Int J of Sci and Math Educ (2017) 15 (Suppl 1):S87–S104
DOI 10.1007/s10763-017-9810-x

Which Techno-mathematical Literacies Are Essential for Future Engineers?

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Received: 28 October 2016 / Accepted: 14 February 2017 / Published online: 20 March 2017

Abstract Due to increased use of technology, the workplace practices of engineers have changed. So-called techno-mathematical literacies (TmL) are necessary for engineers of the 21st century. Because it is still unknown which TmL engineers actually use in their professional practices, the purpose of this study was to identify these TmL. Fourteen semi-structured interviews were conducted with engineers with a background in different educational tracks in higher professional education (e.g. civil, chemical, biotechnical and mechanical engineering). As a result of the data analysis, 7 commonly used TmL are identified: data literacy, technical software skills, technical communication skills, sense of error, sense of number, technical creativity and technical drawing skills. Engineers also noted a discrepancy between their education and workplace needs; they characterized mathematics in their education as an island with limited relevance. These findings lead to recommendations for the future of science, technology, engineering and mathematics (STEM) in higher technical professional education that can help students learn STEM for the future.

Keywords Engineering education · Mathematics education · STEM education · Workplace competencies · Workplace skills

The practices of science, technology, engineering and mathematics (STEM) change, and so should STEM education. But what is this STEM for the future, and what should the future of STEM education look like? This paper is concerned with one aspect of this question, namely which mathematical skills engineers use in practice. Through identifying these skills, the future of mathematics curricula for engineering education can be reshaped in line with the future of STEM.

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Over the past few decades, the use of ICT, digital technology and computer-driven equipment in technical workplaces has changed professional practices (Advisory Committee on Mathematics Education, 2011). Calculations are predominantly performed by computers, and therefore mathematics often remains hidden (Hoyles, Noss, Kent, & Bakker, 2010; Williams and Wake, 2007a). Results of these calculations are not transparent and can be unexpected (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Williams & Wake, 2007b).

The need for mathematical skills that transcend mathematical knowledge and content has been recognised for many years. In her book *What counts as mathematics*, FitzSimons (2002) described a wide spectrum of terms that are used to identify these skills, such as “specific and generic numeracies” (Buckingham, 1997). Broader definitions are also mentioned, for example, “technical competence”, which includes mathematical competence and reflective knowledge (Wedege, 2000). In our study, we specifically focused on the technology-mediated nature of using mathematics in the workplace. Kent, Bakker, Hoyles, & Noss (2005) stated:

> Individuals need to be able to understand and use mathematics as a language that will increasingly pervade the workplace through IT-based control and administration systems as much as conventional literacy (reading and writing) has pervaded working life for the last century. (p. 1)

The ubiquitous use of ICT in all sectors changes the nature of the mathematical skills that are required, but does not reduce the need for mathematics (Hoyles et al., 2002). These new mathematical skills have been labelled Techno-mathematical Literacies (TmL) by Kent et al. (2005). TmL integrate mathematical, workplace and ICT knowledge, and communicative skills (Kent et al., 2005), for example the ability to interpret abstract data (Kent, Noss, Guile, Hoyles, & Bakker, 2007), having a sense of number and a sense of error (Hoyles et al., 2010). It is gradually being acknowledged that employees often lack these skills. Because TmL are rarely learned on the job, it is essential that these skills are developed explicitly (Hoyles, Noss, Kent, Bakker, & Bhinder, 2007).

In higher technical professional education in the Netherlands, there is an ongoing discussion about the content and relevance of mathematics curricula (Van Asselt & Boudri, 2013). Mathematics courses are mostly theoretical, with very few context- and workplace-related examples. Estimation and use of software are rarely a part of these courses, although they are a key part of TmL required in workplaces (Hoyles et al., 2010). Furthermore, students have limited motivation for mathematics because of its perceived lack of relevance. The Dutch situation thus seems similar to international observations (Roth, 2014; Wedege, 1999).

Because of the discrepancy between school and work mathematics (Bakker & Akkerman, 2014), students are insufficiently prepared for their future jobs. Although mathematics education, even at the bachelor level, does not only prepare for future work, it is still important to bridge this gap between what is required in workplaces and addressed in education and to implement TmL as learning goals in mathematics curricula.

