathrm{i}5\kappa^2 H^6 (aa')^2}{72\pi^4} \times \frac{1}{4}\mathcal{L}(y)$$
$$= \frac{\mathrm{i}5\kappa^2 H^2 (H^2aa')^2}{288\pi^4}\left[\mathrm{Li}_2\left(\frac{y}{4}\right) + \ln\left(1 - \frac{y}{4}\right)\ln\left(\frac{y}{4}\right) - \frac{1}{2}\ln^2\left(\frac{y}{4}\right)^2\right]. \tag{20}$$

The dilogarithm seems intimidating but one sees from expression (16) that it is analytic at $y = 0$, so the $++$ and $+-$ contributions cancel,

$$\mathrm{Li}_2\left(\frac{y_{++}}{4}\right) - \mathrm{Li}_2\left(\frac{y_{+-}}{4}\right) = 0. \tag{21}$$

The logarithm of $1 - \frac{y}{4}$ is also analytic at $y = 0$. The only nonzero contribution comes from the logarithms of y,

$$\ln\left(\frac{y_{++}}{4}\right) - \ln\left(\frac{y_{+-}}{4}\right) \longrightarrow 2\pi \mathrm{i}\theta(\Delta\eta - r), \tag{22}$$

$$\ln^2\left(\frac{y_{++}}{4}\right) - \ln^2\left(\frac{y_{+-}}{4}\right) \longrightarrow 4\pi \mathrm{i}\theta(\Delta\eta - r)\ln\left(-\frac{y}{4}\right). \tag{23}$$

Assembling everything gives,

$$(\text{Term 4}) = -\frac{5\kappa^2 H^2 (H^2 aa')^2}{144\pi^3}\,\theta(\Delta\eta - r)\left[\ln\left(1 - \frac{y}{4}\right) - \ln\left(-\frac{y}{4}\right)\right], \tag{24}$$

$$= \frac{\kappa^2 H^2}{72\pi^3} \times -\frac{5}{2}(H^2 aa')^2 \theta(\Delta\eta - r)\ln\left(\frac{(\eta + \eta')^2 - r^2}{\Delta\eta^2 - r^2}\right). \tag{25}$$

3.2. Terms 1–3

What remains is to substitute the various terms from table 1 into the temporal and spatial source integrals on the right hand side of equation (12),

$$\mathcal{S}_k(\eta, k) \equiv -\partial_0 \int \mathrm{d}^4 x'\, \mathrm{i}F^{(1)}_{2,k}(x; x')\partial_0' u_0(\eta', k)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\Delta\vec{x}}, \tag{26}$$

$$\overline{\mathcal{S}}_k(\eta, k) \equiv -k^2 \int \mathrm{d}^4 x'\, [\mathrm{i}F^{(1)}_{2,k}(x; x') + \mathrm{i}G^{(1)}_{2,k}(x; x')]u_0(\eta', k)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\Delta\vec{x}}. \tag{27}$$

The $k = 1$ terms are local and simple to evaluate,

$$\mathcal{S}_1(\eta, k) = \frac{85\kappa^2 H^2}{72\pi^2}\,\partial_0[\ln(a)\partial_0 u_0(\eta, k)],\ \overline{\mathcal{S}}_1(\eta, k) = \frac{5\kappa^2 H^2}{72\pi^2}\,\ln(a)k^2 u_0(\eta, k). \tag{28}$$

Adding the two terms gives,

$$\mathcal{S}_1(\eta, k) + \overline{\mathcal{S}}_1(\eta, k) = \frac{85\kappa^2 H^2}{72\pi^2} \times -\mathrm{i}kHau_0(\eta, k) + O(\ln(a)). \tag{29}$$

All the $k > 1$ terms involve the angular integral,

$$\int \mathrm{d}^3 x' f(r)\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\Delta\vec{x}} = 4\pi \int_0^\infty \mathrm{d}r r^2 f(r)\frac{\sin(kr)}{kr}. \tag{30}$$

The $k = 2$ terms contain a radial delta function which immediately reduces them to single temporal integrations,

$$\mathcal{S}_2(\eta, k) = \frac{\mathrm{i}\kappa^2 H^2 k^2}{4\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} \mathrm{d}\eta' u_0(\eta', k)\sin(k\Delta\eta)\left[\ln\left(\frac{1}{4}aa'\right) + \frac{1}{3} - 2\gamma\right], \tag{31}$$

$$\overline{\mathcal{S}}_2(\eta, k) = -\frac{\kappa^2 H^2 k^3}{12\pi^2}\int_{\eta_i}^{\eta} \mathrm{d}\eta' u_0(\eta', k)\sin(k\Delta\eta)\left[\ln\left(\frac{1}{4}aa'\right) + \frac{1}{3} - 2\gamma\right], \tag{32}$$

where $\Delta\eta \equiv \eta - \eta'$ and $\eta_i \equiv -H^{-1}$ is the initial time. The core expression can be reduced to exponential integrals,

$$\int_{\eta_i}^{\eta} d\eta' e^{-ik\eta'} \sin(k\Delta\eta)\Big[\ln\Big(\frac{aa'}{4}\Big) + C\Big] = \frac{e^{-ik\eta}}{4k}[1 + 2ik\Delta\eta_i - e^{2ik\Delta\eta_i}]\Big[\ln\Big(\frac{a}{4}\Big) + C\Big]$$
$$-\frac{e^{-ik\eta}}{2iH}\Big[1 - \frac{\ln(a)}{a} - \frac{1}{a}\Big] + \frac{\sin(k\eta)}{2ik}\ln(a) + \frac{e^{ik\eta}}{4k}\int_{-2k\eta_i}^{-2k\eta} \frac{dt}{t}[e^{it} - 1], \tag{33}$$

where $\Delta\eta_i \equiv \eta - \eta_i$. The late time ($\eta \to 0^-$) limit of the first line goes like $\ln(a)$, compared with contributions from the second line which approach constants. When the ∂_0 in (31) acts on this logarithm we get,

$$\mathcal{S}_2(\eta, k) = \frac{\kappa^2 H^2}{16\pi^2} \times -ikHau_0(\eta, k)[e^{2ik\Delta\eta_i} - 1 - 2ik\Delta\eta_i] + O(\ln(a)). \tag{34}$$

We can also obtain exact results for term 3,

$$\mathcal{S}_3(\eta, k) = -\frac{i5\kappa^2 H^2}{72\pi^2}\partial_0(\partial_0^2 + k^2)^2 \int_{\eta_i}^{\eta} d\eta' u_0(\eta', k)$$
$$\times \int_0^{\Delta\eta} dr\, r\sin(kr)\Big\{\ln\Big[H^2(\Delta\eta^2 - r^2)\Big] - 1\Big\}, \tag{35}$$

$$\overline{\mathcal{S}}_3(\eta, k) = \frac{5\kappa^2 H^2}{72\pi^2}k(\partial_0^2 + k^2)^2 \int_{\eta_i}^{\eta} d\eta' u_0(\eta', k)$$
$$\times \int_0^{\Delta\eta} dr\, r\sin(kr)\Big\{\ln\Big[H^2(\Delta\eta^2 - r^2)\Big] - 1\Big\}. \tag{36}$$

Three derivatives can be moved inside the integral because the integrand vanishes like $\Delta\eta^3\ln(\Delta\eta)$ for small $\Delta\eta$,

$$(\partial_0^2 + k^2)^2 \int_{\eta_i}^{\eta} d\eta' u_0(\eta', k) \int_0^{\Delta\eta} dr\, r\sin(kr)\{\ln[H^2(\Delta\eta^2 - r^2)] - 1\}$$
$$= 2k(\partial_0 + ik) \int_{\eta_i}^{\eta} d\eta' u_0(\eta', k) \times e^{-ik\Delta\eta}\Big\{\int_0^{2k\Delta\eta} dt\Big[\frac{e^{it} - 1}{t}\Big] + 2\ln(H\Delta\eta)\Big\}, \tag{37}$$

$$= 2ku_0(\eta, k)\Big\{\int_0^{2k\Delta\eta_i} dt\Big[\frac{e^{it} - 1}{t}\Big] + 2\ln(H\Delta\eta_i)\Big\}. \tag{38}$$

Expression (38) is of order one at late times so neither of the # 3 terms contributes at leading order.

3.3. Combining terms 4–7

The factors of aa' in terms 4–7 suggests very strong contributions, but it turns out that these cancel when the terms are summed. We first work out the temporal case. Term 4 requires a partial integration on r,

$$\mathcal{S}_4(\eta,k) = \frac{i5\kappa^2 H^6}{36\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} d\eta' (aa')^2 u_0(\eta',k) \int_0^{\Delta\eta} dr\, r\sin(kr)\ln\!\Bigg[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Bigg],$$

$$= \frac{i5\kappa^2 H^4}{18\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} d\eta'\, aa' \ln\!\Bigg(\frac{aa'}{4}\Bigg) u_0(\eta',k)\sin(k\Delta\eta)$$

$$+ \frac{i5\kappa^2 H^6}{72\pi^2}\,k\partial_0 \int_{\eta_i}^{\eta} d\eta' (aa')^2 u_0(\eta',k) \int_0^{\Delta\eta} dr\cos(kr) \tag{39}$$

$$\times\{[(\eta+\eta')^2 - r^2]\ln[H^2((\eta+\eta')^2 - r^2)] - [\Delta\eta^2 - r^2]\ln[H^2(\Delta\eta^2 - r^2)]\}. \tag{40}$$

The surface term of (40) is partially cancelled by $\mathcal{S}_5(\eta,k)$,

$$\mathcal{S}_5(\eta,k) = -\frac{i5\kappa^2 H^4}{36\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} d\eta'\, aa'\Bigg[\ln\!\Bigg(\frac{aa'}{4}\Bigg) + 2\Bigg] u_0(\eta',k)\sin(k\Delta\eta). \tag{41}$$

The remaining surface term comes from partially integrating $\mathcal{S}_6(\eta,k)$ on r,

$$\mathcal{S}_6(\eta,k) = -\frac{i5\kappa^2 H^6}{18\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} d\eta' (aa')^2 u_0(\eta',k) \int_0^{\Delta\eta} dr\, \frac{r\sin(kr)}{(\eta+\eta')^2 - r^2},$$

$$= -\frac{i5\kappa^2 H^4}{36\pi^2}\,\partial_0 \int_{\eta_i}^{\eta} d\eta'\, aa' \ln\!\Bigg(\frac{aa'}{4}\Bigg) u_0(\eta',k)\sin(k\Delta\eta) \tag{42}$$

$$-\frac{i5\kappa^2 H^4}{36\pi^2}\,k\partial_0 \int_{\eta_i}^{\eta} d\eta' aa' u_0(\eta',k) \int_0^{\Delta\eta} dr\cos(kr)\ln[H^2((\eta+\eta')^2 - r^2)]. \tag{43}$$

