

River coalitions and water trade

By Erik Ansink,^a Michael Gengenbach,^b
and Hans-Peter Weikard^c

^aDepartment of Economic and Social History, Utrecht University and Department of Spatial Economics, Vrije Universiteit Amsterdam

^bDepartment of Social Sciences, Wageningen University

^cDepartment of Social Sciences, Wageningen University, Hollandseweg 1, 6706 KN Wageningen, the Netherlands; e-mail: hans-peter.weikard@wur.nl

Abstract

We analyse coalition stability in a game with a spatial structure. We consider a set of agents located along a river who abstract scarce water for their own benefit. Agents may enter an agreement to mutually acknowledge property rights in river water as a prerequisite for water trade. We find that the potential benefits of water trade may not be sufficient to make all agents in the river cooperate and sign an agreement. Specifically, a complete market for river water may not emerge if there are four or more agents along the river. This result is driven by the spatial structure of our game, in which water that is to be delivered to a downstream coalition member via the territory of an intermediate singleton can be seized.

JEL classifications: C72, D62, Q25.

1. Introduction

We analyse a coalition formation game with a spatial structure to shed light on the stability of river coalitions, partial or grand coalitions, that facilitate water trade. A prerequisite for trade is the mutual acknowledgement of property rights. In the absence of well-defined property rights some agents may seize resources from others. Since [Haavelmo \(1954\)](#), such settings have been analysed using a variety of approaches (e.g., [Bush and Mayer, 1974](#); [Skaperdas, 1992](#); [Muthoo, 2004](#); [Piccione and Rubinstein, 2007](#)). Here, we focus on trans-boundary rivers where inefficient resource allocation is commonplace, precisely because of the lack of well-defined property rights. Water rights are usually contested and the seizing of river water occurs as a result ([Ansink and Weikard, 2009](#)). The inefficiencies associated with deficient property rights can be overcome by an international agreement signed to define water rights. We refer to such agreements (and other types of interaction over trans-boundary water) as river coalitions and examine the conditions for their stability.

The theoretical river sharing literature that emerged after a seminal contribution by [Ambec and Sprumont \(2002\)](#) has used solution concepts from cooperative game theory and

has focused on the grand coalition of all countries located along a river. Some recent contributions to this literature are by [Ambec and Ehlers \(2008\)](#), [van den Brink *et al.* \(2012\)](#), [Ambec *et al.* \(2013\)](#), [Houba *et al.* \(2015\)](#) and [Ansink and Weikard \(2012, 2015\)](#). [Béal *et al.* \(2013\)](#) and [Ansink and Houba \(2015\)](#) provide surveys. Our analysis of water conflict shares its spatial model framework with this literature. Agents are ordered along the river and abstract water for beneficial use, such as irrigation. However, we deviate from these approaches by employing non-cooperative game theory. More specifically, we are interested in specifying the Nash equilibria of a coalition formation game.

Here, a coalition is a subset of agents who mutually acknowledge property rights to river water. They can do so via a formal treaty or an informal agreement.¹ If property rights are mutually acknowledged, water trade can emerge. We argue in this paper that the river structure may obstruct the formation of coalitions with more than three agents and thus the general acknowledgement of property rights. The key reason is that the unidirectionality of water flow implies that upstream agents have the opportunity to take any available water from the river. In the absence of well-defined property rights, they also have the power to do so.²

Our analysis of river water allocation employs the two-stage open-membership cartel game commonly used in the literature on International Environmental Agreements (IEAs); cf. the seminal papers by [Hoel \(1992\)](#), [Carraro and Siniscalco \(1993\)](#) and [Barrett \(1994\)](#) and the recent survey by [Benckroun and Long \(2012\)](#). At stage 1 of this game, dubbed the membership game, agents announce whether or not they will join the agreement. Given these decisions, at stage 2, agents play a resource allocation game which is usually a transboundary pollution game where abatement is a pure public good. In our case, coalition members and the remaining singletons choose their water use. It is generally assumed that the resource allocation game has a unique equilibrium and the analysis of the game focuses on the Nash equilibria of the membership game. These equilibria are referred to as (internally and externally) stable coalitions since no member wants to leave and no singleton wants to join the coalition in equilibrium (cf. [D'Aspremont *et al.*, 1983](#)).

Two particular features distinguish our model from the standard IEA model. First, our model has a spatial structure since the agents are ordered along the river. Water cannot be transported upstream and any water to be delivered to a downstream coalition member via the territory of an intermediate singleton can be seized. We refer to this as leakage. Second, different from the cases discussed in most of the IEA literature, water is not a public good.³ Its (consumptive) use is rivalrous. While water is excludable within an agent's territory, its trade is hampered as long as property rights are not acknowledged. We will see that the potential benefits of water trade may not be sufficient to make all agents in the river cooperate and acknowledge property rights as a prerequisite for trade. Instead, partial markets may emerge where a subset of agents trade river water. This result is driven by leakage.

Spatial structures have been built into transboundary pollution games (e.g., [Folmer and von Mouche, 2000](#)), but their role for the formation and stability of IEAs has not been

- 1 [Giordano *et al.* \(2014\)](#) find that 37% of transboundary river treaties cover the issue of water allocation. [Just and Netanyahu \(1998\)](#) and [Espes and Towfique \(2004\)](#) assess countries' incentives for either formal or informal agreements.
- 2 We follow much of the river sharing literature by assuming that the principle of Absolute Territorial Sovereignty applies. This principle stipulates that, in absence of property rights over river water, every agent has the right to use all river water within its territory.
- 3 Although water quantity is not a public good in most of its uses, water quality is ([Hanemann, 2006](#)).

explored in the literature. An exception is the study by Gengenbach *et al.* (2010), who assess the formation and stability of coalitions for water pollution abatement along a river. The abatement game of that study comprises public goods characteristics, but has a spatial structure. The remarkable finding is that coalition stability is not impacted by the (linear) spatial structure when compared with standard IEA games, except for cases with corner solutions. In this paper, we use a similar approach to analyse river sharing, focusing on water quantity rather than quality. The game we study in this paper is different because water use is rivalrous whereas abatement of water pollution is not. Note that, like Gengenbach *et al.* (2010) and many other papers on river sharing, we only consider the river configuration of through-border ‘linear’ rivers—representing less than half of all transboundary rivers—whilst ignoring other possible river configurations such as border-creator rivers (LeMarquand, 1977; Dinar, 2008). In the concluding section we discuss some challenges associated with applying a coalition model to river geographies that are more complex than the simple through-border river.

