

Landau Levels of Majorana Fermions in a Spin Liquid

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Majorana fermions, originally proposed as elementary particles acting as their own antiparticles, can be realized in condensed-matter systems as emergent quasiparticles, a situation often accompanied by topological order. Here we propose a physical system which realizes Landau levels—highly degenerate single-particle states usually resulting from an orbital magnetic field acting on charged particles—for Majorana fermions. This is achieved in a variant of a quantum spin system due to Kitaev which is distorted by triaxial strain. This strained Kitaev model displays a spin-liquid phase with charge-neutral Majorana-fermion excitations whose spectrum corresponds to that of Landau levels, here arising from a tailored pseudomagnetic field. We show that measuring the dynamic spin susceptibility reveals the Landau-level structure by a remarkable mechanism of probe-induced bound-state formation.

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In 1937, Majorana suggested [1] that a real wave function can describe spin-1/2 quantum particles that are their own antiparticles. Consequently, these particles are charge neutral and decouple from the electromagnetic field, making them hard to detect. While no observation of Majorana fermions as elementary particles has been reported to date, their possible realization as emergent quasiparticles in condensed-matter systems has attracted enormous attention. Specifically, it was realized that superconductors provide a natural habitat, where Bogoliubov quasiparticles at zero energy have properties of Majorana fermions [2]. Moreover, a seminal work of Kitaev [3] demonstrated that dispersive Majorana fermions can emerge as effective degrees of freedom in spin-liquid phases of frustrated quantum magnets [4]. This has triggered an intense search for materials which come close to realizing Kitaev's spin-liquid model on the honeycomb lattice, with $A_2\text{IrO}_3$ ($A = \text{Na, Li}$) (Refs. [5–9]) and $\alpha\text{-RuCl}_3$ (Ref. [10]) currently being the best candidates.

Here we show that the properties of emergent Majorana fermions in quantum magnets can be engineered by controlled lattice distortions. We use spatially inhomogeneous exchange couplings, generated by applying a suitable strain pattern, to transform the linear Dirac-like dispersion of low-energy Majorana fermions in the Kitaev model into a sequence of pseudo-Landau levels. The idea of strain-induced Landau levels was in fact first theoretically proposed [11] and then subsequently realized [12] for electrons in two-dimensional graphene. Our work adapts this concept to the charge-neutral fractionalized quasiparticles of a topological spin liquid. We compute the dynamic spin susceptibility which turns out to display sharp excitations at isolated energies reflecting the Landau-level structure of the strained spin liquid.

Model.—The Kitaev model [3] describes quantum spins 1/2 on a honeycomb lattice subject to spin-anisotropic Ising (or “compass”) interactions. The Hamiltonian, generalized to spatially varying couplings, reads

$$\mathcal{H}_K = - \sum_{\langle ij \rangle_x} J_{ij}^x \hat{\sigma}_i^x \hat{\sigma}_j^x - \sum_{\langle ij \rangle_y} J_{ij}^y \hat{\sigma}_i^y \hat{\sigma}_j^y - \sum_{\langle ij \rangle_z} J_{ij}^z \hat{\sigma}_i^z \hat{\sigma}_j^z, \quad (1)$$

where $\hat{\sigma}^\alpha$ are Pauli matrices and $\langle ij \rangle_\alpha$ denotes an α bond as shown in Fig. 1, with $\alpha = x, y, z$. This model displays an infinite set of constants of motion, which can be interpreted as \mathbb{Z}_2 fluxes through the closed loops of the lattice. Upon representing each spin $\hat{\sigma}$ by four Majorana fermions $\hat{b}^x, \hat{b}^y, \hat{b}^z$, and \hat{c} , with $\hat{\sigma}_i^\alpha = i\hat{b}_i^\alpha \hat{c}_i$, the model (1) takes the form