Term 7 can be re-expressed by moving a factor of $(\partial_0^2 + k^2)$ inside the integral and then performing some partial integrations on r,

$$\mathcal{S}_7(\eta,k) = -\frac{i5\kappa^2 H^2}{72\pi^2}\,\partial_0\Bigg\{a(\partial_0^2 + k^2)\int_{\eta_i}^{\eta} d\eta' a' u_0(\eta',k)$$

$$\times \int_0^{\Delta\eta} dr\, r\sin(kr)[\ln[H^2(\Delta\eta^2 - r^2)] - 1]\Bigg\}, \tag{44}$$

$$= -\frac{i5\kappa^2 H^4}{36\pi^2}\,k\partial_0 \int_{\eta_i}^{\eta} d\eta' aa' u_0(\eta',k) \int_0^{\Delta\eta} dr\cos(kr)\ln[H^2(\Delta\eta^2 - r^2)]. \tag{45}$$

Making some small rearrangements on the sum of (40), (41), (43) and (45) gives,

$$\mathcal{S}_{4-7}(\eta,k) = \frac{i5\kappa^2 H^4}{18\pi^2}\,k\partial_0 \int_{\eta_i}^{\eta} d\eta' aa' u_0(\eta',k) \int_0^{\Delta\eta} dr\cos(kr)$$

$$\times\Bigg\{-1 + \Bigg(\frac{\Delta\eta^2 - r^2}{4\eta\eta'}\Bigg)\ln\!\Bigg[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Bigg] + \frac{1}{2}\ln\!\Bigg[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Bigg]\Bigg\}. \tag{46}$$

The spatial terms follow similar reductions to give,

$$\overline{\mathcal{S}}_{4-7}(\eta,k) = \frac{5\kappa^2 H^4}{18\pi^2} k^2 \int_{\eta_i}^{\eta} d\eta' a a' u_0(\eta',k) \int_0^{\Delta\eta} dr \cos(kr) \Bigg\{ 1 - 2\Big(\frac{\Delta\eta^2 - r^2}{4\eta\eta'}\Big) + 2\Big(\frac{\Delta\eta^2 - r^2}{4\eta\eta'}\Big)^2 \ln\Big[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Big] - \frac{1}{2}\ln\Big[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Big]\Bigg\}. \tag{47}$$

The representations we have achieved in expressions (46) and (47) are effective for taking the late time limit because the logarithms vanish like powers of $\eta\eta'$,

$$\ln\Big[\frac{(\eta+\eta')^2 - r^2}{\Delta\eta^2 - r^2}\Big] = \ln\Big[1 + \frac{4\eta\eta'}{\Delta\eta^2 - r^2}\Big] = \frac{4\eta\eta'}{\Delta\eta^2 - r^2} - \frac{1}{2}\Big(\frac{4\eta\eta'}{\Delta\eta^2 - r^2}\Big)^2 + \dots \tag{48}$$

The expansions of the curly bracketed parts of expressions (46) and (47) are,

$$\Big\{\quad\Big\}_{4-7} = \frac{5}{6}\Big(\frac{4\eta\eta'}{\Delta\eta^2 - r^2}\Big)^2 + \dots, \qquad \Big\{\quad\Big\}_{\bar{4}-\bar{7}} = \frac{1}{6}\Big(\frac{4\eta\eta'}{\Delta\eta^2 - r^2}\Big) + \dots \tag{49}$$

These expansions seem to show that (46) and (47) are finite in the late time limit of $\eta \to 0$, however, this is not quite correct. When the expansion begins to produce inverse powers of $(\Delta\eta^2 - r^2)$ it breaks down at the upper limit of the radial integration, so that the integrals actually grow like $\ln(a)$.

We can obtain analytic forms for the leading growth of (46) and (47) by adding and subtracting to the factor of $\cos(kr)$,

$$\cos(kr) = \cos(k\Delta\eta) + [\cos(kr) - \cos(k\Delta\eta)]. \tag{50}$$

When the square bracketed part of (50) multiplies the curly bracketed parts of expressions (46) and (47) they can be expanded high enough to give a finite limit for $\eta \to 0$. And because the first term of (50) does not depend upon r the radial integration involves only the curly bracketed parts of (46) and (47),

$$\int_0^{\Delta\eta} dr \Big\{\quad\Big\}_{4-7} = -\frac{1}{3}\Delta\eta - \frac{\eta^2}{3\eta'}\ln\Big(\frac{\eta'}{\eta} - 1\Big) + \frac{\eta'^2}{3\eta}\ln\Big(1 - \frac{\eta}{\eta'}\Big),$$

$$\int_0^{\Delta\eta} dr \Big\{\quad\Big\}_{\bar{4}-\bar{7}} = \frac{1}{5}\Delta\eta - \frac{2}{15}\frac{\Delta\eta^3}{\eta\eta'} + \Big[\frac{\eta^3}{5\eta'^2} - \frac{\eta\Delta\eta^2}{3\eta'^2}\Big]\ln\Big(\frac{\eta'}{\eta} - 1\Big) \tag{51}$$

$$+ \Big[\frac{\eta'\Delta\eta^2}{3\eta^2} - \frac{\eta'^3}{5\eta^2}\Big]\ln\Big(1 - \frac{\eta}{\eta'}\Big). \tag{52}$$

We actually need only the leading behaviors for small η,

$$\int_0^{\Delta\eta} dr \Big\{\quad\Big\}_{4-7} = -\frac{1}{2}\eta - \frac{\eta^2}{3\eta'}\ln\Big(\frac{\eta'}{\eta}\Big) + O\Big(\frac{\eta^2}{\eta'}\Big), \tag{53}$$

$$\int_0^{\Delta\eta} dr \Big\{\quad\Big\}_{\bar{4}-\bar{7}} = -\frac{1}{3}\eta\ln\Big(\frac{\eta'}{\eta}\Big) + \frac{2}{3}\eta + O\Big(\frac{\eta^2}{\eta'}\ln\Big(\frac{\eta'}{\eta}\Big)\Big). \tag{54}$$

Substituting in expressions (46) and (47) and performing the temporal integrations gives,

$$\mathcal{S}_{4-7}(\eta, k) = -\frac{5\kappa^2 H^2}{36\pi^2} \times -ikHa\, u_0(\eta, k) + O(\ln(a)), \tag{55}$$

$$\overline{\mathcal{S}}_{4-7}(\eta, k) = O(\ln^2(a)). \tag{56}$$

3.4. Term 8

Table 1 reveals that term 8 has only a spatial part,

$$\overline{\mathcal{S}}_8(\eta, k) = \frac{\kappa^2 H^4}{12\pi^2} ka(\partial_0^2 - k^2) \int_{\eta_i}^{\eta} d\eta' a' u_0(\eta', k) \int_0^{\Delta\eta} dr\, r \sin(kr), \tag{57}$$

$$= \frac{\kappa^2 H^2}{24\pi^2} \times -ikHau_0(\eta, k) \times [e^{2ik\Delta\eta_i} - 1 + 2ik\Delta\eta_i] + O(\ln(a)). \tag{58}$$

One of the peculiarities of this family of exact, de Sitter invariant gauges is that the spatial part makes leading order contributions such as (58). In the noncovariant, average gauge only the temporal part contributes at leading order, and that entirely from the local term analogous to $\mathcal{S}_1(\eta, k)$ [27].

3.5. Total leading spin two contribution

We found leading order contributions from (29), (34), (55) and (58). Their sum is,

$$\mathcal{S}_{1-7}(\eta, k) + \overline{\mathcal{S}}_{1-8}(\eta, k)$$
$$= \frac{\kappa^2 H^2}{48\pi^2} \times -ikHau_0(\eta, k) \times [45 - 2ik\Delta\eta_i + 5e^{2ik\Delta\eta_i}] + O(\ln^2(a)). \tag{59}$$

Substituting (59) in the effective mode equation (12) gives the spin two contribution to the one loop mode function,

$$u_{(1),\text{spin 2}} = \frac{\kappa^2 H^2}{48\pi^2} \frac{ik \ln(a)}{Ha} u_{(0)}(\eta, k) \times [45 - 2ik\Delta\eta_i + 5e^{2ik\Delta\eta_i}] + O\left(\frac{\ln^2(a)}{a^2}\right). \tag{60}$$

That compares with the leading result in the noncovariant gauge [27],

$$u_{(1),\text{noncovariant}} \longrightarrow \frac{\kappa^2 H^2}{48\pi^2} \frac{ik \ln(a)}{Ha} u_{(0)}(\eta, k) \times 6. \tag{61}$$

Both the new (60) and the old (61) results have the same leading time dependence of $\ln(a)/a$. The signs are also the same. However, the new result (60) has a different numerical factor which depends upon the ratio k/H.

4. Spin zero contribution

Here we study the leading order late time one-loop correction of the mode equation arising from the spin zero part of the graviton propagator. The relevant equation to solve is equation (12), where for $iF^{(1)}(x; x')$ and $iG^{(1)}(x; x')$ one inserts the Schwinger–Keldysh versions of spin-zero structure functions (18) and (19). In order to simplify the analysis, equation (12) can be conveniently written as,

$$(\partial_0^2+k^2)u_{(1)}(\eta,k) = u_{(0)}(\eta,k)\Big\{ \mathrm{i}k\,\partial_0 \int \mathrm{d}^4x'\, \mathrm{i}F_0^{(1)}(x;x')\mathrm{e}^{\mathrm{i}k\Delta\eta - \mathrm{i}\vec{k}\cdot\vec{x}} - k^2 \int \mathrm{d}^4x'\, \mathrm{i}G_0^{(1)}(x;x')\mathrm{e}^{\mathrm{i}k\Delta\eta - \mathrm{i}\vec{k}\cdot\vec{x}} \Big\}. \tag{62}$$

The Schwinger–Keldysh spin-zero structure functions in the equation above are constructed from the in-out structure functions (18)–(19) as described in section 3.1, and read,

$$\mathrm{i}F_0^{(1)}(x;x') = \frac{\beta^2\kappa^2H^2}{24\pi^2} \times \Big\{ -\frac{\ln(a)}{8a}\frac{\partial^2}{H^2}\Big[\frac{\delta^4(x-x')}{a'}\Big] + \frac{1}{8}\frac{\partial_0}{aH}\delta^4(x-x') + \frac{(\beta-5)}{12}\ln(a)\delta^4(x-x') + \frac{1}{128\pi a}\frac{\partial^6}{H^2}\Big[\theta\Big(\Delta\eta - \|\Delta\vec{x}\|\Big)\frac{1}{a'}\Big[1-\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big]\Big] + \frac{(\beta-5)}{192\pi}\partial^4\Big[\theta\Big(\Delta\eta - \|\Delta\vec{x}\|\Big)\Big[1-\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big] - \frac{\mathrm{i}H^4}{\pi^2}(aa')^2\mathcal{N}_F(y)\Big]\Big\}, \tag{63}$$

$$\mathrm{i}G_0^{(1)}(x;x') = \frac{\beta^2\kappa^2H^2}{48\pi^2} \times \Big\{ \frac{1}{2}\Big[\ln(a)-1\Big]\delta^4(x-x') + \frac{\mathrm{i}H^4}{\pi^2}(aa')^2\mathcal{N}_G(y)\Big] - \frac{\partial^4}{32\pi}\Big[\theta\Big(\Delta\eta - \|\Delta\vec{x}\|\Big)\Big[1-\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big]\Big]\Big\}, \tag{64}$$

and the Schwinger–Keldysh versions of functions $\mathcal{N}_F$ and $\mathcal{N}_G$ are given at the end of appendix A in (A.14) and (A.15).

