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Self-concept mediates the relation between achievement and emotions in mathematics

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Background. Mathematics achievement is related to positive and negative emotions. Pekrun's control–value theory of achievement emotions suggests that students' self-concept (i.e., self-appraisal of ability) may be an important mediator of the relation between mathematics achievement and emotions.

Aims. The aims were (1) to investigate the mediating role of mathematical self-concept in the relation between mathematics achievement and the achievement emotions of enjoyment and anxiety in a comprehensive model, and (2) to test possible differences in this mediating role between low-, average-, and high-achieving students.

Sample. Participants were ninth-grade students (n = 1,014) from eight secondary schools in the Netherlands.

Methods. Through an online survey including mathematical problems, students were asked to indicate their levels of mathematics enjoyment, anxiety, and self-concept. Structural equation modelling was used to test the mediating role of self-concept in the relation between mathematics achievement and emotions. Multigroup analyses were performed to compare these relations across the three achievement groups.

Results. Results confirmed full mediation of the relation between mathematics achievement and emotions by mathematical self-concept. Furthermore, we found higher self-concepts, more enjoyment and less math anxiety in high-achieving students compared to their average and low-achieving peers. No differences across these achievement groups were found in the relations in the mediational model.

Conclusions. Mathematical self-concept plays a pivotal role in students' appraisal of mathematics. Mathematics achievement is only one factor explaining students' self-concept. Likely also classroom instruction and teachers' feedback strategies help to shape students' self-concept.

Emotions play an important role in educational settings, as they influence cognitive processes and strategies, decision-making, and motivation (Kim & Pekrun, 2014). In recent years, research into the role of emotions in mathematical learning has focused on the relation with achievement and achievement-related choices. Several studies have demonstrated that emotional experiences such as enjoyment and anxiety are reciprocally linked to mathematical achievement and predict the likelihood of career aspirations in science or math (see Ahmed, van der Werf, Kuyper, & Minnaert, 2013;

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Ashcraft, 2002; Goetz, Frenzel, Hall, & Pekrun, 2008). According to Pekrun's control–value theory of achievement emotions, students' academic self-concept is one of the most important antecedents of academic emotions (Bieg, Goetz, & Lipnevich, 2014; Goetz *et al.*, 2008). However, only few studies examined academic emotions and academic self-concept simultaneously (Arens, Yeung, Craven, & Hasselhorn, 2011; Pinxten, Marsh, De Fraine, Van Den Noortgate, & Van Damme, 2014). Moreover, these studies rarely incorporated both positive and negative emotions. In the present study, we test the hypothesis that the relation between achievement and emotions in mathematics, both anxiety and enjoyment, is at least partially mediated by the mathematical self-concept of adolescent students. We expect that not only actual performance but rather the extent to which adolescents feel competent relates to their levels of joy and anxiety.

Anxiety and achievement in mathematics

Most research on emotions and mathematics has focused on negative emotions, anxiety in particular. Several studies found a negative association between math anxiety and performance (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007). A meta-analysis by Ma (1999) has shown an overall correlation of −.27 between math anxiety and achievement in mathematics among elementary and secondary students. Morony, Kleitman, Lee, and Stankov (2013) also found a pancultural correlation of −.27 between math anxiety and achievement in a large international sample of 15-year-olds. Research on the mechanisms underlying this association mainly focused on the role of working memory (i.e., processing efficiency theory, Eysenck & Calvo, 1992) and attention (i.e., attentional control theory, Eysenck, Derakshan, Santos, & Calvo, 2007). Both theories assume that anxiety impairs the processing efficiency of the central executive component of the working memory system, which plays a critical role in mathematical problem-solving. However, although these models explain the impact of math anxiety on performance, they do not explain how anxiety is elicited.

Pekrun's control–value theory of achievement emotions posits that one's beliefs about one's abilities are central in the arousal of achievement emotions like anxiety (Pekrun, 2006). These beliefs, commonly referred to as academic self-concept, are crucial in understanding the relation between academic achievement and emotions. In the mathematical domain, strong cross-sectional correlations ranging from about −.60 to −.70 have been found between self-concept and anxiety (Goetz, Cronjaeger, Frenzel, Lüdtke, & Hall, 2010; Hembree, 1990; Jameson, 2014; Lee, 2009; Morony *et al.*, 2013). Sizeable concurrent and longitudinal relations (around .70) have also been found between mathematics achievement and self-concept (Goetz *et al.*, 2010; Möller, Retelsdorf, Köller, & Marsh, 2011). It is notable that the relation between achievement and anxiety in mathematics appears to be weaker than the relations of mathematical self-concept with both achievement and anxiety. Furthermore, a recent longitudinal study by Ahmed, Minnaert, Kuyper, and van der Werf (2012) showed that prior performance on a national achievement test predicted later math self-concept but not later math anxiety. Moreover, the effect of prior self-concept on later anxiety was considerably larger than the effect of prior anxiety on later self-concept. Together, these findings suggest that the relation between achievement and anxiety in mathematics is, at least partially, mediated by mathematical self-concept.

Enjoyment and achievement in mathematics

Educational research has recently begun to focus also on positive emotions in relation to mathematics achievement. Earlier studies used motivational constructs as a framework, defining intrinsic motivation as the drive to engage in a task for the sake of interest in the task itself (Deci & Ryan, 1985). Intrinsic motivation was found to be moderately related to later mathematics achievement (Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007; Murayama, Pekrun, Lichtenfeld, & vom Hofe, 2013). Later, Pekrun (2006) introduced the concept of academic emotions as a more fine-grained tool to distinguish various negative and positive emotions. In this framework, enjoyment is the positive, activating emotion of liking a specific subject. Several studies have shown a moderate to strong positive relation between enjoyment and achievement in mathematics. For example, a positive concurrent relationship (with r ranging from .39 to .63) between math skills and early math interest, including enjoyment, was found in preschoolers by Fisher, Dobbs-Oates, Doctoroff, and Arnold (2012). Furthermore, their findings indicated that initial math scores predicted later math interest. A positive correlation (with r ranging from .39 to .66) between mathematics achievement and enjoyment was also found in eighth-grade students (Frenzel, Thrash, Pekrun, & Goetz, 2007). Finally, changes in enjoyment were systematically related to changes in mathematics achievement in seventh graders (Ahmed *et al.*, 2013).

