

Planck scale operators, inflation, and fine tuning

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Ultraviolet completion of the standard model plus gravity at and beyond the Planck scale is a daunting problem to which no generally accepted solution exists. Principal obstacles include (a) lack of data at the Planck scale, (b) the nonrenormalizability of gravity, and (c) the unitarity problem. Here we make a simple observation that, if one treats all Planck scale operators of equal canonical dimension democratically, one can tame some of the undesirable features of these models. With a reasonable amount of fine tuning one can satisfy the slow roll conditions required in viable inflationary models. That remains true even when the number of such operators becomes very large.

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I. GRAVITY AS AN EFFECTIVE THEORY

Arguably the simplest way of dealing with gravity below the Planck scale is to treat it as an effective theory, according to which threshold effects from the unknown Planck scale theory at scales sufficiently below the Planck scale can be subsumed to local operators [1,2]. Planck scale operators can be generated both by the threshold effects from Planck scale physics, and by the quantum effects of gravitational and matter fields from sub-Planckian scales. A simple application of gravity as an effective theory results in quantum corrections to gravitational potentials, which are unobservably small on Minkowski background [3–5] but can be much larger on curved backgrounds (on the de Sitter background these corrections can be observably large [6]). In other words, significantly below the Planck scale the effective action of gravity plus matter admits a gradient expansion.¹ Assuming covariance, this then limits the theory to a set of operators which is finite when truncated at any finite canonical dimension. Since we are primarily interested in understanding inflation, here we focus on scalar-tensor theories, for which the fundamental fields are the metric $g_{\mu\nu}$ and a scalar field ϕ , which in inflationary literature is known as the inflaton.²

The fundamental assertion of the effective theory approach to quantum gravity is that operators are classified

according to their canonical (mass) dimension, and generally operators of lower dimension are more important. Here they are, ordered by their canonical dimension:

- (A) Dimension $d = 0$: 1 (also known as the cosmological constant).
- (B) $d = 2$: R (the Hilbert-Einstein term), ϕ^2 (the scalar mass term).
- (C) $d = 4$: R^2 , $R_{\mu\nu}R^{\mu\nu}$, $\phi^2 R$ (the nonminimal coupling of scalar to gravity), GB , ϕ^4 and $(\partial_\mu\phi)(\partial^\mu\phi)$ (the scalar kinetic term).
- (D) $d = 6$: R^3 , $RR_{\mu\nu}R^{\mu\nu}$, ..., $\phi^4 R$, $\phi^2 R^2$, ϕ^6 , $\phi^2(\partial_\mu\phi)(\partial^\mu\phi)$, $R(\partial_\mu\phi)(\partial^\mu\phi)$, $R^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$, etc.,

where R and $R_{\mu\nu}$ denote the Ricci scalar and tensor, respectively; $GB = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ stands for the Gauss-Bonnet term, which is in four space-time dimensions a total derivative, and hence it can be discarded as it does not affect the equations of motion. Note that the number of operators increases rapidly with increasing canonical dimension. Above we list all operators up to $d = 4$ and give a sample of them in $d = 6$. Operators of odd dimension also exist; for example, ϕR is a dimension-3 operator. Here for simplicity we assume that they are not present. One way of excluding them is symmetry. When ϕ is a real scalar, the corresponding symmetry is $O(1) \cong \mathbb{Z}_2$, which requires the action to be invariant under $\phi \rightarrow -\phi$. In the case of N real scalars, the corresponding symmetry is $O(N)$.³

According to the effective theory of gravity, operators of lower dimensionality are more important. If that does not happen, it creates a problem that begs for a theoretical

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¹The well-known working example which Donoghue [1] quotes is the nonlinear sigma model of mesons, which is valid significantly below the scale Λ_{QCD} , the scale of confinement at which QCD strongly couples and below which every perturbative calculation ceases to be valid.

²Some inflationary models comprise more scalar fields. Our considerations are easily generalized to encompass these more general cases. Similarly, our considerations can be generalized to include operators that include other (fermionic and vector) matter fields.

³When an internal symmetry is imposed on the classical level, it typically also survives at the quantum level, which is what we assume is the case in this work. Notable exceptions are spontaneous symmetry breaking by a Higgs-like condensate and “breaking” of gauge symmetry via quantum effects, such as in the scalar electrodynamics on de Sitter space [7,8].

explanation. For example, the fact that the $d = 0$ operator is not important throughout the evolution of the Universe (except possibly in the last seven billion years) is known as the cosmological constant problem, and it is deemed as the most important unsolved hierarchy problem in modern physics [9].

Analogously, during inflation the coefficient of the ϕ^2 term is anomalously small. A possible explanation is conformal symmetry that forbids such a term. The smallness of it can be then explained by a small violation of conformal symmetry generated by quantum effects. The mass scale of the Hilbert-Einstein term R is the Planck scale M_P^2 . It sets the strength of the coupling of gravity to matter (since M_P is large, its coupling to matter $\propto 1/M_P^2$ is weak), and we have no idea why its value is what it is (for a recent discussion on that question, see, e.g., Refs. [10,11]).