When expressions (63) and (64) are inserted into (62) one can express the two principal integrals in terms of 20 relatively simple integrals and seven complicated integrals (over the generalized hypergeometric functions contained in S_F and S_G defined in (A.10)–(A.11)) as follows,

$$\int \mathrm{d}^4x'\mathrm{e}^{\mathrm{i}k\Delta\eta - \mathrm{i}\vec{k}\cdot\Delta\vec{x}}\,\mathrm{i}F_0^{(1)}(x;x') = \frac{\beta^2\kappa^2H^2}{24\pi^2}\Big\{ \frac{1}{192}\Big[-24(I_1-I_2)+16(\beta-5)I_4 + \frac{3}{2}(I_5-I_6)+(\beta-5)\Big(I_7-I_8\Big)\Big] + \frac{\partial}{\partial\beta}\Big[-\frac{q_0B_0}{2}I_9 - \frac{q_0A_0}{2}\Big(I_{10}+I_{11}+I_{12}\Big) + 2q_1B_1I_{13} + 2q_1A_1\Big(I_{14}+I_{15}+I_{16}\Big) - \frac{(\beta-4)(\beta-6)(\beta^2-20\beta+40)}{64\times 5!}I_{13} + \frac{(\beta-4)(\beta-6)(\beta^2-12\beta+40)}{64\times 5!}\mathcal{I}_I - \frac{\beta(\beta-4)(\beta-6)}{8\times 5!}\mathcal{I}_{II} + \frac{5(\beta+6)(\beta-4)(\beta-6)}{16\times 6!}\mathcal{I}_{III}\Big]\Big\}, \tag{65}$$

and

$$\int \mathrm{d}^4x' \mathrm{e}^{\mathrm{i}k\Delta\eta - \mathrm{i}\vec{k}\cdot\Delta\vec{x}}\, \mathrm{i}G_0^{(1)}(x;x') = \frac{\beta^2\kappa^2H^2}{48\pi^2}\Bigg\{ -\frac{1}{2}\Big(I_3 - I_4\Big) - \frac{1}{32}\Big(I_7 - I_8\Big) + \frac{\partial}{\partial\beta}\Bigg[2q_0(2A_0 + B_0)I_{13} + 2q_0A_0(I_{14} + I_{15} + I_{16}) - 2q_1(2B_1 - A_1)I_{17} - 4q_1A_1\big(I_{18} + I_{19} + I_{20}\big) + \frac{\beta(\beta-4)(\beta-6)(\beta-20)}{32\times 5!}I_{17} - \frac{(\beta-4)(\beta-6)(\beta^2-12\beta+40)}{64\times 5!}\mathcal{I}_{IV} - \frac{(\beta-4)(\beta-6)(\beta^2-20\beta-40)}{64\times 5!}\mathcal{I}_V + \frac{\beta(\beta-4)(\beta-6)}{8\times 5!}\mathcal{I}_{VI} - \frac{5(\beta+6)(\beta-4)(\beta-6)}{8\times 6!}\mathcal{I}_{VII}\Bigg]\Bigg\}. \tag{66}$$

The integrals over elementary functions $I_1 - I_{20}$ are defined in appendix B in table B1, and the integrals over generalized hypergeometric functions $\mathcal{I}_I - \mathcal{I}_{VII}$ in table C1 of appendix C.

The simpler integrals from table B1 can be all evaluated exactly, and a procedure how to do that is briefly outlined in appendix B. Here we require only their late time limits given below,

$$I_1 = \mathcal{O}\bigg(\frac{\ln(a)}{a}\bigg), \qquad I_2 = \mathcal{O}(1), \qquad I_3 = \mathcal{O}(1), \qquad I_4 = \ln(a) + \mathcal{O}(1), \tag{67}$$

$$I_5 = \mathcal{O}\bigg(\frac{1}{a}\bigg), \qquad I_6 = \mathcal{O}(1), \qquad I_7 = \mathcal{O}(1), \qquad I_8 = \mathcal{O}(1) \tag{68}$$

$$I_9 = \frac{4\mathrm{i}H}{k}\mathcal{G}\bigg(\frac{k}{H}\bigg)\times a + 8\ln(a) + \mathcal{O}(1), \tag{69}$$

$$I_{10} = \frac{4\mathrm{i}H}{k}\mathcal{G}\bigg(\frac{k}{H}\bigg)a\ln(a) + 8\ln^2(a) + 8\bigg[1 + \mathcal{G}\bigg(\frac{k}{H}\bigg)\bigg]\ln(a) + \mathcal{O}\bigg(\frac{\ln^2(a)}{a}\bigg), \tag{70}$$

$$I_{11} = \frac{4\mathrm{i}H}{k}\mathcal{M}\bigg(\frac{k}{H},0\bigg)a + 4\ln^2(a) + 8\ln(a) + \mathcal{O}(1), \tag{71}$$

$$I_{12} = \frac{4\mathrm{i}H}{k}\bigg[\mathcal{V}\bigg(\frac{k}{H},0\bigg) + \mathcal{M}\bigg(-\frac{k}{H},0\bigg) - 3\mathcal{M}\bigg(\frac{k}{H},0\bigg) + \mathcal{G}\bigg(\frac{k}{H}\bigg)\bigg]a - 8\ln^2(a) - 8\ln(a) + \mathcal{O}(1), \tag{72}$$

$$I_{13} = \frac{2H^2}{k^2}\bigg[\mathcal{G}\bigg(\frac{k}{H}\bigg) + 2 + \frac{\mathrm{i}H}{k}\Big(\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1\Big)\bigg]a^2 + \frac{2H^2}{k^2}\bigg[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H}\bigg]a + 4\ln(a) + \mathcal{O}(1), \tag{73}$$

$$I_{14} = \frac{2H^2}{k^2}\left[\mathcal{G}\left(\frac{k}{H}\right) + 2 + \frac{iH}{k}\left(e^{\frac{2ik}{H}} - 1\right)\right]a^2\ln(a) + \frac{2H^2}{k^2}\left[e^{\frac{2ik}{H}} - 1 - \frac{2ik}{H}\right]a\ln(a)$$
$$+4\ln^2(a) + 2\left[1 + 2\mathcal{G}\left(\frac{k}{H}\right)\right]\ln(a) + \mathcal{O}\left(\frac{\ln^2(a)}{a}\right), \tag{74}$$

$$I_{15} = \frac{2H^2}{k^2}\left[\mathcal{M}\left(\frac{k}{H},0\right) - 2\mathcal{G}\left(\frac{k}{H}\right) - 2 + \frac{iH}{k}\left(1 - e^{\frac{2ik}{H}}\right)\right]a^2 \tag{75}$$

$$+\frac{4iH}{k}\left[\mathcal{G}\left(\frac{k}{H}\right) + 1 - \frac{iH}{2k}\left(1 - e^{\frac{2ik}{H}}\right)\right]a + 2\ln^2(a) + 2\ln(a) + \mathcal{O}(1), \tag{76}$$

$$I_{16} = \frac{2iH^3}{k^3}\left\{2e^{\frac{2ik}{H}} - 2 + e^{\frac{2ik}{H}}\mathcal{G}\left(-\frac{k}{H}\right) - \left(1 + \frac{4ik}{H}\right)\mathcal{G}\left(\frac{k}{H}\right)\right.$$
$$\left. - \frac{ik}{H}\left[\mathcal{M}\left(-\frac{k}{H},0\right) - 3\mathcal{M}\left(\frac{k}{K},0\right) + \mathcal{V}\left(\frac{k}{H},0\right)\right]\right\}a^2 \tag{77}$$

$$I_{17} = -\frac{12H^4}{k^4}\left[1 + \frac{ik}{H} + \frac{ik}{3H}\mathcal{G}\left(\frac{k}{H}\right) + \frac{iH}{2k}\left(e^{\frac{2ik}{H}} - 1\right)\right]a^3$$
$$+\frac{12iH^3}{k^3}\left[1 + \frac{ik}{3H} + \frac{iH}{2k}\left(1 - \frac{2ik}{3H}\right)\left(e^{\frac{2ik}{H}} - 1\right)\right]a^2$$
$$+\frac{4H^2}{k^2}\left[1 - \frac{ik}{H} + \left(1 + \frac{iH}{2k}\right)\left(e^{\frac{2ik}{H}} - 1\right)\right]a + \mathcal{O}(\ln(a)), \tag{78}$$

$$I_{18} = -\frac{12H^4}{k^4}\left[1 + \frac{ik}{H} + \frac{ik}{3H}\mathcal{G}\left(\frac{k}{H}\right) + \frac{iH}{2k}\left(e^{\frac{2ik}{H}} - 1\right)\right]a^3\ln(a)$$
$$+\frac{12iH^3}{k^3}\left[1 + \frac{ik}{3H} + \frac{iH}{2k}\left(1 - \frac{2ik}{3H}\right)\left(e^{\frac{2ik}{H}} - 1\right)\right]a^2\ln(a)$$
$$+\frac{4H^2}{k^2}\left[1 - \frac{ik}{H} + \left(1 + \frac{iH}{2k}\right)\left(e^{\frac{2ik}{H}} - 1\right)\right]a\ln(a) + \mathcal{O}(\ln^2(a)), \tag{79}$$

$$I_{19} = -\frac{3iH^5}{k^5}\left[1 + \frac{4ik}{H} - \frac{6k^2}{H^2} - \frac{4k^2}{H^2}\mathcal{G}\left(\frac{k}{H}\right) + \frac{4k^2}{3H^2}\mathcal{M}\left(\frac{k}{H},0\right) - e^{\frac{2ik}{H}}\left(1 + \frac{2ik}{H}\right)\right]a^3$$
$$-\frac{3H^4}{k^4}\left[1 + \frac{8ik}{3H} - \frac{10k^2}{3H^2} - \frac{4k^2}{3H^2}\mathcal{G}\left(\frac{k}{H}\right) - e^{\frac{2ik}{H}}\left(1 + \frac{2ik}{3H}\right)\right]a^2$$
$$+\frac{iH^3}{k^3}\left[1 + \frac{2k^2}{H^2} + \frac{4k^2}{H^2}\mathcal{G}\left(\frac{k}{H}\right) - e^{\frac{2ik}{H}}\left(1 - \frac{2ik}{H}\right)\right]a + \mathcal{O}(\ln^2(a)), \tag{80}$$

$$
\begin{aligned}
I_{20} = \Bigg\{ & -\frac{4\mathrm{i}H^3}{k^3}\left[\mathcal{V}\left(\frac{k}{H},0\right)+\mathcal{M}\left(-\frac{k}{H},0\right)-3\mathcal{M}\left(\frac{k}{H},0\right)+7\mathcal{G}\left(\frac{k}{H}\right)\right] \\
& +\frac{2\mathrm{i}H^5}{k^5}\left[11+\frac{16\mathrm{i}k}{H}-\frac{10k^2}{H^2}+3\left(1+\frac{2\mathrm{i}k}{H}\right)\mathcal{G}\left(\frac{k}{H}\right)\right. \\
& \left.\left. -\mathrm{e}^{\frac{2\mathrm{i}k}{H}}\left(11+\frac{6\mathrm{i}k}{H}+3\mathcal{G}\left(-\frac{k}{H}\right)\right)\right]\right\}a^3 \\
& +\frac{2H^4}{k^4}\left\{11+\frac{10\mathrm{i}k}{H}-\frac{6k^2}{H^2}+\left[3+\frac{4\mathrm{i}k}{H}-\frac{4k^2}{H^2}\right]\mathcal{G}\left(\frac{k}{H}\right)\right. \\
& \left. -\mathrm{e}^{\frac{2\mathrm{i}k}{H}}\left[11+\left(3-\frac{2\mathrm{i}k}{H}\right)\mathcal{G}\left(-\frac{k}{H}\right)\right]\right\}a^2 \\
& +\frac{2\mathrm{i}H^3}{k^3}\left\{-4-\left(1+\frac{4k^2}{H^2}\right)\mathcal{G}\left(\frac{k}{H}\right)+\mathrm{e}^{\frac{2\mathrm{i}k}{H}}\left[4-\frac{4\mathrm{i}k}{H}+\left(1-\frac{2\mathrm{i}k}{H}\right)\mathcal{G}\left(-\frac{k}{H}\right)\right]\right\}a \\
& +\mathcal{O}(\ln^2(a)).
\end{aligned}
\tag{81}
$$

