Distant Group Responsibility
in Multi-agent Systems

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Abstract. In this paper, we introduce a specific form of graded group responsibility called "distant responsibility" and provides a formal analysis for this concept in multi-agent settings. This concept of responsibility is formalized in concurrent structures based on the power of agent groups in such structures. A group of agents is called responsible for a state of affairs by a number of collective decision steps if there exists a strategy for the agent group to preclude the specified state of affairs in the given number of steps. Otherwise, the group is partially responsible based on its maximum contribution to fully responsible groups. We argue that the notion of distant responsibility is applicable as a managerial decision support tool for allocation of limited resources in multi-agent organizations.

1 Introduction

The emergence of autonomous agents and multi-agent systems requires formal models to represent and reason about the responsibility of agents and agent groups for the outcome of their actions (See [15]). Such models allow to identify agent groups that are responsible for some realised state of affairs, or to support designing agent-based systems with formally specified responsibility for the involved agent groups. Studies in philosophy, e.g., [5,10], and artificial intelligence, e.g., [6,8,11], discuss various aspects of responsibility. Philosophical studies such as [5,10] have focused on the moral and ontological aspect of responsibility while in artificial intelligence, we encounter formalisations for the grade of responsibility [8], for responsibility in organisational settings [11], and for coalitional responsibility [6].

The concept of responsibility also has various dimensions such as individual or group responsibility and backward-looking or forward-looking responsibility. In particular some studies, e.g., [5,8], merely focus on individuals and attribution of responsibility to single agents; while group responsibility is addressed in works that also consider agent groups and ascribe responsibility to a collective of agents, e.g., [6,14]. The second dimension, i.e., backward/forward -looking responsibility, takes into account if the state of affairs is already realized and we are reasoning about it while we are looking back to the past (backward-looking), or whether

the state of affairs that we are reasoning about might eventually take place in the future (forward-looking) [17]. For instance, in [5,8] their responsibility notion is backward-looking, in [6] the focus is on forward-looking responsibility, and in [11], the authors provide notions for both, the forward and backward-looking responsibility.

Existing formal approaches to responsibility focus on either the responsibility of individual agents or one-shot encounters. For example, in [5] the responsibility of an individual agent for a specific state of affairs is explained in terms of the causal relation between the available actions of the involved agents and the resultant outcome, while in [6], a coalition/group is responsible only for the state of affairs that it could preclude by means of its available actions in a one-shot encounter. These approaches, however, do not account for some important and intuitive subtleties of this concept as practised in realistic scenarios such as in political or organisation domains. For instance, in political discourse a party that could avoid the approval of a bill, even via a sequence of interactions, is often seen to be responsible for the bill. Note that the approval of the bill could be formulated as preclusion of its disapproval. In further sections of this paper, we provide a concrete example, i.e., a furnace scenario, via which the nuances of the notion that we have in mind will be displayed.

This paper investigates the general problem of whether and to which extent an arbitrary group of agents is responsible for a state of affairs given the abilities of the involved agents. We aim at addressing this problem by proposing the novel concept of *distant responsibility* that captures the capacity of an agent group to influence the realisation of a state of affairs by a number of collective decision steps. Accordingly, an agent group is responsible for a given state of affairs when it has a collective strategy to avoid the state of affairs by a number of collective decision steps. We differentiate between agent groups that are only able to avoid the state of affairs and those who can maintain their avoidance.

Inspired by [6], we focus on power-based responsibility[1] and formally define an agent group to be responsible for a state of affairs by a number of collective decision steps when it is a minimal group and has the potential to avoid the state of affairs. We deem that it is reasonable to attribute responsibility for a state of affairs to a minimal group whenever the realization of the state of affairs is not possible without the allowance of that agent group. However, we believe that it is not reasonable to attribute any degree of responsibility to a group (for a given state of affairs) that is able to avoid the state of affairs but has imperfect knowledge about its ability. Hence, we assume that all the involved agents have perfect knowledge of the multi-agent system. The concept of distant responsibility is forward-looking in the sense of [17] and not limited to one-shot encounters as it focuses on the potential power of agent groups in a multi-agent setting. Moreover, it allows the assignment of responsibility to arbitrary groups of agents, albeit to a certain quantified degree.

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[1] Other aspects of the concept of responsibility, such as intention of agent groups and their commitment to strategies, are orthogonal to our approach in this paper.

The rest of this paper is organized as follows. Section 2 provides a power-based analysis of the concept of responsibility. Section 3 presents models and preliminary notions for our formalization. In Sects. 4 and 5, we give our definitions for the concept of distant responsibility, introduce formulations for degrees of distant responsibility, and analyse their properties. In Sects. 6 and 7, we provide some discussion of responsibility and related work, respectively. Finally, concluding remarks is presented in Sect. 8.

2   Power-Based Responsibility

Imagine a furnace situated in an industrial firm. The well-functioning of the furnace depends on the actions of the agents a_1, a_2, and a_3 who work on the furnace. They are able to bring units of *fuel* from an illimitable bunker (one unit at a time), make a *spark*, or have a *rest*. While the furnace is active, providing at least two units of fuel is necessary to keep it active. When more than one worker choose to have a rest (or to spark), the furnace is deactivated yet burns out all its available fuel. To activate the furnace, three units of fuel must be provided followed by a spark. We assume that the spark must be provided after (and not simultaneous with) the realization of three units of fuel. The furnace is capable of holding maximum three units of fuel and extras will overflow to the bunker. We write f, s, and r for bringing *fuel*, providing *spark*, and having *rest*, respectively. E.g., while the furnace is inactive and empty, if a_1 and a_2 choose to perform f, and a_3 does s, the furnace will remain inactive. In this case, at least two more rounds are needed to activate the furnace: one to provide a unit of fuel and one to make a spark. In the rest of this paper, we consider the inactivity of the furnace as the (to be avoided) state of affairs.