The first time that an operator that violates unitarity appears is at $d = 4$. This is the $R_{\mu\nu}R^{\mu\nu}$ operator [12,13], and when included into the graviton propagator, it generates a massive ghost field (this is a field whose creation contributes negatively to the energy of the system), thereby destabilizing the theory [14,15]. The operator $R_{\mu\nu}R^{\mu\nu}$ destabilizes gravity even classically via the Ostrogradsky instability [16], and therefore ought to be excluded. In what follows, we assume that operators that violate unitarity are not present in the effective theory, i.e., that they are excluded by some unknown mechanism operative at the Planck scale. However, even if such a mechanism is present at the Planck scale, because gravity is a nonrenormalizable theory [17,18], these types of operators will be generated as a result of quantum fluctuations of matter fields on sub-Planckian scales. The only way of taming these operators is to assume that the resulting effective theory is applicable up to an energy scale sufficiently below the Planck scale, such that these operators do not wreak havoc in the Universe (recall that the ghost mass is Planckian, which suppresses its creation below the Planck scale).

An obvious application of the present work is to the question of fine tuning in inflationary models [19]. One often makes use of an approximate shift symmetry—which is approximately respected by the tree-level cosmological perturbations—and then uses it as an argument to forbid all higher dimensional operators that violate that same shift symmetry. It is however unclear why would such a symmetry would be respected by the Planck scale physics [20]. One notable attempt where such a justification is argued is axion monodromy inflation [21,22], in which there are Goldstone bosons associated with a global symmetry that is softly broken by the exponentially suppressed instanton corrections. This is so because perturbative diagrams generate only derivative couplings to the Goldstone bosons. Some supersymmetric models of inflation [23,24] were

initially conceived to avoid the fine tuning problems of usual inflationary models. However, these types of models tend to suffer from severe tuning problems when Planck scale higher dimensional operators are added [25–27]. The role of quantum corrections and Planck scale operators in Higgs inflation has resulted in an extensive discussion in the literature [28–32] (see also [33]), leading to the conclusion that—unless one forbids the Planck scale operators that violate shift symmetry—the slow roll conditions of Higgs inflation will be quite generically destroyed [34]. At the moment it is unclear, however, why such operators should be forbidden. In passing we note that the reasoning advocated in this work can be fruitfully applied to studying the threshold and quantum corrections in Higgs inflation. The role of the one-loop inflaton quantum corrections in the Higgs inflaton deserves a closer inspection and we intend to address it elsewhere.

The remainder of the paper is divided into three parts. In Sec. II we discuss dimension-4 operators, which is followed by discussion of dimension-6 operators in Sec. III, in which we also remark on the role of operators of dimension $d > 6$. Some concluding remarks are given in Sec. IV.

II. DIMENSION-4 OPERATORS

In this section we consider the effective action for gravity and matter that includes all operators up to canonical dimension 4. For simplicity, for matter we take a real scalar field ϕ and for gravity we take all operators that are consistent with unitarity and we assume that the effective action is consistent with certain symmetries. In our case these are the general covariance of the graviton field and the $O(1) \cong \mathbb{Z}_2$ symmetry of the real scalar. The effective action in four space-time dimensions⁴ is then

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} (R - \Lambda) + \frac{\alpha}{2} R^2 - \frac{\xi}{2} R \phi^2 - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right\}, \quad (1)$$

where $M_P = 1/[8\pi G_N]^{1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass; G_N is the Newton constant; Λ is the cosmological constant; m is the scalar field mass; and α , ξ , and λ are dimensionless couplings of operators of canonical dimension $d = 4$. Here we work in units in which the speed of light $c = 1$ and the reduced Planck

⁴In this paper we are not interested in studying quantum effects of matter and gravitational fields on the effective theory, and therefore we present our results in $D = 4$ space-time dimensions. A generalization of our considerations to general D dimensions—which is a good starting point for studying quantum effects by the method of dimensional regularization—is straightforward.

constant $\hbar = h/(2\pi) = 1$. The action (1) is equivalent to the action⁵

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} (\Phi - \Lambda) + \frac{\alpha}{2} \Phi^2 + \omega^2 (R - \Phi) - \frac{\xi}{2} \Phi \phi^2 - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 \right\}, \quad (2)$$

where we have introduced a new scalar field Φ and a Lagrange multiplier (constraint) field $\omega = \omega(x)$.