The results are expressed in terms of elementary functions and the following integrals,

$$
\mathcal{G}(x) \equiv \int_0^1 \frac{\mathrm{d}\tau}{\tau}[\mathrm{e}^{2\mathrm{i}x\tau}-1] = [\mathrm{ci}(2x)-\gamma_E-\ln(2x)]+\mathrm{i}\left[\mathrm{si}(2x)+\frac{\pi}{2}\right], \tag{82}
$$

$$
\mathcal{M}(x,z) \equiv \int_0^1 \frac{\mathrm{d}\tau}{\tau}[\mathcal{G}(x\tau+z)-\mathcal{G}(z)], \tag{83}
$$

$$
\mathcal{V}(x,z) \equiv \int_0^1 \frac{\mathrm{d}\tau}{\tau}[\mathrm{e}^{2\mathrm{i}x\tau}-1]\mathcal{G}^*(x\tau+z), \tag{84}
$$

where ci and si are the usual cosine-integral and sine-integral functions defined as,

$$
\mathrm{ci}(z) = -\int_z^\infty \mathrm{d}t\frac{\cos(t)}{t} = \int_0^z dt\frac{\cos(t)-1}{t}+\gamma_E+\ln(z) \tag{85}
$$

$$
\mathrm{si}(z) = -\int_z^\infty \mathrm{d}t\frac{\sin(t)}{t} = \int_0^z \mathrm{d}t\frac{\sin(t)}{t}-\frac{\pi}{2}. \tag{86}
$$

The derivation of the late time limit of integrals over hypergeometric functions from table C1 is given in appendix C in some detail. Here we give the final results to relevant order,

$$
\mathcal{I}_I = -\frac{20}{\beta}\frac{\mathrm{i}H}{k}\mathcal{G}\left(\frac{k}{H}\right)a-\frac{40}{\beta}\left[1+\frac{8}{(\beta-4)}\right]\ln(a), \tag{87}
$$

$$
\begin{aligned}
\mathcal{I}_{II} = & -\frac{20}{\beta}\frac{\mathrm{i}H}{k}\mathcal{G}\left(\frac{k}{H}\right)a\ln(a) \\
& -\frac{20}{\beta}\frac{\mathrm{i}H}{k}\left[\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)\mathcal{G}\left(\frac{k}{H}\right)-2\mathcal{M}\left(\frac{k}{H},0\right)+\mathcal{M}\left(-\frac{k}{H},0\right)+\mathcal{V}\left(\frac{k}{H},0\right)\right]a \\
& -\frac{20}{\beta}\ln^2(a)+\frac{40}{\beta}\left[\frac{8}{(\beta-4)}-\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)-\mathcal{G}\left(\frac{k}{H}\right)\right]\ln(a),
\end{aligned}
\tag{88}
$$

$$
\begin{aligned}
\mathcal{I}_{III} = &-\frac{48}{(\beta+6)}\frac{\mathrm{i}H^3}{k^3}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} - \frac{\mathrm{i}k}{H}\mathcal{G}\left(\frac{k}{H}\right)\right]a^2\ln(a)\\
&-\frac{48}{(\beta+6)}\frac{\mathrm{i}H^3}{k^3}\left\{\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} - \frac{\mathrm{i}k}{H}\mathcal{G}\left(\frac{k}{H}\right)\right]\right.\\
&\qquad + \mathrm{e}^{\frac{2\mathrm{i}k}{H}}\left[1+\mathcal{G}\left(-\frac{k}{H}\right)\right] - 1 + \frac{2\mathrm{i}k}{H} + \left[1+\frac{2\mathrm{i}k}{H}\right]\mathcal{G}\left(\frac{k}{H}\right)\\
&\qquad \left. + \frac{\mathrm{i}k}{H}\left[2\mathcal{M}\left(\frac{k}{H},0\right) - \mathcal{M}\left(-\frac{k}{H},0\right) + \mathcal{V}\left(\frac{k}{H},0\right)\right]\right\}a^2\\
&-\frac{48}{(\beta+6)}\frac{H^2}{k^2}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H}\right]a\ln(a)\\
&-\frac{48}{(\beta+6)}\frac{H^2}{k^2}\left\{\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H}\right] + \mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 + \frac{2\mathrm{i}k}{H}\right.\\
&\qquad \left. + \mathrm{e}^{\frac{2\mathrm{i}k}{H}}\mathcal{G}\left(-\frac{k}{H}\right) - \left[1+\frac{2\mathrm{i}k}{H} - \frac{10\mathrm{i}k}{\beta H}\right]\mathcal{G}\left(\frac{k}{H}\right)\right\}a - \frac{48}{(\beta+6)}\ln^2(a)\\
&-\frac{96}{(\beta+6)}\left\{\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right) + \mathcal{G}\left(\frac{k}{H}\right) - 1 - \frac{10}{(\beta-4)}\right\}a
\end{aligned} \tag{89}
$$

$$
\mathcal{I}_{IV} = \frac{320}{\beta(\beta-4)}\frac{\mathrm{i}H}{k}\mathcal{G}\left(\frac{k}{H}\right)a, \tag{90}
$$

$$
\begin{aligned}
\mathcal{I}_{V} = &-\frac{40iH^3}{\beta k^3}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} - \frac{\mathrm{i}k}{H}\mathcal{G}\left(\frac{k}{H}\right)\right]a^2\\
&-\frac{40H^2}{\beta k^2}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} + \frac{8\mathrm{i}k}{(\beta-4)H}\mathcal{G}\left(\frac{k}{H}\right)\right]a\\
\mathcal{I}_{VI} = &-\frac{40\mathrm{i}H^3}{\beta k^3}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} - \frac{\mathrm{i}k}{H}\mathcal{G}\left(\frac{k}{H}\right)\right]a^2\ln(a)\\
&-\frac{40\mathrm{i}H^3}{\beta k^3}\left\{\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H} - \frac{\mathrm{i}k}{H}\mathcal{G}\left(\frac{k}{H}\right)\right]\right.\\
&\qquad + \mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 + \frac{2\mathrm{i}k}{H} + \mathrm{e}^{\frac{2\mathrm{i}k}{H}}\mathcal{G}\left(-\frac{k}{H}\right) - \left(1+\frac{2\mathrm{i}k}{H}\right)\mathcal{G}\left(\frac{k}{H}\right)\\
&\left. + \frac{\mathrm{i}k}{H}\left[2\mathcal{M}\left(\frac{k}{H},0\right) - \mathcal{M}\left(-\frac{k}{H},0\right) - \mathcal{V}\left(\frac{k}{H},0\right)\right]\right\}a^2\\
&-\frac{40H^2}{\beta k^2}\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H}\right]a\ln(a)\\
&-\frac{40H^2}{\beta k^2}\left\{\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,\frac{3}{2}\right)\left[\mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 - \frac{2\mathrm{i}k}{H}\right] + \mathrm{e}^{\frac{2\mathrm{i}k}{H}} - 1 + \frac{2\mathrm{i}k}{H}\right.\\
&\qquad \left. - \left(1 - \frac{2\mathrm{i}k\beta}{(\beta-4)H}\right)\mathcal{G}\left(\frac{k}{H}\right) + \mathrm{e}^{\frac{2\mathrm{i}k}{H}}\mathcal{G}\left(-\frac{k}{H}\right)\right\}a
\end{aligned} \tag{91}
$$

$$
\begin{aligned}
\mathcal{I}_{VII} &= \frac{48iH^5}{(\beta+6)k^5}\left[3e^{\frac{2ik}{H}} - 3 - \frac{6ik}{H} + \frac{6k^2}{H^2} + \frac{2k^2}{H^2}\mathcal{G}\left(\frac{k}{H}\right)\right]a^3\ln(a) \\
&+ \frac{24H^5}{(\beta+6)k^5}\left\{6i\Xi\left(\frac{9}{2}+b_N, \frac{9}{2}-b_N, 7, \frac{3}{2}\right)\left[e^{\frac{2ik}{H}} - 1 - \frac{2ik}{H} + \frac{2k^2}{H^2} + \frac{2k^3}{3H^3}\mathcal{G}\left(\frac{k}{H}\right)\right]\right. \\
&\quad + 19i\left(e^{\frac{2ik}{H}} - 1\right) + \frac{20k}{H} - \frac{6k}{H}e^{\frac{2ik}{H}} + \frac{2ik^2}{H^2} + 6ie^{\frac{2ik}{H}}\mathcal{G}\left(-\frac{k}{H}\right) \\
&\quad + \left(-6i + \frac{12k}{H} + \frac{16ik^2}{H^2}\right)\mathcal{G}\left(\frac{k}{H}\right) \\
&\quad \left. + \frac{4ik^2}{H^2}\left[\mathcal{M}\left(-\frac{k}{H},0\right) - 2\mathcal{M}\left(\frac{k}{H},0\right) + \mathcal{V}\left(\frac{k}{H},0\right)\right]\right\}a^3 \\
&- \frac{48H^4}{k^4(\beta+6)}\left[3 + \frac{4ik}{H} - \frac{2k^2}{H^2} + e^{\frac{2ik}{H}}\left(-3 + \frac{2ik}{H}\right)\right]a^2\ln(a) \\
&+ \frac{24H^4}{(\beta+6)k^4}\left\{2\,\Xi\left(\frac{9}{2}+b_N, \frac{9}{2}-b_N, 7, \frac{3}{2}\right)\left[-3 - \frac{4ik}{H} + \frac{2k^2}{H^2} + e^{\frac{2ik}{H}}\left(3 - \frac{2ik}{H}\right)\right]\right. \\
&\quad + 19\left(e^{\frac{2ik}{H}} - 1\right) - \frac{12ik}{H} + \frac{2k^2}{H^2} - \frac{2ik}{H}e^{\frac{2ik}{H}} + 2e^{\frac{2ik}{H}}\left(3 - \frac{2ik}{H}\right)\mathcal{G}\left(-\frac{k}{H}\right) \\
&\quad \left. + 2\left(-3 - \frac{4ik}{H} + \frac{2k^2}{H^2}\right)\mathcal{G}\left(\frac{k}{H}\right)\right\}a^2 \\
&+ \frac{48iH^3}{(\beta+6)k^3}\left[1 + \frac{2k^2}{H^2} - e^{\frac{2ik}{H}}\left(1 - \frac{2ik}{H}\right)\right]a\ln(a) \\
&- \frac{24H^3}{(\beta+6)k^3}\left\{2i\,\Xi\left(\frac{9}{2}+b_N, \frac{9}{2}-b_N, 7, \frac{3}{2}\right)\left[e^{\frac{2ik}{H}}\left(1 - \frac{2ik}{H}\right) - 1 - \frac{2k^2}{H^2}\right]\right. \\
&\quad + e^{\frac{2ik}{H}}\left(7i + \frac{6k}{H}\right) - 7i - \frac{2ik^2}{H^2} + 2i\,e^{\frac{2ik}{H}}\left(1 - \frac{2ik}{H}\right)\mathcal{G}\left(-\frac{k}{H}\right) \\
&\quad \left. - 2i\left[1 + \frac{2(\beta^2 - 4\beta - 40)}{\beta(\beta-4)}\frac{k^2}{H^2}\right]\mathcal{G}\left(\frac{k}{H}\right)\right\}a, \qquad (92)
\end{aligned}
$$