We have three main results. In Theorem 1 we characterize the equilibria of the river sharing game and, hence, provide insights into the formation and stability of coalitions for river sharing and the emergence of trade of river water. In Theorem 2 we show that there is no leakage in equilibrium. In Theorem 3 we provide a sufficient condition for the (internal) stability of coalitions and we show that large coalitions are not necessarily stable. The latter implies partial rather than complete markets for river water. Specifically, our results show that a coalition with more than three agents may not be stable. As a result, a (complete) market may fail to emerge and an inefficient allocation of water may result. Our paper contributes to explaining the small number of participants in most treaties on river sharing (Just and Netanyahu, 1998; Dinar, 2008; Giordano *et al.*, 2014).

The paper proceeds as follows. In Section 2 we introduce the river sharing game. In Section 3 we present our main results on river coalitions. In Section 4 we provide an example of an unstable grand coalition which proves and illustrates that markets for river water may fail to emerge. In Section 5 we conclude.

2. The river sharing game

2.1 Preliminaries

Consider a set N of $n \geq 2$ agents ordered along a river. Agent 1 is the most upstream and n the most downstream; agent i is upstream of j whenever $i < j$. $U_i \equiv \{1, 2, \dots, i-1\}$ denotes the set of i 's upstream predecessors, while $D_i \equiv \{i+1, i+2, \dots, n\}$ denotes the set of i 's downstream followers. On the territory of each agent, the total amount of water in the river increases by inflow $e_i \geq 0$, which originates from, e.g., rainfall and tributaries. Each agent i abstracts $x_i \geq 0$ units of water from the river. We assume that all abstracted water is used. The amount of water available to an agent depends on water use by upstream agents. Let the total available water on the territory of agent i be denoted by

$$E_i \equiv e_i + \sum_{j \in U_i} (e_j - x_j). \quad (1)$$

Water use cannot exceed availability:

$$x_i \leq E_i, \forall i \in N, \quad (2)$$

but some amount $u_i \geq 0$ of available water may be left in the river and so is unused:

$$u_i \equiv E_i - x_i. \quad (3)$$

Benefits $b_i(x_i)$ of water use—benefits net of abstraction costs—are strictly concave. We assume that $b_i(x_i)$ is differentiable for all $x_i \geq 0$. We also assume that $0 < b'_i(0) \leq \infty$. That is, unlike [Ambec and Sprumont \(2002\)](#) for example, we do *not* assume that the marginal benefits of water use go to infinity as x_i tends to zero. Denote by \hat{x}_i the satiation point of water use for agent i such that $b'_i(\hat{x}_i) = 0$. Without loss of generality ([Ambec and Ehlers, 2008](#), Remark 2) we assume water scarcity by taking satiation points as weakly larger than inflow for all agents:

Assumption 1 $\hat{x}_i \geq e_i \forall i \in N$.

Agents maximize their benefits of water use and may increase these benefits by trading water with others. Water trade implies that water is shared such that the total benefits of water use of the trading agents increase under the constraint that other agents may take some of the river water that passes their territory. This type of trade can be institutionalized in a treaty that defines the property rights to water as well as compensation for water sharing. Since we employ a coalition formation game, such treaty formation forms a natural interpretation of our model. Yet, our model is more general and covers other types of interactions over water too. One closely related example is an informal agreement where historical water use was never formalized but is still respected. Yet another example is a transboundary water market, where the coalition does not consist of members of a formal treaty, but comprises the buyers and sellers in a market for water who tacitly acknowledge property rights (cf. [Dinar and Nigatu, 2013](#)).

Irrespective of the type of coalition considered, the trading of water within the coalition implies that transfers are being paid. These transfers can be interpreted as monetary or in-kind side payments ([Dinar, 2006](#)), as the sharing of the benefits of cooperation ([Sadoff and Grey, 2005](#)), or as market prices for water ([Wang, 2011](#)). The coalition formation game that we develop in this section does not require that we are specific about the levels of such transfers or how the transfers are derived. Instead, in Section 3.1 we will introduce the concept of a sharing rule for the distribution of the coalition payoff. Payoffs are equal to water use benefits corrected for transfers. Any sharing rule thereby implicitly specifies the levels of transfers between the agents in the coalition.

Using our preferred interpretation of the coalition as a treaty, we model the emergence of such agreements as a two-stage open-membership cartel game as is common in the literature on IEAs. At stage 1, each agent decides whether or not to sign the agreement. We denote this choice by $\sigma_i \in \{0, 1\}$, where $\sigma_i = 0$ and $\sigma_i = 1$ mean that i is a non-signatory or a signatory, respectively. The choices of all agents result in a coalition structure $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$ which corresponds to a set of signatories $S = \{i | \sigma_i = 1\}$. We refer to this set as the ‘coalition’ and to signatories as ‘members’. The incentives to join a coalition or to stay outside are examined in Section 3 below. Here we focus on the second stage of the game, the water use decisions for a given coalition structure. At stage 2, both members and singletons choose their water use levels x_i . Coalition members choose their water use level in order to maximize coalition payoff and do so by distributing their available water to the downstream members with the highest marginal benefits, provided this is possible and profitable. Singletons choose their water use levels in order to maximize individual payoffs by using all available water up to their satiation point.

We will say that a coalition S with coalition structure σ is ‘connected’ if for all $i, k \in S$ (that is, $\sigma_i = \sigma_k = 1$) there does not exist $j \notin S$ (that is, $\sigma_j = 0$) with $i < j < k$. If a coalition is not connected, it may be affected by ‘leakage’. Leakage occurs when intermediate singletons (i.e., $i, k \in S, j \notin S$ and $i < j < k$) seize some or all of this passing water, if they are not satiated. For example, in a river sharing game with six agents and a coalition $S = \{1, 2, 4, 5\}$ agent 3 may use water that 2 passes to 4.

Formally, leakage $l_i(u_i)$ consists of member i ’s unused water u_i that does not reach the next downstream member $k = \min(S \cap D_i)$:

$$l_i(u_i) \equiv \min \left(u_i, \sum_{i < j < k} (\hat{x}_j - e_j) \right), \quad i, k \in S, k = \min(S \cap D_i). \quad (4)$$

Obviously, leakage cannot occur in a connected coalition or between neighbouring members in an unconnected coalition. The amount of water received by coalition member k from its nearest upstream member i , is $r_k(u_i) \equiv u_i - l_i(u_i)$. That is, the amount of water passed from i to k equals the amount of water unused by i minus leakage (the water used by intermediate singletons). We have:

$$r_k(u_i) = \begin{cases} 0 & \text{for } 0 \leq u_i \leq \sum_{i < j < k} (\hat{x}_j - e_j) \\ u_i - \sum_{i < j < k} (\hat{x}_j - e_j) & \text{for } u_i > \sum_{i < j < k} (\hat{x}_j - e_j). \end{cases} \quad (5)$$

Equation (5) shows that received water $r_k(u_i)$ is a piecewise function of unused upstream water, with positive amounts only if all intermediate singletons are satiated. Note that for $i, k \in S$ with $k = \min(S \cap D_i)$, we have: $E_k = e_k + r_k(u_i) = x_k + u_k$.