Now upon varying the action (2) with respect to Φ and solving the resulting equation, one obtains

$$\Phi = \frac{1}{\alpha} \left[\omega^2 + \frac{\xi}{2} \phi^2 - \frac{M_P^2}{2} \right]. \quad (3)$$

Inserting this into (2) results in an equivalent action,

$$S = \int d^4x \sqrt{-g} \left\{ \omega^2 R - \frac{M_P^2}{2} \Lambda - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4 - \frac{1}{2\alpha} \left[\omega^2 + \frac{\xi}{2} \phi^2 - \frac{M_P^2}{2} \right]^2 \right\}. \quad (4)$$

In this action the Lagrange multiplier ω appears as a nonminimally coupled scalar. It is hence useful to transform to the Einstein frame by making use of a suitable conformal transformation, $g_{\mu\nu} = \Omega^2(x) g_{\mu\nu}^E$, where Ω is a local (possibly field dependent) conformal function. By making use of the standard conformal transformation for the Ricci scalar and determinant of the metric, we get [our metric signature is $(-, +, +, +)$] (see, e.g., Ref. [35])

$$S = \int d^4x \sqrt{-g_E} \left\{ \Omega^2 \omega^2 \left[R_E - 6 g_E^{\mu\nu} \frac{\nabla_\mu^E \nabla_\nu^E \Omega}{\Omega} - \Omega^2 \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) \right] - \Omega^4 \frac{M_P^2}{2} \Lambda - \Omega^4 \frac{m^2}{2} \phi^2 - \Omega^4 \frac{\lambda}{4!} \phi^4 - \frac{\Omega^4}{2\alpha} \left[\omega^2 + \frac{\xi}{2} \phi^2 - \frac{M_P^2}{2} \right]^2 \right\}. \quad (5)$$

The correct choice for Ω that gets rid of the nonminimal coupling is

$$\Omega^2 = \frac{M_P^2}{2} \frac{1}{\omega^2}. \quad (6)$$

⁵One can easily show the on-shell equivalence of the two actions as follows. Varying Eq. (2) with respect to ω and Φ gives $\omega(R - \Phi) = 0$, $F' - \omega^2 = 0$, $F(\Phi) = \frac{\alpha}{2} \Phi^2 + \frac{M_P^2}{2} (\Phi - \Lambda) - \frac{\xi}{2} \Phi \phi^2$. The nonsingular solutions of these equations are $\Phi = R$ (for $\omega \neq 0$) and $\omega^2 = F'(\Phi)$. Inserting these solutions into the action (2) gives Eq. (1), completing the proof of the on-shell equivalence of the actions (1) and (2), which suffices for our purpose.

Upon partially integrating the second term inside the square brackets in the first line of Eq. (5) and dropping the resulting (presumably irrelevant) boundary term and upon using (6), Eq. (5) simplifies to (for convenience we keep Ω in the action)

$$S = \int d^4x \sqrt{-g_E} \left\{ \frac{M_P^2}{2} R_E - 3 M_P^2 g_E^{\mu\nu} \frac{(\partial_\mu \Omega) (\partial_\nu \Omega)}{\Omega^2} - \Omega^2 \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - \Omega^4 \frac{M_P^2}{2} \Lambda - \Omega^4 \frac{m^2}{2} \phi^2 - \Omega^4 \frac{\lambda}{4!} \phi^4 - \frac{M_P^4}{8\alpha} \left[1 + \frac{\xi}{2} \frac{\phi^2}{M_P^2} \Omega^2 - \Omega^2 \right]^2 \right\}. \quad (7)$$

We have thus gotten rid of the higher dimensional gravitational operator in the original action (1), but the price to pay is the emergence of a second dynamical scalar known as the scalaron [36]. Note that both scalars in (7) have noncanonical kinetic terms. The following simple rescaling brings the scalar kinetic terms into their canonical form,

$$\psi_E = \sqrt{6} M_P \times \ln(\Omega), \quad \phi_E = \Omega \phi \Rightarrow d\phi_E = \Omega d\phi, \quad (8)$$

where we choose the integration constants such that, when $\Omega = 1$ and $\phi = 0$, then $\psi_E = 0$ and $\phi_E = 0$, and we choose the positive root in the first field transformation in (8) (choosing the negative root would lead to a completely equivalent action, which is related to the action obtained below by the transformation $\psi_E \rightarrow -\psi_E$). The implication in (8) holds since Ω and ϕ can be considered as two independent fields [see Eq. (10) below]. With this action (7) becomes the following Einstein frame action,

$$S_E[\psi_E, \phi_E] = \int d^4x \sqrt{-g_E} \left\{ \frac{M_P^2}{2} R_E - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \psi_E) (\partial_\nu \psi_E) - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \phi_E) (\partial_\nu \phi_E) - \frac{M_P^2}{2} \Lambda e^{4\tilde{\psi}_E} - \frac{m^2}{2} \phi_E^2 e^{2\tilde{\psi}_E} - \frac{\lambda}{4!} \phi_E^4 - \frac{M_P^4}{8\alpha} \left[1 + \xi \frac{\phi_E^2}{M_P^2} e^{-2\tilde{\psi}_E} \right]^2 \right\}, \quad (9)$$

where for simplicity we have introduced a rescaled field,

$$\tilde{\psi}_E = \frac{\psi_E}{\sqrt{6} M_P} \equiv \ln(\Omega). \quad (10)$$

We shall now pause to analyze the effective action (9) and its link to inflation. One conventional way of getting inflation is Starobinsky's inflationary model [36], which is obtained in the limit in which $\Lambda = m = \lambda = \xi = 0$. In that case the effective potential in Eq. (9) reduces to

$$(V_E)_{\text{Starobinsky}} = \frac{M_P^4}{8\alpha} [1 - e^{2\tilde{\psi}_E}]^2. \quad (11)$$

The minimum of the potential, $V_E = 0$, is reached when $\tilde{\psi}_E = \psi_E/[\sqrt{6} M_P] = 0$. The potential (11) is suitable for

(large field) inflation as it exhibits a plateau at large negative values of ψ_E , i.e., $\psi_E \ll -M_P$, and inflationary predictions of the Starobinsky model [36] beautifully agree with the measurements [37], provided one chooses α that is consistent with the COBE normalization, $\alpha \simeq 1.1 \times 10^9$. However, as we shall see below, Starobinsky's model is in general unprotected against large corrections arising from higher dimensional Planck scale operators.