These studies suggest that the direct relation with mathematics achievement seems to be stronger for enjoyment than for anxiety. However, high achievement in mathematics does not necessarily coincide with a high level of enjoyment in mathematics (Gottfried, Cook, Gottfried, & Morris, 2005; Gottfried, Marcoulides, Gottfried, & Oliver, 2009). A recent study among ninth-grade students by Andersen and Cross (2014) revealed that only 41% of the high-ability students in mathematics showed high motivation, including enjoyment, in math. This suggests that other factors play a role in determining enjoyment. As with anxiety, Pekrun's control–value theory provides a framework to investigate a possible mediational model in the achievement/enjoyment relation. According to this theory, a positive self-concept implies that the student experiences control over achievement-related activities and, therefore, is able to enjoy the activity (Pekrun, 2006). A number of studies have found significant and strong correlations between enjoyment in mathematics and competence-related measures similar to self-concept (e.g., self-efficacy and competence beliefs) from third-grade to undergraduate students (Andersen & Cross, 2014; Goetz *et al.*, 2008, 2010; Mega, Ronconi, & De Beni, 2014; Pinxten *et al.*, 2014). Moreover, in a longitudinal study from grade 3 to grade 7, Pinxten *et al.* (2014) showed that competence beliefs fully mediated the relation between math enjoyment and achievement. In a cross-sectional sample with 5th to 10th graders, Goetz *et al.* (2008) also confirmed the mediational role of self-concept in the relation between prior math achievement and subsequent enjoyment. In summary, there are clear indications that the relation between achievement and enjoyment in mathematics is, at least partially, mediated by mathematical self-concept.

Ability-related differences

Another question is whether the relations between achievement, self-concept, and academic emotions are equally strong in students of different ability. Two contrasting hypotheses can be formulated. The Dunning–Kruger effect predicts that the association between achievement and self-concept is stronger in higher achieving groups (Dunning, 2011; Dunning, Johnson, Ehrlinger, & Kruger, 2003). Developing a self-concept requires the metacognitive ability of evaluating one's performance, which requires the same expertise that is necessary to perform well. The Dunning–Kruger effect thus predicts that low performers are less able to accurately judge their own performance and may overestimate themselves, whereas high performers are better at judging their performance (Dunning, 2011; Dunning *et al.*, 2003). This view predicts that the relation between achievement and self-concept becomes stronger with increasing ability.

However, evidence to the contrary is provided by the observation in several studies that the majority of talented students in mathematics is not motivated to develop their mathematical abilities during or after secondary school (Plenty & Heubeck, 2013; Riegle-Crumb, Moore, & Ramos-Wada, 2011; Rodriguez, Romero-Canyas, Downey, Mangels, & Higgins, 2013). Perhaps, these low-motivated students had an unrealistically low mathematics self-concept. A possible explanation for the weak association between achievement and self-concept for these students is that they rely on other sources of information than their actual achievement to shape their self-concept. For example, they may compare their achievement to high absolute standards or to other high-achieving peers only.

The present study

The present study has two aims. The first aim was to test to which extent students' mathematical self-concept mediates the relations between achievement and both positive and negative emotions in an integrated model. The conceptual model is shown in Figure 1. The second aim was to examine possible differences in the strength of the relations between the constructs in this model between low-, average-, and high-achieving students.

Method

This study is part of a cross-cultural study on a variety of non-cognitive predictors of mathematics achievement (Morony *et al.*, 2013). The present study was based on data of the Dutch sample and focused exclusively on self-concept and emotions related to mathematics achievement.

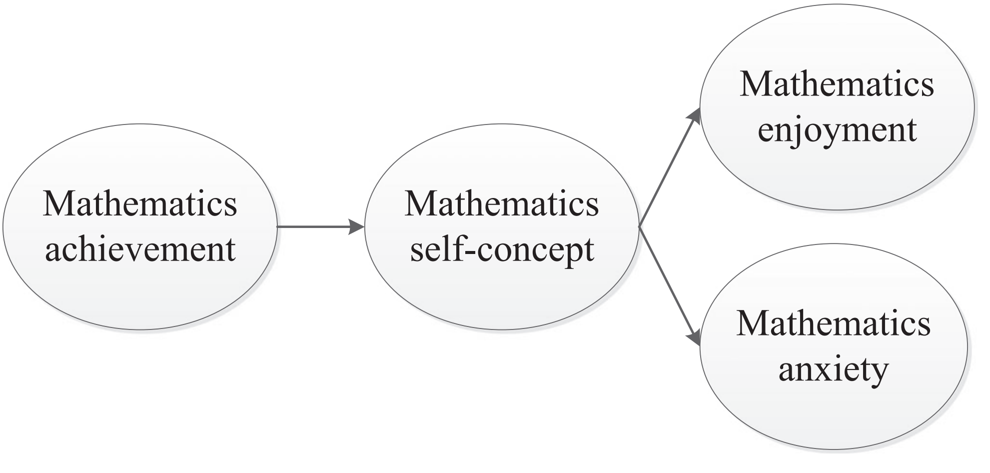

Figure 1. Conceptual model.

Participants

Participants were 1,014 ninth-grade students (55% girls; M_{age} = 15.1 years; SD = 0.58) from eight urban secondary schools in the Netherlands. Ninth grade is the third year of secondary education (there is no middle school). The schools were recruited based on convenience sampling. The Netherlands has a tracked secondary education system, with broadly three tracks. At the end of primary school, an achievement-based recommendation is made to students for the best-fitting track. The sample represented the three tracks of the Dutch secondary school system, including 267 students (53% girls) from VMBO-TL (i.e., lower vocational education at ISCED level 2, European Commission, 2014), 422 students (58% girls) from HAVO (i.e., higher general secondary education at ISCED level 3), and 325 students (54% girls) from VWO (i.e., pre-university education at ISCED level 3).

To answer the second research question, students' mathematics achievement scores (see also the Mathematics achievement section) were standardized by track (VMBO, HAVO, VWO). Subsequently, the group was divided into three approximately equal groups: a low-achieving group (n = 337, 62% girls), an average-achieving group (n = 313, 58% girls), and a high-achieving group (n = 364, 47% girls). Students were taught in separate classrooms with different curricula and we expected self-concept and affective appraisals to be especially influenced by social comparison within the same track (Marsh, 1986). By standardizing the scores, the level of mathematics achievement is compared to peers of the same track instead of peers of the total sample.

Measures

The international research project was coordinated by the National Institute of Education in Singapore. The survey documents were translated from English into Dutch by the authors, based on clear instructions regarding the translation and review. After piloting the survey with students from a school that was not involved in the study, some adaptations were made, as some students indicated that they did not understand certain words. The survey was conducted online using Qualtrics survey software. It consisted of various non-cognitive scales, interspersed with mathematical problems. In the present study, the following scales were used: mathematics achievement, self-concept, anxiety, and enjoyment. The order of the scales and of the items within the scales was randomized. Morony *et al.* (2013) provided an overview of the reliability of the scales in the different countries.