In engineering research, mathematical skills of engineers have been identified (e.g. Gainsburg, 2007; Kent & Noss, 2000, 2001, 2002). In this study, we deploy the concept of techno-mathematical literacies that engineers with a background in higher technical professional education use in their practices. This use in practice is still unknown, yet relevant to know, so as to be able to better prepare students for their future tasks, not only at work but also in future study and as citizens. Hoyles et al. (2010) researched how employees could develop TmL in their workplaces, but how students in higher professional technical education can acquire these skills in mathematics courses is also unknown.

The global research question in the overall study is how mathematics curricula in the technical domain of higher professional education can be developed to help engineers acquire the necessary TmL for their future workplaces. The questions addressed in this article are as follows: (1) which TmL do engineers use in different professional practices and (2) what are their opinions and ideas regarding their own and future mathematics education?

Techno-mathematical Literacies in Engineers' Workplaces

Over the last 50 years, there has been a shift in the Western world from an industrial economy to an information-based economy. Education in the twenty-first century no longer prepares for lifetime employment. Rapid technological change and globalisation ask for individuals who have a broad general education, good communication skills, adaptability and who are committed to lifelong learning (Millar & Osborne, 1998). On all levels, there is a shift from routine to non-routine tasks, and this asks for specific skills in employees. Voogt and Pareja Roblin (2010) identified these as twenty-first-century skills, encompassing problem-solving, creativity, technology skills, critical thinking and complex communication skills.

Mathematics in Technical Workplaces

Because computers perform most calculations, decisions based upon output are therefore prone to serious errors, if the user does not understand the underlying mathematics (Gravemeijer, 2012). On the one hand, it seems that less mathematical knowledge is needed as computers take over a growing number of mathematical tasks. On the other hand, for "mathematics consumers", there is an increased need to be able to handle and understand quantitative information (Gravemeijer, 2013; Levy & Murnane, 2007).

In this study, we consider the whole spectrum of technical domains at the bachelor level. In engineering education research, Kent and Noss (2002) studied the workplaces of civil and structural engineers. Although mathematics plays a central role in engineering, they found that in their own perception, engineers merely use simple mathematics. They practise division of labour by delegating computational aspects of the work to software and by outsourcing to mathematical experts. In the pre-computer era, engineers developed understanding through daily practice of hand calculation. Because of computers, mathematics is more accessible, and engineers now learn by "understanding through use". To understand and use this technology adequately, mathematical literacy is necessary, analogous to language literacy (Kent & Noss, 2001). James (1995) defined the term "literacy" as the ability that enables the process of good communication and that requires a range of experience.

Bergsten, Engelbrecht, and Kågesten (2015) studied the view of engineers towards the relevance and role of "procedural" and "conceptual" mathematical skills. Kent and

Noss (2002) distinguish those two views as “doing” and “understanding” mathematics. Gainsburg (2007) also studied the workplaces of structural engineers and recognised the use of “engineering judgement” which encompasses engineering expertise with mathematical and other capabilities and introduced the term “sceptical reverence”—a critical attitude towards mathematics. Williams, Wake and Boreham (2001) mention “mathematical competence” as the ability to use mathematics needed in workplaces.

To define mathematical needs that transcend mathematical content, a variety of terms is used. For mathematics in technology-based workplace practices, Kent et al. (2005) introduced the term Techno-mathematical Literacies. They wrote: “We use the term as a way of thinking about mathematics as it exists as part of modern, increasingly IT-based workplace practices” (p. 1). In 2007, they elaborated on the choice of this term by explaining that they found “mathematical literacy” a term of more relevance than traditional notions of numerical skills and competences and added the prefix “techno” to stress that mathematics is mediated by technology and literacies—plural—as a wide variety of knowledge, necessary in workplace practices (Kent et al., 2007).

These complex skills go well beyond basic numeracy because they are grounded in data and the context of specific work situations (Bakker, Hoyles, Kent, & Noss, 2006; Hoyles et al., 2002). For example, graphs are often misinterpreted, even by professional scientists, when they do not originate in their own specific domain (Roth, 2003). Interpreting graphs, charts and outcomes with weak connections to data or underlying mathematical relationships is defined as pseudo-mathematics by Hoyles, Noss, Kent, & Bakker (2013). Context and activity in which mathematical understanding takes place are therefore essential, and employees must make mathematical sense of situations that are quite different from their formal mathematical education (Bakker et al., 2006).

Kent et al. (2005) state that dealing with models and taking decisions based on abstract information have always been tasks of highly trained employees, but due to technology, an increasing number of people engage in these systems, which brings a new complexity to the workplace. Furthermore, whereas visible mathematics is mostly associated with routine tasks and less visible mathematics with non-routine tasks, information technology brings an extra layer of invisibility to the processes behind the screen or printout (Hoyles et al., 2013).