The other regime is that of Higgs inflation [38,39]. Higgs inflation is obtained in the limit when $\lambda \simeq 0.5$, $m \approx 0$,⁶ $\alpha = 0$, $\Lambda = 0$, and $|\xi| \gg 1$, $\xi < 0$. In this regime, the following combination of the two fields,

$$1 + \xi \frac{\phi_E^2}{M_P^2} - e^{2\tilde{\psi}_E} \approx 0, \quad (12)$$

must vanish to a high accuracy since the mass associated with that degree of freedom is very large (super-Planckian), implying that degree of freedom decouples and becomes nondynamical. This means that one is left with only one dynamical degree of freedom. To get a better grip of that degree of freedom, we insert the differential form of Eq. (12) into Eq. (9) to obtain the following effective Lagrangian for ψ_E ,

$$(\mathcal{L}_E)_{\text{Higgs inflation}} \approx -\frac{1}{2} \left[1 - \frac{e^{4\tilde{\psi}_E}}{6\xi(1 - e^{2\tilde{\psi}_E})} \right] g_E^{\mu\nu} (\partial_\mu \psi_E) (\partial_\nu \psi_E) - \frac{\lambda M_P^4}{4!\xi^2} (1 - e^{2\tilde{\psi}_E})^2. \quad (13)$$

This form is still not very useful, since the kinetic term is in a noncanonical form. To get it to the canonical form, one would have to make a suitable rescaling of ψ_E which is rather complicated. Rather than doing that, we note that the plateau of the potential (where the relevant part of inflation happens) is attained for large negative values of ψ_E , at which the kinetic term in (13) becomes approximately canonical [this is so because $\exp(4\tilde{\psi}_E) \rightarrow 0$ in (13)]. In conclusion, we have thus shown that inflationary predictions of Higgs inflation and Starobinsky's inflation are identical, provided one makes the identification,

$$\frac{3\xi^2}{\lambda} \leftrightarrow \alpha \simeq 1.1 \times 10^9. \quad (14)$$

Of course, the predictions are not exactly identical, because the noncanonical nature of the kinetic term in Higgs

inflation (13) will play some role at late stages of inflation, making the predictions of the two models slightly different. Nevertheless, these differences are tiny and the predictions of Higgs inflation lie in the sweet spot of the Planck data [37], just as those of Starobinsky's model. And just as in the case of Starobinsky's model, one cannot find a simple argument that would do away with the Planck scale operators [34], which will in general spoil the slow roll conditions of Higgs inflation.

Even though we have shown that, when the appropriate limits are taken, the action (9) contains two very popular and successful inflationary models, here we shall analyze the model (9) from a different perspective. We want to classify the operators appearing in (9) with respect to how they scale with a power of $e^{\tilde{\psi}_E}$, and we refer to these operators as of canonical dimension d_E in the Einstein frame. There are, namely, three classes of terms in (9): those that scale as $e^{4\tilde{\psi}_E}$, that scale as $e^{2\tilde{\psi}_E}$, and constant terms. There are no terms that scale as a negative power of $e^{2\tilde{\psi}_E}$ but—as we shall see in Sec. III—that is simply a consequence of our truncation of the action at canonical dimension $d = 4$. When operators of higher canonical dimension are included, the resulting Einstein frame effective action will contain terms that scale as $e^{(4-d_E)\tilde{\psi}_E}$, with $d_E/2 \in \{0, 1, 2, \dots\} \equiv \mathbb{N} \cup \{0\}$. Because of this property, in the limit of a large and positive ψ_E , only the operators multiplying $e^{4\tilde{\psi}_E}$, $e^{2\tilde{\psi}_E}$, and $e^{0\tilde{\psi}_E} = 1$ will survive; in the limit of a large and negative ψ_E , the operators multiplying $e^{-2n\tilde{\psi}_E}$, with $n = (d_E/2) - 2 \in \{0, 1, 2, \dots\}$ will survive, and there are infinitely many of them. From that observation alone we see that our only hope to get rid of an infinite class of operators is to demand that ψ_E become large and positive (note that both the Higgs and Starobinsky's inflationary model work in the opposite regime). In that case we need to arrange that the operators multiplying positive powers of $e^{2\tilde{\psi}_E}$ are zero (or finely tuned near zero) and that their contribution throughout the history of the Universe is negligible. Let us now discuss how much fine tuning is required to achieve that.