where we have defined

$$\Xi(\alpha_1, \alpha_2, \beta_1, \beta_2) = \psi(\alpha_1 - 1) + \psi(\alpha_2 - 1) - \psi(\beta_1 - 1) - \psi(\beta_2 - 1) - 2\ln(2), \qquad (93)$$

and $\psi(x)$ is the digamma function.

Finally, plugging in all the late time limits of integrals (67)–(81) and (87)–(92) into expressions (65) and (66) gives

$$\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\, i\,F_R^0(x;x') = 0 \qquad \text{to order } \ln(a), \qquad (94)$$

$$\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\, i\,G_R^0(x;x') = 0 \qquad \text{to order } a. \qquad (95)$$

In view of equation (62), this then implies that there is *no* late time one-loop correction that contributes at the leading order as $\propto [\ln(a)/a]u_{(0)}(\eta, k)$ (see equation (60)) to the photon wave function, $u_{(1),\text{spin}\,0} = 0$.

This completes the analysis of the graviton induced one-loop correction to the photon wave function on de Sitter. The analysis shows that, at late times, the leading contribution comes entirely from the spin-two part of the graviton propagator (60), implying that our result is independent on the graviton gauge parameter b (or, equivalently, on the parameter $\beta = (4b-2)/(b-2)$) for $b > 2$.

5. Discussion

Inflation creates an ensemble of gravitons. We have studied the effects that these gravitons have on the propagation of a spatial plane wave photon. What we find is that the one loop electric field grows, relative to the tree order result, by an amount which eventually becomes nonperturbatively strong,

$$F^{(1)}_{0i} \longrightarrow \frac{\kappa^2 H^2}{48\pi^2}\ln(a)\left[45 - \frac{2\mathrm{i}k}{H} + 5\mathrm{e}^{2\mathrm{i}k/H}\right] \times F^{(0)}_{0i}. \tag{96}$$

This comes entirely from the spin-two part of the graviton propagator.

Note that the one loop correction to the photon mode function (60) actually falls off at late times like $\ln(a)/a$. The corresponding electric field involves a *time derivative* of the mode function, so it grows because $\partial_0 \ln(a)/a = H[-\ln(a)+1]$. In contrast, the magnetic field involves only space derivatives of the mode function, so it falls off. This might seem strange but one has to bear in mind that quantum gravitational corrections represent the interaction of photons with virtual graviton fluctuations away from the de Sitter background, and Maxwell's equations do not imply that the electric and magnetic field strengths have the same time average in a general metric [56].

At this stage one might wonder how uncharged gravitons are nonetheless able to induce vacuum polarization. The reason is that photons can pick up 3-momentum from inflationary gravitons. The photon's physical 3-momentum redshifts like $\frac{k}{a(t)}$, whereas inflationary particle production continually replenishes the supply of gravitons with physical 3-momentum $\frac{k}{a(t)} \sim H$. The spin-spin coupling allows these gravitons to continue interacting with the redshifting photon to arbitrarily late times. A scattering is rare—because quantum gravity is weak, even at inflationary scales—but it essentially always adds to the photon's 3-momentum. Because the effect increases in time, but not in space, the electric field strength grows but the magnetic field strength does not.

There are those who argue that the apparent secular growth we find must be a gauge artefact which could be absorbed into a redefinition of the coordinate system [57, 58]. This belief seems strange to us because the best way of understanding any putative quantum field theory effect is as the classical response to the ensemble of virtual particles whose existence and duration are regulated by the energy-time uncertainty principle [14]. If we think in this way no one doubts that inflation creates an ensemble of horizon scale gravitons—that is why there is a tensor power spectrum. If classical photons were required to propagate through a uniform ensemble of gravitational radiation no one doubts that they would occasionally scatter—that is the basis of attempts to detect gravitational radiation by pulsar timing. Nor does anyone doubt that the degree of scattering would increase the longer the photons had to propagate through the ensemble of gravitational radiation. That is the essential physics behind what we are seeing.

However plausible the basic effect may seem, we do need to infer it in a way which does not depend upon the choice of graviton gauge. Checking this was one of the primary motivations for our work, and we did check that the secular growth factor (96) has no dependence on the parameter b which characterizes a general, exact, de Sitter invariant gauge [28]. The vacuum polarization in this gauge depends massively upon b [9], yet we saw in section 4 that none of the b-dependent terms contribute to the secular growth (96). That supports the secular gauge independence conjecture [22].

Unfortunately, our result (96) is not the same as was previously obtained [27] in a noncovariant, average gauge [23]. It has the same sign and spacetime dependence, but the noncovariant average gauge has the factor $45 + 2ik/H + 5e^{2ik/H}$ replaced by just 6. This may mean that the secular gauge independence conjecture is wrong. However, another possibility is that there is an obstacle to imposing the de Sitter breaking, average gauge, just as there has already been shown to be an obstacle to imposing de Sitter invariant, average gauges [59][2].

For typical models of inflation the loop counting parameter is very small $\kappa^2 H^2 \sim 10^{-10}$. This is why perturbation theory remains valid for a huge range of e-foldings. However, our one loop electric field strength (96) grows without bound, and the factor of $\ln(a)$ must eventually overwhelm even this huge suppression factor of $\kappa^2 H^2$ if inflation persists long enough. At that point perturbative techniques become unreliable and one must resort to some other method for making predictions. Although this is a fascinating subject, we have nothing to say about it right now, except to note the intriguing possibility that these sorts of secular effects may betoken an instability of de Sitter space [61–64].

Another important question is whether the secular growth we find can give rise to magnetogenesis by post-inflationary dynamics. The crucial difference between the graviton effect considered here and the effect induced by (light or massless minimally coupled) charged scalars is that charged scalar fluctuations generate a photon mass [44, 65–68], while graviton fluctuations only modify the wave function. Post-inflationary magnetogenesis depends crucially on the photon mass [69–71], which exponentially boosts the photon vacuum energy during inflation, then disappears after inflation, causing that vacuum energy to produce huge numbers of real photons. In our case the electric field (96) gets amplified but the dispersion relation of the photon is unchanged. Hence the energy density in super-horizon photons is suppressed by a factor of k^4, and only enhanced by the comparatively weaker factor of $\ln(a)$ on the electric field strength. Post-inflationary physics will transfer energy from the electric to the magnetic field, eventually reaching equipartition (if that is not prevented by the large conductivity that could be generated during post-inflationary thermalization). However, the result will be tiny primordial magnetic fields on cosmological scales.

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[2] One might also worry about the invariant gauge propagator. An independent computation using just the spatial, transverse-traceless graviton modes claims a different result, even a different sign, for the 'infrared divergence' [60]. However, there does not seem to be any contradiction. The different sign derives from a confusion between infrared divergence (due to the small k end of the mode sum) and secular growth (due to the large $k \sim Ha(t)$) part of the mode sum. That is, the same logarithmic integral gives a minus sign from the lower limit and a plus sign from the upper limit. The different numerical factor derives from the fact that a covariant gauge propagator of the sort we are using includes constrained degrees of freedom (which are important because they mediate forces) as well as the dynamical graviton modes.

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Appendix A. The functions $\mathcal{N}_F(y)$ and $\mathcal{N}_G(y)$

The functions $\mathcal{N}_F(y)$ and $\mathcal{N}_G(y)$ in expressions (18)–(19) can be represented by the following series,

$$\mathcal{N}_F(y) = \frac{\partial}{\partial\beta}\Big[-\frac{q_0A_0}{y}\ln\Big(\frac{y}{4}\Big) - \frac{q_0(A_0+B_0)}{y} + \frac{q_1A_1}{2}\ln^2\Big(\frac{y}{4}\Big) + q_1B_1\ln\Big(\frac{y}{4}\Big) + S_F(y)\ln\Big(\frac{y}{4}\Big) + \widetilde{S}_F(y)\Big], \tag{A.1}$$

$$\begin{aligned}\mathcal{N}_G(y) = \frac{\partial}{\partial\beta}\Big[&-\frac{q_0A_0}{2}\ln^2\Big(\frac{y}{4}\Big) - q_0(2A_0+B_0)\ln\Big(\frac{y}{4}\Big)\\ &+q_1A_1y\ln^2\Big(\frac{y}{4}\Big) + q_1(2B_1-A_1)y\ln\Big(\frac{y}{4}\Big)\\ &+q_1(A_1-B_1)y + S_G(y)\ln\Big(\frac{y}{4}\Big) + \widetilde{S}_G(y)\Big],\end{aligned} \tag{A.2}$$

where the four power series are,

$$S_F(y) = \sum_{n=0}^{\infty}\frac{q_{n+2}A_{n+2}}{(n+1)}y^{n+1}, \tag{A.3}$$

$$S_G(y) = \sum_{n=0}^{\infty}\frac{(n+3)q_{n+2}A_{n+2}}{(n+1)(n+2)}y^{n+2}, \tag{A.4}$$

$$\widetilde{S}_F(y) = \sum_{n=0}^{\infty}\frac{q_{n+2}}{(n+1)}\Big[B_{n+2} - \frac{A_{n+2}}{(n+1)}\Big]y^{n+1}, \tag{A.5}$$

$$\widetilde{S}_G(y) = \sum_{n=0}^{\infty}\frac{(n+3)q_{n+2}}{(n+1)(n+2)}\Big[B_{n+2} - \frac{n^2+6n+7}{(n+1)(n+2)(n+3)}A_{n+2}\Big]y^{n+2}, \tag{A.6}$$

and the coefficients are,

$$q_n = \frac{\Gamma\Big(\frac{5}{2}+b_N+n\Big)\Gamma\Big(\frac{5}{2}-b_N+n\Big)}{4^{n+1}(n+1)!\,(n+2)!\,\Gamma\Big(\frac{1}{2}+b_N\Big)\Gamma\Big(\frac{1}{2}-b_N\Big)}, \tag{A.7}$$

$$A_n = \frac{(n+1)}{8(n+3)(n+4)\beta}[n(n-1)\beta^2 - 4(n-1)(3n+2)\beta + 40n(n+1)], \tag{A.8}$$

$$B_n = A_n\left[\psi\left(\frac{5}{2} + b_N + n\right) + \psi\left(\frac{5}{2} - b_N + n\right) - \psi(n+2) - \psi(n+3)\right] + \frac{1}{8(n+3)^2(n+4)^2\beta}\left[\beta^2(n^4 + 14n^3 + 37n^2 - 12)\right. \tag{A.9}$$

$$\left. -4\beta(3n^4 + 42n^3 + 125n^2 + 52n - 22) + 40(n+1)(n^3 + 13n^2 + 36n + 12)\right].$$