Mathematics in Technical Education

To prepare students for their future jobs, a significant change in education is essential. Schools focus on conventional skills, facts and procedures, whilst learning for the twenty-first century is about using and integrating knowledge in a problem-oriented interactive curriculum (Fadel, Honey, & Pasnik, 2007). Wagner (2008) mentions an achievement gap between curricula in school and what students need to succeed in a global economy. Students do not learn the skills that matter in today’s and tomorrow’s world. He adds that our system and methods of education were created in a previous century with other needs.

There is a growing interest in implementing twenty-first-century skills as educational goals. For example, the OECD Programme for International Student Assessment is currently undergoing significant changes. Last year, the assessment of 15-year-olds was expanded with more skill-related questions, involving for instance collaborative problem-solving.

STEM education, in general, has received increasing attention in recent years (Kuenzi, 2008), and there is a widespread call for improvement in higher education.

Research on STEM learning in the past decades has provided a substantial knowledge base of effective pedagogies and instructional strategies on how students learn (Singer, Nielsen, & Schweingruber, 2012). However, the implementation rates, adoption and scale-up of this knowledge remain low. One reason for this is that STEM educators in higher education are not used to letting educational change processes be informed by theory or literature (Borrego & Henderson, 2014).

As for mathematics education, Garfunkel (2011) states that different sets of mathematical skills are useful for different careers. He also emphasises the importance of learning mathematics in the context of science and providing both useable knowledge and abstract skills. He identifies the mathematics that is needed as "quantitative literacy" and "mathematical modelling".

The use of mathematics in workplaces is rather different from conventional mathematics education. According to Steen (2003), school mathematics is complex, but used in simple problems, whereas workplace mathematics is simple, but is used in complex problems. Alpers (2010) states that mathematics in engineering education has two major goals: "It should enable students to understand, set up and use the mathematical concepts, models, and procedures that are used in the application subjects" and "to provide students with a sound mathematical basis for their future professional life" (p. 2). Whilst content is still essential in engineering education, Cardella (2008) states the importance of "mathematical thinking". The curriculum documents of the SEFI Mathematics Working Group are changing "from contents to outcomes to competencies". In the latest edition of 2013 (Alpers et al.), the main message is that "although contents are still important, they should be embedded in a broader view of mathematical competencies". The document follows the mathematical categories identified by Niss (2003), comprising thinking mathematically, posing and solving mathematical problems, modelling mathematically, reasoning mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating about mathematics and making use of aids and tools. Kent and Noss (2000) add that, in this computer era, the conventional approach of content first and application later in school and work, where the engineer and student apply tools made by others, is no longer adequate.

A gradual shift from mathematical content to a broader definition of mathematical needs can be seen. An example of an initiative regarding mathematics in context and use of technology is the design of a software tool to help students learning laboratory techniques in secondary vocational education develop a better understanding of proportional reasoning in dilution (Bakker, Groenveld, Wijers, Akkerman, & Gravemeijer, 2014). In engineering education, model-eliciting activities (MEAs) are increasingly used at the introductory course level. MEAs are case study problems that small groups solve over one or two class periods and that form a bridge between mathematics and engineering (Hamilton, Lesh, Lester, & Brilleslyper, 2008). In this article, we focus on TmL used by engineers in their workplace practices to specify and elaborate STEM skills to be addressed in STEM education of the future.

Method

The answers to the two research questions were addressed by means of a task-oriented interview study. In 14 semi-structured interviews, the engineers' tasks and practices were used as prompts to reveal the TmL for the first research question. Additionally, the ideas and opinions of the engineers about their previous education and mathematics education, in general, were asked to answer the second research question.

This method was chosen for the following reasons. When asked what mathematics they use, employees often reply “nothing”, even if researchers, whilst observing these employees at work, think otherwise (cf. Duchhardt & Vollstedt, 2016; Noss & Hoyles, 1996). Because employees often do not recognise the mathematics in their work, using questionnaires was not an adequate method as they would not provide valid information about the TmL used by engineers, which is the first research question in this study. Also, ethnographic observation was not an efficient choice because this method would take too much time to cover all technical domains of higher professional education. The interviews, each taking 1.5–2 h, were conducted in the authentic workplaces of engineers in the technical domains of applied science, ICT, built environment, and mechanical and electrical engineering. All participants (eight male and six female) had a background in higher technical professional education (HBO) and originated in the whole spectrum of technical domains. The participants, presented in Table 1, were recruited on a voluntary basis via LinkedIn and other available professional networks and had working experience varying from 2 to 40 years.

