Physics Letters B 769 (2017) 281–288

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gravitational microlensing in Verlinde's emergent gravity

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ARTICLE INFO

Article history:
Received 6 December 2016
Received in revised form 29 March 2017
Accepted 29 March 2017
Available online 31 March 2017
Editor: M. Trodden

ABSTRACT

We propose gravitational microlensing as a way of testing the emergent gravity theory recently proposed by Eric Verlinde [1]. We consider two limiting cases: the dark mass of maximally anisotropic pressures (Case I) and of isotropic pressures (Case II). Our analysis of perihelion advancement of a planet shows that only Case I yields a viable theory. In this case the metric outside a star of mass M_* can be modeled by that of a point-like global monopole whose mass is M_* and a deficit angle $\Delta = \sqrt{(2GH_0M_*)/(3c^3)}$, where H_0 is the Hubble rate and G the Newton constant. This deficit angle can be used to test the theory since light exhibits additional bending around stars given by, $\alpha_D \approx -\pi\Delta/2$. This angle is independent on the distance from the star and it affects equally light and massive particles. The effect is too small to be measurable today, but should be within reach of the next generation of high resolution telescopes. Finally we note that the advancement of periastron of a planet orbiting around a star or black hole, which equals $\pi\Delta$ per period, can be also used to test the theory.

1. Global monopole metric

In a recent paper Eric Verlinde [1] has proposed a novel emergent gravity theory. The most important claim of the theory is that dark matter has no particle origin but instead it is an emergent manifestation in modified gravity. Assuming spherical symmetry Verlinde obtains,

$$\int_0^r \frac{GM_D(r')^2}{r'^2}dr' = \frac{cH_0M_B(r)r}{6}, \tag{1}$$

where $H_0 = 2.36 \times 10^{-18}\ \mathrm{s}^{-1} \simeq \sqrt{\Lambda/3}$ is the current Hubble parameter, Λ is the cosmological constant (whose value is determined by the current dark energy density), $G = 6.674 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg\,s}^2)$ is the Newton's constant, $c \approx 3 \times 10^8$ m/s is the speed of light, $M_B(r)$ ($M_D(r)$) is the baryonic mass (dark mass) inside a sphere of radius r.

Eq. (1) implies that for a star of uniform density ρ_*, $M_* = \frac{4\pi}{3}r^3\rho_*$, inside the star,

$$M_D(r) = \sqrt{\frac{2cH_0M_*r^5}{3GR_*^3}} \propto r^{5/2}, \qquad r < R_*, \tag{2}$$

where R_* denotes star's radius. On the other hand, outside the star, $M_D \propto r$, and we have,

$$M_D(r) = \sqrt{\frac{cH_0M_*}{6G}} \times r, \qquad r \geq R_*. \tag{3}$$

The main goal of this paper is to construct the metric tensor that consistently incorporates (1) within the Verlinde's emergent gravity theory and to investigate how that metric can be used to test the theory. The fundamental assumption we make is that the theory admits metric formulation that can be obtained by solving suitably modified Einstein's equations (23). In the Appendix we perform a detailed analysis of such a theory. Unfortunately, we do not have all of the information needed to fully specify the metric. A reasonable assumption is that the modified stress energy tensor is diagonal, $T_\mu{}^\nu = \mathrm{diag}[-\rho, P_r, P_\theta, P_\varphi]$, see (24). In the weak gravitational field regime (which is of our principal concern here) that should be justified. This leaves us with four unknown functions: energy density ρ (which we can determine from (1)) and three unknown pressures: P_r, P_θ, P_φ. For spherically symmetric mass distribution the two angular pressures must be equal, $P_\theta = P_\varphi \equiv P_\perp$. The remaining pressures are unknown, but are nevertheless tightly constrained by the TOV equation (41), however not enough to be completely specifiable. Rather than attempting to extend Verlinde's theory to obtain a relationship between the energy density and pressures, here we consider two simple and plausible *Ansätze*:

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http://dx.doi.org/10.1016/j.physletb.2017.03.061

Case I: Field-like dark mass: $P_\perp = 0$;
Case II: Particle-like dark mass: $P_\perp = P_r \equiv P$.

In addition, in Case II, we assume that inside a star (where except at very small radii baryonic contribution dominates) baryonic matter is non-relativistic, and hence $P_B \ll \rho_B$, implying also $P \ll \rho$.

The extensive analysis in the Appendix (*cf.* Eqs. (39), (50) and (67)) shows that the metric tensor is of the form,

$$ds^2 = -\left(1 - \Delta - \frac{1-w'}{2}\frac{H_0^2 r^2}{c^2} - \frac{2GM_*}{c^2 r}\right)\left(\frac{r}{r_H}\right)^{(1+w')\Delta} c^2 dt^2 + \frac{dr^2}{1 - \Delta - \frac{H_0^2 r^2}{c^2} - \frac{2GM_*}{c^2 r}} + r^2 d\Omega_2^2\,, \qquad (4)$$

where $d\Omega_2^2 = d\theta^2 + \sin^2(\theta) d\varphi^2$ is the metric of the two-dimensional unit sphere ($\theta \in [0,\pi]$, $\varphi \in [0, 2\pi)$), $w' = P/\rho = -1$ is the equation of state parameter for Case I ($P_r = P$, $P_\perp = 0$) and $w' = 0$ for Case II ($P_r = P = P_\perp$). This then implies that outside the star for Case I the metric can be written as that of a point-like global monopole on de Sitter background,[1] and Δ is the deficit solid angle defined by,

$$\Delta = \sqrt{\frac{2GH_0 M_*}{3c^3}}\,. \qquad (5)$$

That Δ in (5) indeed represents a deficit solid angle that cannot be removed by a coordinate transformation can be shown as follows. Observe firstly that the volume (surface area) of a two sphere of radius r is $\Omega(S^2(r)) = 4\pi r^2$, which defines the coordinate r (these coordinates are similar to those used in the Schwarzschild metric). Now, one can try to remove Δ by the following coordinate transformations,

$$\tilde r = \frac{r}{\sqrt{1-\Delta}}\,, \qquad \tilde t = (1-\Delta)^{\frac{1}{2}[1+(1+w')\Delta]} \times t\,,$$
$$(1-\Delta) d\Omega_2^2 = d\tilde\Omega_2^2 \qquad (6)$$

after which Δ seems to disappear from the metric (4). Indeed, the equivalent metric is,

$$ds^2 = -\left(1 - \frac{1-w'}{2}\frac{H_0^2 \tilde r^2}{c^2} - \frac{2G\tilde M_*}{c^2 \tilde r}\right)\left(\frac{\tilde r}{r_H}\right)^{(1+w')\Delta} c^2 d\tilde t^{\,2} + \frac{d\tilde r^2}{1 - \frac{H_0^2 r^2}{c^2} - \frac{2G\tilde M_*}{c^2 r}} + \tilde r^2 d\tilde\Omega_2^2\,, \qquad (7)$$

where

$$\tilde M_* = \frac{M_*}{(1-\Delta)^{3/2}}\,. \qquad (8)$$

However, Δ does not entirely disappear since in the new coordinates,

$$d\tilde\Omega_2^2 = d\tilde\theta^2 + \sin^2\left(\frac{\tilde\theta}{\sqrt{1-\Delta}}\right) d\tilde\varphi^2 \qquad (9)$$