Responsible Groups by Distance: Let us assume that the furnace is inactive and empty. Attributing responsibility to the groups of agents that are able to preclude the inactivity of the furnace (i.e., the state of affairs) by means of their collective strategy, introduced at [6], suggests that all nonempty groups are responsible for the state of affairs, but in different number of steps. E.g., the group a_2a_3 can provide three units of fuel in at least two rounds and then make a spark in order to activate the furnace. Therefore, we see that responsible agent groups can be characterized by the minimum number of steps they need to be a minimal group that possesses the preclusive power over the state of affairs. E.g., assuming inactive and empty furnace, a_1 is a minimal group that is able to preclude the inactivity in at least four steps, a_1a_2 is a minimal group that is able to do the same in at least three steps, and $a_1a_2a_3$ is a minimal group that is responsible for the state of affairs in two steps. Note that $a_1a_2a_3$ is not responsible in three steps due to the minimality condition because any of its two member subsets, i.e., a_1a_2, a_1a_3, and a_2a_3, are responsible in three steps. We see that the preclusive power of a group, together with the minimality and the length of the collective strategy, are sufficient elements to characterize the notion of *distant group responsibility*. The rationale behind this concept of group responsibility is that in real scenarios (e.g., from the industrial and

political context) it enables the beneficiary parties (e.g., managers and lobbyists) to balance and decide how to invest their limited resources in the agent groups involved in the multi-agent system (e.g., investing on minimum number of agents with least number of interactions).

Two Types of Responsibility: We distinguish agent groups that are able to preclude a state of affairs in some steps, *responsible groups*, from those that are able to maintain their preclusion as well, *strictly responsible groups*. E.g., assuming inactive and empty furnace, singleton groups could preclude the inactivity in at least four steps, but they are not able to maintain their preclusion afterwards. Instead, two-member groups are able to preclude the inactivity in at least three steps and maintain their preclusion afterwards. We call the latter agent groups with maintenance ability *strictly responsible* groups for the state of affairs. This distinction can be meaningful for a manager who aims at keeping the furnace active (and not only activating it). In this case, we believe that it is reasonable to allocate relatively larger investment in the groups that are able to preclude the inactivity and maintain it in comparison to those that are only able to activate the furnace.

Responsibility Degrees: The proposed notions of responsibility can be used to assign a *responsibility degree* to groups. Consider the furnace in the inactive and empty state. Although singleton groups cannot preclude the inactivity in three steps, they contribute to the groups a_1a_2, a_1a_3, and a_2a_3 that enjoy such a preclusive power in three steps. Based on this observation and in continuation of the notion of *structural degree of responsibility* in [22], we assign a responsibility degree in some given d steps to any group that shares member(s) with responsible groups in d steps. This degree reflects the maximum contribution of the group in question to the groups that possess a strategy towards preclusion of the state of affairs in the given number of steps. E.g., a_1 contributes to a_1a_2 and a_1a_3, but not to a_2a_3. If we shift to two steps, two member groups have a larger share in $a_1a_2a_3$ (which has a two step preclusion power) than any singleton group. Thus, the proportion of contribution of a group to responsible groups is the key element in the formulation of our responsibility degree. Such a gradation provides a measure that enables the reasoner to make quantitative distinction among non-responsible groups for a state of affairs.

3   Models and Preliminary Notions

We use *Concurrent Structures* to model the behaviour of multi-agent systems [3].

Definition 1 (Concurrent Structure). *A concurrent structure is a tuple $M = (N, Q, Act, d, o)$, where $N = \{a_1, ..., a_k\}$ is a set of agents, Q is a non-empty finite set of states with typical element $q \in Q$, Act is a non-empty finite set of atomic actions, $d : N \times Q \rightarrow \mathcal{P}(Act)$ is the function that determines the actions available to any agent $a \in N$ in state $q \in Q$, and o is a deterministic and partial transition function that assigns a state $q' = o(q, \bar{\alpha})$ to a state $q \in Q$ and*

action profile $\bar{\alpha} = \langle \alpha_1, ..., \alpha_k \rangle \in d(a_1, q) \times ... \times d(a_k, q)$. *We use* $d_a(q)$ *instead of* $d(a, q)$, *and* $d(q)$ *instead of* $d(a_1, q) \times ... \times d(a_k, q)$.

For the sake of readability, we use N (Q, *Act*, etc.) to denote the set of agents (states, actions, etc.) in M, without explicitly referring to a concurrent structure M. A *path* in M is an infinite sequence $\lambda = q_0, q_1, ...$ of states such that $q_i \in Q$ ($i \geq 0$) and there is a transition between each q_i, q_{i+1}. For a path λ, $\lambda[i] = q_i$ denotes the ith state ($i \geq 0$) of λ and $\Lambda(q)$ denotes the set of all paths that start in q. A perfect information (memoryless) *strategy* of agent a is a function $s_a : Q \to Act$ such that $s_a(q) \in d_a(q)$. Set of such functions will be denoted by Σ_a. A collective strategy s_C for a group $C \subseteq N$ is a tuple of individual strategies for all agents $a \in C$. The *outcome* of strategy s_C in state $q \in Q$ is defined as the set of all paths that may result from execution of s_C: $out(q, s_C) = \{\lambda \in \Lambda(q) \mid \forall i \in \mathbb{N}_0 \exists \bar{\alpha} = \langle \alpha_1, ..., \alpha_k \rangle \in d(\lambda[i]) \forall a \in C$ $(\alpha_a = s_C^a(\lambda[i]) \wedge o(\lambda[i], \bar{\alpha}) = \lambda[i+1])\}$, where s_C^a denotes the individual strategy of agent a in the collective strategy s_A. A *state of affairs* refers to a set $S \subseteq Q$ and $\bar{S}$ denotes the set $Q \setminus S$.

Our multi-agent furnace scenario is modelled as the concurrent structure $M = (N, Q, Act, d, o)$, where $N = \{a_1, a_2, a_3\}$, $Q = \{q_0, ..., q_4\}$, $Act = \{f, s, r\}$, $d_a(q) = Act$ for all $a \in N$ and $q \in Q$ (Fig. 1). The inactivity of the furnace, considered as the state of affairs $S = \{q_0, q_1, q_2, q_3\}$.
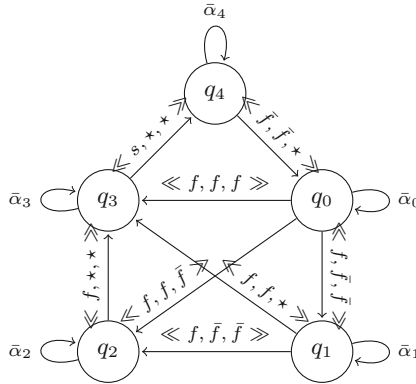

Fig. 1. State q_4 is the only state where the furnace is active. In $q_i \in \{q_0, q_1, q_2, q_3\}$ the furnace is inactive with i unit(s) of fuel. For convenience $\bar{f}$ denotes either s or r, and $\ll f, \bar{f}, \bar{f} \gg$ denotes the set of action profiles involving one single action f, i.e., $\{\langle f, \bar{f}, \bar{f} \rangle, \langle \bar{f}, f, \bar{f} \rangle, \langle \bar{f}, \bar{f}, f \rangle \mid \bar{f} \in \{s, r\}\}$ (similar for others). Moreover, $\bar{\alpha}_i$ denotes any unspecified action profile in $q_i \in Q$ and $\star \in Act$ denotes any available action. The outcome function is as displayed by the accessibility relation in the figure, e.g., $o(q_0, \ll f, f, f \gg) = q_3$ is illustrated by the arrow from q_0 to q_3.