River sharing is severely constrained by leakage. In Section 3 we will see that leakage may obstruct the emergence of water trade when gains from trade are insufficient to satisfy intermediate agents’ demands.

2.2 Equilibrium water use

With these preliminaries we turn to the analysis of the second stage of the game. Coalition S is given at this stage and agents take their water use decisions. The solution to this sub-game is a Partial Agreement Nash Equilibrium (Chander and Tulkens, 1995, 1997). The coalition allocates water efficiently by maximizing the joint welfare of its members. The singletons are maximizing their own welfare. The water use choice $x_i^*(S, E_i)$ of singleton $i \notin S$ solves:

$$\max_{x_i(S, E_i)} b_i(x_i(S, E_i)). \quad (6)$$

That is, each singleton maximizes his individual benefits of water use, subject to water availability. Note that, by Assumption 1, programme 6 is equivalent to $x_i^*(S, E_i) = \min\{\hat{x}_i, E_i\}$.

Optimal water use by coalition members is slightly more involved. Optimal water use x_i^* (S, E_i) of coalition members $i \in S$ solves:

$$\max_{x_i(S, E_i)} b_i(x_i(S, E_i)) + \sum_{j \in D_i \cap S} b_j[x_j^*(S, E_j(x_i(S, E_i)))]. \quad (7)$$

Coalition members will take into account potential benefits of water use by their downstream fellow members.

Technically, programme 6 and programme 7 for all singletons and coalition members respectively, can be found using backward induction, analogous to [Ambec and Ehlers \(2008\)](#). The result yields a unique vector of water use for all agents, denoted $x^* = (x_1^*, \dots, x_n^*)$. Using (e_1, \dots, e_n) we obtain the equilibrium available water, $E_i^* \equiv e_i + \sum_{j \in U_i} (e_j - x_j^*)$, and the equilibrium unused water, $u_i^* = E_i^* - x_i^*$, for each agent.

2.3 Equilibrium properties

The following lemma presents two causal relations between water passing and the amount of water received, which we use to prove Theorems 1 and 2, introduced below.

Lemma 1 For $i, k \in S$ with $k = \min(S \cap D_i)$, in equilibrium we have:

$$\begin{aligned} u_i &= 0 \Leftrightarrow r_k(u_i) = 0, \text{ or, equivalently,} \\ u_i &> 0 \Leftrightarrow r_k(u_i) > 0. \end{aligned}$$

Proof By (5) we have both $u_i = 0 \Rightarrow r_k(u_i) = 0$ and $r_k(u_i) > 0 \Rightarrow u_i > 0$. The reverse implications are derived as follows. By (6) and (7), all agents aim to maximize the benefits of water use, either individually or as a coalition. By Assumption 1, though, water is scarce. It follows that all water will be used; it will not be wasted. Using (5), no waste implies both $r_k(u_i) = 0 \Rightarrow u_i = 0$ and $u_i > 0 \Rightarrow r_k(u_i) > 0$. \square

Lemma 1 simply states that the coalition passes water if this is beneficial to the coalition and otherwise not. One consequence of the second part of Lemma 1 is that whenever $u_i > 0$, we know that $u_i > l_i(u_i)$ and that any intermediate singleton j with $i < j < k$ must be satiated so that $x_j = \hat{x}_j$. Note that leakage does not necessarily prevent coalition formation or destabilize cooperation within the coalition. As long as leakage is sufficiently small there are gains from water trade by forming a coalition (but see Theorem 2 below). Water passing has implications for the payoffs of both coalition members and singletons. Suppose that, given an unconnected coalition S , there exists a singleton $j \notin S$ and members $i, k \in S$ with $k = \min(S \cap D_i)$ such that $i < j < k$. In case $u_i = 0$, member k does not receive any water from i even though this may have increased the coalition payoff. In this case, the two subsets of coalition members $(i \cup U_i \cap S)$ and $(k \cup D_k \cap S)$ do not increase their payoff beyond the sum of what they could achieve independently. In case $u_i > 0$, however, water passed from i to k increases the coalition payoff. In addition, singleton j benefits from this water passing, if he was not satiated before.

Lemma 1 suffices to establish the following theorem which is inspired by [Kilgour and Dinar \(2001\)](#) and generalizes insights by [Ambec and Sprumont \(2002\)](#). The theorem describes equilibrium properties of the river sharing game, in terms of the marginal benefits of water use and water passing.

Theorem 1 Consider any coalition $S \subseteq N$ with $i, k \in S$ such that $k = \min(S \cap D_i)$. In equilibrium, exactly one of the following statements is true:

- $b'_i(x_i) = b'_k(x_k)$ and $r_k(u_i) \geq 0$,
- $b'_i(x_i) < b'_k(x_k)$ and $r_k(u_i) > 0$ with $x_i = 0$,
- $b'_i(x_i) < b'_k(x_k)$ and $r_k(u_i) = 0$ with $x_i = E_i$,
- $b'_i(x_i) > b'_k(x_k)$ and $r_k(u_i) = 0$ with $x_i = E_i$.

Proof Consider any coalition $S \subseteq N$ in the river sharing game with $i, k \in S$ such that $k = \min\{l \in S \cap D_i\}$. We prove the mutually exclusive statements (a), (b), (c) and (d) consecutively.

- Consider an equilibrium water allocation vector x such that $b'_i(x_i) = b'_k(x_k)$. With equal marginal benefits, agent i has no incentive to change his water use level x_i irrespective of $r_k(u_i) = 0$ or $r_k(u_i) > 0$. This establishes statement (a).

Suppose that (a) does not hold. There are two possibilities, either $b'_i(x_i) < b'_k(x_k)$ or $b'_i(x_i) > b'_k(x_k)$. We analyse both cases, starting with the first.