Mathematics achievement

Mathematics achievement was measured with accuracy scores (0 or 1) on 12 items spanning several mathematical subdomains: 11 items from the PISA 2003 assessment (Salz & Figueroa, 2009) and one item from the TIMSS 2007 assessment (Mullis, Martin, & Foy, 2008, p. 107). In the structural equation modelling (research question 1), mathematics achievement was included as a latent variable. A confirmatory factor analysis (CFA) based on all 12 mathematics items showed factor loadings below .3 for two items, which were therefore excluded from further analyses. The final CFA with 10 items revealed only significant factor loadings (p < .001) ranging from .30 to .69 and showed excellent fit to the data: comparative fit index (CFI) = 1.00 and root mean square error of approximation (RMSEA) = 0.00. This latent factor was used to answer research question 1. For the multigroup analyses (research question 2), the latent factor mathematics achievement was included as a manifest variable. This was necessary because the computational complexity of the full model proved to be too high for the software. This mathematics score was standardized for each track separately.

Mathematics enjoyment

After the final mathematics item had been completed, students were asked to indicate how much they like mathematics on a 5-point Likert scale (with scale points 'not at all', 'a little', 'somewhat', 'quite a bit', 'very much', scored 1–5, respectively). Because this question was always on one of the final pages and not all students finished the questionnaire within the available time, there were missing values (15%). ANOVAs comparing students who did and did not fill out this question showed no differences in mathematics anxiety, F (1, 993) = 0.51, p = .477, and mathematics self-concept, F (1, 993) = 3.73, p = .054. However, students who did not answer the question showed a significantly higher mathematics achievement score, F (1, 1012) = 9.84, p = .002 (medium to strong effect size with Cohen's d = 0.65, Cohen, 1992). Further examination showed that in some classes, not enough time had been scheduled for the students to finish the questionnaire. Moreover, it is plausible that high achievers spent more time on the mathematical problems and therefore could not finish the final questions. No further correction or control was applied because this indicates missing at random (MAR) and the percentage of missing values was considered acceptable. Note that only one item was used to assess mathematics enjoyment. Therefore, no further scaling procedure was applied. Using only one item was considered adequate because the concept measured was very straightforward and including multiple items would therefore have resulted in asking the same question in different words, which would have yielded a high reliability but not a truly richer measurement of enjoyment.

Mathematics anxiety and self-concept

Mathematics anxiety (15 items) and self-concept (10 items) were assessed with items from PISA 2003, together with items that were extensively piloted with Singaporean students in 2009 (see Morony *et al.*, 2013). Students responded on a 4-point scale ('strongly disagree', 'disagree', 'agree', 'strongly agree', scored 1–4, respectively). To evaluate the structure of the mathematics anxiety and self-concept factors and to explore the relation between both factors (i.e., to test the separation between affect and competence, Arens *et al.*, 2011; Pinxten *et al.*, 2014), we carried out two exploratory factor analyses (EFA; principal axis analysis with direct oblimin rotation). Based on the first EFA, five anxiety items were excluded from further analyses because of weak factor loadings (<.3) or cross-loadings. The second EFA, based on the remaining 10 anxiety items and 10 self-concept items, revealed two factors (see Table 3 for the items of the scales). The two factors clearly represented mathematics self-concept (factor loadings ranged from .52 to .88) and mathematics anxiety (factor loadings ranged from .56 to .83) and in combination explained 59% of the variance. The factor intercorrelation was −.580. This shows that mathematics self-concept and anxiety are related but distinguishable factors. The two factors were used for construction of the measurement model described below and were included as latent variables in the structural equation models (SEM) and multigroup analyses, and as manifest variables (based on the factor scores) in the descriptive statistics.

Procedure

Data collection took place in spring 2011 and was conducted by graduate students. They were trained and supervised by the first two authors. Participants completed the survey in groups during a mathematics class of approximately 45 min. The survey took place in computer laboratories in their schools.

Statistical analyses

The statistical analyses were carried out in four steps. First, we examined descriptive statistics and correlations. Second, we conducted a CFA to estimate a measurement model for mathematics anxiety and mathematics self-concept. Third, we compared two SEMs to examine our first research question concerning the mediational role of self-concept in the relation between mathematics achievement and emotions. Fourth, we conducted a multigroup SEM to answer the second research question about the differences between low, average, and high achievers. CFA and SEM were performed using Mplus (Version 7; Muthén & Muthén, 1998–2012). The maximum-likelihood estimator was used for the CFA. Because of the categorical variable mathematics enjoyment, the weighted least squares mean and variance-adjusted estimator (WLSMV) was used for the SEM. A disadvantage of the WLSMV estimator is that the chi-square (χ^2) test of model fit cannot be unambiguously evaluated in models with categorical variables. Therefore, we used the CFI and RMSEA rather than the chi-square test to evaluate model fit and used an adapted chi-square difference test, as recommended by Muthén and Muthén (1998–2012), to compare different models. CFI is considered good if >.95 and acceptable if >.90; RMSEA is considered good if <.05 and acceptable if <.08 (Kline, 2005).

Results

Descriptive statistics

Table 1 provides the means and standard deviations of the measured variables of the total group and the three achievement-defined subgroups. The association between gender and track was not significant, χ^2 (2, N = 1,014) = 2.14, p = .344. However, there was a significant association between gender and group classification, χ^2 (2, N = 1,014) = 17.38, p < .001. Univariate ANOVAs showed that girls reported a higher level of mathematics anxiety than boys, F (1, 993) = 15.63, p < .001, a lower mathematical self-concept, F (1, 993) = 22.67, p < .001, and a lower level of mathematics enjoyment, F (1, 864) = 5.00, p = .026. As the effect sizes were small (respectively, Cohen's d = 0.25, d = 0.31, and d = 0.15), analyses were performed on the entire sample.