In the following definitions, we omit $M = (N, Q, Act, d, o)$ as it is clear from the context that we are always focused on a given multi-agent system. Thus,

references to elements of M should be seen as elements of a given concurrent structure M that is modelling the multi-agent system. For instance, we simply write (the set of states) Q instead of Q *in M*.

Definition 2 (Ability to achieve/maintain). *Let $q \in Q$ and $S \subseteq Q$ a state of affairs. Group $C \subseteq N$ can q-achieve S in $d \in \mathbb{N}_1$ steps iff there is $s_C \in \Sigma_C$ such that $\lambda[d] \in S$ for all $\lambda \in out(q, s_C)$ and C cannot q-achieve S in $d' < d$ steps. Moreover, group $C \subseteq N$ can q-maintain S in $d \in \mathbb{N}_1$ steps iff there is $s_C \in \Sigma_C$ such that $\lambda[i] \in S$ for all $\lambda \in out(q, s_C)$ and $i \geq d$, and C cannot q-maintain S in $d' < d$ steps.*

Assuming inactive and empty furnace (from now state q_0), groups $a_1 a_2$, $a_1 a_3$, and $a_2 a_3$ can activate the furnace in three steps but not less. These groups can also q_0-maintain the activity of the furnace in three steps. However, group $a_1 a_2 a_3$ can q_0-maintain the activity by two steps. Note that a group that can q-achieve/maintain S in d steps cannot do so in $d' < d$. Also, a group C might be able to q-achieve S in d steps by a strategy while it can q-maintain S in $d' \geq d$ steps by means of a different strategy.

Proposition 1 (Maintain implies achieve). *For $q \in Q$, if $C \subseteq N$ can q-maintain $S \subseteq Q$ in d steps, then C can q-achieve S in $d' \leq d$ steps.*

Proof. The ability to q-maintain S in d steps necessitates the existence of a collective strategy s_C that guarantees that among all the paths in $out(q, s_C)$, from state $\lambda[d]$ on, all states are a member of S (Definition 2). Hence, achieving S in d steps is guaranteed. As C may have another strategy s'_C that could guarantee S in $d' < d$ steps, C can q-achieve S in $d' \leq d$ steps.

Note that the ability to achieve does not imply the ability to maintain. So, the other way does not hold in general. The next property shows that adding new members to a group that is able to achieve/maintain a state of affairs, preserves both of the abilities. This would be in correspondence with *monotonicity of power* in [12]. In other words, adding new members to an agent group does not have any negative influence on the ability of the group to achieve/maintain a state of affairs from a given source state and in a specific number of steps. In the following, whenever it is clear from the context, we may omit the phrase "from a given source state and in a specific number of steps".

Proposition 2 (Preservation of abilities). *For $q \in Q$, if $C \subseteq N$ can q-achieve/maintain $S \subseteq Q$ in d steps, then C' can q-achieve/maintain S in $d' \leq d$ steps for $C \subseteq C' \subseteq N$.*

Proof. C has a strategy s_C to q-achieve/maintain S in d steps, regardless of the actions of agents in $N \setminus C$. So, either the group $C' \supseteq C$ has a different strategy $s_{C'}$ to q-achieve/maintain S in $d' < d$ steps or the subgroup $C \subseteq C'$ can execute the former strategy s_C and q-achieve/maintain S in d steps while agents in $C' \setminus C$ are executing an arbitrary action. So, in both cases the claim is justified.

4 Distant Group Responsibility

The concept of *distant* responsibility that we have in mind is *forward-looking*, *local*, and *minimal*. Our approach is forward-looking in the sense of [17] as we merely appraise the potential of groups to avoid a state of affairs and consider that the state of affairs cannot be realized without the group's allowance. However, this does not suggest that a responsible group necessarily practices its preclusive power and prevents the state of affairs. Secondly, our responsibility notion is local in the sense that the preclusive power of groups is considered with respect to a given state and not globally in the whole multi-agent system. Hence, a group that is responsible for a state of affairs from the current state in some given number of steps, might be non-responsible for the same state of affairs from another state in the given number of steps. Finally, a responsible group for a state of affairs in a given number of steps is minimal in the sense that the group is a smallest possible group that has the power to avoid the state of affairs in the given number of steps. In the following definition we omit concurrent structure $M = (N, Q, Act, d, o)$ as we assume it is clear from the context.

Definition 3 (Distant responsibility). *For $q \in Q$, group $C \subseteq N$ is q-responsible for $S \subseteq Q$ in $d \in \mathbb{N}_1$ steps iff C is a minimal group that can q-achieve $\bar{S}$ in d steps. The set of all q-responsible groups C for S in d steps is denoted by $\delta(q, d, S)$.*

Definition 3 allows two distinct groups being q-responsible for one and the same state of affairs by the same or even different number of steps. According to the following proposition, any two distinct responsible groups for one and the same state of affairs in the same number of steps could not be a subgroup of each other.

Proposition 3 (Incomparability). *For $q \in Q$, let $C \neq C'$ be two distinct q-responsible groups from N for $S \subseteq Q$ in d steps. Then, $C \not\subset C'$ and $C' \not\subset C$.*

Proof. Suppose either $C \subset C'$ or $C' \subset C$. The former case contradicts with the minimality of C' as a q-responsible for S in d steps and the latter contradicts with the minimality of C as a q-responsible for S in d steps.

Due to the minimality, we have the following corollary of Proposition 3.

Corollary 1. *For $q \in Q$, if $C \subseteq N$ is q-responsible for $S \subseteq Q$ in d steps, then for $C' \subseteq N$ neither $C' \subset C$ nor $C' \supset C$ are q-responsible for S in d steps.*

In case a group is responsible for a state of affairs in d steps, it would not be responsible by any number of steps other than d. So, in case of existence, this distance has the uniqueness property.