[1] Global monopoles are topological solutions of classical equations of motion of a scalar field theory with 3 real scalar fields, $\vec\Phi = (\Phi_1, \Phi_2, \Phi_3)^T$ whose Lagrangian is $O(3)$ symmetric and whose potential exhibits a spontaneous symmetry breaking, $V(\vec\Phi) = (\lambda/4)(\vec\Phi^T\cdot\vec\Phi - \Phi_0^2)^2$. One can show that in this case the solution with topological charge 1 [2] will backreact on the metric such to induce a solid deficit angle, $\Delta = 8\pi G\Phi_0^2$ ($c = 1$), see e.g. [3,4]. From the gravitational point of view compact star-like dense objects (black hole mimickers) built out of topologically charged scalar matter [5] resemble ordinary stars in Verlinde's theory.

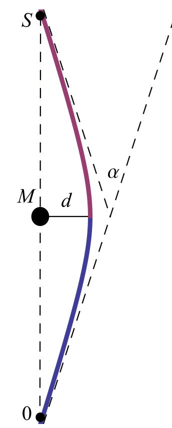

Fig. 1. Light deflection around a star of mass $M_* = M$. The closest distance to the center of the star is d. The deflection angle α can be calculated by integrating (12) along the path of light from the source S to the observer O.

and $\tilde\varphi$ and $\tilde\theta$ take values in the intervals,

$$\tilde\theta \in [0, \pi\sqrt{1-\Delta}\,]\,, \qquad \tilde\varphi \in [0, 2\pi\sqrt{1-\Delta}\,)\,. \qquad (10)$$

It is an easy exercise to calculate the surface area of the two dimensional sphere of radius $\tilde r$ in these new coordinates,

$$\Omega(S^2(\tilde r)) = 4\pi(1-\Delta)\tilde r^2\,. \qquad (11)$$

From this result it is obvious that the sphere contains a solid angle deficit of, $\delta\Omega = -4\pi\Delta$, completing the proof. In the following section we discuss the physical significance of this result.

2. Gravitational lensing

In this section we consider the lensing in a metric given by (4) and (7). The usual weak (linearized) lensing formula for the deflection angle (in radians),

$$\alpha = -\frac{1}{c^2}\int \nabla_\perp\big[\phi(\vec x) + \psi(\vec x)\big] d\ell\,, \qquad (12)$$

where ℓ is the path along the light geodesic (from the source to the observer, see Fig. 1), $\nabla_\perp$ is the gradient operator in the plane orthogonal to the propagation of light and ϕ and ψ are the two gravitational potentials (corresponding to the g_{00} and g_{rr} metric perturbations). Outside the star these potentials can be read off from (4),

$$\phi(r) = -\frac{GM_*}{r} - \frac{1-w'}{2}\frac{H_0^2}{2} r^2 + \frac{c^2(1+w')\Delta}{2}\ln\left(\frac{r}{r_H}\right) - \frac{c^2\Delta}{2}\,,$$
$$\psi(r) = -\frac{GM_*}{r} - \frac{H_0^2}{2} r^2 - \frac{c^2\Delta}{2}\,. \qquad (13)$$

The lensing formula (12) can be used for the first three parts of the potential (the one induced by the star mass, by the Universe's expansion and the logarithmic piece), but it cannot be used for the constant contribution, $\phi_D = -c^2\Delta/2$, from the dark mass simply because, $\nabla_\perp\phi_D = 0$ (how to calculate light deflection due to ϕ_D is discussed below). For that reason it is better to use the second form of the metric (7), in which case the gravitational potential is,

$$\tilde\phi = -\frac{G\tilde M_*}{\tilde r} - \frac{1-w'}{2}\frac{H_0^2}{2}\tilde r^2 + \frac{c^2(1+w')\Delta}{2}\ln\left(\frac{\tilde r}{r_H}\right),$$
$$\tilde\psi = -\frac{G\tilde M_*}{\tilde r} - \frac{H_0^2}{2}\tilde r^2 \qquad (14)$$

Inserting this into (12) gives for the lensing angle,

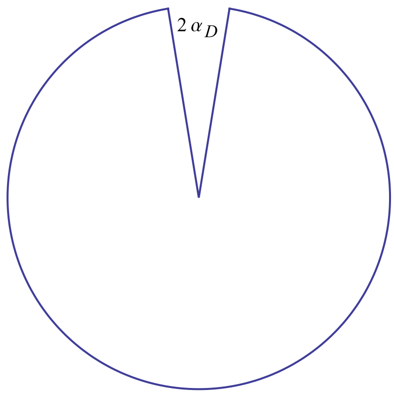

Fig. 2. The total deficit angle $2\alpha_D = 2\pi(1-\sqrt{1-\Delta})$ in the equatorial plane $\theta = \pi/2$ around a star of mass $M_* = M$. The two rays emanating from the origin are identified. Due to the angle deficit, a light ray from a distant star and observed by a distant observer exhibits a 'change' in direction given by α_D.

$$\alpha_1 = -\frac{4GM_*}{c^2(1-\Delta)^{3/2}d} + \frac{3-w'}{2}\frac{H_0^2\ell d}{c^2} - \frac{\pi}{2}(1+w')\Delta\,, \tag{15}$$

where $\ell = \overline{SO}$ is the distance from the source S to the observer O and d is the closest distance of light from the star center, see Fig. 1. The first integral was evaluated by assuming that the source and the observer are infinitely far from the star, which is for small lensing angles an excellent approximation. It is instructive to compare the first two contributions in (15). Note first that the second part is accurate only when $H_0\ell \ll c$, because otherwise the linear lensing formula (12) fails. This means that the second contribution is $\ll 2H_0dc$. The two contributions will be approximately equal when $2GM_*/d \sim H_0d \times (H_0\ell) \ll H_0dc$, that means when the distance d expressed in units of the Schwarzschild radius of the star R_S becomes comparable to the distance expressed in units of the Hubble distance d_H, *i.e.* when $d \simeq \sqrt{R_S d_H}\sqrt{d_H/\ell}$.