- Consider an equilibrium water allocation vector x such that $b'_i(x_i) < b'_k(x_k)$. There are two options. If $r_k(u_i) > 0$, any intermediate singletons are satiated so that agent i continues passing water until marginal benefits are equated (i.e., statement [a] applies) or until $u_i = E_i$ (that is, $x_i = 0$) but still we have $b'_i(x_i) < b'_k(x_k)$. This establishes statement (b).
- Continuing on the former case, if $r_k(u_i) = 0$ so that water passing is no option (if not, statement [a] or [b] would apply), agent i uses any available water himself, up to his satiation point. Assumption 1 combined with benefit maximization by any singletons and coalition members upstream of agent i implies that no water is wasted: $E_i \leq \hat{x}_i$. As a result, we have $x_i = E_i$. This establishes statement (c).
- Consider an equilibrium water allocation vector x such that $b'_i(x_i) > b'_k(x_k)$. If $r_k(u_i) > 0$, then the difference in marginal benefits prescribes that water passing should be reduced until marginal benefits are equated (i.e., statement [a] applies) or until $u_i = 0$ and therefore $r_k(u_i) = 0$. As a result, we have $x_i = E_i$. This establishes statement (d). \square

Theorem 1 outlines the importance of water availability, water passing, and leakage for the marginal benefits of neighbouring coalition members. In case (a) marginal benefits are equalized. In this case there may or may not be water passing and leakage. In the next two cases the downstream member has higher marginal benefits and marginal benefits cannot be equalized through water passing. In case (b) because of water shortage and in case (c) because leakage prevents agent i from efficient water passing to j . Case (c) extends the theorem by Kilgour and Dinar (2001) by taking into account the impact of leakage in addition to water shortage as in statement (b). Note that it is not possible to have $b'_i(x_i) < b'_k(x_k)$ with $0 < x_i < E_i$ and $r_k(u_i) > 0$ as the latter implies that intermediate singletons are satiated so that an additional unit of passed water would increase the coalition payoff. Case (d) holds whenever the upstream partner has higher marginal benefits but cannot increase his water use due to the water availability constraint (2). The coalition cannot improve payoff by passing water.

Observe that marginal benefits do not necessarily decrease (weakly) when moving downstream. In case (b) of Theorem 1, marginal benefits actually increase due to water shortage. This is because, unlike Ambec and Sprumont (2002) for example, we do not

assume that the marginal benefits of water use go to infinity as x_i tends to zero. In case (c) of Theorem 1, marginal benefits may increase due to a sufficiently large leakage threshold. Such leakage blocks the passing of water that would have increased the coalition payoff.

2.4 Two benchmark situations

Using programmes (6) and (7), we first present two extreme benchmark situations, All Singletons and the Grand Coalition.

In the All Singletons benchmark, the coalition structure is $\sigma = (0, \dots, 0)$ so that $S = \emptyset$ and programme (6) applies to all agents. Each agent only takes into account his individual benefits in choosing his water use, constrained only by water availability. The equilibrium of this game is found by applying programme (6) recursively to each agent, starting upstream with agent 1 who solves the programme subject to $x_1 \leq E_1 = e_1$. Water use $x_1(\emptyset, E_1)$ determines E_2 according to (1) and the allocation to agent 2 is found by solving the programme subject to $x_2 \leq E_2 = e_1 + e_2 - x_1(\emptyset, E_1)$, and so on. We call this allocation the All Singletons allocation.

Remark 1 Assumption 1 states that $\hat{x}_i \geq e_i \forall i \in N$. Therefore application of programme (6) implies that $\hat{x}_i \geq x_i(\emptyset, E_i) = e_i \forall i \in N$ and therefore $E_i = e_i \forall i \in N$. Hence, by Assumption 1, in the All Singletons allocation there exists no water passing, and agents are only satiated in case Assumption 1 holds with equality.

In the Grand Coalition benchmark, the coalition structure is $\sigma = (1, \dots, 1)$ so that $S = N$ and programme (7) applies to all agents. Each agent takes into account his individual benefits as well as benefits to all downstream agents in choosing his water use. Unlike in the All Singletons benchmark, here the water use of agent i depends on downstream marginal benefits. The solution to this game is found by solving the sub-game starting with agent 1, who takes into account subsequent decisions by downstream agents. We call this allocation the Grand Coalition allocation.

Clearly, given the difference between the two maximization programmes (6) and (7), water use levels in the grand coalition are different from those in the All Singletons benchmark. In general, water is efficiently allocated in the Grand Coalition because water is passed from agents with low marginal benefits to agents with higher marginal benefits subject only to the water availability constraint (2). Only for agents 1 and n we can generally establish differences in water use between the All Singletons and Grand Coalition equilibria. Irrespective of marginal benefits we have $x_1(N, E_1) \leq x_1(\emptyset, E_1)$ and $x_n(N, E_n) \geq x_n(\emptyset, E_n)$. That is, water use by agent 1 in the Grand Coalition is weakly lower than his water use in the All Singletons benchmark, while the reverse holds for agent n .

Notice that all agents (weakly) prefer any coalition S over the All Singletons benchmark. To see why, recall that coalition members maximize the coalition payoff. By (7), the coalition payoff is at least as large as the sum of payoffs if all coalition members behave as singletons. Assuming an appropriate sharing rule for the coalition payoff it can be guaranteed that each coalition member prefers the water use equilibrium under coalition S over the All Singletons benchmark. We formalize this argument in Condition 1, introduced below in Section 3.1. A related observation can be made for singletons. In the All Singletons benchmark, each agent uses $x_i = e_i$. In the river sharing game, the only difference for singletons is that there may be water passing between coalition members so that leakage

may increase their water use to $x_i(S, E_i) > e_i$. Hence, singletons too (weakly) prefer the equilibrium of the river sharing game over the All Singletons benchmark.

3. River coalitions

A priori there is no reason why either the All Singletons or the Grand Coalition would result as an equilibrium of the river sharing game. Some agents in the Grand Coalition may prefer to free-ride (internal instability) while some singletons may prefer cooperation to the All Singletons benchmark (external instability). Hence, we now turn to the analysis of partial markets for river water using the tools of coalition theory. We analyse coalition stability by applying the concepts of internal and external stability and we employ the equilibrium water use levels determined by programmes (6) and (7). This allows us to derive the next results. Theorem 2 establishes that stable coalitions do not suffer from leakage. Finally, Theorem 3 shows that every coalition with two or three agents is internally stable, but larger coalitions may not be internally stable. This implies that partial rather than complete markets for river water may emerge as equilibria in the river sharing game.