In order to perform the computation here we need to resum the series (A.3)–(A.6). For the purpose of this paper, in which we need the retarded vacuum polarization, it suffices to sum only the series $S_F(y)$ and $S_G(y)$ which multiply $\log(y)$ in equations (A.1) and (A.2). The results can be expressed in terms of generalized hypergeometric functions,

$$\begin{aligned} S_F(y) = &-\frac{(\beta-4)(\beta-6)(\beta^2-20\beta+40)}{128\times 5!} \\ &+\frac{(\beta-4)(\beta-6)(\beta^2-12\beta+40)}{128\times 5!}\,{}_2F_1\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N\right\}, \left\{6\right\}, \frac{y}{4}\right) \\ &-\frac{\beta(\beta-4)(\beta-6)}{16\times 5!}\,{}_3F_2\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N, 1\right\}, \left\{6, 2\right\}, \frac{y}{4}\right) \\ &+\frac{5(\beta+6)(\beta-4)(\beta-6)}{32\times 6!}\,y\times{}_4F_3\left(\left\{\frac{9}{2}+b_N, \frac{9}{2}-b_N, 1, 1\right\}, \left\{7, 2, 2\right\}, \frac{y}{4}\right), \end{aligned} \tag{A.10}$$

$$\begin{aligned} S_G(y) = &-\frac{\beta(\beta-4)(\beta-6)(\beta-20)}{64\times 5\,!}\,y \\ &+\frac{(\beta-4)(\beta-6)(\beta^2-12\beta+40)}{128\times 5!}\,y\times{}_2F_1\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N\right\}, \left\{6\right\}, \frac{y}{4}\right) \\ &+\frac{(\beta-4)(\beta-6)(\beta^2-20\beta-40)}{128\times 5!}\,y\times{}_3F_2\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N, 1\right\}, \left\{6, 2\right\}, \frac{y}{4}\right) \\ &-\frac{\beta(\beta-4)(\beta-6)}{16\times 5!}\,y\times{}_4F_3\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N, 1, 1\right\}, \left\{6, 2, 2\right\}, \frac{y}{4}\right) \\ &+\frac{5(\beta+6)(\beta-4)(\beta-6)}{16\times 6!}\,y^2\times{}_4F_3\left(\left\{\frac{9}{2}+b_N, \frac{9}{2}-b_N, 1, 1\right\}, \left\{7, 2, 2\right\}, \frac{y}{4}\right), \end{aligned} \tag{A.11}$$

where

$$\beta = 2\frac{2b-1}{b-2}, \qquad b_N = \sqrt{\frac{25}{4} - \beta} = \sqrt{\frac{3(3b-14)}{4(b-2)}}. \tag{A.12}$$

The third line in equation (A.11) can also be written as,

$$y\times{}_3F_2\left(\left\{\frac{7}{2}+b_N, \frac{7}{2}-b_N, 1\right\}, \{6, 2\}, \frac{y}{4}\right) = \frac{20}{\beta}\left[{}_2F_1\left(\left\{\frac{5}{2}+b_N, \frac{5}{2}-b_N\right\}, \{5\}, \frac{y}{4}\right) - 1\right]. \tag{A.13}$$

The Schwinger–Keldysh form of functions $\mathcal{N}_F(y)$ and $\mathcal{N}_G(y)$, obtained by procedure described in section 3.1, is

$$
\begin{aligned}
\mathcal{N}_F(y) = \frac{\mathrm{i}\pi^2}{H^4(aa')^2}\frac{\partial}{\partial\beta}\Bigg\{ & -\frac{q_0 B_0}{2}\times\frac{H^2}{\pi}aa'\,\partial^2\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& -\frac{q_0 A_0}{2}\times\frac{H^2}{\pi}aa'\ln(aa')\,\partial^2\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& -\frac{q_0 A_0}{2}\times\frac{H^2(aa')}{\pi}\partial^2\left[\ln\left(-\frac{H^2}{4}\Delta x^2\right)\theta(\Delta\eta-\|\Delta\vec{x}\|)\right] \\
& +2q_1 B_1\times\frac{H^4}{\pi}(aa')^2\,\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& +2q_1 A_1\times\frac{H^4}{\pi}(aa')^2\ln(aa')\,\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& +2q_1 A_1\times\frac{H^4}{\pi}(aa')^2\ln\left(-\frac{H^2}{4}\Delta x^2\right)\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& +2\times\frac{H^4}{\pi}(aa')^2 S_F(y)\,\theta(\Delta\eta-\|\Delta\vec{x}\|)\Bigg\},
\end{aligned}
\tag{A.14}
$$

and

$$
\begin{aligned}
\mathcal{N}_G(y) = \frac{-\mathrm{i}\pi^2}{H^4(aa')^2}\frac{\partial}{\partial\beta}\Bigg\{ & 2q_0(2A_0+B_0)\times\frac{(H^2aa')^2}{\pi}\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& +2q_0 A_0\times\frac{H^4}{\pi}(aa')^2\ln\left(-\frac{H^2}{4}aa'\Delta x^2\right)\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& -2q_1(2B_1-A_1)\times\frac{H^6}{\pi}(aa')^3\Delta x^2\,\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& -4q_1 A_1\times\frac{H^6}{\pi}(aa')^3\Delta x^2\ln\left(-\frac{H^2}{4}aa'\Delta x^2\right)\theta(\Delta\eta-\|\Delta\vec{x}\|) \\
& -2\times\frac{H^4}{\pi}(aa')^2 S_G(y)\,\theta(\Delta\eta-\|\Delta\vec{x}\|)\Bigg\}.
\end{aligned}
\tag{A.15}
$$

Appendix B. The integrals over elementary functions

In this appendix we give in table B1 the definition of 20 simpler integrals appearing in (65) and (66), and some basic steps of their evaluation. Their late time limit is given in (67)–(81) of section 4.

The first four integrals in table B1 are very simple, and we do not discuss them here. The remaining integrals have the following general form,

$$
\begin{aligned}
\mathcal{J}_{\vec{n}}\left(\frac{k}{H},a\right) = & \frac{a^{n_1}[\ln(a)]^{n_2}H^4}{\pi}\int \mathrm{d}^4x'\,\mathrm{e}^{\mathrm{i}k\Delta\eta-\mathrm{i}\vec{k}\cdot\Delta\vec{x}} \\
& \times\frac{\partial^{2n_3}}{H^{2n_3}}\left[\theta(\Delta\eta-r)(a')^{n_1}[\ln(a')]^{n_2}(H\Delta x)^{2n_4}\left[\ln\left(\frac{H^2\Delta x^2}{4}\right)\right]^{n_5}\right],
\end{aligned}
\tag{B.1}
$$

where $(\vec{n})^T = (n_1,n_2,n_3,n_4,n_5)$ with $n_1=-1,0,1,2,3$, $n_2=0,1$, $n_3=0,1,2,3$, $n_4=0,1$ and $n_5=0,1$ and

Table B1. Integrals over elementary functions appearing in spin 0 contributions (65) and (66) to the source for the photon mode function.

n	I_n definition
1	$\frac{\ln(a)}{a}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\frac{\partial^2}{H^2}\Big[\frac{\delta^4(x-x')}{a'}\Big]$
2	$\frac{1}{a}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\frac{\partial_0}{H}\delta^4(x-x')$
3	$\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\delta^4(x-x')$
4	$\ln(a)\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\delta^4(x-x')$
5	$\frac{1}{\pi a}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\frac{\partial^6}{H^2}\Big[\theta(\Delta\eta - \|\Delta\vec{x}\|)\frac{1}{a'}\Big]$
6	$\frac{1}{\pi a}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\frac{\partial^6}{H^2}\Big[\theta(\Delta\eta - \|\Delta\vec{x}\|)\frac{1}{a'}\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big]$
7	$\frac{1}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^4\theta(\Delta\eta - \|\Delta\vec{x}\|)$
8	$\frac{1}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^4\Big[\theta(\Delta\eta - \|\Delta\vec{x}\|)\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big]$
9	$\frac{aH^2}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^2[\theta(\Delta\eta - \|\Delta\vec{x}\|)\,a']$
10	$\frac{a\ln(a)H^2}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^2[\theta(\Delta\eta - \|\Delta\vec{x}\|)\,a']$
11	$\frac{aH^2}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^2[\theta(\Delta\eta - \|\Delta\vec{x}\|)\,a'\ln(a')]$
12	$\frac{aH^2}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\partial^2\Big[\theta(\Delta\eta - \|\Delta\vec{x}\|)\,a'\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)\Big]$
13	$\frac{a^2H^4}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^2$
14	$\frac{a^2\ln(a)H^4}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^2$
15	$\frac{a^2H^4}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^2\ln(a')$
16	$\frac{a^2H^4}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^2\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)$
17	$\frac{a^3H^6}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^3\Delta x^2$
18	$\frac{a^3\ln(a)H^6}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^3\Delta x^2$
19	$\frac{a^3H^6}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)\,(a')^3\ln(a')\Delta x^2$
20	$\frac{a^3H^6}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\theta(\Delta\eta - \|\Delta\vec{x}\|)(a')^3\Delta x^2\ln\Big(-\frac{H^2}{4}\Delta x^2\Big)$

$$\Delta x^2 = -(\Delta\eta)^2 + r^2, \qquad r = \|\vec{x} - \vec{x}'\|, \qquad \partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu = -\partial_0^2 + \partial_i^2. \tag{B.2}$$

The first step is to extract the derivatives in (B.1) in front of the integral. In order to do that, one can use the following equlity,

$$e^{-ik\cdot\Delta x}\partial^2 = (\partial^2 + 2ik\cdot\partial - k^2), \quad k\cdot\Delta x = \eta_{\mu\nu}k^\mu\Delta x^\nu = -k^0\Delta\eta + \vec{k}\cdot(\vec{x} - \vec{x}')$$
$$k^0 = \|\vec{k}\|, \quad k^2 = \eta_{\mu\nu}k^\mu k^\nu = 0. \tag{B.3}$$