We conducted a MANOVA to investigate the differences in mathematical self-concept, anxiety, and enjoyment between the low-, average-, and high-achieving students. The differences in mean scores between the three groups were significant, Λ = 0.89, F(6, 1722) = 16.66, p < .001. Univariate analyses showed that the three groups differed significantly on all three variables: mathematics self-concept, F(2, 863) = 49.86, p < .001, η^2 = .10, mathematics anxiety, F(2, 863) = 19.54, p < .001, η^2 = .04, and enjoyment in mathematics, F(2, 863) = 32.91, p < .001, η^2 = .07. Note that the effect sizes were small to medium. *Post-hoc* analyses with Bonferroni correction showed that all group means differed significantly, except for the difference in mathematics anxiety and enjoyment between low and average achievers. As shown in Table 1, the higher the math achievement, the higher the level of mathematical self-concept. Furthermore, high-

Table 1. Descriptive statistics

	Total group			Low achievers			Average achievers			High achievers		
	N	*M* (range)	*SD*	*N*	*M* (range)	*SD*	*N*	*M* (range)	*SD*	*N*	*M* (range)	*SD*
Achievement[a]	1,014	0.00 (−3.69 to 3.58)	1.00	337	−1.09 (−3.69 to −0.48)	0.50	313	−0.06 (−0.47 to 0.42)	0.26	364	1.06 (0.43–3.58)	0.51
Self-concept	995	0.00 (−1.67 to 1.31)	0.65	334	−0.22 (−1.67 to 1.31)	0.63	303	−0.08 (−1.67 to 1.31)	0.62	358	0.27 (−1.54 to 1.31)	0.60
Anxiety	995	0.00 (−0.72 to 1.75)	0.47	333	0.11 (−0.72 to 1.58)	0.48	308	0.02 (−0.72 to 1.64)	0.45	354	−0.12 (−0.72 to 1.75)	0.45
Enjoyment	866	3.04 (1–5)	1.26	302	2.73 (1–5)	1.26	262	2.88 (1–5)	1.26	302	3.49 (1–5)	1.12

Notes. [a]Factor scores standardized by track.

achieving students reported a significantly lower level of mathematics anxiety and a significantly higher level of enjoyment than average- and low-achieving students.

Table 2 presents the correlations between the measured variables in the total sample and in the three subsamples. In the total sample, all variables were significantly interrelated and correlations varied from weak to strong. Note that self-concept correlates more strongly with mathematics emotions (i.e., anxiety and enjoyment) than with mathematics achievement.

Measurement model

A CFA was conducted to create the measurement model that underlies the structural model. In the measurement model, the construction of the latent variables was optimized. In the structural model described below, interrelations between the variables were examined. In Figure 2, the final measurement model is presented. All factor loadings were significant at the $p < .001$ level, ranging from .54 to .83 for mathematics self-concept and from .64 to .80 for mathematics anxiety. Based on the modification indices[1] and inspection of the content of the items, we allowed some error terms to correlate in order to improve model fit. Typically, these items contained the same (Dutch) words or phrases (see Figure 2 and Table 3). The final measurement model demonstrated good fit with CFI = .97 and RMSEA = .046.

Table 2. Intercorrelations between the measured variables

	1.	2.	3.
Total group			
1. Mathematics achievement	–		
2. Mathematics self-concept	.34**	–	
3. Mathematics anxiety	−.22**	−.60**	–
4. Mathematics enjoyment	.28**	.73**	−.46**
Low achievers			
1. Mathematics achievement	–		
2. Mathematics self-concept	.14*	–	
3. Mathematics anxiety	−.09	−.58**	–
4. Mathematics enjoyment	.12*	.70**	−.42**
Average achievers			
1. Mathematics achievement	–		
2. Mathematics self-concept	.07	–	
3. Mathematics anxiety	−.08	−.58**	–
4. Mathematics enjoyment	.05	.70**	−.37**
High achievers			
1. Mathematics achievement	–		
2. Mathematics self-concept	.20**	–	
3. Mathematics anxiety	−.10	−.58**	–
4. Mathematics enjoyment	.19*	.72**	−.49**

Note. *$p < .05$; **$p < .001$.

[1] *The modification index in Mplus provides the expected decrease in chi-square if the parameter is freely estimated (M.I. > 10).*

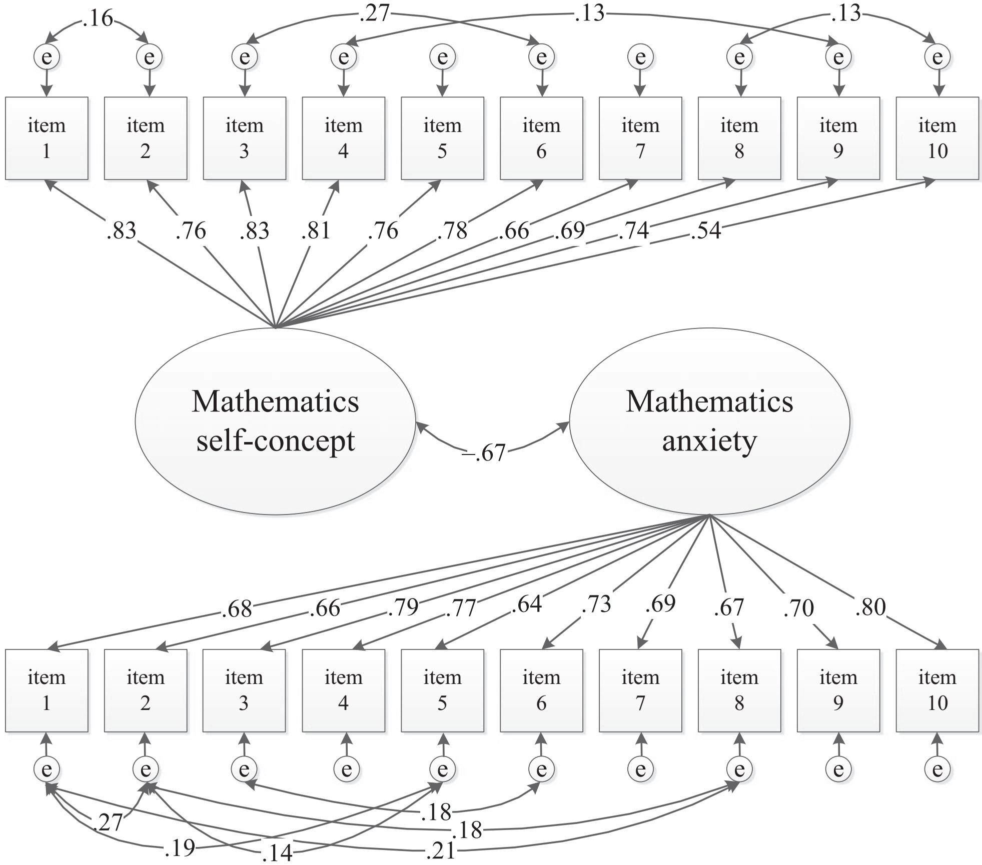

Figure 2. Measurement model of mathematics self-concept and anxiety based on confirmatory factor analysis. All factor loadings were significant with $p < .001$.