3.1 Stability conditions

In Section 2.2 we established for every coalition structure a unique vector of water use levels x^* . Each x^* induces a partition function $v(S)$ that establishes the coalition payoff and individual payoffs to singletons. Singletons' payoffs equal the benefits of water use and are denoted by $v_i(S) = b_i(x_i(S, E_i))$, $i \notin S$. Coalition payoffs are $v_S(S) = \sum_{i \in S} b_i(x_i(S, E_i))$ and may be redistributed among coalition members. The partition function in combination with a sharing rule induces a valuation function that gives a payoff vector $(v_i(S))_{i \in N}$ for all $S \subseteq N$. For coalition members, payoffs equal the benefits of water use corrected for transfers within the coalition. We assume that the coalition uses a sharing rule for distributing the coalition payoff that satisfies a Weak Claim Rights Condition:⁴

Condition 1 (Weak Claim Rights Condition). Payoffs are shared such that for all $S \subseteq N$ we have $v_i(S) \geq v_i(S_{-i})$ for all $i \in S$ if $v_S(S) \geq \sum_{i \in S} v_i(S_{-i})$.

Condition 1 says that each coalition member receives at least his outside option payoff (his 'claim') if the coalition payoff is large enough to satisfy all claims.

At stage 1 of the coalition formation game, we are interested in analysing Nash equilibria. A Nash equilibrium at the coalition formation stage is found by applying the concepts of internal and external stability (D'Aspremont *et al.*, 1983). A coalition is internally stable when no coalition member $i \in S$ wants to leave the coalition; it is externally stable when no singleton $j \notin S$ wants to join the coalition:

Condition 2 (Internal stability). $v_i(S) \geq v_i(S_{-i}) \forall i \in S$.

Condition 3 (External stability). $v_i(S) > v_i(S_{+i}) \forall i \notin S$.

Our external stability condition is somewhat non-standard as it uses a strict inequality sign. Implicitly we assume here that agents who are indifferent between joining the coalition and being a singleton would always join. This simplifies the analysis and can be justified if being a singleton, playing non-cooperatively, is associated with a (small) reputational

4 The condition is a weaker version of the Claim Rights Condition introduced by Weikard (2009).

loss. It is clear that no coalition member has an incentive to leave the coalition under any sharing rule that meets Condition 1, provided the coalition payoff exceeds the sum of claims. Internal stability is therefore guaranteed whenever it can possibly be obtained. Note that the two stability concepts are linked (Weikard, 2009, Lemma 1):

Lemma 2 Under Condition 1, a coalition S is externally unstable if there exists $j \in N \setminus \{S\}$ such that coalition S_{+j} is internally stable.

We will invoke Lemma 2 in Section 3.2 since it allows us to focus on internal stability only.

For the stability analysis that follows below it will also be useful to notice that the river sharing game is super-additive:

Definition 1 (Super-additivity). A partition function v is super-additive if for all $S \subset N$ and all $i \in N \setminus \{S\}$ it holds that $v_{S_{+i}}(S_{+i}) \geq v_S(S) + v_i(S)$.

Remark 2 The river sharing game is super-additive. If an additional agent joins a coalition, water allocation would improve—in terms of efficiency—or remain unaltered. Hence the joint payoff of the initial coalition and the additional agent in the enlarged coalition cannot fall short of their initial payoffs.

3.2 Stable coalitions

We now turn to the stability analysis of coalitions and show how leakage is obstructing the emergence of water trade since it limits the size of stable coalitions.

First, we show how the river sharing game differs from standard IEA games. A common assumption in the IEA literature is that agents are homogeneous, which allows analytical solutions (Barrett, 1994). In this paper, this assumption would imply that agents have the same benefit functions and water inflow, but they obviously differ in their location along the river.

Remark 3 In a river sharing game with homogeneous agents such that $e_i = e \forall i \in N$ and $b_i(x_i) = b(x_i) \forall i \in N$, there are no possible gains from water trade. Therefore, each agent is indifferent between joining and not joining the coalition. By Condition (3) the Grand Coalition forms, but this coalition is not welfare-improving.

Our second main result is established in the next theorem.

Theorem 2 In the river sharing game, there does not exist a stable coalition with leakage.

Proof The proof is by contradiction. Consider a stable coalition $S \subseteq N$ and two of its coalition members $i, k \in S$ such that $k = \min(S \cap D_i)$, and consider leakage. By Lemma 1 and using (4), leakage implies that there is a singleton $j : i < j < k$ with parameters e_j and \hat{x}_j such that $u_i > 0$ and $r_k(u_i) > 0$. Agent j is satiated with payoff $v_j(S) = b_j(\hat{x}_j)$. Now consider coalition S_{+j} . By super-additivity of the game, $v_{S_{+j}}(S_{+j}) \geq v_S(S) + v_j(S)$. By Condition 1, coalition S_{+j} is internally stable and by Lemma 2, this implies that coalition S is externally unstable, a contradiction. \square

Theorem 2 indicates that leakage will not occur in equilibrium. One option is that water-seizing singletons will always join the coalition since the forgone benefits due to

leakage are sufficient to compensate them. In other words, coalitions tend to be connected. This option is proven for two-agent coalitions in part (ii) of Theorem 3 below. Alternatively, leakage may prevent a coalition from being stable. Therefore, absence of leakage in equilibrium does not imply that leakage is an unimportant aspect of the river sharing game.

For the remaining results it is useful to introduce the following definition of effectiveness.

Definition 2 (Effectiveness). Coalition S is effective for agent $k \in S$ if and only if, for $i = \max\{l \in S \cap U_k\}$ we have $r_k(u_i) > 0$.

Effectiveness refers to member k receiving water from the nearest upstream coalition member. It allows us to distinguish between two types of claims c_j , see Condition 1, as established in the following lemma.

Lemma 3 Claim c_j by agent j in coalition $S \subseteq N$ is either low, c_j^L , or high, c_j^H . We have $c_j^L = b_j(e_j)$ and $c_j^H = b_j(\hat{x}_j)$.

Proof A claim by agent j in coalition $S \subseteq N$ is based on his outside option payoff which he would receive under coalition S_{-j} . Clearly, if agent j is either the most upstream or most downstream member of S , then by programmes (6) and (7), no water remains unused upstream, so that agent j has a low claim $c_j^L = b_j(e_j)$.