Because the goal of these interviews was to make an inventory of TmL in use, the interview scheme of the task-based interviews was inspired by the sparsely available literature on this topic (e.g. Hoyles et al., 2013). It consisted of sets of questions of the following categories: the tasks of the engineer; the engineer was asked to explain and show all of his/her tasks, including software and computer-driven equipment. As a result, in the data analysis, the data could be searched for technological and mathematical elements. Furthermore, there were questions about what mathematics skills were used in their opinion, communication with colleagues, management and customers, and division of labour. Finally, to answer the second research question, questions about the

Table 1 Educational background and workplace settings of the participants

Participant	Education	Workplace setting	Gender
1	Construction	Sales engineer	F
2	Chemical engineering	Quality assurance engineer	F
3	Environmental science	Permit/license advisor	M
4	Mechanical engineering	Sales engineer	M
5	Civil engineering	Department manager	M
6	Mechanical engineering	Calculator	F
7	Chemistry	Research assistant	M
8	Mechatronics	Quality and process manager	M
9	Computer science	Technical consultant	F
10	Biology and applied medical laboratory technology	Senior research assistant	M
11	Electrical engineering	Application lead	M
12	Electrical engineering	Technical writer	M
13	Business mathematics	Web analyst	F
14	Business engineering	Marketing and communication officer	F

engineers' previous education and ideas about how to improve mathematics education at HBO were also addressed.

For the data collection, audio was recorded and screenshots from software usage were taken and the interviews were transcribed verbatim. The goal of the analysis was to identify and define TmL categories and to be exhaustive for these categories, but we did not consider them to be exclusive in advance, and they did, in fact, turn out to overlap somewhat. We used Atlas TI™ to support open, axial and finally selective coding (Boeije, 2005).

Based on the TmL definitions and examples provided in the literature (Bakker et al., 2006; Hoyles et al., 2002; Hoyles et al., 2013), to establish a list of TmL, the data were analysed in cycles, bottom-up and top-down, for mathematics and mathematical skills, technical skills, technical communication and software usage. Codes and sub-codes, with a unit of analysis of fragments with one topic, were assigned and grouped in families. First, technological and mathematical elements were given provisional codes. After consulting experts, combinations of these sub-codes were grouped into TmL categories, which together formed a code family. For example, provisional codes such as "analysing information", "drawing conclusions" and "searching information" were assembled in the TmL category *data literacy*. Another family consisted of the engineers' opinions about their previous mathematics education and ideas for mathematics education in general. Together with a family of miscellaneous codes, this resulted in a codebook of three families. Two of the 14 interviews were also coded by an external coder, who was provided with the codebook and the full transcription of the interviews and asked to assign TmL categories to the transcripts. The interrater reliability, Cohen's kappa, was .84, suggesting that the coding procedure was reliable.

Additionally, the frequencies were determined. These frequencies do not show the use of TmL categories of a representative sample of the population of engineers but provide an indication of the TmL use of a set of engineers from various domains. We do, however, think that the frequencies provide a sense of which TmL may be more common across engineering fields (generating rather than confirming hypotheses).

Results

TmL Used by the Engineers

In answer to the first research question on the TmL used in engineers' practices, the data analysis led to the identification of seven main categories of techno-mathematical literacies used by the engineers, presented in Table 2. Because the tasks of engineers are complex, we often observed the use of multiple TmL categories in a task and also some overlap between the categories.

To illustrate and substantiate the TmL categories used by engineers, we now shall present each TmL category with an exemplary excerpt, followed by a few examples of combinations of TmL categories.

TmL 1: Data Literacy

Data literacy concerns the ability to handle textual, numerical and graphical data sensibly. The engineer has to analyse, interpret, draw conclusions and take action

Table 2 TmL categories with descriptions and frequencies

	TmL category	Description	Frequency
1	Data literacy	The ability to analyse and interpret technical data and graphical representations, draw conclusions and take action accordingly	29
2	Technical software skills	The ability to use professional software, e.g. Excel™, as a calculation tool	80
3	Technical communication skills	The ability to communicate technical information with colleagues, customers, supervisors and other parties	32
4	Sense of error	The ability to check and verify data and detect errors	27
5	sense of number	The ability to handle and interpret numbers sensibly	14
6	Technical creativity	The ability to produce creative solutions to puzzles and problems (by using, e.g. cleverness or experience)	20
7	Technical drawing skills	The ability to understand and produce technical drawings (by using, e.g. spatial insight)	12

accordingly. One of the tasks of the marketing and communication officer (P14) was to gain as many clicks as possible on advertisements in her company's campaigns in Google AdWords™.