Structural model

Research question 1: Examination of the conceptual model

We compared two structural models (see Figure 3) to test our hypothesis that the relation between mathematics achievement and mathematic emotions (i.e., anxiety and enjoyment) is mediated by mathematics self-concept. In Model 1, mathematics anxiety and enjoyment were predicted by both mathematics achievement and mathematics self-concept. Model 1 showed good fit to the data: CFI = .97 and RMSEA = .025. Note, however, that the regression coefficients of the paths from mathematics achievement to mathematics enjoyment and anxiety were nearly zero.

To test the mediation hypothesis, in Model 2, the direct effects of mathematics achievement on mathematics anxiety and enjoyment, and the covariance of anxiety and enjoyment were constrained to zero. The results showed good model fit, with CFI = .97 and RMSEA = .024. Constraining the paths to zero did not significantly worsen model fit, $\chi^2_{\text{diff}}(1) = 3.82, p = .051$. As this more parsimonious model still fitted the data well, Model 2 is preferred over Model 1. The Delta method (MacKinnon, 2008) showed significant indirect effects of mathematics achievement via self-concept on mathematical enjoyment (standardized indirect effect = .35, $p < .001$) and anxiety (standardized indirect

Table 3. Items of the mathematics self-concept and anxiety scales (provided to support interpretation of the measurement model as depicted in Figure 2)

Mathematics self-concept	Mathematics anxiety
1. I learn Mathematics quickly	1. I get very nervous doing Mathematics problems
2. In my Mathematics class, I understand even the most difficult work	2. I get very tense when I have to do Mathematics homework
3. I get good marks in Mathematics	3. I often worry that it will be difficult for me in Mathematics classes
4. I am just not good in Mathematics (R)	4. I feel helpless when doing a Mathematics problem
5. I have always believed that Mathematics is one of my best subjects	5. I become physically uncomfortable when I have to go to Mathematics class
6. I usually do well in Mathematics	6. It scares me to think that I will be taking advanced Mathematics classes in the future
7. I can do almost all the work in Mathematics class if I don't give up	7. I am worried about being called on in Mathematics class
8. Even if the work in Mathematics is hard, I can learn it	8. The harder I work on Mathematics, the more nervous I get
9. Mathematics is harder for me than for many of my classmates (R)	9. When the teacher is handing out Mathematics tests, I feel like I am going to be sick
10. I am sure I can learn the skills taught in Mathematics class well	10. I'm afraid I won't be able to keep up with the rest of the Mathematics class

Note. R = reverse-coded.

effect = $-.29$, $p < .001$). In sum, Model 2 showed full mediation of the relation between mathematics achievement and both mathematic emotions (i.e., anxiety and enjoyment) by mathematics self-concept.

Research question 2: Differences between low, average, and high achievers

A multigroup analysis was carried out in three steps. In the first step, Model 2 from the previous section was fitted to the three groups as an unconstrained model. This model tested whether the overall factor structure fits all three groups, while parameters were allowed to vary between groups. This model of *configural invariance* showed suboptimal fit to the data, with CFI = .90 and RMSEA = .054. Table 4 presents for each group the standardized parameter estimates of this unconstrained model.

In the second step, measurement invariance across the three groups was tested. In this step, it was analysed whether the items measured the constructs in the same way in each group. In a fixed sequence (Vandenberg & Lance, 2000), the following parameters were consecutively constrained to be equal across groups: factor loadings of mathematics self-concept and anxiety, intercepts of mathematics self-concept and anxiety, and thresholds of mathematics enjoyment. At each step, model fit as tested with the chi-square difference test did not significantly worsen. The final model showed *scalar invariance* (i.e., full measurement equivalence across the three groups) and fitted the data well, with CFI = .95 and RMSEA = .034, indicating that the items indeed measured the constructs in the same way in each group.

Finally, *structural invariance* was tested. Now also the regression coefficients of the three structural paths between the constructs were constrained to be equal across groups. The model showed a good fit to the data, CFI = .96 and RMSEA = .029. There was no