The next step is to integrate over the angles, resulting in,

$$\mathcal{J}_{\vec{n}}\left(\frac{k}{H}, a\right) = 4a^{n_1}[\ln(a)]^{n_2} \frac{(\partial^2+2\mathrm{i}k\cdot\partial)^{n_3}}{H^{2n_3-4}} \int_{-1/H}^{-1/(Ha)} \mathrm{d}\eta' \mathrm{e}^{\mathrm{i}k\Delta\eta}(a')^{n_1}[\ln(a')]^{n_2} \times \int_0^{\Delta\eta} drr^2 \frac{\sin(kr)}{kr}(H^2\Delta x^2)^{n_4}\left[\ln\left(\frac{H^2\Delta x^2}{4}\right)\right]^{n_5}. \tag{B.4}$$

In what follows we perform a substitution of variables to dimensionless quantities,

$$\tau \equiv -H\eta' = \frac{1}{a'}, \quad \Delta\tau = H\Delta\eta = \tau - \frac{1}{a}, \quad \rho = \frac{r}{\Delta\eta}, \quad K = \frac{k}{H}, \tag{B.5}$$

upon which (B.4) becomes,

$$\mathcal{J}_{\vec{n}}\left(\frac{k}{H}, a\right) = 4a^{n_1}(-1)^{n_4}[\ln(a)]^{n_2} \frac{(-\partial_0^2-2\mathrm{i}k\partial_0)^{n_3}}{KH^{2n_3}} \int_{1/a}^{1} \mathrm{d}\tau \mathrm{e}^{\mathrm{i}K\Delta\tau}\tau^{-n_1}[-\ln(\tau)]^{n_2} \times(\Delta\tau)^{2+2n_4}\int_0^1 \mathrm{d}\rho\rho\, \sin(K\Delta\tau\rho)(1-\rho^2)^{n_4}\left[\ln\left(\frac{1}{4}\Delta\tau^2(1-\rho^2)\right)\right]^{n_5}, \tag{B.6}$$

where we made use of the fact that the integral depends on time but not on space. The ρ-integral is doable in all cases ($n_4, n_5 = 0, 1$), and the result can be expressed in terms of elementary functions and the function $\mathcal{G}(z)$ defined in (82). Indeed, if we define,

$$\mathcal{K}_{(n_4,n_5)}(K\Delta\tau) \equiv (K\Delta\tau)^2 \int_0^1 \mathrm{d}\rho\rho\, \sin(K\Delta\tau\rho)(1-\rho^2)^{n_4}[\ln(1-\rho^2)]^{n_5}, \tag{B.7}$$

we have

$$\mathcal{K}_{(0,0)}(z) = \sin(z) - z\cos(z) \tag{B.8}$$

$$\mathcal{K}_{(1,0)}(z) = \frac{2}{z^2}[(3-z^2)\sin(z) - 3z\cos(z)] \tag{B.9}$$

$$\begin{aligned}\mathcal{K}_{(0,1)}(z) &= [\sin(z) - z\cos(z)][\mathrm{ci}(2z) - \gamma_E - \ln(z/2)] \\ &\quad - [\cos(z) + z\sin(z)]\left[\mathrm{si}(2z) + \frac{\pi}{2}\right] + 2\sin(z) \\ &= \left[\frac{\mathrm{i}-z}{2}(\mathcal{G}(z) + \ln(4)) + \mathrm{i}\right]\mathrm{e}^{-\mathrm{i}z} + \mathrm{c.c.}\end{aligned} \tag{B.10}$$

$$\begin{aligned}\mathcal{K}_{(1,1)}(z) &= \frac{2}{z^2}\Big\{[(3-z^2)\sin(z) - 3z\cos(z)][\mathrm{ci}(2z) - \gamma_E - \ln(z/2)] \\ &\quad - [(3-z^2)\cos(z) + 3z\sin(z)]\left[\mathrm{si}(2z) + \frac{\pi}{2}\right] \\ &\quad + (11-z^2)\sin(z) - 5z\cos(z)\Big\} \\ &= \frac{1}{z^2}[(3\mathrm{i} - 3z - \mathrm{i}z^2)(\mathcal{G}(z) + \ln(4)) + (11i - 5z - \mathrm{i}z^2)]\mathrm{e}^{-\mathrm{i}z} + \mathrm{c.c.}\end{aligned} \tag{B.11}$$

The remaining τ integral can also be done exactly for all of the integrals in table B1, and the results can be expressed in terms of elementary functions and the special functions $\mathcal{G}(z)$, $\mathcal{M}(x, z)$ and $\mathcal{V}(x, z)$ defined by the integrals (82)–(84). Finally, upon taking the late time limit

($a \to \infty$ and $k/(aH) \ll 1$) of these integrals one obtains the results given in the main text in equations (67)–(81).

Appendix C. The integrals over hypergeometric functions

Here we give the details of the computation of the late time limit ($a \gg 1$ and $k/(aH) \ll 1$) of integrals over generalized hypergeometric functions appearing in (65) and (66). We need to evaluate seven specific integrals, defined in table C1, all of which are captured by the master integral,

$$\mathcal{I}_{q,N}(\{\lambda_i\},\{\sigma_i\}) = \frac{a^2H^4}{\pi}\int d^4x'\, e^{ik\Delta\eta - i\vec{k}\cdot\Delta\vec{x}}\,\theta(\Delta\eta - \|\Delta\vec{x}\|)(a')^2 y^N\, {}_{q+1}F_q\left(\{\lambda_i\},\{\sigma_i\},\frac{y}{4}\right), \tag{C.1}$$

Here the following quantities are defined,

$$\Delta\eta = \eta - \eta', \qquad \Delta\vec{x} = \vec{x} - \vec{x}', \qquad y = H^2aa'[\|\Delta\vec{x}\|^2 - \Delta\eta^2]. \tag{C.2}$$

First we integrate over the spatial angular coordinates, where we introduced $\vec{r} = \Delta\vec{x}$ and $r = \|\vec{r}\,\|$ (so now $y = H^2aa'[r^2 - \Delta\eta^2]$),

$$\mathcal{I}_{q,N} = 4a^2H^4\int_{-1/H}^{-1/(aH)} d\eta'\, e^{ik\Delta\eta}(a')^2\int_0^{\Delta\eta} dr\, r^2\frac{\sin(kr)}{kr}\, y^N\, {}_{q+1}F_q\left(\{\lambda_i\},\{\sigma_i\},\frac{y}{4}\right). \tag{C.3}$$

Next we switch to a dimensionless time integration variable τ,

$$\tau \equiv -H\eta', \qquad \Delta\tau \equiv H\Delta\eta = \tau - \frac{1}{a}, \tag{C.4}$$

and to a dimensionless radial integration variable ρ,

$$r = \rho\Delta\eta = \frac{\rho\Delta\tau}{H}, \tag{C.5}$$

and define the shorthand notation for a dimensionless momentum,

$$K = \frac{k}{H}. \tag{C.6}$$

This turns the integral (C.3) into

$$\mathcal{I}_{q,N} = (-1)^N 4a^{2+N}\int_{1/a}^1 d\tau\, e^{iK\Delta\tau}\,\tau^{-2-N}(\Delta\tau)^{3+2N} \times \int_0^1 d\rho\,\rho^2(1-\rho^2)^N\,\frac{\sin(K\rho\Delta\tau)}{K\rho\Delta\tau}\,{}_{q+1}F_q\left(\{\lambda_i\},\{\sigma_i\},-\frac{a(\Delta\tau)^2}{4\tau}[1-\rho^2]\right). \tag{C.7}$$

Trigonometric functions are uniformly convergent on the whole real line, so we may expand them in a power series, and interchange the summation and integration operations in (C.7). The power series is

$$\frac{\sin(K\rho\Delta\tau)}{K\rho\Delta\tau} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(K\rho\Delta\tau)^{2n},$$

Table C1. Integrals over generalized hypergeometric functions appearing in spin 0 contributions (65) and (66) to the source for the photon mode function. The integrals are defined in terms of the master integral (C.1).

A	$\mathcal{I}_A = \mathcal{I}_{q,N}(\{\lambda_i\},\{\sigma_i\})$
I	$\mathcal{I}_{1,0}\Big(\{\frac{7}{2}+b_N,\frac{7}{2}-b_N\},\{6\}\Big)$
II	$\mathcal{I}_{2,0}\Big(\{\frac{7}{2}+b_N,\frac{7}{2}-b_N,1\},\{6,2\}\Big)$
III	$\mathcal{I}_{3,1}\Big(\{\frac{9}{2}+b_N,\frac{9}{2}-b_N,1,1\},\{7,2,2\}\Big)$
IV	$\mathcal{I}_{1,1}\Big(\{\frac{7}{2}+b_N,\frac{7}{2}-b_N\},\{6\}\Big)$
V	$\mathcal{I}_{2,1}\Big(\{\frac{7}{2}+b_N,\frac{7}{2}-b_N,1\},\{6,2\}\Big)$
VI	$\mathcal{I}_{3,1}\Big(\{\frac{7}{2}+b_N,\frac{7}{2}-b_N,1,1\},\{6,2,2\}\Big)$
VII	$\mathcal{I}_{3,2}\Big(\{\frac{9}{2}+b_N,\frac{9}{2}-b_N,1,1\},\{7,2,2\}\Big)$

and we can write the integral (C.7) as

$$\mathcal{I}_{q,N} = 4(-1)^N a^{2+N}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}K^{2n}\int_{1/a}^{1}\mathrm{d}\tau\;\mathrm{e}^{\mathrm{i}K\Delta\tau}\tau^{-2-N}(\Delta\tau)^{3+2N+2n}$$
$$\times\int_0^1\mathrm{d}\rho\;\rho^{2+2n}(1-\rho^2)^N\;{}_{q+1}F_q\bigg(\{\lambda_i\},\{\sigma_i\},-\frac{a(\Delta\tau)^2}{4\tau}[1-\rho^2]\bigg). \tag{C.8}$$

Next, making a substitution of variable,

$$\nu = 1-\rho^2, \tag{C.9}$$

puts the integral (C.8) in the form

$$\mathcal{I}_{q,N} = 2(-1)^N a^{2+N}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}K^{2n}\int_{1/a}^{1}\mathrm{d}\tau\;\mathrm{e}^{\mathrm{i}K\Delta\tau}\tau^{-2-N}(\Delta\tau)^{3+2N+2n}$$
$$\times\int_0^1\mathrm{d}\nu(1-\nu)^{n+\frac{1}{2}}\nu^N\;{}_{q+1}F_q\bigg(\{\lambda_i\},\{\sigma_i\},-\frac{a(\Delta\tau)^2}{4\tau}\nu\bigg), \tag{C.10}$$

where now the integral over ν can be done exactly[3],

$$\mathcal{I}_{q,N} = 2(-1)^N(N!)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}K^{2n}\frac{\Gamma\Big(n+\frac{3}{2}\Big)}{\Gamma\Big(n+N+\frac{5}{2}\Big)}a^{2+N}\int_{1/a}^{1}\mathrm{d}\tau\;\mathrm{e}^{\mathrm{i}K\Delta\tau}\tau^{-2-N}$$
$$\times(\Delta\tau)^{3+2N+2n}\;{}_{q+2}F_{q+1}\bigg(\{\lambda_i,1+N\},\bigg\{\sigma_i,n+N+\frac{5}{2}\bigg\},-\frac{a(\Delta\tau)^2}{4\tau}\bigg). \tag{C.11}$$

[3] One can make use of the integral 7.512.12 from [72]. However, that integral requires $aa'\Delta\tau^2/4<1$, which is equivalent to $\frac{a'}{a}+\frac{a}{a'}<6$, which is broken at early times when t' is much before t (more precisely when $a'/a<3-\sqrt{8}$). Since the result of integration is proportional to a hypergeometric function, it is reasonable to assume that the result applies in the whole region of integration in the sense that the hypergeometric function in (C.10) is defined on the whole complex plane (except on the cuts).