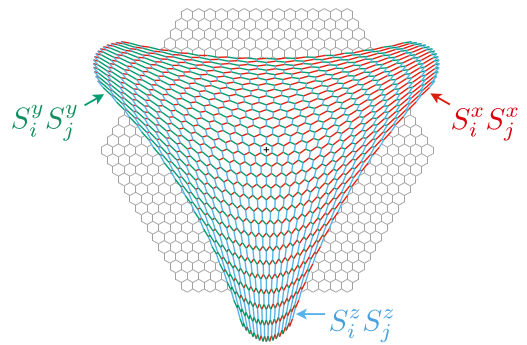


FIG. 1. Lattice setup: Honeycomb-lattice flake with $N = 675$ unit cells where bonds with different color correspond to Ising interactions J^α of different spin components α . The distorted lattice visualizes the displacement according to Eq. (4) arising from triaxial strain; the unstrained hexagonal flake is shown in the back in gray. Longer (shorter) bonds correspond to weaker (stronger) exchange couplings J_{ij}^α .

$$\mathcal{H}_{\hat{u}} = i \sum_{\langle ij \rangle} J_{ij}^{\alpha} \hat{u}_{ij} \hat{c}_i \hat{c}_j, \quad (2)$$

where $\hat{u}_{ij} \equiv i \hat{b}_i^{\alpha_{ij}} \hat{b}_j^{\alpha_{ij}}$ and $\hat{u}_{ij} = -\hat{u}_{ji}$. The operators \hat{u}_{ij} represent conserved quantities with eigenvalues of $u_{ij} = \pm 1$ which determine the values of the \mathbb{Z}_2 fluxes. Hence, the Hamiltonian $\mathcal{H}_{\hat{u}}$ (2) represents a nearest-neighbor hopping problem for the c (or “matter”) Majorana fermions which are coupled to a static \mathbb{Z}_2 gauge field. Its exact solution can be written in terms of canonical-fermion excitation modes with non-negative energies ϵ_m . For homogeneous and isotropic couplings $J_{ij}^{\alpha} \equiv J$, it describes a \mathbb{Z}_2 spin liquid with matter-fermion excitations displaying a gapless Dirac spectrum.

For our case of inhomogeneous couplings, we shall solve the Kitaev model numerically on finite-size lattices. To this end, Eq. (2) is rewritten into the following bilinear Hamiltonian for the matter Majorana fermions:

$$\mathcal{H}_u = \frac{i}{2} (\hat{c}_A^T \hat{c}_B^T) \begin{pmatrix} 0 & M \\ -M^T & 0 \end{pmatrix} \begin{pmatrix} \hat{c}_A \\ \hat{c}_B \end{pmatrix}, \quad (3)$$

where M is an $N \times N$ matrix with elements $M_{ij} = J_{ij}^{\alpha} u_{ij}$ and $\hat{c}_{A(B)}$ is a vector of length N of Majorana operators on the $A(B)$ sublattice, with N the number of unit cells. We construct the matrix M for a given set of couplings $\{J_{ij}^{\alpha}\}$ and diagonalize the problem via a singular-value decomposition which yields the excitation energies ϵ_m and the eigenmodes. From these we calculate the local density of states (LDOS) as the local spectral density of the c fermions; for details, see Supplemental Material [13].

Majorana Landau levels.—In the context of graphene, it has been theoretically shown [11,15] that a spatial modulation of hopping energies mimics the effect of a vector potential in Dirac fermion systems. Near the Dirac energy, this emergent vector potential can be expressed through the strain tensor u_{ij} as $\vec{A} \propto \pm(u_{xx} - u_{yy}, -2u_{xy})^T$, with opposite sign for electrons belonging to the two Dirac cones (or “valleys”) located at $\vec{K} = (4\pi/(3\sqrt{3}), 0)$ and $\vec{K}' = (-4\pi/(3\sqrt{3}), 0)$ (with the lattice constant a_0 set to unity), such that time-reversal symmetry is preserved. If the resulting pseudomagnetic field, $\vec{B} = \text{rot} \vec{A}$, is sufficiently homogeneous—applying, e.g., to triaxial strain patterns as shown in Fig. 1—it can induce single-particle pseudo-Landau levels very similar to Landau levels in a physical magnetic field. The system then displays a so-called quantum valley Hall effect [11]; i.e., it combines a chiral and an antichiral quantum Hall effect at the valleys \vec{K} and \vec{K}' , respectively. For triaxial strain the displacement vector is given by [11,16]

$$\vec{U}(x, y) = \bar{C}(2xy, x^2 - y^2), \quad (4)$$

where \bar{C} (measured in units of $1/a_0$) parameterizes the distortion and $u_{ij} = (\partial_i U_j + \partial_j U_i)/2$. This displacement yields—to first order in \bar{C} —a homogeneous pseudomagnetic field whose strength is proportional to \bar{C} .