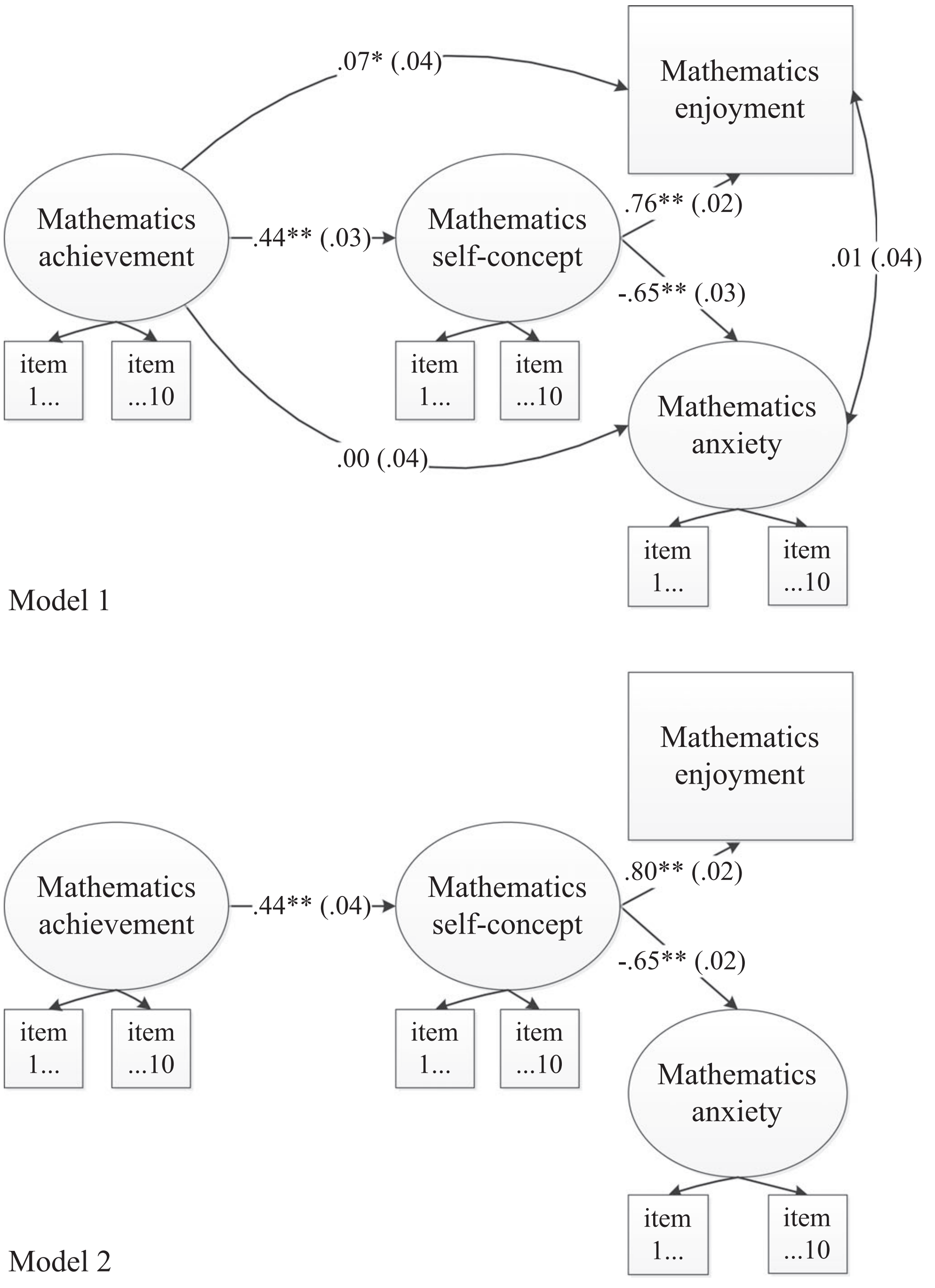

Figure 3. Standardized parameter estimates (structural coefficients and standard errors) of the initial (Model 1) and final (Model 2) structural model. $*p < .05$; $**p < .001$.

significant change in model fit between this and the previous model, $\chi^2_{\text{diff}}(6) = 3.18$, $p = .786$. These results indicate that the relations between the constructs did not differ between the three groups.

Table 4. Standardized parameter estimates for low, average and high achievers (unconstrained model)

	Low achievers			Average achievers			High achievers		
	β	*SE*	*p*	β	*SE*	*p*	β	*SE*	*p*
Achievement → self-concept	.14	0.06	.015	.09	0.06	.149	.21	0.05	<.001
Self-concept → enjoyment	.77	0.03	<.001	.75	0.04	<.001	.81	0.02	<.001
Self-concept → anxiety	−.64	0.04	<.001	−.62	0.04	<.001	−.63	0.03	<.001

Discussion

In this study, we investigated academic achievement, academic emotions, and academic self-concept of adolescent students in an integrative framework. The main objective was to test the hypothesis that the relation between achievement and emotions in mathematics (i.e., enjoyment and anxiety) is mediated by students' mathematical self-concept.

Research question 1: The mediational role of self-concept

We found significant indirect effects of achievement through self-concept on both enjoyment and anxiety. The results support the hypothesis of full mediation of the achievement/emotion relations by self-concept, in line with Pekrun's control–value theory. This theory posits that achievement emotions are evoked based on appraisals of subjective control over achievement activities and their outcomes, and on the subjective value of these activities and outcomes (Pekrun, 2006). In the present study, we focused on the first type of appraisals and, corroborating previous research (Ahmed *et al.*, 2012; Bieg *et al.*, 2014; Goetz *et al.*, 2008, 2010), we found suggestive evidence for their pivotal role in determining emotions regarding mathematics. Moreover, our results showed that mathematics self-concept could also completely account for the strong negative association between anxiety and enjoyment in mathematics, attesting to the central importance of students' mathematics self-concept in the affective appraisal of mathematics achievement.

Our analysis showed that the relations between mathematics self-concept and emotions were stronger than the relation between self-concept and achievement, with the relation between achievement and self-concept being moderate. Apparently, the cognitive and emotional aspects of self-appraisal are only to a limited degree predicted by achievement on a test administered at the same time as the measurement of the appraisals. Put differently, the process of emotion induction during achievement situations in mathematics is controlled by the self-perceived ability of the student, and not (directly) by the students' actual ability. This supports the idea that students do not only rely on their immediate performance but also on previous experience and other sources of information, such as teachers' feedback and pedagogical strategies, to develop their academic self-concept (Marsh, 1986; Möller *et al.*, 2011; Pekrun, 2006). For example, self-concept has been shown to be affected by differentiated instruction strategies (Marsh, Trautwein, Lüdtke, & Köller, 2008; Roy, Guay, & Valois, 2015), teachers' perceptions of their pupils' ability (Upadyaya & Eccles, 2014), and the students' perceived social support from their teacher (Ahmed, Minnaert, van der Werf, & Kuyper, 2010; Sakiz, Pape, & Hoy, 2012).

Research question 2: Differences between the different achievement groups

High-achieving students had a more positive mathematics self-concept and expressed more enjoyment and less anxiety than average- and low-achieving students. However, there were no differences between low-, average-, and high-achieving students in the strengths of the achievement/self-concept and self-concept/emotion relations. These findings are in line with previous studies showing higher self-perceptions and higher self-reported grade point averages (GPAs) in gifted students than in students from the general population, but at the same time invariance across these groups in the strength of the relation between academic self-perceptions and GPA (McCoach & Siegle, 2001, 2003).

On the other hand, recent research suggests that the relation between mathematics achievement and emotion (i.e., anxiety) may be curvilinear (Wang *et al.*, 2015). We inspected scatter plots of the bivariate relations of mathematics achievement and emotions, but found no indication that curvilinear relationships could explain the current results. Nonetheless, in further research on subgroups of students such as high achievers, it is important to keep in mind the possibility of nonlinear relationships between achievement, emotions, and competence-related appraisals. Furthermore, it has been shown that high achievers tend to underestimate their ability when they compare themselves to peers, while at the same time accurately evaluating their performance with regard to absolute standards (Ehrlinger, Johnson, Banner, Dunning, & Kruger, 2008). Therefore, future research on the academic self-concept of high achievers could differentiate between evaluative measures that rely on social comparison versus more objective measures (e.g., how many questions did you answer correctly).

Given the central role of self-concept in relation to mathematics achievement and emotions, one important implication of the present study should be increased awareness among educators of the ways in which they contribute to the development of students' self-concept (Lim & Chapman, 2013; Urdan & Schoenfelder, 2006). For example, the quality of instruction and feedback teachers provide to students influences both appraisals of subjective control and subjective value of achievement activities, which in combination define the emotions students experience (Pekrun, 2006). As emotions play an important role in learning achievements and educational career decisions, teachers should be stimulated to foster positive emotions in the mathematics classroom by enhancing the mathematics self-concepts of their students. In particular, high achievers who underestimate their abilities, do not enjoy mathematics, and/or suffer from math anxiety may need positive performance feedback to enhance their self-concept. By adjusting assessment and feedback procedures, mathematics in school will be more attractive and valued and students' self-concept and liking of mathematics will be enhanced. Intervention studies to test this hypothesis are worthwhile.