Proposition 4 (Responsibility distance). *For $q \in Q$, if $C \subseteq N$ is q-responsible for $S \subseteq Q$ in d and d' steps, then $d = d'$.*

Proof. Suppose the contrary. According to Definition 3, C is a minimal group that can q-achieve $\bar{S}$ in both d and d' steps with either $d < d'$ or $d > d'$. Both cases contradict the final part of Definition 2 which states that a group that can q-achieve S in d steps cannot q-achieve S in $d' < d$ steps.

4.1   Strictly Responsible Groups by Distance

A group that, in addition to having the power to preclude the realization of a state of affairs in a certain number of steps, has the power to maintain the preclusion afterwards is called *strictly responsible group*. E.g., groups a_1, a_2, and a_3 in our furnace scenario are able to preclude the inactivity in four steps, but they are unable to maintain their preclusion. In contrast, groups a_1a_2, a_1a_3, a_2a_3, and $a_1a_2a_3$ are able to preclude the inactivity in three steps and maintain their preclusion afterwards.

Definition 4 (Strict responsibility). *For $q \in Q$, group $C \subseteq N$ is strictly q-responsible for $S \subseteq Q$ in $d \in \mathbb{N}_1$ steps iff C is a minimal group that can q-maintain $\bar{S}$ in d steps. The set of all strictly q-responsible groups C for S in d steps is denoted by $\sigma(q, d, S)$.*

Intuitively, this notion attributes the responsibility for a state of affairs S to a group of agents that can preclude S in some steps, has control on holding the preclusion of the state of affairs, and all its members are necessary for this performance.

Example 1 (Responsible Groups). Following our furnace scenario and using Definitions 3 and 4, we have $\delta(q_0, 4, S) = \{a_1, a_2, a_3\}$, $\delta(q_0, 3, S) = \{a_1a_2, a_1a_3, a_2a_3\}$, $\delta(q_0, 2, S) = \{a_1a_2a_3\}$, $\sigma(q_0, 3, S) = \{a_1a_2, a_1a_3, a_2a_3\}$ and $\sigma(q_0, 2, S) = \{a_1a_2a_3\}$. We note that singleton groups, i.e., a_1, a_2, and a_3, are not able to maintain their preclusion of inactivity. Hence, they are q_0-responsible for the inactivity of the furnace in 4 steps but are not strictly q_0-responsible for such a state of affairs in any number of steps. This is due to their inability, i.e., lack of sufficient members, to keep the furnace active while it is activated.

Although all strictly responsible groups possess the combined ability of precluding the state of affairs and maintaining their preclusion in some steps, due to the minimality concern, it is not necessary that a strictly responsible group be also a responsible group by a distance.

Proposition 5 (Two forms of responsibility). *For $q \in Q$, if $C \subseteq N$ is strictly q-responsible for $S \subseteq Q$ in d steps, C is not necessarily a distantly q-responsible group for S.*

Proof. We provide a counter example. Consider $S = \{q_0, q_1, q_3\}$ as the state of affairs in the furnace scenario. Then, $a_1a_2a_3$ is strictly q_0-responsible for S in 1 step as it is a minimal group that can q_0-achieve $q_2 \in \bar{S}$ in 1 step from q_0 (by selecting action profiles $\langle f, f, \bar{f} \rangle$) and stay in q_2 for ever (by selecting action profile $\langle \bar{f}, \bar{f}, \bar{f} \rangle$). Note that due to the minimality condition the set of q_0-responsible groups for S in 1 step contains only a_1a_2, a_1a_3, and a_2a_3.

4.2   Responsibility for Contingent Situations

We circumscribe the set of states of affairs by excluding two classes of *impossible* and *necessary* states of affairs and introducing our *contingency* postulate. To demonstrate the rationale behind this, consider again the furnace scenario. In this scenario, precluding $S = \{q_0, q_1, q_2, q_3\}$ from the state q_0 in 1 step is not possible. In other words, S is a necessity in 1 step and precluding it in 1 step is seen as an impossibility. In contrast, precluding the state of affairs $S' = \{q_4\}$ in 1 step from q_0 is a necessity as there always exists a strategy (e.g., $s_\varnothing \in \Sigma_\varnothing$) that succeeds in preclusion of S' in 1 step. This is due to the fact that for all possible $s_N \in \Sigma_N$ it holds that $\lambda[1] \in \bar{S}'$ for all $\lambda \in out(q_0, s_N)$. Thus, S' is an impossibility in 1 step and its avoidance in 1 step is inherently necessary. We believe that in either of the cases, attributing responsibility to any group $C \in N$ is not a meaningful imputation because in both cases the achievement or avoidance of the state of affairs does not depend on the agents' actions.

Definition 5 (Contingency postulate). *For $q \in Q$, a state of affairs $S \subseteq Q$ is q-contingent in $d \in \mathbb{N}_1$ steps iff N can q-achieve $\bar{S}$ in $d' \leq d$ steps and $\varnothing$ cannot q-achieve $\bar{S}$ in $d'' \leq d$ steps.*

By excluding necessities, we omit all states of affairs S that are not avoidable in d steps. So, any q-contingent state of affairs S in d steps would be avoidable by N in d steps or less, and moreover, S should not be an impossibility in d steps or less (i.e., $\bar{S}$ should not be a necessity, and thus achievable by the empty group, in d steps or less). In the following proposition, we show that for any contingent state of affairs, there exists at least a (minimal) non-empty group that is responsible for it in at most $d \in \mathbb{N}_1$ steps. This matches the intuition that when a state of affairs S is reachable but not necessary within some rounds of collective actions, at least one group of involved agents must be able to preclude it. Hence, in case S occurs, its occurrence took place by means of allowance of such a group.

Proposition 6 (Existence of responsible group). *For $q \in Q$, if $S \subseteq Q$ is q-contingent in d steps, there exists a non-empty q-responsible group $C \subseteq N$ for S in $d' \leq d$ steps.*

Proof. According to Definition 5, for any q-contingent S in d steps, we have that N can q-achieve $\bar{S}$ in $d' \leq d$ steps. So, if N is a minimal group that can q-achieve $\bar{S}$ in d' steps, based on Definition 3, $C = N$ would be q-responsible for S in $d' \leq d$ steps. Otherwise, via exclusion of excess members, we reach a minimal subgroup $C \subset N$ that is q-responsible for S in $d' \leq d$ steps. Note that according to the second condition for q-contingency of S (Definition 5), C could not be empty. Thus, a nonempty group $C \subseteq N$ would be q-responsible for S in $d' \leq d$ steps.