In order to do that analysis, consider the Einstein frame action (9) that includes operators up to canonical dimension 4, $d \leq 4$. The operators that in the Einstein frame contribute as those of canonical dimension $d_E = 0$ to the Lagrangian density are

$$\Delta_0 \mathcal{L}_E = -\frac{M_P^2}{2} \left(\Lambda + \frac{M_P^2}{4\alpha} \right) e^{4\tilde{\psi}_E}, \quad (15)$$

while the operators of canonical dimension $d_E = 2$ are

$$\Delta_2 \mathcal{L}_E = \left[\frac{M_P^4}{4\alpha} - \frac{1}{2} \left(m^2 - \frac{\xi M_P^2}{2\alpha} \right) \phi_E^2 \right] e^{2\tilde{\psi}_E}. \quad (16)$$

From Eq. (15) we see that by choosing the cosmological constant Λ such that it cancels the latter contribution, i.e.,

⁶The Higgs mass $m = m_H$ is completely negligible during inflation. With our convention for λ , $\lambda = 2(m_H/v)^2 \simeq 0.5$, where $m_H = 125$ GeV and $v = 246$ GeV are the Higgs mass and its vacuum expectation value, respectively. The simplistic analysis presented above neglects the running of the Higgs self-coupling λ , which is important since it significantly changes with the renormalization group scale. For our purpose this simplified analysis suffices because Higgs inflation is not the main focus of our work.

$\Lambda = -M_{\text{P}}^2/(4\alpha)$, removes the problematic dimension-0 contribution to the Lagrangian, resulting in

$$\Delta_0 \mathcal{L}_E = 0. \quad (17)$$

However, the tuning does not completely work for the $d_E=2$ operator (16). Namely, upon choosing $m^2 = \xi M_{\text{P}}^2/(2\alpha)$ in Eq. (16) one is still left with the following offending $d_E = 2$ contribution,

$$\Delta_2 \mathcal{L}_E = \frac{M_{\text{P}}^4}{4\alpha} e^{2\tilde{\psi}_E}, \quad (18)$$

which—as far as we can see—cannot be removed completely by fine tuning of the parameters in the effective action (9).⁷ Another trouble with that term is that its contribution to the effective potential in the Einstein frame is negative for any value of ψ_E and for $\alpha > 0$. One way of arguing away that term is to choose α negative and sufficiently large. As we will see in a moment, that choice is not acceptable if we are to build inflation from the action (9). Fortunately, this problem is unique to the truncation $d \leq 4$ and does not persist when higher dimensional operators are included.

Assuming that $\Delta_0 \mathcal{L}_E$ and $\Delta_2 \mathcal{L}_E$ can be neglected, the remaining terms contributing to the effective potential in (9) are those of canonical dimension $d_E = 4$,

$$-\Delta_4 \mathcal{L}_E \supset \Delta_4 V_E = \frac{\lambda}{4!} \phi_E^4 + \frac{M_{\text{P}}^4}{8\alpha} \left[1 + \xi \frac{\phi_E^2}{M_{\text{P}}^2} \right]^2. \quad (19)$$

This effective potential can generate a viable inflationary model provided λ is small enough and α is large enough. Indeed, in that case the potential exhibits an approximate plateau, $\Delta_4 V_E = M_{\text{P}}^4/(8\alpha)$. The value of α is fixed by the COBE constraint, $\alpha \approx 1.1 \times 10^9$, and with $\lambda = 0$, ξ negative, and $|\xi| \ll 1$ [this is the opposite limit from Higgs inflation; see Eq. (12)], one gets a viable inflationary model (whose slow roll parameters are to leading order those of the inverted quadratic model [40]). In this model the inflaton condensate grows during inflation towards $1 + \xi \phi_E^2/M_{\text{P}}^2 = 0$, at which point the potential energy minimizes at zero, $\Delta_4 V_E = 0$ [here we have assumed that the contribution from (18) stays negligibly small during inflation]. Note that the potential (19) is flat along ψ_E , so

one could also foresee (by a suitable choice of initial conditions) a period of ultraslow roll inflation [41,42], followed by an inverted quadratic inflation along ϕ_E .

As we have seen in this section, including all operators up to the canonical dimension $d = 4$ has some undesirable features; in particular there remains a dimension-2 operator (18) that cannot be canceled. For that reason, to get a more complete understanding of the role of higher dimensional operators during inflation, it is necessary to analyze dimension-6 operators, which is what we do next.

