

# SUPPORTING PROBLEM SOLVING THROUGH HEURISTIC TREES IN AN INTELLIGENT TUTORING SYSTEM

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*In this article we address how to teach the use of heuristics in problem solving. We present an approach using a new support model within an online tutoring system. The outcomes of a pilot study conducted in the Netherlands are discussed. On the basis of these outcomes we make suggestions to improve the model and its implementation.*

*Keywords: problem solving, heuristics, heuristic tree, digital learning environment*

## THEORETICAL BACKGROUND

To solve a mathematical problem, one needs to combine mathematical skills and activities that have already been mastered. So one needs to make strategic decisions on a cognitive level that transcends the procedural. A way to guide a student in such strategic decisions is by providing heuristics. A heuristic (Pólya, 1945) is a general strategy to address a problem, e.g., *investigate special cases*. Heuristics are a form of support on the high end of the cognitive spectrum. On the other end are concrete hints that reveal steps towards a solution.

In this research we study how the delivery of heuristics and hints to learners should be structured. Schoenfeld claims (1985, p. 73): “many heuristic labels subsume half a dozen strategies or more. Each of these more precisely defined strategies needs to be fully explicated before it can be used reliably by students”.

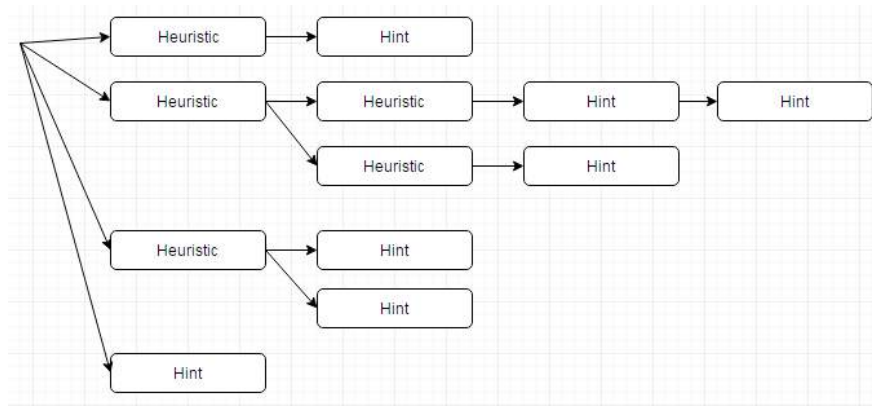
So how should the support using heuristics be structured? The phases suggested by Pólya (1945) and elaborated by Schoenfeld (1985) provide structure. Additionally, several studies suggest a role for fading in various ways, in particular using Intelligent Tutoring Systems. Renkl et al. (2004) begin instruction with worked examples of multi-step problems. In each subsequent problem they remove more solution steps from the example and end up with problems to be solved unguided. This strategy proves to be effective in their experiments. Bokhove and Drijvers (2012) report that even though gradually fading the available feedback on a task causes the performance on that task to gradually decrease as well, the overall effect of the course seems to increase. Roll, Baker, Alevan and Koedinger (2014) discuss an intelligent tutoring system that influences help-seeking patterns. They find that “overusing help is associated with lower learning gains” and “on steps for which students lack basic knowledge, failed attempts are more productive than seeking help”.

## SUPPORT FOR PROBLEM SOLVING THROUGH HEURISTIC TREES

For this study we designed a series of problem solving tasks in an online tutoring system (the Digital Mathematics Environment, recently renamed Numworx<sup>1</sup>). All the problems needed Pythagoras' Theorem in some way.

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<sup>1</sup> <http://www.numworx.nl/en/>



**Figure 1: the structure of a heuristic tree**

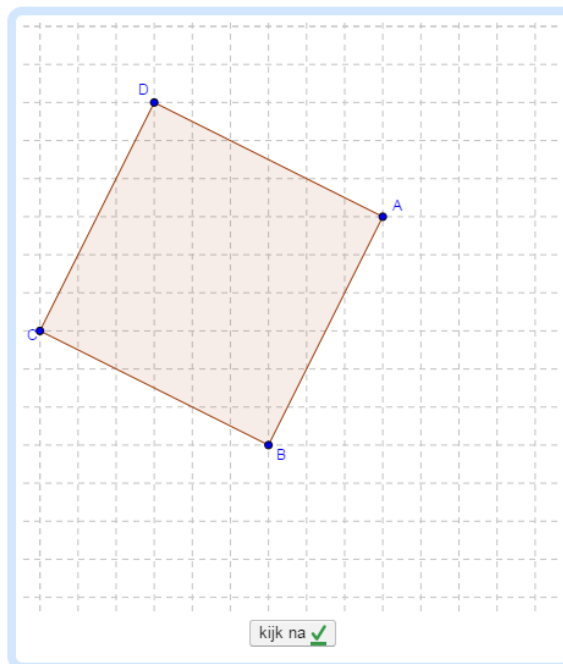
Each problem comes with a *heuristic tree* (see Figure 1). Along each branch a heuristic is node-by-node translated into concrete hints. Each branch represents either a phase or stepping stone in the problem solving process. The learner can choose to access the next node along a branch, but, to avoid overuse, is discouraged to do so by forfeiting a point for each step. This way the fading effect is in the hands of the learner. The goal is a self-regulated transition from procedural thinking to conceptual thinking about the problems involving Pythagoras theorem.