5   Degrees of Distant Responsibility

We attributed the distant responsibility for a state of affairs to agent groups that can preclude the state of affairs by a given number of steps. Thus, a group that

only misses one member (in comparison to a responsible group by distance) will be simply considered as a non-responsible group. However, in realistic scenarios, parties with interests in preclusion of a state of affairs are often prepared to invest their limited resources even in such non-responsible groups of agents, albeit proportional to the contribution they can have in the responsible groups. We therefore formulate the degree of responsibility with respect to the contributory share of agent groups in responsible groups.

5.1   Two Responsibility Degrees

Consider again the furnace scenario. For a manager who wants to activate the furnace with the least number of actions (from state q_0), it would be reasonable to invest more resources on two member groups than in singleton groups, although none are q_0-responsible for the inactivity of the furnace in 2 steps. So, despite the fact that two member groups are not able to preclude the inactivity in 2 steps, they have larger contribution than singleton groups to the group $a_1a_2a_3$ which is the q_0-responsible group in 2 steps. Note that the inactivity could not be avoided by shorter distances from q_0. We apply the methodology of [22] for formulating the notion of *structural degree of responsibility* and deem that attributing a degree of responsibility that reflects the grade of preclusive power of agent groups would be a reasonable notion for gradation of distant responsibility. Note again that we omit the repetition of M in the following as it is clear from the context that we are focused on a given multi-agent system.

Definition 6 (Degrees of responsibility). *For $q \in Q$, the degree of q-responsibility of $C \subseteq N$ for $S \subseteq Q$ in $d \in \mathbb{N}_1$ steps defined as $\mathcal{DRD}(C, q, d, S) = \max\limits_{\hat{C} \in \delta(q,d,S)} (\{i \mid i = 1 - \frac{|\hat{C} \setminus C|}{|\hat{C}|}\})$. In case $\delta(q, d, S) = \varnothing$, $\mathcal{DRD}(C, q, d, S)$ is undefined. Moreover, the degree of strict q-responsibility of C for S in $d \in \mathbb{N}_1$ steps is defined as $\mathcal{DSD}(C, q, d, S) = \max\limits_{\hat{C} \in \sigma(q,d,S)} (\{i \mid i = 1 - \frac{|\hat{C} \setminus C|}{|\hat{C}|}\})$. In case $\sigma(q, d, S) = \varnothing$, $\mathcal{DSD}(C, q, d, S)$ is undefined.*

Note that the degrees are bounded in the range of $[0, 1]$: degree 1 is assigned to the responsible groups and degree 0 is assigned to the groups that have no contribution to the responsible groups. It should be noted that attribution of distant responsibility (degrees) to non-contingent states of affairs is not meaningful. According to the following proposition, the addition of new members to a group could not have negative influence on the responsibility degrees. This is in accordance with the concept of *monotonicity of power* [12].

Proposition 7 (Monotonicity of degrees). *Let $q \in Q$, $d \in \mathbb{N}_1$ and $C \subseteq C' \subseteq N$. We have that $\mathcal{DRD}(C, q, d, S) \leq \mathcal{DRD}(C', q, d, S)$ and $\mathcal{DSD}(C, q, d, S) \leq \mathcal{DSD}(C', q, d, S)$.*

Proof. Based on Definition 6, both degrees of responsibility reflect the maximum contribution of C to all responsible groups. This leads to a degree in range of

[0, 1] for C. So, all elements in $C' \setminus C$ are either influential in increasing the share of C' in a responsible group by distance or have no influence. Hence, the two degrees might only increase after absorption of some new members.

According to the next proposition, responsible groups by a distance for a given state of affairs and their supersets, have the full degree of responsibility, equal to one, for the state of affairs by the specified distance.

Proposition 8 (Full degrees of responsibility). *For $q \in Q$, let $C \subseteq N$ be a q-responsible group for $S \subseteq Q$ in $d \in \mathbb{N}_1$ steps. Then, for all $C' \supseteq C$, $\mathcal{DRD}(C', q, d, S) = \mathcal{DRD}(C, q, d, S) = 1$. Analogously, for a strictly q-responsible group C for S in $d \in \mathbb{N}_1$ steps, for all $C' \supseteq C$ we have that $\mathcal{DSD}(C', q, d, S) = \mathcal{DSD}(C, q, d, S) = 1$.*

Proof. Based on Definition 6, the degree of (strict) responsibility of a responsible group C in d steps is equal to 1. This is due to fact that C has the maximum possible contribution to C itself. As value of 1 is the maximum possible value for both degrees and according to the monotonicity of degrees (Proposition 7), all super-groups of responsible groups by distance will be assigned with responsibility degree 1.

Example 2 (Responsibility Degrees). According to Definition 3, for the furnace scenario we have $\delta(q_0, d, S) = \varnothing$ for $d \leq 1$ and $d \geq 5$ such that $\mathcal{DRD}(C, q_0, d, S)$ is undefined for all groups C when $d \leq 1$ or $d \geq 5$. For all singleton groups $A \in \{a_1, a_2, a_3\}$, $\mathcal{DRD}(A, q_0, 2, S) = 1/3$, $\mathcal{DRD}(A, q_0, 3, S) = 1/2$, and $\mathcal{DRD}(A, q_0, 4, S) = 1$. Moreover, for two member groups $B \in \{a_1 a_2, a_1 a_3, a_2 a_3\}$, $\mathcal{DRD}(B, q_0, 2, S) = 2/3$ and $\mathcal{DRD}(B, q_0, 3, S) = \mathcal{DRD}(B, q_0, 4, S) = 1$. Finally, we have $\mathcal{DRD}(a_1 a_2 a_3, q_0, d, S) = 1$ and $\mathcal{DRD}(\varnothing, q_0, d, S) = 0$ for all $d \in \{2, 3, 4\}$. When we move to strict degrees of q_0-responsibility for S, for $d \leq 1$ and $d \geq 4$, $\sigma(q_0, d, S) = \varnothing$. Accordingly, for any group C, $\mathcal{DSD}(C, q_0, d, S)$ is undefined for all $d \leq 1$ and $d \geq 4$. We have $\sigma(q_0, 2, S) = \{a_1 a_2 a_3\}$ and $\sigma(q_0, 3, S) = \{a_1 a_2, a_1 a_3, a_2 a_3\}$. So, in distances 2 and 3, for all singleton groups $A \in \{a_1, a_2, a_3\}$, $\mathcal{DSD}(A, q_0, 2, S) = 1/3$ and $\mathcal{DSD}(A, q_0, 3, S) = 1/2$. Furthermore, for all two member groups $B \in \{a_1 a_2, a_1 a_3, a_2 a_3\}$, $\mathcal{DSD}(B, q_0, 2, S) = 2/3$ and $\mathcal{DSD}(B, q_0, 3, S) = 1$. Finally, we have $\mathcal{DSD}(a_1 a_2 a_3, q_0, d, S) = 1$ and $\mathcal{DRD}(\varnothing, q_0, d, S) = 0$ for $d \in \{2, 3\}$.