III. DIMENSION-6 OPERATORS

Even though conceptually straightforward, a complete analysis of dimension-6 operators is rather tedious, simply because there are many of them. For simplicity, in what follows we analyze only the operators that do not modify kinetic terms and do not violate unitarity. The $d = 6$ operators which contribute to the action are then

$$\Delta S^{(6)} = \int d^4x \sqrt{-g} \left\{ \frac{\beta_6}{3} \frac{R^3}{M_{\text{P}}^2} + \frac{\alpha_6}{2} \frac{R^2 \phi^2}{M_{\text{P}}^2} - \frac{\xi_6}{2} \frac{R \phi^4}{M_{\text{P}}^2} - \frac{g_6}{6!} \frac{\phi^6}{M_{\text{P}}^2} \right\}. \quad (20)$$

Examples of the $d = 6$ operators we are not considering are the following kinetic operators: $R g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi)$, $\phi^2 g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi)$, $G^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi)$, etc., where $G^{\mu\nu} = R^{\mu\nu} - (1/2) g^{\mu\nu} R$ is the Einstein tensor. One can show that including these operators would not in any essential way change our analysis. In addition, there are $d = 6$ operators that violate unitarity, such as $RR_{\mu\nu} R^{\mu\nu}$, $R \square R$, etc., which we assume to be absent. As in Sec. II, we can introduce a Lagrange multiplier field ω^2 that enforces $\Phi = R$. Varying with respect to Φ gives a quadratic equation [cf. Eq. (3)],

$$\frac{\Phi^2}{M_{\text{P}}^4} + \frac{\alpha + \alpha_6 \phi^2/M_{\text{P}}^2}{\beta_6} \frac{\Phi}{M_{\text{P}}^2} + \frac{1}{2\beta_6} \left[1 - \left(2 \frac{\omega^2}{M_{\text{P}}^2} + \xi \frac{\phi^2}{M_{\text{P}}^2} + \xi_6 \frac{\phi^4}{M_{\text{P}}^4} \right) \right] = 0, \quad (21)$$

which is solved by

$$\frac{\Phi_{\pm}}{M_{\text{P}}^2} = -\frac{\alpha + \alpha_6 \phi^2/M_{\text{P}}^2}{2\beta_6} \pm \sqrt{\frac{1}{4\beta_6^2} \left[\alpha + \alpha_6 \frac{\phi^2}{M_{\text{P}}^2} \right]^2 - \frac{1}{2\beta_6} \left[1 - \left(2 \frac{\omega^2}{M_{\text{P}}^2} + \xi \frac{\phi^2}{M_{\text{P}}^2} + \xi_6 \frac{\phi^4}{M_{\text{P}}^4} \right) \right]}. \quad (22)$$

When transforming to the Einstein frame, just as in Sec. II, one gets

⁷One can try to remove that operator by suitably choosing ϕ_E . That is not satisfactory since ϕ_E is a dynamical field, and we would like to keep it that way and use it if needed for inflation.

$$\omega^2 = \frac{M_{\text{P}}^2}{2} e^{-2\tilde{\psi}_E}; \quad \tilde{\psi}_E = \ln(\Omega) \equiv \frac{\psi_E}{\sqrt{6}M_{\text{P}}};$$

$$\phi_E = \phi e^{\tilde{\psi}_E} \equiv M_{\text{P}} \tilde{\phi}_E. \quad (23)$$

It is now convenient to rewrite Eq. (22) in the Einstein frame as

$$\tilde{\Phi} \equiv \frac{\Phi}{M_{\text{P}}^2} = -\frac{\alpha}{2\beta_6} - \frac{\alpha_6}{2\beta_6} \tilde{\phi}_E^2 e^{-2\tilde{\psi}_E} + \frac{\text{sign}[\alpha]}{2\beta_6} \sqrt{\alpha^2 - 2\beta_6} \sqrt{1 + \frac{2}{\alpha^2 - 2\beta_6} [\beta_6 + (\alpha\alpha_6 + \beta_6\xi)\tilde{\phi}_E^2] e^{-2\tilde{\psi}_E} + \frac{1}{\alpha^2 - 2\beta_6} (\alpha_6^2 + 2\beta_6\xi_6)\tilde{\phi}_E^4 e^{-4\tilde{\psi}_E}}, \quad (24)$$

where we have picked the solution that in the limit $\beta_6 \rightarrow 0$ reduces to (3) and we assumed $\alpha^2 - 2\beta_6 \geq 0$. When expressed in terms of $\tilde{\Phi}$, the Einstein frame action can be written as

$$S_E^{(6)} \equiv S_E + \Delta S_E^{(6)} = \int d^4x \sqrt{-g_E} \left[\frac{M_{\text{P}}^2}{2} R_E - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \psi_E)(\partial_\nu \psi_E) - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \phi_E)(\partial_\nu \phi_E) \right. \\ \left. + \frac{M_{\text{P}}^4}{2} \left\{ \left[-\frac{\Lambda}{M_{\text{P}}^2} + \left(\alpha + \frac{2\beta_6}{3} \tilde{\Phi} \right) \tilde{\Phi}^2 \right] e^{4\tilde{\psi}_E} + \left[-\tilde{\Phi} + \left(\tilde{\Phi}(-\xi + \alpha_6 \tilde{\Phi}) - \frac{m^2}{M_{\text{P}}^2} \right) \tilde{\phi}_E^2 \right] e^{2\tilde{\psi}_E} \right. \right. \\ \left. \left. - \left(\frac{\lambda}{12} + \xi_6 \tilde{\Phi} \right) \tilde{\phi}_E^4 - \frac{g_6}{360} \tilde{\phi}_E^6 e^{-2\tilde{\psi}_E} \right\} \right]. \quad (25)$$