Next, consider the remaining case where agent j is not the most upstream or downstream member of S . Consider coalition members i and k with $i < j < k$ and such that $i = \max\{l \in S \cap U_j\}$ while $k = \min\{l \in S \cap D_j\}$. In words, there are no other coalition members located between i and j nor between j and k . Consider coalition S_{-j} . There are two options:

- a. If $u_i = 0$, then $E_j = e_j$ so that agent j has a low claim: $c_j^L = b_j(e_j)$.
- b. If $u_i > 0$, then by Lemma 1 we have $r_k(u_i) > 0$ and therefore $E_j \geq \hat{x}_j$, so that agent j has a high claim: $c_j^H = b_j(\hat{x}_j)$. \square

Lemma 3 implies that agent j has a high claim only when coalition S_{-j} is effective for agent k , who is the nearest downstream coalition member. Otherwise, agent j has a low claim. The next theorem shows that coalitions of various types are all internally stable, with the exception of coalitions of at least four agents.

Theorem 3 The following is true in the river sharing game:

- i. Every two-agent coalition is internally stable.
- ii. For every two-agent coalition $S = \{i, k\}$ with $i < k$ and S is effective for k , it holds that $S' = \{i, i + 1, \dots, k - 1, k\}$ is internally stable.
- iii. Every three-agent coalition is internally stable.
- iv. A coalition S of at least four agents is not necessarily internally stable.

Proof The proof is for each part separately.

Part (i): This follows immediately from super-additivity of the game combined with Condition 1.

Part (ii): By effectiveness of S for k we have $r_k(u_i) > 0$ and all singletons j such that $i < j < k$ are satiated. By Part (i) we know that S is internally stable such that claims of agents i and k are satisfied. We can now add any agent j such that $i < j < k$ and by super-additivity $v_{S+j}(S+j) \geq v_S(S) + v_j(S)$. Claims in S do not change when j joins. By super-additivity, and using Condition 1, these claims can be met by $S+j$ such that the internal stability Condition 2 holds. The argument can be repeated for all $j \in \{i+1, \dots, k-1\}$.

Part (iii): Consider coalitions $S = \{i, j, k\}$ and $S' = \{i, k\}$ with $i < j < k$. There are two cases. If S' is not effective for k , then $r_k(u'_i) = 0$, which implies that claims are c_i^L, c_j^L, c_k^L . By super-additivity, and using Condition 1, these claims can be met by S such that the internal stability Condition 2 holds. If S' is effective for k , then $r_k(u'_i) > 0$, which implies that claims are c_i^L, c_j^H, c_k^L . Since $v_j(S') = c_j^H$. By super-additivity, and using Condition 1, these claims can be met by S such that the internal stability Condition 2 holds.

Part (iv): We prove this part by an example of a four-agent coalition that is internally unstable even though the sharing rule satisfies Condition 1. The example is provided in Section 4. \square

Combining parts (i) and (iii) of Theorem 3 with Lemma 2, we have that every one-agent (i.e., All Singletons) and every two-agent coalition is externally unstable (note that this last result requires $n \geq 3$).

The arguments used in the proof of parts (i) to (iii) of Theorem 3 do not generalize to larger coalitions. The reason is that the sum of claims of the intermediate coalition members (i.e., those coalition members that are not the most upstream or downstream member of S) may be too high for the coalition to compensate. By Condition 1, this implies that internal stability is not satisfied. Specifically, such a situation may arise if: (i) the coalition S consists of more than three agents; (ii) the claims of the intermediate members are high (Lemma 3); and (iii) the coalition $S' = \{\min\{S\}, \max\{S\}\}$, is not effective for $\max\{S\}$ (part [ii] of Theorem 3). An example, provided in the next section, illustrates how the determinants of stability interact and proves our result that internal stability for river sharing games with more than three agents cannot be guaranteed.

4. Example of an unstable Grand Coalition

In this section, we develop an example that proves part (iv) of Theorem 3. We will show that a coalition of more than three agents may not be internally stable and, hence, that there is no guarantee that a river coalition would include all agents in the river. In other words, water trade may fail to emerge or may not reach efficient levels such that some gains from trade remain unexploited. Similarities of our example with Example 1 in [Ambec and Ehlers \(2008\)](#) are discussed at the end of this section.

We construct our example using the three conditions identified at the end of Section 3.2. That is, we consider a river sharing game with four agents and the Grand Coalition $S = N = \{1, 2, 3, 4\}$. We choose our parameter values such that both the claims of the intermediate members are high and the coalition $\{1, 4\}$ is not effective. Clearly, if $\{1, 4\}$ were effective, then by part (ii) of Theorem 3, N is stable and the river coalition would cover all four agents. Given that $\{1, 4\}$ is ineffective, the condition of high claims for intermediate members assures that the overall gains from water trade are insufficient to satisfy all claims.

Table 1. An example river sharing game with four agents: levels of inflow, satiation points, and marginal benefits for the All Singletons benchmark.

i	e_i	\hat{x}_i	$b'_i(e_i)$
1	2.00	3	0.22
2	0.75	1	0.50
3	0.75	1	0.50
4	0.55	1	0.90

We construct our example using the following quadratic benefit function:

$$b_i(x_i) = \frac{1}{\hat{x}_i^2} \cdot (2\hat{x}_i x_i - x_i^2). \tag{8}$$

Recall that \hat{x}_i is the satiation level of agent i . To simplify our calculations, we normalize the satiation benefits such that $b_i(\hat{x}_i) = 1$.

Parameter values for e_i and \hat{x}_i in our example are provided in Table 1. Column 4 of Table 1 indicates that there are potential gains from water trade. Water used by agent 1 can be profitably passed to agents 2–4 in order to increase the total benefits of water use. In the Grand Coalition, the marginal benefits of water use can be equalized among all agents.

We proceed by comparing coalition $\{1, 3, 4\}$ and the Grand Coalition which suffices to prove part (iv) of Theorem 3. Consider coalition $\{1, 3, 4\}$. For this coalition we obtain the equilibrium water allocation $x = (1.40, 1.00, 0.82, 0.82)$ with coalition payoff $v_{\{1,3,4\}} = 0.72 + 0.97 + 0.97 = 2.65$. Singleton agent 2 benefits from leakage and is completely satiated at $x_2 = \hat{x}_2 = 1$. This coalition is effective for agent 3 (but note that part [ii] of Theorem 3 does not apply here) since $v_{\{1\}} + v_{\{3,4\}} = 2.64 < 2.65 = v_{\{1,3,4\}}$. That is, if the coalition would not deliver water to agent 3, the coalition payoff would be lower. Therefore, agent 2 has a high claim in the Grand Coalition, which equals $c_2^H = b_2(\hat{x}_2) = 1$. A similar argument can be made for coalition $\{1, 2, 4\}$, which establishes that agent 3 has a high claim in the Grand Coalition, which equals $c_3^H = b_3(\hat{x}_3) = 1$. Agents 1 and 4 have low claims in the Grand Coalition (see proof of Lemma 3). Their claims are $c_1^L = b_1(e_1) = 0.89$ and $c_4^L = b_4(e_4) = 0.80$. Hence, the sum of claims is $0.89 + 1 + 1 + 0.80 = 3.69$.