Adapting the idea of strain-induced artificial gauge fields to spin systems, we study the Kitaev model (1) with spatially modulated exchange couplings. Given that the Majorana-fermion hopping matrix elements in Eq. (2) are given by the exchange couplings J_{ij}^{α} , we choose

$$J_{ij}^{\alpha} = J^{\alpha} [1 - \beta(|\vec{\delta}_{ij}|/a_0 - 1)], \quad (5)$$

where we calculate the distance $\vec{\delta}_{ij} = \vec{R}_i + \vec{U}_i - \vec{R}_j - \vec{U}_j$ using the $\vec{U}(x, y)$ from Eq. (4) evaluated at the lattice positions \vec{R}_i of the undistorted honeycomb lattice. The factor β encodes the strength of magnetoelastic coupling, and the parameter (dubbed “strain” below) $C = \bar{C}\beta$ will enter our simulations as a measure of the modulations of the J_{ij}^{α} . Throughout this Letter, we choose $\beta = 1$; larger values of β (hence, smaller \bar{C}) reduce nonlinearities in the strain pattern but are less realistic for transition-metal oxides. We note that, for a real material, Eq. (5) represents a linear approximation to the full dependence of the exchange constant on the bond length [16].

The inhomogeneous Kitaev model (1) with couplings given by Eq. (5) is expected to display a \mathbb{Z}_2 spin-liquid phase with an extremely unusual excitation spectrum: While the \mathbb{Z}_2 fluxes remain static, the matter Majorana fermions will form highly degenerate Landau levels. Given that a physical magnetic applied to a Kitaev magnet has an entirely different effect—it induces flux dynamics and leads to a gapped matter spectrum [3]—strain represents a unique way to generate Majorana Landau levels.

This is well borne out by our numerical results, obtained for finite-size Kitaev systems of hexagonal shape (Fig. 1) with open zigzag boundaries and lattice sizes up to $2N = 15000$ sites. We focus on the case of isotropic couplings and choose $J^{\alpha} = J$ as the energy unit. We limit the strain C to be smaller than C_{\max} , the latter being defined as the largest C where all $J_{ij} > 0$ [13]. For $C < C_{\max}$, we find that the ground state is located in the flux-free sector where all u_{ij} can be chosen to be $+1$. We monitor the LDOS of the matter fermions [13] which is shown in the top panels in Fig. 2: In the unstrained case $C = 0$, this is the familiar honeycomb-lattice DOS [Fig. 2(a)], with $\rho(\omega) \propto \omega$ at low energies, with the difference to graphene that only half of the spectrum corresponding to $\omega \geq 0$ is realized due to the Majorana nature of the matter fermions. Importantly, the results for finite strain show clear Landau-level peaks at low energies highlighted by the black arrows in Figs. 2(d) and 2(g). In particular, the Landau-level energies ϵ_n display the scaling $\epsilon_n \propto \sqrt{nC}$ with the lowest Landau level (LLL) located at zero energy (Fig. 3), characteristic of honeycomb-lattice Dirac fermions [18].