In order to get an insight into the scaling of the action (25) with $e^{\tilde{\psi}_E}$, it is useful to expand $\tilde{\Phi}$ in (24) in powers of $e^{-2\tilde{\psi}_E}$ as follows:

$$\tilde{\Phi} = \tilde{\Phi}_0 + \tilde{\Phi}_{-2} e^{-2\tilde{\psi}_E} + \tilde{\Phi}_{-4} e^{-4\tilde{\psi}_E} + \tilde{\Phi}_{-6} e^{-6\tilde{\psi}_E} + \mathcal{O}(e^{-8\tilde{\psi}_E}), \quad (26)$$

where

$$\tilde{\Phi}_0 = -\frac{\text{sign}[\alpha]}{2\beta_6} \left[|\alpha| - \sqrt{\alpha^2 - 2\beta_6} \right] \quad (27)$$

$$\tilde{\Phi}_{-2} = -\frac{\alpha_6}{2\beta_6} \tilde{\phi}_E^2 + \frac{\text{sign}[\alpha]}{2\beta_6 \sqrt{\alpha^2 - 2\beta_6}} \{ \beta_6 + (\alpha\alpha_6 + \beta_6\xi) \tilde{\phi}_E^2 \} \quad (28)$$

$$\tilde{\Phi}_{-4} = \frac{\text{sign}[\alpha]}{4\beta_6 (\alpha^2 - 2\beta_6)^{3/2}} \{ -[\beta_6 + (\alpha\alpha_6 + \beta_6\xi) \tilde{\phi}_E^2]^2 + (\alpha^2 - 2\beta_6) [\alpha_6^2 + 2\beta_6\xi_6] \tilde{\phi}_E^4 \} \quad (29)$$

$$\tilde{\Phi}_{-6} = \frac{\text{sign}[\alpha]}{4\beta_6 (\alpha^2 - 2\beta_6)^{5/2}} [\beta_6 + (\alpha\alpha_6 + \beta_6\xi) \tilde{\phi}_E^2] \{ [\beta_6 + (\alpha\alpha_6 + \beta_6\xi) \tilde{\phi}_E^2]^2 - (\alpha^2 - 2\beta_6) [\alpha_6^2 + 2\beta_6\xi_6] \tilde{\phi}_E^4 \}. \quad (30)$$

Upon inserting (26) into the action (25), one can resolve terms multiplying different powers of $e^{2\tilde{\psi}_E}$. The fastest growing terms are the terms multiplying $e^{4\tilde{\psi}_E}$. They contribute to the Lagrangian density as the cosmological constant, and hence we demand that to a high accuracy those terms vanish,

$$\Delta_4 \mathcal{L} = \frac{M_{\text{P}}^4}{2} \left[-\frac{\Lambda}{M_{\text{P}}^2} + \left(\alpha + \frac{2\beta_6}{3} \tilde{\Phi}_0 \right) \tilde{\Phi}_0^2 \right] e^{4\tilde{\psi}_E} \approx 0. \quad (31)$$

This can be achieved, e.g., by the appropriate choice of Λ . This fine tuning corresponds to the usual tuning of the cosmological constant, and we have nothing new to add here. The second set of terms is those multiplying $e^{2\tilde{\psi}_E}$,

$$\Delta_2 \mathcal{L} = \frac{M_{\text{P}}^4}{2} \left\{ -\tilde{\Phi}_0 + 2(\alpha + \beta_6 \tilde{\Phi}_0) \tilde{\Phi}_0 \tilde{\Phi}_{-2} \right. \\ \left. + \left[\tilde{\Phi}_0(-\xi + \alpha_6 \tilde{\Phi}_0) - \frac{m^2}{M_{\text{P}}^2} \right] \tilde{\phi}_E^2 \right\} e^{2\tilde{\psi}_E} \approx 0. \quad (32)$$

As indicated in this equation, we also demand that this term vanish to a sufficiently high accuracy. As Eq. (32) contains two types of terms—those that do not contain any power of $\tilde{\phi}_E$ and those multiplying $\tilde{\phi}_E^2$ —that amounts to two more fine tunings of the parameters. We have quite a few parameters at our disposal (in that sense the situation becomes better when more higher dimensional operators are included). For example, a suitable choice of the mass term m^2 and of β_6 can do the job (the COBE normalization is now set by $\alpha^2 - 2\beta_6$, so we can choose at will either α or

β_6 to fine tune the $d_E = 4$ term). Assuming that the terms that contain a positive power of $e^{2\tilde{\psi}_E}$ are so fine tuned that they can be neglected, we are left with the constant term ($d_E = 4$) and with the terms that scale with negative powers of $e^{2\tilde{\psi}_E}$ ($d_E \geq 6$), i.e., the terms that decay for large values of