> I:So you watch these numbers [of clicks on advertisements]. And then what do you think or do?
>
> P14:I think: All right, this is not going well. This one here, which is very low, must be removed. But I can also leave it as it is. I watch the numbers and think: OK, this one is very low, but do I mind that? No, because the fact that they show is enough. They don't have to click it [click on the advertisement].

She analysed the numbers (which numbers are low and whether that is a problem) and drew conclusions (the fact that they show is enough), which are important elements of *data literacy*. Working with both textual or numerical data and with graphical representations is a part of data literacy. For example, a technical writer, whose job was to write manuals for machines that produce digital chips (P12), emphasised that his skills to produce insightful graphics are very important.

TmL2: Technical Software Skills

The second TmL category, *technical software skills*, is the most frequently observed category in this study and relates to the ability to use technical software. It includes working not only with general (e.g. Excel™) but also domain-specific technical company software. A relevant issue regarding technical software skills is the level of transparency. One can distinguish three levels of transparency, called white box, grey box or black box. In the case of a white box, the user knows exactly which calculations and mathematics are behind the interface of the software and understands these. When software is perceived as a grey box, the user understands only a part of the processes, and in the case of a black box, none (Kent & Noss, 2002, Williams & Wake, 2007a).

We have observed these three levels of transparency in the tasks of the engineers in the interviews and shall present some examples. Furthermore, we found that Excel™ as a calculation and planning tool, in particular, was employed in every technical domain, and some engineers also programmed in this application.

An example of programming in Excel™ was presented by the sales engineer (P4), who sells stabilisation fins for yachts. To calculate the price of a fin for the customers, he created an Excel™ calculation tool to save time, thus emphasising the potential gain in efficiency through software tools:

> At one point, I looked at how to make the calculations easier. Because every time, they searched for lists, what did we get last time, what were the costs from the beginning all over again? I considered that insensible, so at one point I made something [an Excel™ calculation tool for price calculation], with a purpose to make things easier for my colleague and myself.

For all three levels of transparency, an example is now given. For a department manager in the field of civil engineering (P5), the calculations in his Excel™ calculation tool that converts rain in millilitres to cubic metres, were completely transparent to him (white box) because he considered them "basic knowledge". For a chemical engineer who works at a plant for potato products (P2), the software she uses is sometimes less transparent (grey box). She could understand the integrals behind a software tool she uses to calculate the amount of reducing sugar in the potato product with infrared spectrometry, but she does not have to. However, she does have to know what the numbers in the outcome mean. The research assistant (P7) in chemistry perceives his software for the spectrometer as a black box, which is not a problem because of alternative checks. When measuring the spectrum of an enzyme, he knows what the shape of the spectrum generally should look like, then checks with a small ruler and draws conclusions by reasoning back.

TmL 3: Technical Communication Skills

The third category of TmL, *technical communication skills*, includes communication with various parties. One can distinguish horizontal communication (with colleagues and other departments) and vertical communication (with management, customers and employees). A broad variety of these skills is important, as explained by the quality and process manager (P8). He stated that his educational background, mechatronics, supports these skills: "I can talk with technical people as well as management". Sometimes it is important that simple language is used. The license advisor in environmental engineering (P3) asks for plain "Miffy" language to facilitate mutual understanding (Miffy is a famous Dutch rabbit figure for toddlers made by Dick Bruna, with very simple language use).

Interaction with customers requires more advanced technical communication skills. The sales engineer (P1) who designs climate ceilings that can both cool and heat the room explained how she asks her customer's specific additional questions. For the interviewer, she used an analogue (cars and colours) to vividly elaborate these questions:

> When we calculate the ceiling, we have to deal with the design. The customer often says: this [climate] ceiling should feel like such and such and good luck.

> And then we ask: You want a white car, but do you want a Fiat or a BMW? That is a big difference. And then we mostly succeed and know which other party we have to involve. In the end, the customer says: I want the BMW.