The next proposition illustrates a case in which a singleton group exclusively possesses the preclusive power over a state of affairs; hence, is the unique (strictly) responsible group for the state of affairs from a given source state in a specific number of steps. The existence of such a *dictator* agent, polarizes the space of (strict) responsibility degrees of all the possible groups for the state of affairs in the specified distance.

Proposition 9 (Polarizing dictatorship). *For $q \in Q$, let $\hat{C} \subseteq N$ be a unique singleton q-responsible group for $S \subseteq Q$ in $d \in \mathbb{N}_1$ steps. Then, for any arbitrary $C \subseteq N$, $\mathcal{DRD}(C, q, d, S) \in \{0, 1\}$ such that $\mathcal{DRD}(C \in I, q, d, S) = 1$ and $\mathcal{DRD}(C \in O, q, d, S) = 0$ where $I = \{C \subseteq N \mid C \supseteq \hat{C}\}$ and $O = \{C \subseteq N \mid$*

$C \not\supseteq \hat{C}\}$. *Moreover, for* $q \in Q$, *let* $\hat{C} \subseteq N$ *be a unique singleton strictly q-responsible group for* $S \subseteq Q$ *in* $d \in \mathbb{N}_1$ *steps. Then, for any arbitrary* $C \subseteq N$, $\mathcal{DSD}(C, q, d, S) \in \{0, 1\}$ *such that* $\mathcal{DSD}(C \in I, q, d, S) = 1$ *and* $\mathcal{DSD}(C \in O, q, d, S) = 0$ *where* $I = \{C \subseteq N \mid C \supseteq \hat{C}\}$ *and* $O = \{C \subseteq N \mid C \not\supseteq \hat{C}\}$.

Proof. For any arbitrary $C \subseteq N$, we have that either $C \in I$ or $C \in O$. Based on Proposition 8, for all the supersets of a q-responsible group for a state of affairs in a given number of steps, the degree of q-responsibility is equal to one (for the same state of affairs and in the specified number of steps). So, for all the groups C in $I = \{C \subseteq N \mid C \supseteq \hat{C}\}$, we have that $\mathcal{DRD}(C \in I, q, d, S) = 1$. Moreover, in case a group C does not include the dictator $\hat{C}$, there exists no other q-responsible group to contribute to. Therefore, the degree of q-responsibility for all the groups C in $O = \{C \subseteq N \mid C \not\supseteq \hat{C}\}$ would be equal to zero. By an analogous line of proof, we will have the second part of the proposition for the degree of strict q-responsibility of any arbitrary $C \subseteq N$.

   This proposition illustrates that in existence of a uniquely responsible agent, responsibility becomes an all-or-nothing concept. Hence, any arbitrary agent group will be either responsible for the state of affairs (from a source state and in a given number of steps) or non-responsible. I.e., no agent group will be *partially* responsible. This is due to the aggregation of preclusive power in a unique agent.

5.2   Responsibility Degrees for Collaborative Situations

In this section we focus on a specific class of states of affairs, called *collaborative states of affairs*. The realization of a collaborative state of affairs in a given number of steps depends on all agents in the multi-agent system. For these states of affairs, the grand coalition N is the unique (strictly) q-responsible group for some d steps. For instance, in the furnace scenario, the grand coalition $a_1 a_2 a_3$ is the only (strictly) q_0-responsible group for the inactivity of the furnace in 2 steps.

Definition 7 (Collaborative situations). *For* $q \in Q$, *a state of affairs* $S \subseteq Q$ *is q-collaborative in* $d \in \mathbb{N}_1$ *steps iff* $\delta(q, d, S) = \{N\}$. *Moreover, a state of affairs* S *is strictly q-collaborative in* $d \in \mathbb{N}_1$ *steps iff* $\sigma(q, d, S) = \{N\}$.

   The following lemma focuses on degrees of distance responsibility for collaborative situations and illustrates the proportionality of degrees to the group size.

Lemma 1 (Proportionality). *For* $q \in Q$, *if* $S \subseteq Q$ *is a q-collaborative state of affairs in* $d \in \mathbb{N}_1$ *steps then for any* $C \subseteq N$ *we have that* $\mathcal{DRD}(C, q, d, S) = \frac{|C|}{|N|}$. *Moreover, If* S *is a strictly q-collaborative state of affairs in* $d \in \mathbb{N}_1$ *steps then for any* $C \subseteq N$ *we have that* $\mathcal{DSD}(C, q, d, S) = \frac{|C|}{|N|}$.

Proof. First, we note that $N = \{a_1, \ldots, a_k\}$. Based on Definition 7, grand coalition N is the unique q-responsible group for S in d steps, i.e., $\delta(q, d, S) = \{N\}$. Hence, for any group $C \subseteq N$ the degree of q-responsibility for S in d steps (Definition 6) can be reformulated as $\mathcal{DRD}(C, q, d, S) = 1 - \frac{|N \setminus C|}{|N|}$ which is equal to $\frac{|C|}{|N|}$. Proof of the second claim follows the same line of reasoning in which the assumption that $\sigma(q, d, S) = \{N\}$ implies that $\mathcal{DSD}(C, q, d, S) = \frac{|C|}{|N|}$ for any $C \subseteq N$.

Based on this lemma, in case the grand coalition N is the unique (strictly) responsible group for a specific state of affairs S in d steps, for any group $C \subseteq N$, the degree of (strict) q-responsibility for S in d steps is directly proportional to the size of C. For a collaborative state of affairs the two functions of responsibility degree, i.e., degree of responsibility by distance and degree of strict responsibility by distance, are both additive and scalable.