$\tilde{\psi}_E$. Demanding a sufficiently large ψ_E can render all higher order terms irrelevant, such that during inflation only the constant term (and possibly the term $\propto e^{-2\tilde{\psi}_E}$) are of any importance. The terms that remain in the Einstein action (25) are to a good approximation

$$\begin{aligned}
 S_E^{(6)} \approx & \int d^4x \sqrt{-g_E} \left\{ \frac{M_P^2}{2} R_E - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \psi_E) (\partial_\nu \psi_E) - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \phi_E) (\partial_\nu \phi_E) \right. \\
 & + \frac{M_P^4}{2} \left[-\tilde{\Phi}_{-2} + \alpha(\tilde{\Phi}_{-2}^2 + 2\tilde{\Phi}_0\tilde{\Phi}_{-4}) + 2\beta_6\tilde{\Phi}_0(\tilde{\Phi}_0\tilde{\Phi}_{-4} + \tilde{\Phi}_{-2}^2) + \tilde{\Phi}_{-2}(-\xi + 2\alpha_6\tilde{\Phi}_0)\tilde{\phi}_E^2 - \left(\frac{\lambda}{12} + \xi_6\tilde{\Phi}_0 \right) \tilde{\phi}_E^4 \right] \\
 & + \frac{M_P^4}{2} \left[\tilde{\Phi}_{-4} + 2\alpha(\tilde{\Phi}_0\tilde{\Phi}_{-6} + \tilde{\Phi}_{-2}\tilde{\Phi}_{-4}) + 2\beta_6 \left(\tilde{\Phi}_0^2\tilde{\Phi}_{-6} + 2\tilde{\Phi}_0\tilde{\Phi}_{-2}\tilde{\Phi}_{-4} + \frac{1}{3}\tilde{\Phi}_{-2}^3 \right) \right. \\
 & \left. \left. + (-\xi\tilde{\Phi}_{-4} + \alpha_6(2\tilde{\Phi}_0\tilde{\Phi}_{-4} + \tilde{\Phi}_{-2}^2))\tilde{\phi}_E^2 - \xi_6\tilde{\Phi}_{-2}\tilde{\phi}_E^4 - \frac{g_6}{360}\tilde{\phi}_E^6 \right] e^{-2\tilde{\psi}_E} \right\}, \quad (33)
 \end{aligned}$$

where the second line contains the $d_E = 4$ terms that are independent of ψ_E and the last two lines contain the $d_E = 6$ terms that scale as $e^{-2\tilde{\psi}_E}$. One can show that a similar analysis can be performed when other (kinetic) operators of dimension 6 are included. More importantly, an analogous analysis can be carried out when even higher dimensional operators are included (albeit the analysis becomes more technically involved). One can easily convince oneself that, truncating at an arbitrary but finite order $d \geq 6$, in general three fine tunings suffice to get rid of all the terms that contain positive powers of $e^{2\tilde{\psi}_E}$. In that sense the analysis presented in this section is generic. It would be incorrect to think that, every time one adds new higher dimensional operators, one ought to retune. Nature has chosen (unknown) operators that carry information about the Planck scale physics, and therefore fine tuning needs to be done only once for the set of operators that nature has picked.

The next question we need to address is whether the action (33) is suitable for inflationary model building. The analysis of inflation is in fact very similar to that for a dimension-4 effective potential (19) that we did in Sec. II and therefore here we only present an analysis in broad brush strokes. From the current cosmic microwave background observations we know that the constant term in the second line in (33) ought to be tuned to accord with the COBE normalization, and slow rolling can be either along the ϕ_E direction (which corresponds to a decrease in potential energy) and/or along an increasing ψ_E . Depending on what the values of the masses of the two scalar fields are, inflation will either occur in the ψ_E direction (when the scalaron is the lighter field) or in the scalar field direction (when ϕ_E is the lighter field). When the masses of the two fields are comparable, one can get a genuine two-field inflationary model. In the former case (rolling along ψ_E) one gets a

Starobinsky-like inflationary model, with the important difference that the scalaron rolls in the direction of an increasing scalaron condensate. That is not a problem since the evolution of the scalaron and Ricci scalar become decoupled in this model since the scalaron and graviton become independent degrees of freedom in the limit when $\psi_E \rightarrow \infty$. In the latter case (when ϕ_E is rolling) inflation can be similar to that of an inverted quadratic potential. Finally we mention that one can start inflation with nonattractor initial conditions that at early stages yield an ultra-slow-roll inflation along ψ_E , which at a later stage turns into a slow-roll inflation along ϕ_E .

IV. CONCLUSIONS AND DISCUSSION

In this paper we have studied the role of Planck scale operators that arise as threshold corrections from the unknown Planck scale physics. The central result of this work is the following: provided one permits a reasonable amount of fine tuning (precisely three fine tunings are needed), one can get a flat enough effective potential in the Einstein frame to grant inflation whose predictions are consistent with observations. This remains true independently on how many Planck scale operators one adds (provided there are sufficiently many of them). Canonical quantization of the effective theory in general results in changed values of the couplings for each of the operators, but does not in any essential way change the conclusions reached in this work.

Our hope (and the principal motivation for this work) is that our insights will help to advance the understanding of quantum (loop) corrections during inflation. In particular, we hope that we will be able to understand whether, how, and under what conditions these quantum effects can become detectable.

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