In order to get the lensing generated by the deficit angle Δ, note that it is convenient to assume that the plane shown in Fig. 1 corresponds to the equatorial plane $\theta = \pi/2$, or equivalently $\tilde{\theta} = (\pi/2)\sqrt{1-\Delta}$. In that plane the azimuthal angle takes values in the interval, $\tilde{\phi} \in [0, 2\pi\sqrt{1-\Delta})$. The geometry is flat and can be represented by a plane in which the wedge whose angle equals to the deficit angle $2\pi[1-\sqrt{1-\Delta}]$ is cut out, and the opposite sides of the wedge are identified, representing conical geometry (of a global string or a point mass in two spatial directions), see Fig. 2. Consider now a ray in this conical geometry propagating from a distant source (on one side of the central point where the star is located) to a distant observer (on the other side of the central point). Due to the conical geometry, to a very good approximation that ray will exhibit an angle deflection of one half of the total deficit angle, *i.e.*

$$\alpha_D = -\pi\left(1-\sqrt{1-\Delta}\right) \approx -\frac{\pi}{2}\Delta\,. \tag{16}$$

The total deflection angle is then simply the sum of (15) and (16),

$$\alpha = \alpha_1 + \alpha_D = -\frac{4GM_*}{c^2(1-\Delta)^{3/2}d} + \frac{3-w'}{2}\frac{H_0^2\ell d}{c^2} - \frac{\pi}{2}(1+w')\Delta - \pi\left(1-\sqrt{1-\Delta}\right). \tag{17}$$

Let us now see whether α_D can be large enough to be measurable for typical stars. Consider first our Sun for which,

$$M_* = M_\odot = 1.989\times 10^{30}\ \mathrm{kg}\,,\quad R_* = R_\odot = 6.957\times 10^{8}\ \mathrm{m}\,,\quad H_0 = 2.36\times 10^{-18}\ \mathrm{s}^{-1}\,, \tag{18}$$

and therefore from (17),

$$\alpha_\odot = \Big(-1.74\frac{R_\odot}{d} + 2.26\times 10^{-12}\frac{3-w'}{4}\frac{d}{R_\odot}\frac{\ell}{d_H} - 9\times 10^{-7}(2+w')\Big)[\mathrm{arcsec}] \qquad (\mathrm{Sun})\,. \tag{19}$$

At first sight the second and third contributions look desperately small. Note however that, while the first contribution to α in (19) drops with the distance from the Sun, the second contribution grows and the third contribution stays constant. Let us now compare these numbers with the sensitivity of modern observational probes. For example, the ESA's GAIA mission [6–8] (whose purpose is to make 3 dimensional optical image of about one billion stars in the Milky Way by measuring parallaxes of stars) has a sensitivity of about 20 μarcsec for stars of magnitude 15 or larger and 7 μarcsec for stars of magnitude 10 or larger, which is about a factor 20 (7) too low to be able to observe α_D induced by the Sun. Analogously, the Event Horizon Telescope (EHT) [9,10] – whose goal is to make a radio map (in wavelengths of about 1 mm) of the neighborhood of the Milky Way black hole located near the galactic center in Sag A^* – will have angular resolution of about 10 μarcsec, a factor of 10 above the resolution required to see α_D generated by the Sun. From this analysis we see that the next generation of even more precise observatories will probably reach the required precision to be able to see the microlensing induced by the dark potential ϕ_D of the Sun.

Let us now have a closer look at our Milky Way black hole, whose mass is about $M_{\rm BH} = 4.3\times 10^6\ M_\odot$ and whose Schwarzschild radius is, $R_S = 2GM_{\rm BH}/c^2 = 1\times 10^{10}$ m,

$$\alpha_{\rm BH} = -\frac{2R_S}{d}[\mathrm{rad}] + \Big(9.6\times 10^{-18}\frac{3-w'}{4}\frac{d}{R_S}\frac{\ell}{d_H} - 1.7\times 10^{-3}(2+w')\Big)[\mathrm{arcsec}] \quad (\mathrm{SagA^*\ black\ hole})\,. \tag{20}$$

The result for α_D is within the reach of EHT, but it is not clear whether the EHT mission will be able to measure so small deflection angles in the vicinity of the black hole.

Note that the angular deflection induced by α_D is equal for all objects, independently on how fast they move. That means that (massive) objects that move with a speed $v < c$ will exhibit the same 'dark' deflection angle α_D as light. This is not true for the usual gravitational lensing. Indeed, by solving the geodesic equation for ultra-relativistic particles one obtains,[2]

$$\alpha_m = \left(1+\frac{1}{\gamma^2}\right)\Big(-\frac{2GM_*}{c^2(1-\Delta)^{3/2}d} + \frac{1-w'}{2}\frac{H_0^2\ell d}{c^2} - \frac{\pi}{2}(1+w')\Delta\Big) - \frac{2GM_*}{c^2(1-\Delta)^{3/2}d} + \frac{H_0^2\ell d}{c^2} - \pi\left(1-\sqrt{1-\Delta}\right) \tag{21}$$

where $\gamma = [1-(v/c)^2]^{-1/2} \gg 1$ and we have neglected the change of velocity parallel to the motion. In the limit when $\gamma \to \infty$, Eq. (21) reduces to Eq. (17), as it should. By comparing the angle of deflection of light (17) with that of relativistic particles (21)

[2] The relevant geodesic equations for the four velocity, $u^\mu = (u^0, \vec{u})$, $\vec{u} = (\vec{u}_\perp, \vec{u}_\parallel)$ are,

$$\frac{du_\perp}{d\lambda} + \frac{(u^0)^2}{c^2}\nabla_\perp\tilde{\phi} + \frac{(\vec{u})^2}{c^2}\nabla_\perp\tilde{\psi} = 0\,,\quad \frac{d\ell}{d\lambda} = \|\vec{u}\| \equiv u\,,\quad (u^0)^2 = \vec{u}^2 + c^2 = \gamma^2c^2\,.$$

The deflection angle is then, $\alpha \approx u_\perp/u$. By solving the geodesic equations for $u_\perp/u$ and adding the deficit angle contribution one arrives at (21).