One important limitation of the present study was its correlational design, making it impossible to establish causality. Longitudinal studies, especially on the predictive role of self-concept and emotions in later career choices, are therefore desirable, especially given the declining interest in pursuing mathematical careers. Note, however, that the present results are fully in line with recent longitudinal studies on this topic (Ahmed *et al.*, 2012, 2013; Goetz *et al.*, 2008; Pinxten *et al.*, 2014).

Another limitation, due to the fact that existing data from a cross-cultural study were used, is the use of only one item to measure mathematics enjoyment. The reliability of this measure may have been rather low and this could have attenuated the relationships with the other constructs. Moreover, one item may not be sufficient to cover all relevant facets of a construct. Note, however, that mathematics enjoyment is a rather straightforward concept and that the relations that were found with the other constructs were of considerable size. Therefore, the use of only one item for measuring enjoyment does not seem to have seriously threatened the validity of the present results. Another limitation concerning mathematics enjoyment was the relatively high amount of missing values on this variable. As examination of the missingness indicated MAR, this has also not seriously threatened the results.

In conclusion, the results of this study showed a full mediation effect of mathematical self-concept in the concurrent relation between mathematics achievement and the emotions of anxiety and enjoyment in adolescents, which were equivalent across ability groups. The moderate relationships between mathematics achievement and self-concept, and the full mediation by self-concept of the relationships between achievement and emotions that was found, although all constructs were measured in the same situation using a single questionnaire, is convincing evidence that self-concept is a student characteristic that is crucially important for appraising mathematics and that is shaped by multiple factors, likely also including teachers' instruction and feedback strategies.

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References

Ahmed, W., Minnaert, A., Kuyper, H., & van der Werf, G. (2012). Reciprocal relationships between math self-concept and math anxiety. *Learning and Individual Differences*, *22*, 385–389. doi:10.1016/j.lindif.2011.12.004

Ahmed, W., Minnaert, A., van der Werf, G., & Kuyper, H. (2010). Perceived social support and early adolescents' achievement: The mediational roles of motivational beliefs and emotions. *Journal of Youth and Adolescence*, *39*, 36–46. doi:10.1007/s10964-008-9367-7

Ahmed, W., van der Werf, G., Kuyper, H., & Minnaert, A. (2013). Emotions, self-regulated learning, and achievement in mathematics: A growth curve analysis. *Journal of Educational Psychology*, *105*, 150–161. doi:10.1037/a0030160

Andersen, L., & Cross, T. L. (2014). Are students with high ability in math more motivated in math and science than other students? *Roeper Review*, *36*, 221–234. doi:10.1080/02783193.2014.945221

Arens, A. K., Yeung, A. S., Craven, R. G., & Hasselhorn, M. (2011). The twofold multidimensionality of academic self-concept: Domain specificity and separation between competence and affect components. *Journal of Educational Psychology*, *103*, 970–987. doi:10.1037/a0025047

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*, 181–185. doi:10.1111/1467-8721.00196

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, *130*, 224–237. doi:10.1037/0096-3445.130.2.224

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*, 243–248. doi:10.3758/BF03194059

Bieg, M., Goetz, T., & Lipnevich, A. A. (2014). What students think they feel differs from what they really feel: Academic self-concept moderates the discrepancy between students' trait and state emotional self-reports. *PLoS One*, *9*, e92563. doi:10.1371/journal.pone.0092563

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155–159. doi:10.1037/0033-2909.112.1.155

Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behavior*. New York, NY: Springer Science & Business Media.

Dunning, D. (2011). The Dunning-Kruger effect: On being ignorant of one's own ignorance. In J. M. Olson & M. P. Zanna (Eds.), *Advances in experimental social psychology, 44* (pp. 247–290). Amsterdam, the Netherlands: Academic Press.

Dunning, D., Johnson, K., Ehrlinger, J., & Kruger, J. (2003). Why people fail to recognize their own incompetence. *Current Directions in Psychological Science*, *12*, 83–87. doi:10.1111/1467-8721.01235

Ehrlinger, J., Johnson, K., Banner, M., Dunning, D., & Kruger, J. (2008). Why the unskilled are unaware: Further explorations of (absent) self-insight among the incompetent. *Organizational Behavior and Human Decision Processes*, *105*, 98–121. doi:10.1016/j.obhdp.2007.05.002

European Commission. (2014). *The structure of the European education systems 2014/15: Schematic diagrams*. Retrieved from http://eacea.ec.europa.eu/education/eurydice/documents/facts_and_figures/education_structures_EN.pdf

Eysenck, M. W., & Calvo, M. G. (1992). Anxiety and performance: The processing efficiency theory. *Cognition & Emotion*, *6*, 409–434. doi:10.1080/02699939208409696

Eysenck, M. W., Derakshan, N., Santos, R., & Calvo, M. G. (2007). Anxiety and cognitive performance: Attentional control theory. *Emotion*, *7*, 336–353. doi:10.1037/1528-3542.7.2.336

Fisher, P. H., Dobbs-Oates, J., Doctoroff, G. L., & Arnold, D. H. (2012). Early math interest and the development of math skills. *Journal of Educational Psychology*, *104*, 673–681. doi:10.1037/a0027756

Frenzel, A. C., Thrash, T. M., Pekrun, R., & Goetz, T. (2007). Achievement emotions in Germany and China. A cross-cultural validation of the Academic Emotions Questionnaire—Mathematics. *Journal of Cross-Cultural Psychology*, *38*, 302–309. doi:10.1177/0022022107300276

Goetz, T., Cronjaeger, H., Frenzel, A. C., Lüdtke, O., & Hall, N. C. (2010). Academic self-concept and emotion relations: Domain specificity and age effects. *Contemporary Educational Psychology*, *35*, 44–58. doi:10.1016/j.cedpsych.2009.10.001

Goetz, T., Frenzel, A. C., Hall, N. C., & Pekrun, R. (2008). Antecedents of academic emotions: Testing the internal/external frame of reference model for academic enjoyment. *Contemporary Educational Psychology*, *33*, 9–33. doi:10.1016/j.cedpsych.2006.12.002

Gottfried, A. W., Cook, C. R., Gottfried, A. E., & Morris, P. E. (2005). Educational characteristics of adolescents with gifted academic intrinsic motivation: A longitudinal investigation from school entry through early adulthood. *Gifted Child Quarterly*, *49*, 172–186. doi:10.1177/001698620504900206

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., & Oliver, P. H. (2009). A latent curve model of parental motivational practices and developmental decline in math and science academic intrinsic motivation. *Journal of Educational Psychology*, *101*, 729–739. doi:10.1037/a0015084

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., Oliver, P. H., & Guerin, D. W. (2007). Multivariate latent change modeling of developmental decline in academic intrinsic math motivation and achievement: Childhood through adolescence. *International Journal of Behavioral Development*, *31*, 317–327. doi:10.1177/0165025407077752

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*, 33–46. doi:10.3102/00346543058001047

Jameson, M. M. (2014). Contextual factors related to math anxiety in second-grade children. *The Journal of Experimental Education*, *82*, 518–536. doi:10.1080/00220973.2013.813367

Kim, C., & Pekrun, R. (2014). Emotions and motivation in learning and performance. In J. M. Spector, M. D. Merrill, J. Elen & M. J. Bishop (Eds.), *Handbook of research on educational communications and technology* (pp. 65–75). New York, NY: Springer.