**Square**

In the grid the horizontal and vertical distance between the grid points is 1.

**Assignment**

Move the points A, B, C en D in such a way that the square ABCD has area 45.



**Hints**

How could you make a start with this problem?  
Hint:

What seem to be important prerequisites given the question and the data?  
Hint:

What is the problem?  
What is the difficulty with 45 and the area of a square)?  
Hint:

How do you find the relative position of point A and B by translating the problem to a different branch of mathematics?  
Hint:

Where do you put point C and D?  
Hint:

**Figure 2: screen shot of DME's problem course**

Our hypothesis is that learners' progress along branches gradually decreases as they learn to employ the strategies indicated by the heuristics without further explication. The research question is whether this self-regulated top down approach of teaching heuristics in a digital learning environment improves students' ability to apply heuristics in problem solving and improves their problem solving results.

## DESIGN PRINCIPLES FOR HEURISTIC TREES

We would like to design a heuristic tree structure that both supports learners in problem solving and teaches them to use heuristics. Navigating the heuristic tree should be intuitive and logical for the learners. To this purpose we used the following design principles:

1. The structure of the tree should represent the logical order of reasoning within a solution model.
2. The various branches should also be ordered following the various stages of problem solving. In general: orientation, planning and acting, reflection.
3. The structure of the tree should also match with the intuitive approach of the problem taken by learners.
4. The order along a branch should be *from* heuristics *to* more concrete hints, thereby explaining the use of the heuristic.
5. The help offered in different branches should be independent stepping stones, in the sense that for the help offered in one branch no information in any of the other branches should be needed.
6. Each click should not give more help than asked for.
7. The formulation before the click should not yet give away the heuristic or hint, but give an indication of what can be obtained.

It is a challenge to simultaneously satisfy principles 1,2 and 3, because they not always agree. Principles 5, 6 and 7 are meant to ensure that learners do not receive more help than desired. Designing a heuristic tree that satisfies all these principles is a challenge.

## THE ROLE OF HEURISTIC TREES

The ideal heuristic tree should offer help that is as well suited to the learner as the help offered by a real life teacher. The student must self-diagnose what help is needed, whereas in traditional classroom situations it is the teacher who makes that diagnosis

The concept of the heuristic tree was conceived for the implementation of heuristic problem solving training in digital learning environments. A defining characteristic of problems is that students working on them get stuck. Providing help during a problem solving session with a big group of students can therefore be very demanding for a teacher. The digital environment can provide relief and the possibility to monitor the students' progress in the use of heuristics. In our implementation the teacher can track which heuristics and hints have been used by individual learners.

This suggests that designing a heuristic tree for a problem is useful preparation for a teacher who wants to use the problem even in a classroom without a digital learning environment. In two recent workshops teachers were given the task of designing a heuristic tree for a given problem. It sparked engaged didactical discussions on how to support the students in their problem solving. The heuristic tree provides a structure for the teachers' thoughts and discussions and highlights both the phasing of problem solving and the tension between hints and heuristics.

## PILOT STUDY OUTCOMES AND OUTLOOK

Pilot studies took place in one grade 8 and one grade 11 class at a secondary school in the Netherlands. The students had about 45 minutes to work on the problems. They first received a

short instruction in how to navigate the Digital Mathematics Environment and the problem solving tutor.

The general conclusion is that, in focusing on the structure of heuristic trees, this first version did not pay enough attention to important factors of problem solving as discussed in, for example, Schoenfeld (1985): control (self-regulation) and belief (motivation). These aspects will have to be implemented in version 2.0. Before conclusions can be drawn regarding the hypothesis, these issues and the problems with navigation the heuristic tree and help abuse will have to be addressed.

*Self-regulation.* Learners either used no hints or used them all at once. They either wanted to solve the problem without help or search the whole tree for the golden hint. This fits in with two main forms of help abuse as described by Aleven et al. (2006): clicking through the hints, and help avoidance. They suggest that the learner should actively be taught how to use (digital) assistance. To this purpose they designed a “help-seeking tutor”, but they also presented the learner with a video explaining what they consider the ideal way to seek help.

*Motivation.* Many learners struggled with the difficulty of the problems. This caused loss in motivation for some. In version 2.0 the problems will have a wider range of levels, beginning with easier ones and building up from there. Different avenues within problems or a choice of problems should be offered.

*Another issue: navigation.* Learners find it hard to navigate through the heuristic tree. They do not realize that it is structured on the phases of the solving process or that the heuristics precede the hints. Learning to use it properly should be part of the lesson, as well as learning about heuristics. 45 minutes was not long enough: one would probably need a short series of lessons. The phases should perhaps be based less on Pólya’s phases or the steps of the solution and more on the intuitive approaches of students; on the questions that come to them naturally while working on the problem.

The next step in this research project will be to improve the online course on the points mentioned above. This will be followed by a larger scaled field test over a longer period of time, hopefully giving us data to draw conclusions on our main hypothesis concerning the use of heuristic trees.

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