(such as cosmic rays which have a large γ factor) one could in principle isolate the component that is independent on the speed of motion, thereby testing the Verlinde formula (1). The measurement will not be easy, but it is not impossible.

Finally, we point out that – due to the deficit angle – the dark mass M_D will cause an addition advancement of perihelion/periastron of a planet, which is per orbit,[3]

$$\Delta\phi_{\rm periastron} = \pi\Delta - \pi(1+w')\Delta\frac{c^2L^2}{2G^2M_*^2}\times\left(1-\frac{G^2M_*^2}{c^2L^2}-\frac{2G^4M_*^4}{c^4L^4}\right)[\text{per orbit}]\,, \qquad (22)$$

where L is the angular momentum per unit mass of a planet. Note that the method used to derive (22) reproduces correctly only the linearized part of the exact expression for the geometric effect, which equals: $2\pi(1-\sqrt{1-\Delta})$. For a planet in the solar system the first (geometric) term produces a tiny effect. Indeed, for any planet in the Solar system the perihelion advancement is, $\Delta\phi_{\rm perihelion,1} = \pi\Delta = 1.8$ μarcsec per orbital period. This is to be contrasted, for example, with the general relativistic advancement of perihelion of Mercury, $\Delta\phi_{\rm GR} = 0.1$ arcsec per period, which is about 50000 times larger. However, the second term in (22) is large. For example, for Mercury the classical radius is $r_c = L^2/(GM_\odot) = 5.5\times10^{10}$ m and the Schwarzschild radius of the Sun is, $r_s = 2GM_\odot/c^2 \simeq 2950$ m and thus, $\Delta\phi_{\rm perihelion,2\ Mercury} \approx -\pi\Delta\times r_c/r_s = -34$ arcsec per orbital period, which is much larger than the general relativistic effect (for other planets in the Solar system the effect is even larger). Based on this observation alone, one concludes that either (1) Verlinde's emergent gravity is ruled out, or (2) the dark mass in Verlinde's theory is field-like and produces a highly anisotropic pressure (Case I). If latter is true (which is the one we favor) $w' = -1$ in Eq. (22), and the only effect that survives is the first (geometric) term which produces a tiny effect that is equal for all planets in the solar system. Next, recall that the accuracy of current measurements for Mercury is at the level of $\sim 1\% = 10^{-2}$, which is still far above the sensitivity ($\sim 10^{-5}$) required to measure the perihelion advancement generated by the geometric term in (22) (Case I when $w' = -1$). For larger stars and large black holes however, the advancement of periastron is much larger and therefore potentially easier to observe.

3. Discussion

In this letter we construct the metric tensor associated with spherically symmetric distribution of matter in Verlinde's emergent gravity [1]. We consider two cases. In Case I (maximally anisotropic pressures) we assume that the angular pressures vanish and (outside the star) obtain the metric tensor of a global monopole. In Case II we assume isotropic pressures. The metric tensor containing both cases is given in Eq. (4), where different cases are expressed through the equation of state parameter, $w' = P/\rho$ (in Case I, $w' = -1$ and in Case II, $w' = 0$). Our analysis of advancement of perihelion/periastron (22) shows that only Case I represents a viable model. In that case the metric (4) exhibits a solid angle deficit (5). Here we suggest that this deficit angle can be used to test the theory *via* gravitational lensing and periastron advancement. We consider two cases: the metric of the Sun and that of our Galactic black hole. Even though the effect is tiny, the next generation of observatories are expected to reach the angular sensitivity needed to be able to measure it.

Furthermore, we note that the effect of dark mass (1)–(3) can be tested by comparing the deflection angle of relativistic particles (21) with that of light (17) and by precisely measuring the advancement of periastron (22) of planets orbiting around stars.

The extensive experience acquired in microlensing used for tracking down MACHOs and for discovering new (Earth like) extra-Solar planets might be of use for detecting dark mass [11–13].

Since the effect of dark mass is cumulative, a much larger effect is generated by galaxies and clusters of galaxies, and a preliminary discussion of that effect (that mimics dark matter) can be found in the original reference [1]. Even though galaxies and clusters are expected to contribute much stronger to the lensing than planets in the solar system, we are reluctant to make any quantitative claims on how light will be affected by Verlinde's emergent gravity in these systems. This is because large systems are not yet virialized and therefore the role of Universe's expansion can be essential for obtaining the correct answer for the lensing of light. Presumably, the correct formalism is to treat galaxies and clusters as small sized perturbations on top a spatially homogeneous expanding Universe and we hope to address in more detail that question in a future publication.

After the first version of this work several references have appeared [14,15] and [16] that also discuss how to test Verlinde's emergent gravity. Ref. [14] uses a large set of galaxies and clusters to check Verlinde's theory, and find that it gives results consistent with observations. Brower et al. [14] do point out that they apply the formula of Verlinde in the region where it is strictly speaking not applicable (because of a large deviation from spherical symmetry). Milgrom and Sanders [16] point out that MOND is currently better tested and constrained and that the purpose of papers like the present is to sharpen the critical assessment of Verlinde's theory. Finally, Ref. [15] remarks that Solar system measurements are reaching the accuracy needed to test Verlinde's theory. In particular, the measurements of the advancement of perihelion of Mars are at the level that Verlinde's theory can be marginally tested.

Acknowledgements

We acknowledge inspiring discussions with Colin Norman and Michael Kopp. This work is in part supported by the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW). Lei-Hua Liu is funded by the Chinese Scholarschip Council (CSC).

