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Letter to the Editor

Reply to Knobloch

I am sorry to hear that Professor Eberhard Knobloch (Knobloch, 2017) found my paper (Blåsjö, 2017) a disagreeable polemic. It goes without saying that I, like any Leibniz scholar, am greatly indebted to Professor Knobloch's work. It is his tireless and meticulous editorial efforts that made Leibniz's treatise accessible to the scholarly community in the first place. This is an invaluable service to the field for which I have nothing but respect. I regret that I did not convey this clearly in my paper. However, critical discussion of interpretative issues is surely a vital and constructive form of historical scholarship. This is the spirit in which my paper was intended.

I am perplexed by the accusation that I have "fabricated citations." Professor Knobloch raises this charge in two instances. One is that, whereas he spoke only of the foundations of infinitesimal geometry, I on the other hand conflate this with the foundations of the infinitesimal calculus, and project this misconception onto him. I do indeed consider the two to be closely related, and it seems to me that Professor Knobloch's own statements have always indicated this as well. In my paper I quote his claim that "in modern terms Leibniz demonstrated the integrability of a huge class of functions." Elsewhere he has written that "Satz 6 gibt eine strenge Grundlegung der Leibniz'schen Integrationstheorie" (Leibniz, 2016, 283). It does not seem unreasonable to me to take these statements to pertain to the foundations of the calculus.

Professor Knobloch finds the second case of a "fabricated citation" where I write that it is "misleading to speak of a 'rigorous' or 'Riemannian' proof of the 'integrability of a huge class of functions'." Professor Knobloch objects that he never said "Riemannian proof." Indeed he did not, and that is why the word "proof" is not in quotation marks in my paper. Professor Knobloch did say, however, that Leibniz gave a very general, fully rigorous proof that is equivalent to proving "the integrability of a huge class of functions by means of Riemannian sums." It does not seem unreasonable to me to use "Riemannian proof" as a ballpark paraphrase of this view (while also explicitly referencing and reproducing Professor Knobloch's exact words, so that readers can judge for themselves whether I am accurately characterising his view).

Professor Knobloch also maintains that I give an "impossible" reading of Leibniz's phrase, when discussing Proposition 6, that "it serves to lay the foundations for the whole method of indivisibles in the soundest possible way." I proposed that we must understand "it" to refer to the method of proof rather than the proposition as such. Professor Knobloch maintains that, grammatically speaking, "it" absolutely must refer to Proposition 6. I do not deny this, but the meaning cannot be decided by grammar alone. As seen in the full passage quoted in my paper ("Prop. 6. est spinosissima in qua morose demonstratur ..."), Leibniz clearly uses this grammatical construction (i.e., Proposition 6 as the subject) when referring to the proof. That is to say, Leibniz clearly uses this phraseology to refer to this section of text broadly, rather than to the propositional statement as such. Therefore, I do not think my interpretation is at all impossible.

Professor Knobloch's view that "Theorem 6 concerns only the curve passing through the points D of figure 3, not the curve passing through the points C" is clearly at odds with Leibniz's wording. Leibniz's

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proposition is formulated in terms of the latter curve. The interpretation I gave in my paper makes perfect sense of this fact.

Professor Knobloch claims that Proposition 6 is used in Proposition 46. It is not, in my view. Leibniz makes no reference to it in this connection. Nor does Professor Knobloch in the 1993 paper he cites. Professor Knobloch's view that Proposition 6 must be involved because curved figures and rectangles are interchanged seems to me a consequence of his interpretation of Proposition 6 rather than evidence for it. (Leibniz's phrase "quadratura absolute geometrica" refers to the fact that the proof is "geometrical" in the sense of Descartes, i.e., involves only algebraic curves—a point that has no bearing on the issue at hand.)

If we look instead at the propositions beyond the 7th where Leibniz does explicitly refer to Proposition 6, we find that he never invokes it as a blanket license to replace curved figures by approximating rectangles, but rather says that the validity of such steps *can* be demonstrated *in the manner of* Proposition 6, in perfect agreement with my interpretation. ("severe demonstrari posset, ex illis quae prop. 6. diximus" (XII); "areas autem figurarum esse ut summas applicatarum normalium, ex methodo indivisibilium constat, et ad modum propositionis 6. severe demonstrari potest" (XVI); "methodo indivisibilium ad modum prop. 6. demonstrata" (XVII); "ad modum eorum quae prop. 6. 7. 8. diximus facile fieri posse constat" (XXIII).)

Leibniz's self-promoting passages in the scholia to Propositions 22 and 23, which Professor Knobloch brings up, must be read in context. These are propositions regarding quadratures of conic sections, which is, after all, the actual subject of Leibniz's treatise (as its title says). Leibniz's references to novelty and generality concern this, not foundations, it seems to me. The phrase about "new light" is sweepingly addressed to "ignorant" (Leibniz's term) people who reject the use of infinitely small quantities altogether. I do not think this should be read as a careful articulation of the foundational novelty of what Leibniz did seventeen proposition earlier. In my view, it is more natural to read it as an indication that Leibniz wants to use the infinitely small quite freely, as he does in Propositions 22 and 23, but feels obligated to include things like Proposition 6 and this remark about the "ignorant" as a way of preempting anticipated critiques of an overly pedantic nature, much as I argued in my paper.

Professor Knobloch names determinant and elimination theory as examples of important work by Leibniz that remained unpublished during his lifetime. A notable difference with the case at hand is that, as I stated in my paper, Leibniz "had many occasions to write on the foundations of the calculus in print and correspondence"—indeed he was specifically challenged on this exact point and wrote considered replies (without ever naming his old Proposition 6 as providing the definitive answer).

Finally, nothing could be further from my intent than denigrating Leibniz's mathematical achievements. I love Leibniz. I wrote a book on Leibniz which is full of nothing but admiration for his work. My only desire is to understand Leibniz in his own terms.

Viktor Blåsjö
Mathematical Institute
Utrecht University
Budapestlaan 6
3584 CD Utrecht
The Netherlands
E-mail address: V.N.E.Blasjo@uu.nl

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