Next we start approximating the integral under the sum in (C.11), which we denote as,

$$\mathcal{J}^{n}_{q,N} = a^{2+N}\int_{1/a}^{1} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}K\Delta\tau}\tau^{-2-N}(\Delta\tau)^{3+2N+2n} \times {}_{q+2}F_{q+1}\left(\{\lambda_i, 1+N\}, \left\{\sigma_i, n+N+\frac{5}{2}\right\}, -\frac{a(\Delta\tau)^2}{4\tau}\right). \tag{C.12}$$

We do not know how to evaluate the full integral, instead we seek to find the late time behavior for $a \gg 1$ and $k/(aH) \ll 1$. In particular, we want to isolate the late time growing terms up to order $\ln(a)$ or a, depending whether it appears in the integral over F or over G, respectively.

Let us now make a variable substitution,

$$\tau = \frac{t}{a} \;\Rightarrow\; t = \frac{a}{a'}, \tag{C.13}$$

which puts the integral (C.12) into the form

$$\mathcal{J}^{n}_{q,N} = a^{-2n}\int_{1}^{a} \mathrm{d}t\, \mathrm{e}^{\frac{\mathrm{i}K(t-1)}{a}} t^{-2-N}(t-1)^{3+2N+2n} \times {}_{q+2}F_{q+1}\left(\{\lambda_i, 1+N\}, \left\{\sigma_i, n+N+\frac{5}{2}\right\}, -\frac{(t-1)^2}{4t}\right). \tag{C.14}$$

Because of the factor a^{-2n} outside, we need to identify only the the contributions to the remaining integral that grow as a^{2n} or faster in the late time limit. We will do that by approximating the integrand by a much simpler function, which we will be able to integrate over. It is of no relevance if we retain some terms that contribute to subleading orders at late times (i.e. that grow slower than a^{2n}), since in the end we will neglect them anyway. What is important is just for the new approximated integrand to capture correctly the relevant late time terms.

The smallest parameter in the first set of parameters of the hypergeometric function in the integrand can be 1. Therefore, the leading behavior of the hypergeometric function for large arguments is $\sim t^{-1}$ (and possibly times some integer powers of $\ln(t)$, which does not change the argument). Therefore, for large t, the leading behavior of the integrand is $\sim t^{N+2n}$ (the phase factor $\exp[iK(t-1)/a]$ does not change the argument either). This means that the leading late time behavior is (not counting the powers of logarithms),

$$\mathcal{J}^{n}_{q,N} \sim a^{-2n}\int^{a} \mathrm{d}t\, t^{N+2n} \sim a^{1+N}, \tag{C.15}$$

and is independent of parameter n. Therefore, by extracting the first $N+2$ (recall that $N = 0, 1, 2$) terms in the asymptotic expansion of the hypergeometric function we obtain all the relevant contributions to $\mathcal{J}^{n}_{q,N}$ at late times. The asymptotic expansion of hypergeometric functions will take the form (16.11.6 and 16.11.2 from [73]),

$${}_{q+2}F_{q+1}\left(\{\lambda_i, 1+N\}, \left\{\sigma_i, n+N+\frac{5}{2}\right\}, -\frac{(t-1)^2}{4t}\right) \approx \sum_{l=1}^{N+1+s_*} t^{-l} \times c^{n,l}_{q,N}(\lambda_i, \sigma_i, \ln(t)) \equiv \mathcal{C}^{n}_{q,N}(\{\lambda_i\}, \{\sigma_i\}, t), \tag{C.16}$$

where the c-coefficients can contain some integer powers of $\ln(t)$. Therefore, replacing the hypergeometric function by its asymptotic form (C.16) captures correctly the late time limit of the integral (C.14) and we can write,

$$\mathcal{J}^n_{q,N} \approx a^{-2n} \int_1^a \mathrm{d}t\, \mathrm{e}^{\frac{\mathrm{i}K(t-1)}{a}} t^{-2-N}(t-1)^{3+2N+2n}\, \mathcal{C}^n_{q,N}(\{\lambda_i\}, \{\sigma_i\}, t) + \mathcal{O}(a^{s_i}), \tag{C.17}$$

where $\mathcal{C}$ is defined in (C.16) and $s_{i=F} = 0$ and $s_{i=G} = 1$ (for notational simplicity we do not expressly include logarithmic corrections in the order of the estimate). The resulting integrals can all be performed. But before doing that we find it far more convenient to switch back to the integration variable $\tau = t/a$ and switch the order of integration and summation over n in (C.11),

$$\mathcal{I}_{q,N} \approx 2(-1)^N (N!) a^{2+N} \int_{1/a}^1 \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}K\Delta\tau} \tau^{-2-N} (\Delta\tau)^{3+2N} \times \sum_{n=0}^{\infty} \frac{(-1)^n (K\Delta\tau)^{2n}}{(2n+1)!} \frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma\left(n+N+\frac{5}{2}\right)} \mathcal{C}^n_{q,N}(\{\lambda_i\}, \{\sigma_i\}, a\tau) + \mathcal{O}(a^{s_i}). \tag{C.18}$$

Now equations (16.11.2)–(16.11.6) of [73] allow us to express the relevant coefficients of asymptotic expansions of the hypergeometric functions (below we define $X(t) = (t-1)^2/t$) as follows,

$$\mathcal{C}^n_{1,0} = \left(\left\{\frac{7}{2} + b_N, \frac{7}{2} - b_N\right\}, \{6\}, t\right) = \frac{10(2n+3)}{\beta} \left\{\frac{1}{X(t)} - \frac{8(2n+1)}{(\beta-4)} \frac{1}{tX(t)}\right\}, \tag{C.19}$$

$$\mathcal{C}^n_{2,0}\left(\left\{\frac{7}{2} + b_N, \frac{7}{2} - b_N, 1\right\}, \{6, 2\}, t\right) = \frac{10(2n+3)}{\beta} \left\{\frac{\ln[X(t)]}{X(t)} + \Xi\left(\frac{7}{2} + b_N, \frac{7}{2} - b_N, 6, n + \frac{5}{2}\right) \frac{1}{X(t)} + \frac{8(2n+1)}{(\beta-4)} \frac{1}{tX(t)}\right\}, \tag{C.20}$$

$$\mathcal{C}^n_{3,1}\left(\left\{\frac{9}{2} + b_N, \frac{9}{2} - b_N, 1, 1\right\}, \{7, 2, 2\}, t\right) = \frac{12(2n+5)}{(\beta+6)} \left\{\frac{\ln[X(t)]}{X(t)} + \Xi\left(\frac{9}{2} + b_N, \frac{9}{2} - b_N, 7, n + \frac{7}{2}\right) \frac{1}{X(t)} + \frac{10(2n+3)}{\beta} \frac{1}{X(t)^2} - \frac{40(2n+3)(2n+1)}{\beta(\beta-4)} \frac{1}{tX(t)^2}\right\}, \tag{C.21}$$

$$\mathcal{C}^n_{1,1}\left(\left\{\frac{7}{2} + b_N, \frac{7}{2} - b_N\right\}, \{6\}, t\right) = \frac{80(2n+5)(2n+3)}{\beta(\beta-4)} \frac{1}{X(t)^2}, \tag{C.22}$$

$$\mathcal{C}^n_{2,1}\left(\left\{\frac{7}{2} + b_N, \frac{7}{2} - b_N, 1\right\}, \{6, 2\}, t\right) = \frac{10(2n+5)}{\beta} \left\{\frac{1}{X(t)} - \frac{8(2n+3)}{(\beta-4)} \frac{1}{X(t)^2}\right\}, \tag{C.23}$$

$$
\begin{aligned}
&\mathcal{C}^{n}_{3,1}\left(\left\{\frac{7}{2}+b_N,\frac{7}{2}-b_N,1,1\right\},\{6,2,2\},t\right)\\
&\quad=\frac{10(2n+5)}{\beta}\left\{\frac{\ln[X(t)]}{X(t)}+\Xi\left(\frac{7}{2}+b_N,\frac{7}{2}-b_N,6,n+\frac{7}{2}\right)\frac{1}{X(t)}\right.\\
&\qquad\left.+\frac{8(2n+3)}{(\beta-4)}\frac{1}{X(t)^2}\right\},
\end{aligned}
\tag{C.24}
$$

$$
\begin{aligned}
&\mathcal{C}^{n}_{3,2}\left(\left\{\frac{9}{2}+b_N,\frac{9}{2}-b_N,1,1\right\},\{7,2,2\},t\right)\\
&\quad=\frac{6(2n+7)}{(\beta+6)}\left\{\frac{\ln[X(t)]}{X(t)}+\left[1+\Xi\left(\frac{9}{2}+b_N,\frac{9}{2}-b_N,7,n+\frac{9}{2}\right)\right]\frac{1}{X(t)}\right.\\
&\qquad\left.+\frac{40(2n+5)(2n+3)}{\beta(\beta-4)}\frac{1}{X(t)^3}\right\}.
\end{aligned}
\tag{C.25}
$$

When these expressions are inserted into equation (C.18), the corresponding series over n can be performed. For example, for the first integral from table C1 one obtains,

$$
\begin{aligned}
&\mathcal{I}_{1,0}\left(\left\{\frac{7}{2}+b_N,\frac{7}{2}-b_N\right\},\{6\}\right)\\
&\quad\approx\frac{40a}{\beta K}\int_{1/a}^{1}\frac{\mathrm{d}\tau}{\tau^2}\,\mathrm{e}^{\mathrm{i}K\Delta\tau}\left[\tau\,\sin(K\Delta\tau)-\frac{8(K\Delta\tau)}{a(\beta-4)}\cos(K\Delta\tau)\right].
\end{aligned}
\tag{C.26}
$$

This integral can be evaluated and expressed in terms of elementary functions and $\mathcal{G}$. When expanded in powers of $1/a$, one obtains equation (87).

An analogous procedure can be utilized to evaluate the rest of table C1, yielding the remaining integrals (88)–(92). We do not present here the details of that evaluation.

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