Appendix A

In this appendix we solve the Einstein equation sourced by the dark mass. We solve the Einstein equation,

$$G_\mu{}^\nu + \delta_\mu{}^\nu\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}\,, \qquad (23)$$

[3] The result (22) follows from the conservation equation for motion of a planet,

$$\left(\frac{dx}{d\varphi}\right)^2 + (1-\Delta)\frac{c^2L^2}{G^2M_*^2} - 2x + (1-\Delta)x^2 - \frac{2G^2M_*^2}{c^2L^2}x^3 = \frac{E^2L^2}{c^2G^2M_*^2}\left(\frac{x}{x_H}\right)^{(1+w')\Delta},$$

where $x = L^2/(GM_*r)$. Taking a derivative with respect to φ and expanding the solution around the classical one $x_0 1 + e\cos(\varphi)$, as $x(\varphi) = x_0 + x_1$, where e denotes ellipticity, one obtains,

$$x_1 = x_{GR} + \Delta\left(1+\frac{e}{2}\varphi\sin(\varphi)\right) + \frac{E^2L^2}{2c^2G^2M_*^2}\left(1+\frac{e^2}{2}-\frac{e}{2}\varphi\sin(\varphi)-\frac{e^2}{6}\cos(2\varphi)+\mathcal{O}(e^3)\right),$$

where x_{GR} denotes the general relativistic correction, and E is the energy per unit mass, which for a circular orbit, $E^2 = c^4\left(1-\frac{G^2M_*^2}{c^2L^2}-\frac{2G^4M_*^4}{c^4L^4}\right)$. The advancement of periastron is caused by the term proportional to $\varphi\sin(\varphi)$, and the coefficient of that term yields Eq. (22).

where Λ is the (emergent) cosmological constant $G_\mu{}^\nu$ is the Einstein tensor, $\delta_\mu{}^\nu$ is the Kronecker delta, and $T_\mu{}^\nu$ is the energy–momentum tensor which we assume here to be diagonal and of the form,

$$T_\mu{}^\nu = \mathrm{diag}\left[-\rho_B(r) - \rho_D(r), P_r, P_\perp, P_\perp\right], \tag{24}$$

where $\rho_B(r)$ is the usual matter contribution, which for a star of constant density $\rho_* = (3M_*c^2)/(4\pi R_*^3)$ and radius R_*,

$$\rho_B(r) = \begin{cases} \frac{3M_*c^2}{4\pi R_*^3} & \text{for} \quad r \le R_*, \\ 0 & \text{for} \quad r > R_*, \end{cases} \tag{25}$$

and $\rho_D(r) = [c^2/(4\pi r^2)]dM_D/dr$ is the energy density of dark mass (2)–(3) given by,

$$\rho_D(r) = \begin{cases} \frac{5c^2}{4\pi}\sqrt{\frac{cH_0M_*}{6GR_*^3 r}} & \text{for} \quad r \le R_*, \\ \frac{c^2}{4\pi r^2}\sqrt{\frac{cH_0M_*}{6G}} = \frac{c^4}{8\pi G r^2}\sqrt{\frac{r_S}{3r_H}} & \text{for} \quad r > R_*, \end{cases} \tag{26}$$

where $r_S = 2GM/c^2$, $r_H = H_0/c$. Eq. (24) allows for a contribution from the pressure $P_r = P_r(r)$ and angular pressures, $P_\theta = P_\varphi = P_\perp(r)$ (the two angular pressures must be equal by the symmetry of the problem and more formally by the angular component of the covariant conservation for $T_{\mu\nu}$). How to calculate these pressures is not clear from Verlinde's paper [1]. Here we shall consider two simple cases, namely:

(I) Field-like dark mass: $P_r = P(r)$, $P_\perp = 0$ and
(II) Particle-like dark mass: $P_r = P_\perp = P(r)$.

Note that in the case of global monopoles [3] and at large distances we have $P_r = -\rho$, and $P_\perp = 0$, which is therefore analogous (not surprisingly) to the field-like case. Even though we do not know what the pressure in Verlinde's theory is, we know that the covariant conservation of the energy–momentum tensor must hold,

$$\nabla_\mu T^\mu{}_\nu = 0\,. \tag{27}$$

As we show below this equation contains a very useful information on the problem.

For a spherically symmetric matter distribution we can assume a static, diagonal metric tensor of the form,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\mathrm{e}^{2\alpha(r)}dt^2 + \mathrm{e}^{2\beta(r)}dr^2 + r^2 d\Omega_2^2\,,$$
$$d\Omega_2^2 = d\theta^2 + \sin^2(\theta)d\varphi^2\,, \tag{28}$$

where $g_{\mu\nu}$ denotes the metric tensor. Inserting (28) into (23) gives the following three equations,

$$G_t{}^t = \mathrm{e}^{-2\beta}\left(\frac{1}{r^2} - \frac{2\beta'}{r}\right) - \frac{1}{r^2}$$
$$= -\Lambda - \frac{8\pi G}{c^4}\left(\rho_B(r) + \rho_D(r)\right) \tag{29}$$

$$G_r{}^r = \mathrm{e}^{-2\beta}\left(\frac{1}{r^2} + \frac{2\alpha'}{r}\right) - \frac{1}{r^2} = -\Lambda + \frac{8\pi G}{c^4}P_r(r) \tag{30}$$

$$G_\theta{}^\theta = G_\varphi{}^\varphi = \mathrm{e}^{-2\beta}\left(\frac{\alpha' - \beta'}{r} + \alpha'' + (\alpha')^2 - \alpha'\beta'\right)$$
$$= -\Lambda + \frac{8\pi G}{c^4}P_\perp(r)\,, \tag{31}$$

where $\alpha' = d\alpha/dr$, $\beta' = d\beta/dr$. The last equation does not provide an independent information since it is implied by the contracted Bianchi identity, $\nabla_\mu G^\mu{}_\nu = 0$.

The relevant information in the conservation law (27) is in the $\nu = r$ equation (the $\nu = \theta$ equation tells us that $P_\theta = P_\varphi = P_\perp$),

$$P_r' + \left(\alpha' + \frac{2}{r}\right)P_r - \frac{2}{r}P_\perp = -\alpha'\rho\,. \tag{32}$$

This equation and equations (29)–(31) represent the set of equations we ought to solve for the metric functions $\alpha(r)$ and $\beta(r)$.

The first equation (29) can be solved by noting that $r^2 G_t{}^t = \left[r\mathrm{e}^{-2\beta}\right]' - 1$, and hence,

$$\mathrm{e}^{-2\beta(r)} = 1 - \frac{\Lambda}{3}r^2 - \frac{8\pi G}{c^4 r}\int_0^r \left(\rho_B(\tilde r) + \rho_D(\tilde r)\right)\tilde r^2 d\tilde r\,, \tag{33}$$

where we set an integration constant to zero (this is because in the absence of matter metric tensor must reduce to the Minkowski metric). The integral in (33) is to be understood such that, at $r = R_*$, β is continuous (as it is required by Eq. (29)), *i.e.* the integral (33) is continuous at $r = r_*$.