Kline, R. B. (2005). *Principles and practice of structural equation modeling*. New York, NY: The Guilford Press.

Lee, J. (2009). Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences*, *19*, 355–365. doi:10.1016/j.lindif.2008.10.009

Lim, S. Y., & Chapman, E. (2013). Identifying affective domains that correlate and predict mathematics performance in high-performing students in Singapore. *Educational Psychology*, *35*, 747–764. doi:10.1080/01443410.2013.860221

Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, *30*, 520–540. doi:10.2307/749772

MacKinnon, D. P. (2008). *Introduction to statistical mediation analysis*. New York, NY: Taylor & Francis.

Marsh, H. W. (1986). Verbal and math self-concepts: An internal/external frame of reference model. *American Educational Research Journal*, *23*, 129–149. doi:10.3102/00028312023001129

Marsh, H. W., Trautwein, U., Lüdtke, O., & Köller, O. (2008). Social comparison and big-fish-little-pond effects on self-concept and other self-belief constructs: Role of generalized and specific others. *Journal of Educational Psychology*, *100*, 510–524. doi:10.1037/0022-0663.100.3.510

McCoach, D. B., & Siegle, D. (2001). A comparison of high achievers' and low achievers' attitudes, perceptions, and motivations. *Academic Exchange Quarterly*, *5*, 71–76.

McCoach, D. B., & Siegle, D. (2003). The structure and function of academic self-concept in gifted and general education students. *Roeper Review*, *25*, 61–65. doi:10.1080/02783190309554200

Mega, C., Ronconi, L., & De Beni, R. (2014). What makes a good student? How emotions, self-regulated learning, and motivation contribute to academic achievement. *Journal of Educational Psychology*, *106*, 121–131. doi:10.1037/a0033546

Möller, J., Retelsdorf, J., Köller, O., & Marsh, H. W. (2011). The reciprocal internal/external frame of reference model an integration of models of relations between academic achievement and self-concept. *American Educational Research Journal*, *48*, 1315–1346. doi:10.3102/0002831211141964

Morony, S., Kleitman, S., Lee, Y. P., & Stankov, L. (2013). Predicting achievement: Confidence vs self-efficacy, anxiety, and self-concept in Confucian and European countries. *International Journal of Educational Research*, *58*, 79–96. doi:10.1016/j.ijer.2012.11.002

Mullis, I. V., Martin, M. O., & Foy, P. (2008). TIMSS 2007. *International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Eighth and Fourth Grades*. Chestnut Hill, MA: Boston College.

Murayama, K., Pekrun, R., Lichtenfeld, S., & vom Hofe, R. (2013). Predicting long-term growth in students' mathematics achievement: The unique contributions of motivation and cognitive strategies. *Child Development*, *84*, 1475–1490. doi:10.1111/cdev.12036

Muthén, L. K., & Muthén, B. O. (1998–2012). *MPlus user's guide* (7th ed.). Los Angelos, CA: Author.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review*, *18*, 315–341. doi:10.1007/s10648-006-9029-9

Pinxten, M., Marsh, H. W., De Fraine, B., Van Den Noortgate, W., & Van Damme, J. (2014). Enjoying mathematics or feeling competent in mathematics? Reciprocal effects on mathematics achievement and perceived math effort expenditure. *British Journal of Educational Psychology*, *84*, 152–174. doi:10.1111/bjep.12028

Plenty, S., & Heubeck, B. G. (2013). A multidimensional analysis of changes in mathematics motivation and engagement during high school. *Educational Psychology*, *33*, 14–30. doi:10.1080/01443410.2012.740199

Riegle-Crumb, C., Moore, C., & Ramos-Wada, A. (2011). Who wants to have a career in science or math? Exploring adolescents' future aspirations by gender and race/ethnicity. *Science Education*, *95*, 458–476. doi:10.1002/sce.20431

Rodriguez, S., Romero-Canyas, R., Downey, G., Mangels, J. A., & Higgins, E. T. (2013). When school fits me: How fit between self-beliefs and task benefits boosts math motivation and performance. *Basic and Applied Social Psychology*, *35*, 445–466. doi:10.1080/01973533.2013.823613

Roy, A., Guay, F., & Valois, P. (2015). The big-fish–little-pond effect on academic self-concept: The moderating role of differentiated instruction and individual achievement. *Learning and Individual Differences*, *42*, 110–116. doi:10.1016/j.lindif.2015.07.009

Sakiz, G., Pape, S. J., & Hoy, A. W. (2012). Does perceived teacher affective support matter for middle school students in mathematics classrooms? *Journal of School Psychology*, *50*, 235–255. doi:10.1016/j.jsp.2011.10.005

Salz, S., & Figueroa, D. T. (2009). *Take the test: Sample questions from OECD's PISA assessments*. Paris: OECD Publishing.

Upadyaya, K., & Eccles, J. (2014). Do teachers' perceptions of children's math and reading related ability and effort predict children's self-concept of ability in math and reading? *Educational Psychology*, *35*, 110–127. doi:10.1080/01443410.2014.915927

Urdan, T., & Schoenfelder, E. (2006). Classroom effects on student motivation: Goal structures, social relationships, and competence beliefs. *Journal of School Psychology*, *44*, 331–349. doi:10.1016/j.jsp.2006.04.003

Vandenberg, R. J., & Lance, C. E. (2000). A review and synthesis of the measurement invariance literature: Suggestions, practices, and recommendations for organizational research. *Organizational Research Methods*, *3*, 4–70. doi:10.1177/109442810031002

Wang, Z., Lukowski, S. L., Hart, S. A., Lyons, I. M., Thompson, L. A., Kovas, Y., . . . Petrill, S. A. (2015). Is math anxiety always bad for math learning? The role of math motivation. *Psychological Science*, *26*, 1863–1876. doi:10.1177/0956797615602471

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