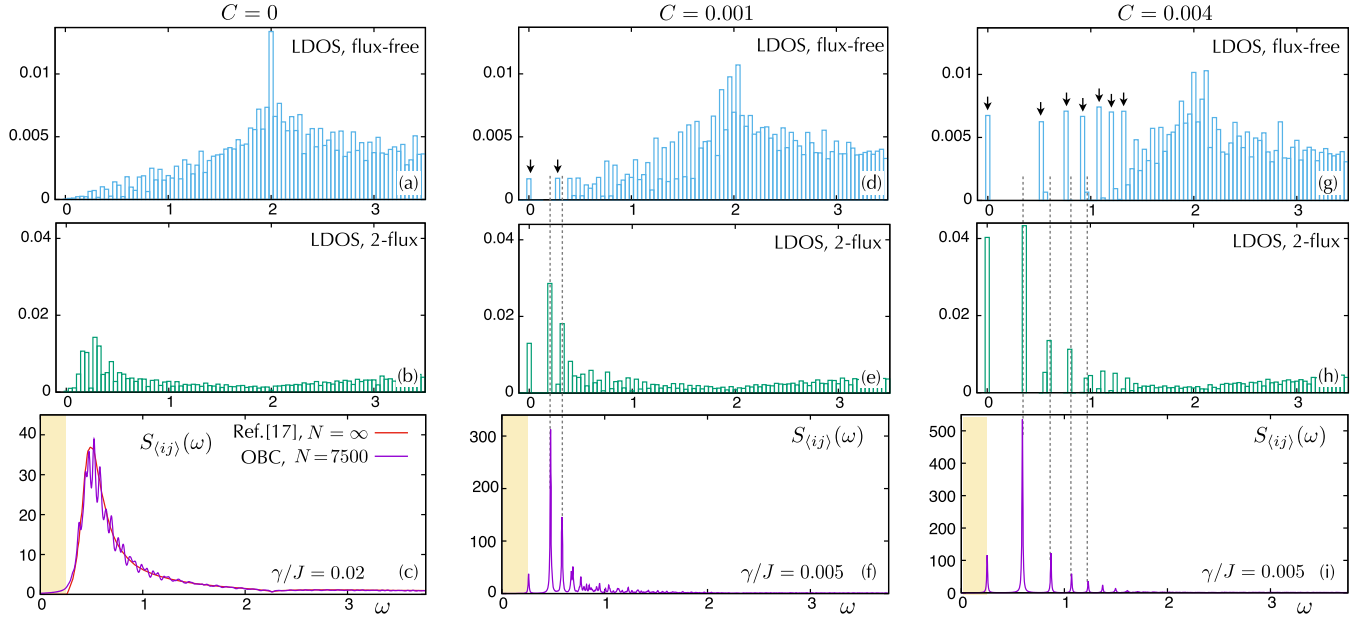


FIG. 2. Simulation results: Histogram of the matter-fermion LDOS in the flux-free (top, blue) and two-flux sectors (middle, green), together with the dynamical spin correlator $S_{ij}(\omega)$ with $S_{ij} = S_{ii} + S_{jj} + 2S_{ij}$ (bottom, purple), for the cases of (a)–(c) unstrained ($C = 0$), (d)–(f) moderate strain ($C = 1 \times 10^{-3}$), and (g)–(i) large strain ($C = 4 \times 10^{-3}$). All quantities are measured in the center of a sample with $2N = 15\,000$ sites. The $S(\omega)$ plots employ a Lorentzian broadening of width γ ; moreover, the energy axis has been shifted with respect to the LDOS plots by the amount of the local flux gap [13] Δ_{ij} (yellow region). (c) also shows $S(\omega)$ for the infinite homogeneous system [17]. While the LDOS of the flux-free sector [(d) and (g)] nicely shows the emergence of Majorana Landau levels (black arrows), the two-flux sector features bound states in the gaps between the Landau levels [(e) and (h)]—these are detected in $S_{ij}(\omega)$ as sharp peaks (dashed lines are guides to the eye).

Dynamic structure factor.—In order to detect the Landau-level structure, we propose to measure dynamic spin correlations

$$S_{ij}^{\alpha\beta}(t) = \langle \hat{\sigma}_i^\alpha(t) \hat{\sigma}_j^\beta(0) \rangle \quad (6)$$

whose Fourier transform is—in a magnet—accessible by neutron-scattering experiments. We restrict our attention to zero temperature where $S_{ij}^{\alpha\beta}(\omega)$ is proportional to the imaginary part of the dynamic spin susceptibility. The evaluation of the dynamic spin correlations in the Kitaev model has been first discussed in Ref. [17]. Specifically, the application of the operator $\hat{\sigma}_i^\alpha = i\hat{b}_i^\alpha \hat{c}_i$ changes the \mathbb{Z}_2 fluxes in the plaquettes adjacent to the α bond emanating from site i . Starting from the ground state $|0\rangle$, which is flux-free, the intermediate state $|\lambda\rangle$ is then a state with a locally excited flux pair; i.e., the hopping problem of matter fermions in the intermediate state involves a local “flux impurity” $\hat{V} = -2iJ_{ij}^\alpha \hat{c}_i \hat{c}_j$. The fact that fluxes are static implies that all correlators beyond nearest-neighbor sites are strictly zero.