By integrating (33) we can obtain the solution for $\beta(r)$ inside the star ($r \le R_*$),

$$\mathrm{e}^{-2\beta(r)} = 1 - \frac{2GM(r)}{c^2 r} - \frac{\Lambda}{3}r^2\,, \tag{34}$$

where

$$M(r) = M_B(r) + M_D(r)\,, \quad M_B(r) = M_*\left(\frac{r}{R_*}\right)^3,$$

$$M_D(r) = \sqrt{\frac{2cH_0M_*r^5}{3GR_*^3}} \quad (r < R_*). \tag{35}$$

Next let us consider the gravitational field outside the star, $r > R_*$. We can split the integral in (33) to $0 \le r \le R_*$ and $r > R_*$. The first integral yields a constant,

$$M(R_*) = \frac{4\pi}{c^2}\int_0^{R_*}\left(\rho_B(\tilde r) + \rho_D(\tilde r)\right)\tilde r^2 d\tilde r = M_* + \sqrt{\frac{2cH_0M_*}{3G}}R_*$$
$$= M_* + \frac{c^2}{2G}\Delta \times R_*\,. \tag{36}$$

From this we see that ρ_D inside a star generates a potential that at the surface of the star generates a solid deficit angle,

$$\Delta \equiv \sqrt{\frac{2H_0GM_*}{3c^3}}\,. \tag{37}$$

The required continuity of β at $r = R_*$ implies that the deficit angle is inherited by an exterior metric, *i.e.* the interior of a star in Verlinde's emergent gravity has the same effect as the core of a global magnetic monopole and generates a boundary condition (at star's surface) that corresponds to that of a global monopole. With this in mind the total integral gives,

$$M(r) = \frac{4\pi}{c^2}\int_0^r \left(\rho_B(\tilde r) + \rho_D(\tilde r)\right)\tilde r^2 d\tilde r$$
$$= M_* + \frac{c^2}{2G}\Delta \times r\,, \qquad (r > R_*)\,. \tag{38}$$

When this is inserted into (33) one obtains,

$$\mathrm{e}^{-2\beta(r)} = 1 - \Delta - \frac{\Lambda}{3}r^2 - \frac{2GM_*}{c^2 r}\,, \qquad \Delta = \sqrt{\frac{2H_0GM_*}{3c^3}} = \sqrt{\frac{r_S}{3r_H}}\,, \tag{39}$$

where $r_S = 2GM/c^2$ and $r_H = H_0/c = \sqrt{\Lambda/3}$ denote the Schwarzschild and Hubble radius, respectively. The main result up to now is that the radial part of the metric tensor in Verlinde's emergent gravity is generally of the form (34), where inside the star $M(r)$ is given by (35), while outside the star $M(r)$ is given in (38), such that the exterior metric (39) exhibits a solid deficit angle Δ defined in (37). From the main text we know that Δ signifies a solid deficit angle. Eqs. (34) and (39) are exact solutions for $\beta(r)$ when a star is of constant energy density.

Next we consider $\alpha = \alpha(r)$ which is determined by Eqs. (30) and (34),

$$\alpha'(r) = \frac{1}{2}\frac{-\frac{2}{3}\Lambda r + \frac{2GM(r)}{c^2r^2} + \frac{8\pi G}{c^4} r P_r(r)}{1-\frac{\Lambda}{3}r^2 - \frac{2GM(r)}{c^2 r}}\,, \tag{40}$$

with $M(r)$ given by Eqs. (34)–(35) inside the star and by Eqs. (38)–(39) outside the star. Since Eq. (40) is sourced by pressure, one has to consider Eq. (40) together with the conservation equation (32). It is convenient to use Eq. (40) to get rid of α' in (32), resulting in the Tolman–Oppenheimer–Volkoff (TOV) equation for hydrostatic equilibrium,

$$P_r' = -\frac{P_r+\rho}{2} \times \frac{-\frac{2}{3}\Lambda r + \frac{2GM(r)}{c^2r^2} + \frac{8\pi G}{c^4} r P_r(r)}{1 - \frac{2GM(r)}{c^2 r} - \frac{\Lambda}{3}r^2} - \frac{2}{r}(P_r - P_\perp)\,, \tag{41}$$

which can be solved for P_r (if one knows $P_\perp$). When the solution of this equation is inserted into (40) one can solve for $\alpha(r)$.

Let us consider more closely the TOV equation (41). It is convenient to introduce dimensionless pressures, $p_r = [8\pi G/c^4] r_H^2 P_r$, $p_\perp = [8\pi G/c^4] r_H^2 P_\perp$, energy density $\tilde{\rho} = [8\pi G/c^4] r_H^2 \rho$ and distance, $x = r/r_H$ ($r_H = H_0/c$). The TOV equation can then be written as a Riccati differential equation,

$$\frac{dp}{dx} + \left[\frac{2\epsilon}{x} + \frac{1}{2}\frac{-2x + \frac{\Delta}{x} + \frac{3\Delta^2}{x^2} + x\tilde{\rho}(x)}{A(x)}\right]p + \frac{x}{2A(x)}p^2 = -\frac{-2x + \frac{\Delta}{x} + \frac{3\Delta^2}{x^2}}{2A(x)}\tilde{\rho}(x)\,, \tag{42}$$

where

$$A(x) = 1 - \Delta - x^2 - \frac{3\Delta^2}{x} \qquad (x > x_* = r/R_*)$$

and where

$$\epsilon = \begin{cases} 1 & \text{for Case I: } p_r = p,\ p_\perp = 0\,, \\ 0 & \text{for Case II: } p_r = p_\perp = p\,. \end{cases} \tag{43}$$

We are primarily interested in solving for $\alpha(r)$ outside the star, and hence from (26) we see that,

$$\tilde{\rho} = \frac{8\pi G r_H^2}{c^4}\rho_D = \frac{\Delta}{x^2} \quad (r > R_*). \tag{44}$$

Furthermore, since we are interested in the metric at sub-Hubble distances, $x = r/r_H \ll 1$, and in the weak field regime, $x \gg r_S/r_H = 3\Delta^2$, one can approximate $A(x)$ in all cases of interest $A(x) \approx 1$ in the TOV equation (42). In what follows we consider separately Cases I and II.