To communicate effectively with customers, one has to consider what representations to use. The web analyst (P13) was aware that she cannot use tables for her customers. She argued: "I think presenting a lot of tables is very adequate, but for most people, that is not the case. People cannot work with those". She recognises that her level of *data literacy* does not always correspond with that of a client, so she adapts her *technical communication*.

Division of labour and collaboration are evidenced in multiple references to asking each other questions, interacting with colleagues from different disciplines and seeking advice. Of course, a junior often asks for advice from a senior, but the other way around is also important, according to our senior application lead (P11): "I can still read C+ [a programming language], but of course my knowledge is outdated, and then I ask one of the youngsters".

TmL4: Sense of Error

As a fourth TmL category, we distinguished having a *sense of error*, which encompasses the ability to detect errors in all sorts of data. Because errors—also small ones—can have a large effect, this is an important skill. The license advisor (P3) often scrutinises the reports he receives: "We have to do something with that when we see something conspicuous in such a report. Maybe it is a sloppy mistake; we have to detect that and pass it on". When errors in numbers are detected, this TmL category has some overlap with the next category, *sense of number*. We have often observed the combination of these two TmL categories in the engineers' tasks. In the next section, a combination of TmL categories, we will provide an example of this.

TmL5: Sense of Number

Handling numbers sensibly is very important for engineers. It is essential to understand what a certain number means and how to interpret such numbers. When using software, for example, the input of the right numbers and interpreting the output numbers correctly are crucial. The technical writer (P12), with a background in electrical engineering, speaks about a kind of "numeracy" that is needed, which includes handling units, for example, to know the difference between m (milli) and μ (micro). *Sense of number* not only often combines with *sense of error* but also with *technical software skills*. In the combined TmL section, we discuss this.

TmL 6: Technical Creativity

The sixth TmL category, *technical creativity*, is a particular skill which encompasses cleverness, experience and puzzle-solving abilities, especially for engineers who design. The mechanical engineer of large cooling systems (P6) solves a lot of puzzles in her work: "We are specialised in solving puzzles. It is puzzling and boggling and calculating. Is it correct, is this all?" When programming software, technical creativity plays an important role, she explained: “My colleague, for example, designed a small programme in Excel™ to calculate the costs of pipe lengths. Considering all the variables, it is not so difficult in a mathematical sense; it is just puzzling with formulas”.

TmL 7: Technical Drawing Skills

The seventh and last TmL category entails *technical drawing skills*. These skills include understanding and interpreting technical drawings and, for some engineers, producing them as well. An important component of these skills is spatial insight. Although this skill is merely domain-specific and applies, for example, more to the domain of built environment (e.g. construction) than to the domain of applied science (e.g. chemistry), the permit advisor with a background in environmental engineering (P3) also needs this skill when he interprets technical drawings.

> I need spatial insight so that I understand what this thing looks like from the top, side, and front. Because I know that when there is a driveway on the right in the drawing, it is on the other side of the section drawing. You have to be able to think in planes and think spatially.

Combination of TmL Categories

As mentioned before, because the tasks of engineers are often complex, multiple TmL categories were assigned to many fragments. The TmL category *sense of error*, for example, often combines with a *sense of number*. The sales engineer (P4) who sells stabilisation fins for yachts explained how he checks the numerical data of the yachts (e.g. sizes) for errors because “one-tenth or two-tenth can have a big influence on the outcome”. The research assistant (P7) states the importance of context for a *sense of number* with the analogue of the speed of a car to detect an error (*sense of error*).

> When you calculate the speed of a car and the outcome is 6000 km/h, then you know there must be a mistake somewhere in the calculation. When it’s about bare numbers, it is more complicated to detect errors.

Having *data literacy* does not mean the engineer always understands all the details, as explained by the technical writer (P12) of manuals for machines that produce digital chips. He combines *data literacy* with a *sense of error* when receiving input and explains that, as a technical writer, he knows a little about a lot. Looking at graphs and formulas of polynomials, for example, he often does not know which formula corresponds with a certain graph. However, he learns by doing; when seeing something strange (*sense of error*), he puzzles and searches until he understands (*data literacy*).

The sales engineer (P4) of the stabilisation fins for yachts integrates TmL categories *data literacy*, *technical software skills* and *technical communication skills* in his task of calculating power versus fin size. His Excel™ calculation tool, with endless formulas, embodies years of experience. When another company offers a fin of 3.5 m^2, where his company can offer only 2.94 m^2, he concludes that the competition is wrong or has a better solution. He then starts thinking and sparring with his colleagues (*technical* *communication skills*): “How can the competition offer this size for this amount of power (*data literacy* and *technical software skills*)?”