Next consider the Grand Coalition. The optimal water allocation is achieved when the cheap water from upstream agent 1 is delivered to agent 4. The resulting water allocation is $x = (1.54, 0.84, 0.84, 0.84)$ and the coalition payoff is $v_N = 0.76 + 0.97 + 0.97 + 0.97 = 3.68$. The Grand Coalition payoff (3.68) is insufficient to meet the sum of claims (3.69) and, hence, the Grand Coalition is not stable. Inflow, passed water, and water use for coalitions $\{1, 3, 4\}$ (with and without water passing and leakage) and the Grand Coalition are illustrated in Fig. 1.

Although our modelling framework is different, the above example is similar to Example 1 in Ambec and Ehlers (2008) who, using a cooperative game theory model, demonstrate that ‘when there are more than three agents all distributions may violate the cooperative core lower bounds’. The reason is a similarity between the cooperative core, usually called δ -core, and the internal stability solution concepts. Both consider that after a deviation, i.e., after a withdrawal from the coalition, the other agents will continue their membership and behave accordingly. Despite the similarity of the proofs of Theorem 2 of Ambec and Ehlers (2008) and of part (iv) of our Theorem 3 the scope of the theorems is

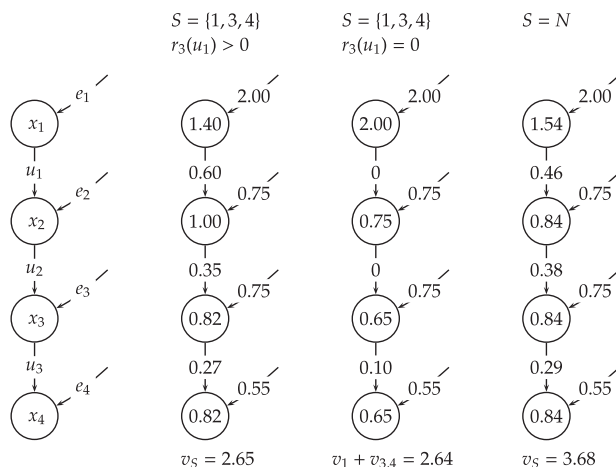


Fig. 1. Inflow, passed water, and water use for $S = N$ and $S = \{1, 3, 4\}$ (with and without water passing and leakage); nodes are agents and arrows indicate water flows. Note that the numbers for water use provided may not sum to total available water $\sum_{i \in N} e_i = 4.05$ due to rounding.

quite different. Using a cooperative approach, [Ambec and Ehlers \(2008\)](#) are not interested in settings where partial coalitions emerge. By contrast our result extends to all partial coalitions with at least four agents in any river sharing game.

5. Conclusion

Our analysis has shown that the potential benefits of water trade in transboundary rivers may not be sufficient to make all agents in the river cooperate and acknowledge the property rights that are a prerequisite for trade. However, a partial agreement with three agents will always be stable. Then a partial market emerges, in which a subset of agents trades river water, while others choose not to join the agreement. One of our main results shows that leakage—or water seizing—does not occur in equilibrium. In essence, if, in an equilibrium, an upstream agent sells water to a downstream agent, the cooperation of all agents located in-between can be ‘bought’. They receive compensation for abstaining from water seizing.

The results of our paper contribute to two different and so far largely unconnected strands of the literature. First, we contribute to the analysis of IEAs by adding a spatial structure. In the river setting location is important to determine the outside option payoffs that are decisive for coalition stability. There are other types of IEAs where location, or the spatial structure in general, is an important component of the analysis. One example is IEAs on biodiversity or marine conservation where the location of habitats plays a crucial role. Another example is IEAs on non-uniformly mixing pollutants like sulphur emissions or water pollution, where transport and dispersion of pollutants is key. This last topic has been picked up by the literature on transboundary pollution games ([Folmer and von Mouche, 2000](#)) using a transport matrix whose coefficients reflect the dispersion of pollution from source to target locations. In the river setting of our paper, this would correspond to a ‘water balances’ matrix as in [Ansink and Houba \(2012\)](#). Assessing more general matrices in the context of a coalition formation game appears to be challenging, however, since this would introduce (even more) asymmetry between the agents in the game.

Second, in the light of network economics the river structure is a simple (directed) network where in-between agents have a decisive position that can facilitate or block upstream–downstream cooperation (cf. Currarini *et al.*, 2016). The application of networks to the analysis of river sharing games is largely restricted to papers that use cooperative game theory. Non-cooperative game theory has seen fewer applications to the river sharing game, mostly confined to bargaining analyses in a two-agent setting. With the exception of Gengenbach *et al.* (2010), we are not aware of other papers that have applied a coalition formation game in a river structure.

Given that river water is not a public good, one would expect transboundary trade in river water to emerge spontaneously. In this paper we show that this is not necessarily true, mainly due to the constraints imposed by unidirectional river flow. Our results help to explain the predominance of ‘small’ treaties on water sharing in transboundary rivers and the difficulties often seen in negotiations on river sharing (cf. Just and Netanyahu, 1998). For example, Giordano *et al.* (2014) find that only a quarter of all transboundary river treaties include all countries in the river basin to which they apply. While the failures of river sharing negotiations have often been explained using arguments based on political feasibility (LeMarquand, 1977; Dinar, 2000; Zawahri *et al.*, 2016), in this paper we show that ‘large’ treaties may be economically unattractive from (some) individual countries’ perspective. In addition, our results help to explain the lack of substantial transboundary water trade. An applied model that confirms this result was recently published by Dinar and Nigatu (2013), who modelled the possible gains from trade among three countries that share the Blue Nile. Their results show that such trade-agreements are not core-stable, indicating the fragility of cooperation in transboundary basins (see also Nigatu and Dinar, 2016). Summarizing, our results put recent research on river sharing in a broader perspective and demonstrate that the benefits of cooperative water use may not be reaped easily.

Funding

This work was supported by the European Research Council [grant number 339647] to EA.

Acknowledgements

We thank two reviewers and seminar participants at various conferences for constructive feedback.