A.1. Case I: Field-like dark mass: $P_r = P$, $P_\perp = 0$

In Case I and outside the star Eq. (42) can be simplified as,

$$\frac{dp}{dx} + \left[\frac{2}{x} - x + \frac{\Delta}{x} + \frac{3}{2}\frac{\Delta^2}{x^2}\right]p + \frac{x}{2}p^2 = \left(x - \frac{\Delta}{2x} - \frac{3}{2}\frac{\Delta^2}{x^2}\right)\frac{\Delta}{x^2}\,, \tag{45}$$

where $\Delta = \sqrt{r_S/(3r_H)}$. Note that the terms $dp/dx + 2p/x$ dominate the equation for any $x < 1$ and therefore they determine the form of the solution,

$$p = \frac{p_0}{x^2}\,, \tag{46}$$

where p_0 is an integration constant we wish to determine. Since (46) solves $dp/dx + 2p/x = 0$, the remaining terms in (42) combine to an algebraic equation that also must be satisfied. We shall now show that this algebraic equation determines p_0,

$$\left(-x + \frac{\Delta}{x} + \frac{3}{2}\frac{\Delta^2}{x^2}\right)p_0 + \frac{p_0^2}{2x} = \left(x - \frac{\Delta}{2x} - \frac{3}{2}\frac{\Delta^2}{x^2}\right)\Delta\,. \tag{47}$$

At first sight this does not appear as a consistent equation since it contains a baroque x dependence. A closer look at (47) reveals however that it is consistent. To see that let us split (47) into two equations and require that each of them be separately satisfied,

$$-\left(x - \frac{\Delta}{2x} - \frac{3}{2}\frac{\Delta^2}{x^2}\right)p_0 = \left(x - \frac{\Delta}{2x} - \frac{3}{2}\frac{\Delta^2}{x^2}\right)\Delta\,,$$
$$\frac{\Delta}{2x}p_0 + \frac{p_0^2}{2x} = 0\,. \tag{48}$$

The first equation is solved for $p_0 = -\Delta$ while the second equation is satisfied when $p_0 = 0$ or $p_0 = -\Delta$, implying that both equations are solved when,

$$p_0 = -\Delta \;\Rightarrow\; P = -\rho\,. \tag{49}$$

We emphasize that (49) solves (45) for all x satisfying, $\Delta^2 \ll x \ll 1$ ($r_S \ll r \ll r_H$), which is also the range in which the static metric (7) is valid. Note that the solution (49) is imposed in different ranges of x by different terms in Eq. (45) or (47). Indeed, when $1 \gg x \gg \Delta^{1/2}$ ($r \gg (r_S r_H^3)^{1/4}$) the first term in (47) enforces the solution (49), when $\Delta^{1/2} \gg x \gg \Delta$ ($(r_S r_H^3)^{1/4} \gg r \gg (r_S r_H)^{1/2}$) the third term in (47) enforces (49), and finally when $\Delta^2 \ll x \ll \Delta$ ($r_S \ll r \ll (r_S r_H)^{1/2}$) the second term in (47) enforces (49). It is remarkable that *all of these terms* impose the *same* solution for p_0. That is of course no coincidence and the structure of the dominant solution can be traced back to Eq. (41), from which we see that when $P_r = -\rho$, P_r solves the simple equation, $P_r' = -2P_r/r$ which is solved by $P_r = P_0/r^2$.

When (49) is inserted into (40) one sees that, in the exterior of a star, the negative pressure contribution cancels the Δ dependent contribution from $2GM(r)/c^2 = \Delta \times r + 2GM_*/c^2$ and one obtains,

$$e^{2\alpha(r)} = 1 - \Delta - \frac{\Lambda}{3}r^2 - \frac{2GM_*}{c^2 r} = e^{-2\beta(r)}\,, \qquad (r > R_*)\,. \tag{50}$$

This solution is valid everywhere in the exterior of a star in the weak field regime, $r_S \ll r \ll r_H$ and it is equivalent to the metric of a global monopole in the exterior of the monopole core. Next we consider Case II, in which dark mass is assumed to be particle-like.

A.2. Case II: Particle-like dark mass: $P_r = P = P_\perp$

In this fully isotropic case the TOV equation (42) reduces to a Riccati equation ($|A(x) - 1| \ll 1$),

$$\frac{dp}{dx} + \left[-x + \frac{\Delta}{x} + \frac{3}{2}\frac{\Delta^2}{x^2}\right]p(x) + \frac{x}{2}p^2 = \left(x - \frac{\Delta}{2x} - \frac{3}{2}\frac{\Delta^2}{x^2}\right)\frac{\Delta}{x^2}\,. \tag{51}$$

The simplest way to solve a Riccati equation for $p = p(x)$,

$$p' + q_1(x)p + q_2(x)p^2 = q_0(x) \tag{52}$$

is to introduce substitutions,

$$Q_1 = q_1 - \frac{q_2'}{q_2} = -x - \frac{1-\Delta}{x} + \frac{3}{2}\frac{\Delta^2}{x^2},$$

$$Q_0 = -q_0 q_2 = -\frac{\Delta}{2} + \frac{\Delta^2}{4x^2} + \frac{3}{4}\frac{\Delta^3}{x^3}, \quad p = \frac{u'}{q_2 u},$$

upon which Eq. (52) reduces to a second order linear differential equation,

$$u'' + Q_1 u' + Q_0 u = 0. \tag{53}$$

A further substitution, $v = a(x)u$, with $a'/a = Q_1(x)/2$, reduces (53) to,

$$v'' + \left[-\frac{a''}{a} + Q_0\right]v = 0, \tag{54}$$

which in our case becomes,

$$v'' + \left[-\frac{x^2}{4} + \frac{3\Delta^2}{4x} - \frac{3-4\Delta}{4x^2} + \frac{9\Delta^2}{4x^3} - \frac{9\Delta^4}{16x^4}\right]v = 0. \tag{55}$$

Notice that once we know $v = v(x)$, then the pressure can be calculated by,

$$p = \frac{2}{x}\left[-\frac{Q_1}{2} + \frac{v'}{v}\right] = 1 + \frac{1-\Delta}{x^2} - \frac{3}{2}\frac{\Delta^2}{x^3} + \frac{2}{x}\frac{v'}{v}. \tag{56}$$