Proposition 10 (Semilinearity). *For $q \in Q$, if $S \subseteq Q$ is a q-collaborative state of affairs in $d \in \mathbb{N}_1$ steps then (1.1) for $C, C' \subseteq N$ such that $C \cap C' = \varnothing$ we have that $\mathcal{DRD}(C \cup C', q, d, S) = \mathcal{DRD}(C, q, d, S) + \mathcal{DRD}(C', q, d, S)$ and (1.2) for $a \in \mathbb{Q}_{\geq 0}$ and $C, C' \subseteq N$ such that $|C'| = a.|C|$ we have that $\mathcal{DRD}(C', q, d, S) = a.\mathcal{DRD}(C, q, d, S)$. Moreover, If S is a strictly q-collaborative state of affairs in $d \in \mathbb{N}_1$ steps then (2.1) for $C, C' \subseteq N$ such that $C \cap C' = \varnothing$ we have that $\mathcal{DSD}(C \cup C', q, d, S) = \mathcal{DSD}(C, q, d, S) + \mathcal{DSD}(C', q, d, S)$ and (2.2) for $a \in \mathbb{Q}_{\geq 0}$ and $C, C' \subseteq N$ such that $|C'| = a.|C|$ we have that $\mathcal{DSD}(C', q, d, S) = a.\mathcal{DSD}(C, q, d, S)$.*

Proof. "(1.1 and 2.1) Additivity": According to Lemma 1, as $\delta(q, d, S) = \{N\}$ we have that $\mathcal{DRD}(C \cup C', q, d, S) = \frac{|C \cup C'|}{k = |N|}$. Considering that $C \cap C' = \varnothing$ we can reformulate it as $\frac{|C|}{k} + \frac{|C'|}{k}$ which is equal to $\mathcal{DRD}(C, q, d, S) + \mathcal{DRD}(C', q, d, S)$. An analogous line of proof shows that if $\sigma(q, d, S) = \{N\}$ it holds that $\mathcal{DSD}(C \cup C', q, d, S) = \mathcal{DSD}(C, q, d, S) + \mathcal{DSD}(C', q, d, S)$. Additionally, we can also entail that for any arbitrary group C and partition $P = \{C_1, \ldots, C_n\}$ of C, we have $\sum_{i=1}^{n} \mathcal{DRD}(C_i, q, d, S) = \mathcal{DRD}(C, q, d, S)$ if $\delta(q, d, S) = \{N\}$. Moreover, $\sum_{i=1}^{n} \mathcal{DSD}(C_i, q, d, S) = \mathcal{DSD}(C, q, d, S)$ if $\sigma(q, d, S) = \{N\}$. "(1.2 and 2.2) Scaling behaviour": Based on Lemma 1 and the assumption that $|C'| = a.|C|$, we have that $\mathcal{DRD}(C', q, d, S) = \frac{|C'|}{k = |N|} = a.\frac{|C|}{k = |N|}$ which is equal to $a.\mathcal{DRD}(C, q, d, S)$. Analogously for Part 2.2.

6   Discussion

Although the concept of responsibility is extensively studied in philosophy and AI, there is no consensus on a general (in)formal definition or about semantics for this concept. We believe this is due to various dimensions of responsibility such as causality, knowledge, intentionality, morality, etc. As a result, various studies have focused on different dimensions of responsibility (see Sect. 1). In this work,

we focused on the *power dimension* of responsibility ignoring other dimensions such as knowledge dimension. Hereby, we discuss the relation between these two concepts, i.e., power and knowledge, and our notion of responsibility. We are aware that our formal exposition of responsibility ignores various dimensions of this concept. This is by purpose as our concern is to investigate the power dimension of responsibility. We believe that formalizing this dimension captures some (but not all) intuitive subtleties of responsibility and can be applied in some real-world scenarios such as strategic planning, reasoning in political context, and design of resource sharing mechanisms in multi-agent systems as we will explain later in this section. Note that focusing on a specific dimension of a phenomenon such as responsibility is a common practice. E.g., Chokler and Halpern [8] focus merely on causal aspect of responsibility ignoring other issues.

As framed by [16], *"power is a capacity or potential"* which might remain unexercised. If a group of agents is able to preclude a state of affairs, it is not justified to entail that they will necessarily do so. We do not claim that a group is responsible if collective actions take place. Conversely, we consider forward-looking responsibility (in sense of [17]). Roughly speaking, possessing power does not imply that the group necessarily exercise its power. As we only analyze possibilities, groups that possess collective strategies towards a preclusion are not committed to execute it (see [1] for an in-detail analysis and an ATL-based formalization of group strategies that come without (or with) commitment). Our analysis applies before the coalition formation process and considers the possibilities of potential groups/coalitions. Our notion of responsibility is formulated by assuming that agents have perfect knowledge about the system. By means of emphasizing our approach to formulating responsibility in terms of *power* and our *perfect knowledge* assumption, a possible misunderstanding of our forward-looking notion of group responsibility can be pointed out. This is to apply our notions in scenarios from legal domain. We believe that in assessing culpability, it is the case that the reasoning is about an already realized state of affairs (in past), where backward-looking responsibility is applicable. Moreover, we follow [8] and believe that for attribution of liability, blameworthiness, and in principles such as *contributory negligence* in the legal domain, level of knowledge of agents plays a significant role. Therefore, responsibility notions that take into account the imperfect knowledge are applicable while we consider perfect knowledge. Moreover, we remind that our conception of responsibility is free of any moral overtone.

Our notion of distant responsibility can be applied to design and analyse task-allocation mechanisms and resource-sharing protocols in multi-agent systems. As argued in [13], a task-decomposition procedure that takes the potentials of involved agent groups into account can enhance the applicability of the task-allocation mechanisms. Consider a decision-maker who is faced with a complex task (e.g., to avoid the inactivation of an industrial furnace) and is able to compute the degrees of distant responsibility of all the possible agent groups in the system for various combinations of sub-tasks. This simply enables the decision-maker to allocate each sub-task to an agent group with highest degree

of distant responsibility in the least number of steps. In the ideal cases where for each sub-task a fully responsible group does exist, this task allocation mechanism guarantees the fulfillment of the complex task. And in other cases, it is guaranteed that each sub-task is allocated to the most capable agent group.