The permit advisor, with a background in environmental engineering (P3), whilst interpreting technical drawings also uses *data literacy* and *sense of number*. He explained: “In supporting reports, we have to be able to interpret the drawings. All these parameters, for example, 50 m^3, how much water is that? All these pipes, we have to understand how it works”. To interpret the technical drawings, he needs *data literacy* (how pipes work) and *sense of number* (how much water is 50 m^3).

Engineers’ Ideas and Opinions

With respect to the second research question on the ideas and opinions of the engineers, we discuss the findings in subsections regarding their previous mathematics education, the benefits of mathematics in general and the future of mathematics.

Mathematics: Island or Mainland

All engineers’ previous education had been theoretical, mostly without context, professional tasks or products. They perceived their mathematics courses as islands, with no relation to the rest of their education. They asked themselves (and their teachers): “Why do we have to learn this? What has this to do with my major?” This had a significant influence on the motivation for mathematics, as the sales engineer (P4) explained that he perceived mathematics as something that just had to be done. It was not his favourite subject; he found other, applied, courses much more interesting.

Almost all interviewees thought mathematics should be taught in context to enhance student motivation and to recognise its purpose (only two interviewees did not spontaneously say this). They stated that their mathematics was completely separated from the rest of the courses and a real obstacle for many students. After all, higher technical professional education is about application! The engineer with a background in mechatronics (P8) once had a teacher who made a drawing of a car crash with a tree as an example to illustrate movement and to explain why the driver hit the steering wheel, and then he thought: this is useful!

Besides the advantage of context in mathematics for motivation, the sales engineer with a background in construction (P1) emphasised the importance of context for better understanding.

> I experience technical mathematics as hard because, at university, we had plain exercises in a book. It would have been better if there had been practice examples because I understand $water \times air$ much better than $a \times b$. But I still have the problem that when we calculate a tennis court, I do not recognise the same rectangle in a soccer field.

The technical writer with a background in electrical engineering (P12) also wished he had more knowledge of applied mathematics. He engaged in advanced mathematics during his education, but one time, during a presentation when someone demonstrated calculated forces in a forklift truck, he did not understand the calculations and wished he had been trained in applications “back then”.

Mathematics: Useful for Cognitive Skills

Although engineers say they do not use advanced mathematics, the engineers who were former high achievers in mathematics emphasised the general importance of mathematics for analytic and logical thinking, learning and interpretation abilities. The business engineer (P14) recognised an indirect profit of her mathematics education. She said that she had not solved any equation since university, but her mathematical background guaranteed a certain level in the way of looking at things.

Mathematics: Even More Technology in the Future

For the future, the interviewees think that the skills they need will be the same as now, but that there will be even more computer tools and technology. They state that mathematics will always be very important because technical developments rely on it completely. The chemical engineer (P2) thinks one has to know and understand the basics in mathematics education, but the focus should be on comprehension of the software and not on performing complex calculations by hand. The sales engineer (P4) added:

> I have to say, calculus and such, I have never used it. Most of the time it is hidden in the software, and it would be nonsense to let someone calculate for a whole day what a computer can do in a minute.

The technical writer of electrical engineering (P12) took it a step further and provided an example of trial and error with a PID controller instead of understanding the software or calculating by hand.

> A proportional–integral–derivative controller (PID-controller) calculates with derivative and integral terms. There are an entry and an exit, and you try to set the exit in a way that the controller does what you want, based on the entry. But you do not have to calculate PID areas! There are just some variables you can set, and then you watch what happens. Trial and error. Nobody builds these controllers; you just buy them.

What implications do these results bear for the education of engineers with a background in higher technical professional education?

Conclusion and Discussion

The first research question concerned the identification of the TmL used in the professional practices of engineers with a background in higher professional education. In reply, seven main categories of TmL, summarised in Table 2, were identified. This inventory is in line with the generic categories of Bakker et al. (2006), Hoyles et al. (2002) and Hoyles et al. (2013), who found that the ability to interpret data and graphs (TmL1, *data literacy*), the ability of calculation and estimation, having a sense of error (TmL4, *sense of error*, and TmL5, *sense of number*) and the ability to communicate (e.g. technical information; TmL3, *technical communication skills*) are common TmL in workplaces within the technical and economic domains.