Let us solve (55) for $v(x)$. A careful look at all terms in (55) reveals that it is the fourth term ($\propto 1/x^2$) inside the square brackets that dominates for the relevant range of coordinates, $\Delta^2 \ll x \ll 1$ ($r_S \ll r \ll r_H$) However, that does not mean that one can neglect other terms. It is hard to solve (55) in full generality. Nevertheless, one can find an approximate solution as follows. Let us first rewrite (55) as,

$$v'' + \left(\frac{F_2}{x^2} + \frac{F_3}{x^3} - \frac{F_4}{x^4}\right)v = 0,$$

$$F_2(x) = -\frac{x^4}{4} + \frac{3\Delta^2}{4}x - \frac{3-4\Delta}{4},$$

$$F_3 = \frac{9\Delta^2}{4}, \qquad F_4 = \left(\frac{3\Delta^2}{4}\right)^2, \tag{57}$$

where $F_2(x)$ is an adiabatic function of x on the whole interval, $\Delta^2 \ll x \ll 1$, in the sense that, $F_2'/F_2 \ll F_2$ (attempting to include F_3/x and/or F_4/x^2 into F_2 would break adiabaticity when $\Delta^2 \ll x \ll \Delta$). The following substitutions, $v(x) \to v(y(x))$ with $y = 1/x$ and $w(y) = yv(y)$ reduce (57) to the Whittaker differential equation,

$$\frac{d^2 w}{dy^2} + \left(\frac{F_2}{y^2} + \frac{F_3}{y} - F_4\right)w(y) = 0, \tag{58}$$

whose two linearly independent solutions are given by the Whittaker functions,

$$w(y) \sim M_{\frac{F_3}{2\sqrt{F_4}},\, \frac{1}{2}\sqrt{1-4F_2}}\left(2\sqrt{F_4}y\right),$$

$$W_{\frac{F_3}{2\sqrt{F_4}},\, \frac{1}{2}\sqrt{1-4F_2}}\left(2\sqrt{F_4}y\right). \tag{59}$$

These functions are related to the confluent hypergeometric function by standard relations,

$$M_{\nu,\mu}(z) = e^{-z/2} z^{\frac{1}{2}+\mu} \times {}_1F_1\left(\frac{1}{2}+\mu-\nu; 1+2\mu; z\right),$$

$$W_{\nu,\mu}(z) = e^{-z/2} z^{\frac{1}{2}-\mu} \frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu-\nu)} \times {}_1F_1\left(\frac{1}{2}-\mu-\nu; 1-2\mu; z\right) + e^{-z/2} z^{\frac{1}{2}+\mu} \frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2}-\mu-\nu)} \times {}_1F_1\left(\frac{1}{2}+\mu-\nu; 1+2\mu; z\right). \tag{60}$$

By making use of (60) and (59) one gets two linearly independent solutions for $v(x) = xw(1/x)$,

$$v_\pm(x) \sim e^{-3\Delta^2/(4x)} x^{\frac{1}{2}(1\mp\delta_v)} \times {}_1F_1\left(-1 \pm \frac{\delta_v}{2}; 1 \pm \delta_v; \frac{3\Delta^2}{2x}\right), \tag{61}$$

where we have defined,

$$\delta_v \equiv \sqrt{1-4F_2} = \sqrt{4(1-\Delta) + x^4 - 3x\Delta^2} \approx 2\sqrt{1-\Delta} + \frac{x^4}{8\sqrt{1-\Delta}}. \tag{62}$$

The physical solution is a linear combination of v_+ and v_- in (61),

$$v = C\left[v_+(x) + cv_-(x)\right], \tag{63}$$

where C, c are unknown (integration) constants. When (63) is inserted into (56) one obtains,

$$p = 1 + \frac{1-\Delta}{x^2} - \frac{3\Delta^2}{2x^3} + \frac{1}{1+cx^{\delta_v}}\left\{\left[\frac{1-\delta_v}{x^2} + \frac{3\Delta^2}{2x^3}\frac{3}{1+\delta_v}\right] + cx^{\delta_v}\left[\frac{1+\delta_v}{x^2} + \frac{3\Delta^2}{2x^3}\frac{3}{1-\delta_v}\right]\right\}, \tag{64}$$

where we have neglected derivatives of $\delta_v(x)$ (which is justified in the adiabatic approximation we are using). One way of determining which is the case is to solve for pressure inside the star and then continuously match at the exterior solution. A detailed analysis[4] shows that v_+ dominates, *i.e.* $|c|x^{\delta_v} \ll 1$ and Eq. (64) evaluates to,

$$p = 1 + \frac{x^2}{8} + \frac{\Delta^3}{2x^3}. \tag{65}$$

With this in mind and upon inserting (65) into (40) one obtains the following (approximate) equation for the metric outside a star ($r > R_*$),

$$\frac{d\alpha}{dx} = \frac{1}{2}\frac{d}{dx}\ln\left(1 - \Delta - x^2 - \frac{3\Delta^2}{x}\right) + \frac{1}{2}\left(\frac{\Delta}{x} + x + \frac{x^3}{8} + \frac{\Delta^3}{2x^2}\right). \tag{66}$$

Let us focus on the terms in the second parentheses. The second term changes (halfs) the cosmological constant while the third

[4] The TOV equation can be solved inside the star ($x < R_*/r_H$). One can show that the approximate solution for pressure is of the form,

$$p(x) = -\frac{5}{9}\tilde{\rho}_* x + p_0 + \text{higher order terms},$$

where $p_0 \ll \tilde{\rho}_*$ is the pressure at the center of the star (since the star is assumed to be non-relativistic, it is reasonable to assume that the pressure at the center of the star is much less than the energy density) and $\tilde{\rho}_* = 3r_H^2 r_S/R_*^3 = 8\phi_*^3/(3\Delta^4) \gg 1$ is the rescaled energy density (25) and we introduced the dimensionless potential at the star surface, $\phi_* \equiv r_S/(2R_*)$. For our Sun $\phi_* \sim 10^{-6}$ and $\Delta \sim 3 \times 10^{-11} \ll \phi_*$. To get the pressure at the star surface, we need to compare $\Delta/x_* = 2\phi_*/(3\Delta) \gg 1$ with $-x_* p(x_*) \simeq 10\phi_*/3 - 4(p_0/\rho_*)\phi_*^2 \ll 1$ and hence the pressure contribution just outside the star must be negligible when compared to Δ/x^2. This means that the v_+ contribution in (63)–(64) must dominate, thus justifying Eq. (65).

term is a small corrections that is negligible except at very large distances and the last (third) term is a small (order Δ) correction to the Newtonian term. Upon neglecting the last two terms, Eq. (66) can be integrated to obtain,

$$e^{2\alpha(r)} = \left(1 - \Delta - \frac{\Lambda}{6}r^2 - \frac{2GM}{r}\right)\left(\frac{r}{r_H}\right)^{\Delta} . \tag{67}$$

This result is used in the main text to write the metric tensor (4).

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