Concerning the resource allocation process, sharing resources among agent groups and applying justifiable methods for resource allocation could be challenging (see [7]). Based on degrees of responsibility of agent groups for (un-)desired states of affairs in a given distances, the decision maker(s) can categorize the agent groups that are influential for realization of a state of affairs concerning the cost of the groups (e.g., the group size) or the quality of group's available strategy regarding the state of affairs (e.g., length of the strategy). We see that such a categorization establishes a justifiable base for prioritizing the agent groups for resource allocation. Our proposed framework could also be applied to decide whether a specific resource assignment (in a given multi-agent system) ensures that a state of affairs is avoidable. For such a purpose, we can model a certain scenario in our framework where we specify the resource assignment in terms of available actions for each agent in each state (the d function in the concurrent structure). Then the avoidability of a given state of affairs could be verified based on our notion of distant responsibility. For instance, if for all states $q \in Q$ there exists at least one q-responsible group for the given state of affairs in one step, we can verify that the specified resource assignment guarantees the avoidability of S in one step. Applying the concept of responsibility for verifying system specifications is an already exploited methodology (see [8,9]).

The other domain in which we see applicability for the notion of distant responsibility is in analysis of industrial supply chain and specifically as a method for ascribing *extended product responsibility* in Life-Cycle Assessment (LCA). The so called *extended product responsibility* mainly concerns the extent of responsibility of involved actors in the business and industry sector, e.g., producer, middle-customer, and consumer, for the environmental consequences of the whole life-cycle of a product (see [21]). We deem that in case (for instance) the producer and a set of customers have a joint strategy to avoid the incidence of an undesired environmental situation, e.g., release of a specific amount of a hazardous gas, they are responsible for such a situation (distant responsibility) while each of the involved agents/groups in such a responsible group are partially responsible (degree of distant responsibility).

7   Related Work

The proposed notions of responsibility are closely related to the forward-looking notions in [6]. More precisely, the notion of (weakly) q-responsible in [6] is identical to distant responsibility in 1 step. Another study that investigates both backward- and forward-looking responsibility is [11]. They formalize forward-looking responsibility in terms of the set of organizational plans that define the agents' obligations. Our work is also related to studies such as [19,20] that provide qualitative degrees for the concept of responsibility in comparison to our quantitative degrees.

One noticeable work that defines a qualitative degree of responsibility is [8] which has a causality-based approach. They build their graded notion of responsibility on the critically degree of a setting regarding an already materialized event in the past (backward-looking) while our notions are power-based and regard the eventualities in the future. However, one main similarity between our approach and [8] is that both the studies provide a quantitative degree of responsibility while most works on the concept of responsibility, either introduce qualitative degrees of responsibility, e.g., crucial or necessary coalitions in [6], or basically conceptualize responsibility as an all-or-nothing notion and refuse to grade it.

Two other studies that focus on aspects of responsibility that we ignored in our conception are [5,14]. In [5], an agent is *morally* responsible for an outcome in case all the three conditions: agency, causal relevancy, and avoidance opportunity are fulfilled. Besides their main focus on *moral* responsibility (in comparison to our power-based responsibility), our approach to formulating the concept of responsibility is distinguishable from their study regarding the three following aspects. Firstly, their notion is merely focused on a single agent while we address agent groups. Secondly, in their formalization, causal relations play the main role while we base our notions on strategic abilities of agents. And thirdly, they claim that attribution of responsibility requires both (1) the causal relation between actions of the agent and the realized outcome and (2) the avoidance opportunity for the agent in question; while we consider the forward-looking precluding power, a sufficient condition. In [14], STIT logic is used to provide a logical analysis of the concept of responsibility and attribution of responsibility. There are three main differentiating points between our study and their approach. Firstly, they investigate the relation between responsibility and attribution emotions, e.g., moral disapproval, where we focus on possibilities of potential agent groups. Secondly, their study regards already materialized state of affairs and formulate backward-looking responsibility; while we have a forward-looking approach. And finally, they consider different "time of choice" and regard the level of knowledge of agents about the choice of other agents while we have local notions for each state of the multi-agent system and assume the perfect knowledge of agents on available actions for each agent and possible state transitions in the system.

Our conception of *distant responsibility* investigates whether an agent group has the strategic power to influence the materialization of a situation. So, we briefly compare our approach with the Banzhaf index [4] and the Shapley-Shubik index [18] as the two well-established power indices. Firstly, in our conception, we consider agent groups while both the indices are focused on the power of an individual agent. Secondly, in the formulation of our degree of distant responsibility we follow the methodology of [22] and regard the maximum contribution of agent groups (to a responsible group) where Banzhaf has a probabilistic approach. Finally, our focus is merely on the preclusive power (in sense of [16]) of agent groups while both the Banzhaf measure and the Shapley-Shubik index consider the ability of agents to determine the final outcome which we see more related to the combined ability to be able to both preclude and provide a situation.

8   Conclusion and Future Work

We proposed various notions of group responsibility and for each notion explained how the degree of the responsibilities for arbitrary groups of agents can be determined. The presented notions allows one to analyse and reason about the potential of an arbitrary group of agents and differentiate between agent groups with respect to their (1) responsibility attribution, (2) type of responsibility, and (3) degree of responsibility. Our notions are motivated by intuitive and desirable properties, e.g., an agent group which only misses one member to become responsible for a state of affairs receives a higher responsibility degree than one that misses more members. The presented notions of responsibility are forward-looking and local in the sense that they capture the potential of agent groups regarding the realization of a given state of affairs within the current state.

Although the attribution of *responsibility* and the *degree of responsibility* are addressed in this paper, the question about supremacy order among *responsible* groups (by the same distance) or within the set of *partially responsible* groups (with similar degrees) is a domain-specific question that could be answered with respect to characteristics of the application domain. Hence, we are aiming to enhance our *responsibility notions* by an additional *cost function* that regards the balancing between two parameters: *group size* and *responsibility distance*. This extended *responsibility framework* could provide a ranking among the set of (partially) responsible groups of agents and be used as an analysis tool for reasoning in collective decision making scenarios such as multi-step election scenarios in political domain or in analysing the dynamics of system behaviour and process executions in multitasking computer systems. We believe that our approach in formalizing the forward-looking responsibility in terms of power, is also applicable in conceptualizing the backward-looking responsibility and related notions such as *blameworthiness* and *accountability*. However, following [8,11], we see that for these concepts and in particular for the concept of *blame*, one prerequisite is to allow the variety in *knowledge* of the involved agents in the multi-agent system and to consider the epistemic state of agents. Finally, we aim to enrich our responsibility framework by providing logical characterization of the proposed notions in the coalitional logic with quantification [2].

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