References

- Ambec, S., Dinar, A., and McKinney, D. (2013) Water sharing agreements sustainable to reduced flows, *Journal of Environmental Economics and Management*, **66**, 639–55.
- Ambec, S. and Ehlers, L. (2008) Sharing a river among satiable agents, *Games and Economic Behavior*, **64**, 35–50.
- Ambec, S. and Sprumont, Y. (2002) Sharing a river, *Journal of Economic Theory*, **107**, 453–62.
- Ansink, E. and Houba, H. (2012) Market power in water markets, *Journal of Environmental Economics and Management*, **64**, 237–52.
- Ansink, E. and Houba, H. (2015) The economics of transboundary river management, in A. Dinar and K. Schwabe (eds) *Handbook of Water Economics*, Edward Elgar, Cheltenham.
- Ansink, E. and Weikard, H.-P. (2009) Contested water rights, *European Journal of Political Economy*, **25**, 247–60.

- Ansink, E. and Weikard, H.-P. (2012) Sequential sharing rules for river sharing problems, *Social Choice and Welfare*, 38, 187–210.
- Ansink, E. and Weikard, H.-P. (2015) Composition properties in the river claims problem, *Social Choice and Welfare*, 44, 807–31.
- Barrett, S. (1994) Self-enforcing international environmental agreements, *Oxford Economic Papers*, 46, 878–94.
- Béal, S., Ghintran, A., Rémila, E., and Solal, P. (2013) The river sharing problem: A survey. *International Game Theory Review*, 15, 1340016.
- Benchenkroun, H. and Long, N.V. (2012) Collaborative environmental management: A review of the literature, *International Game Theory Review*, 14, 1240002.
- Bush, W. and Mayer, L. (1974) Some implications of anarchy for the distribution of property, *Journal of Economic Theory*, 8, 401–12.
- Carraro, C. and Siniscalco, D. (1993) Strategies for the international protection of the environment. *Journal of Public Economics*, 52, 309–28.
- Chander, P. and Tulkens, H. (1995) A core-theoretic solution for the design of cooperative agreements on transfrontier pollution, *International Tax and Public Finance*, 2, 279–93.
- Chander, P. and Tulkens, H. (1997) The core of an economy with multilateral environmental externalities. *International Journal of Game Theory*, 26, 379–401.
- Currarini, S., Marchiori, C., and Tavoni, A. (2016) Network economics and the environment: insights and perspectives, *Environmental and Resource Economics*, 65, 159–89.
- D'Aspremont, C., Jacquemin, A., Gabszewicz, J., and Weymark, J. (1983) On the stability of collusive price leadership, *Canadian Journal of Economics*, 16, 17–25.
- Dinar, S. (2000) Negotiations and international relations: a framework for hydropolitics, *International Negotiation*, 5, 375–407.
- Dinar, S. (2006) Assessing side-payment and cost-sharing patterns in international water agreements: the geographic and economic connection, *Political Geography*, 25, 412–37.
- Dinar, S. (2008) *International Water Treaties: Negotiation and Cooperation Along Transboundary Rivers*, Routledge, London.
- Dinar, A. and Nigatu, G. (2013) Distributional considerations of international water resources under externality: the case of Ethiopia, Sudan and Egypt on the Blue Nile, *Water Resources and Economics*, 2–3, 1–16.
- Espey, M. and Towfique, B. (2004) International bilateral water treaty formation, *Water Resources Research*, 40, W05S051–58.
- Folmer, H. and von Mouche, P. (2000) Transboundary pollution and international cooperation, in T. Tietenberg and H. Folmer (eds) *The International Yearbook of Environmental and Resource Economics 2000/2001*, Edward Elgar, Cheltenham.
- Gengenbach, M., Weikard, H.-P. and Ansink, E. (2010) Cleaning a river: an analysis of voluntary joint action, *Natural Resource Modeling*, 23, 565–90.
- Giordano, M., Drieschova, A., Duncan, J.A., Sayama, Y., de Stefano, L., and Wolf, L. (2014) A review of the evolution and state of transboundary freshwater treaties. *International Environmental Agreements: Politics, Law and Economics*, 14, 245–64.
- Haavelmo, T. (1954) *A Study in the Theory of Economic Evolution*, North-Holland, Amsterdam.
- Hanemann, M. (2006) The economic conception of water, in P. Rogers, M. Llamas, and L. Martinez-Cortina (eds) *Water Crisis: Myth or Reality*, Taylor & Francis, London.
- Hoel, M. (1992) International environment conventions: the case of uniform reductions of emissions, *Environmental and Resource Economics*, 2, 141–59.
- Houba, H., van der Laan, G., and Zeng, Y. (2015) International environmental agreements for river sharing problems, *Environmental and Resource Economics*, 62, 855–72.
- Just, R. and Netanyahu, S. (1998) International water resource conflicts: experience and potential, in R. Just and S. Netanyahu (eds) *Conflict and Cooperation on Transboundary Water Resources*, Kluwer Academic Publishers, Boston, MA.

- Kilgour, D. and Dinar, A. (2001) Flexible water sharing within an international river basin, *Environmental and Resource Economics*, **18**, 43–60.
- LeMarquand, D. (1977) *International Rivers: The Politics of Cooperation*. Westwater Research Centre, Vancouver.
- Muthoo, A. (2004) A model of the origins of basic property rights, *Games and Economic Behavior*, **49**, 288–312.
- Nigatu, G. and Dinar, A. (2016) Economic and hydrological impacts of the Grand Ethiopian Renaissance Dam on the Eastern Nile River Basin, *Environment and Development Economics*, **21**, 532–55.
- Piccione, M. and Rubinstein, A. (2007) Equilibrium in the jungle, *Economic Journal*, **117**, 883–96.
- Sadoff, C. and Grey, D. (2005) Cooperation on international rivers: a continuum for securing and sharing benefits, *Water International*, **30**, 420–27.
- Skaperdas, S. (1992) Cooperation, conflict, and power in the absence of property rights, *American Economic Review*, **82**, 720–39.
- van den Brink, R., van der Laan, G., and Moes, N. (2012) Fair agreements for sharing international rivers with multiple springs and externalities, *Journal of Environmental Economics and Management*, **63**, 388–403.
- Wang, Y. (2011) Trading water along a river, *Mathematical Social Sciences*, **61**, 124–30.
- Weikard, H.-P. (2009) Cartel stability under an optimal sharing rule, *The Manchester School*, **77**, 575–93.
- Zawahri, N., Dinar, A., and Nigatu, G. (2016) Governing international freshwater resources: an analysis of treaty design, *International Environmental Agreements: Politics, Law and Economics*, **16**, 307–31.