The earlier finding that mathematical skills are almost always mediated by technology is confirmed in this study (Hoyles et al., 2010). The category with the highest frequency is the second TmL, *technical software skills*, and it is assumed to be the most important TmL. Hoyles et al. (2013) identified TmL in the financial sector. A similar TmL appreciates the existence of a mathematical model underlying computer output. The software is then experienced as a white box (an element of TmL2, *technical software skills*). In engineers' workplaces, this is often, but not always, the case. Sometimes, the calculation tool is a black box, which is not always a problem, as we saw in the example of the Research Assistant and his spectrometer software and the example of the PID controller.

Some TmL mentioned in the literature were not found in our current study on engineers. For example, the TmL of model making is hardly used by engineers with a background in higher professional education, but is used at the academic level. On the lower end of the educational levels, Hoyles et al. (2002) found the precise entry of data as a TmL in the manufacturing sector. This was not found in the workplace of engineers either. We attribute this to the level of the interviewees' positions and educational backgrounds: They had attended higher professional education (bachelor's level), but were not academically trained (master's level; cf. Frejd & Bergsten, 2016). Furthermore, even within the same educational level, TmL are not merely generic skills. This study also points out that specific TmL are required for particular domains. *Technical creativity* and *technical drawing skills* are TmL that are specifically found in the technical domain and that, to our knowledge, have not been described in TmL literature before.

In answer to the second research question on the opinions and ideas of engineers about their previous education and mathematics education in general, we found foremost that the engineers thought that mathematics should be taught in the context of professional tasks. This is not only important for student motivation but also to gain experience with, and to better understand, applied mathematics as it is used in the workplace.

For the future, they still see a core role for the TmL skills as identified above, but using, even more, calculation tools, software and technology. Most engineers pursue extra education for specific work areas such as statistics, software and project management and say they will continue to need to do so.

Several engineers advise that mathematics education should focus on using and understanding technical software. Students have to know the basics, but should not perform complex calculations that computers can do instantly by hand. To understand underlying software and to uncover the black box, it can be useful to use mechanical tools as an intermediate step to digital technology (Bakker, Wijers, Jonker, & Akkerman, 2011). The Advisory Committee on Mathematics Education (2011) states that employers think employees should study mathematics at a higher level than they use in their practices to provide them with confidence and versatility to use mathematics in new situations at work.

Although the engineers state that they do not use the advanced mathematics of their education, they emphasise the general importance of mathematics for analytic and logical thinking and also recognise technology to rely completely on mathematics. Gainsburg (2007) advocates a realistic view of mathematics. Bialik and Kabbach (2014) agree with this view and state that it is most likely that higher-order thinking skills support mathematical skills and not the reverse. However, Rajagukguk and Simanjuntak (2015) found improvement in students' critical thinking ability using integrated problem-based mathematics teaching kits implemented with ICT. Also, Huang, Ricci, & Mnatsakanian (2016) recommend "thinking through math" with meaningful mathematical experiences to enhance critical thinking.

The identified TmL and the information obtained from the remarks about the engineer's previous education and their ideas about mathematics education in general lead to some recommendations for mathematics curricula in higher technical professional education. In our view, mathematics education should be rooted in professional tasks and products, and TmL should be amongst the main learning goals because today's and tomorrow's engineers need these new skills in the modern workplace which is equipped with more and more advanced technology. Therefore, subsequent (design) research of applied mathematics courses in engineering education that aims to foster these TmL is necessary.

Compliance with Ethical Standards

Funding This research is funded by Avans University of Applied Science.

Appendix

Table 3 Interview scheme

Subject	Question	Sub-question
Job	What is your job description?	
	What are your tasks?	
	Do you have collaborations and/or division of labour in your job?	
	What mathematical skills do you use in your job?	Do you use diagrams and/or tables?
	What software and equipment do you use?	What is the purpose of your software and equipment usage?
	Can you give examples of your tasks and demonstrate them, including software usage?	
Communication	With which parties do you communicate?	How is this communication?
	Do you ask for advice or aid?	From whom?
Previous education	What mathematical skills did you learn?	What are the most important skills that you have learned?
	What software and equipment did you use?	
	What are your experiences regarding your own mathematics education?	Did you use software and/or pen and paper?
	What are your opinions and ideas about mathematics education for higher technical professional education in general?	
Education at work	What mathematical and technical skills did you learn on the job?	Did you use software and/or pen and paper?
Future	Which skills do you need in 5 years' time?	
	What are your ideas about your lifelong learning?	

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