The numerical calculation of a zero-temperature dynamic structure factor $S_{ij}(\omega)$ involves two diagonalizations: one in the flux-free sector with all $u = 1$ and one in the excited-state flux sector with all $u = 1$ except $u_{ij} = -1$ on the bond involving the measured site(s). The

eigenvectors in both flux sectors are then used to construct the matrix elements entering $S_{ij}(\omega)$. We employ the single-mode approximation [17], with details given in Supplemental Material [13].

For the structure factor of the strained Kitaev model, a central observation is that the impurity \hat{V} induces a sequence of single-particle bound states which energetically lie between the Landau levels. This can be nicely seen in Figs. 2(e) and 2(h), which show the LDOS in the two-flux sector, measured on the flipped bond. This LDOS displays essentially the same Landau levels as the corresponding flux-free system but shows additional sharp peaks of even higher weight at energies E_n in between the Landau levels—these peaks arise from isolated states localized near the flipped bond. We note that related bound states have been reported recently in a study of spin excitations in the field-induced non-Abelian phase of the Kitaev model [19]. More broadly, we recall that in-gap bound states occur frequently in one- and two-dimensional systems, because the real part of the single-particle Green’s function diverges at gap edges. In fact, recent theory work on topological band insulators has suggested that the *generic* occurrence of impurity-induced bound states in a two-dimensional gapped system is a signature of its topological nature [20]. For the present case of bond impurities in the triaxially strained honeycomb lattice, we have checked that bound

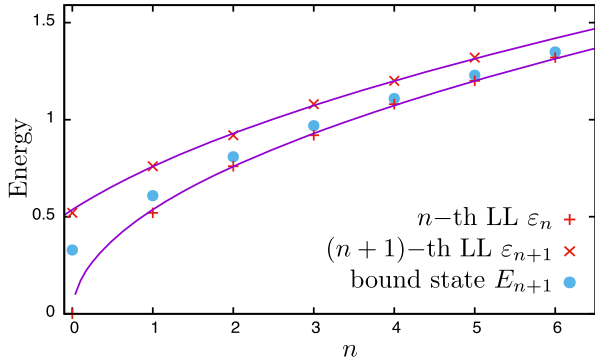


FIG. 3. Energy positions of the Landau levels ε_n as observed in the matter-fermion LDOS of the flux-free sector, shown together with the bound-state energies E_n in the two-flux sector; the E_n are identical to the peak positions in the correlator $S_{\langle ij \rangle}(\omega)$ with the local flux gap Δ_{ij} subtracted [except for the lowest peak, which originates from the LLL; hence, our numbering for E_n starts with $n = 1$]. The data correspond to that of Figs. 2(g) and 2(i) with $C = 0.004$. The lines are fits of ε_n to a \sqrt{n} dependence; both ε_n and ε_{n+1} are shown for visualization purposes such that the Landau-level gap is visible for each n .

states between the Landau levels occur indeed generically, i.e., independent of the impurity strength [13], hinting at the topological character of the matter-fermion sector.

Remarkably, these Majorana-fermion bound states enable a unique detection of the Landau-level structure: The largest matrix elements for the correlator $S_{\langle ij \rangle}(\omega)$ arise from these bound states in the two-flux sector, such that $S_{\langle ij \rangle}(\omega)$ displays sharp peaks at $E_n + \Delta_{ij}$, where Δ_{ij} is the local flux gap [13,21]. As can be seen in the lower panels in Fig. 2, these bound-state peaks dominate over the peaks arising from the Landau levels themselves.

In Fig. 3, we show the energetic positions of the bound-state peaks in $S_{\langle ij \rangle}(\omega)$, with the local flux gap Δ_{ij} subtracted, and compare them to the energies of the Landau levels found in the matter-fermion LDOS of the flux-free system. The plot shows that the bound-state energies track the energy dependence $\varepsilon_n \propto \sqrt{nC}$ of the Landau levels, approaching the lower edge of the respective gap with increasing n . Given that sharp bound states can arise only in gapped systems, the observation of a series peaks in $S_{\langle ij \rangle}(\omega)$ with a \sqrt{n} energy dependence (after subtracting a constant corresponding to Δ_{ij}) represents a direct signature of the peculiar excitation spectrum—Landau levels separated by gaps—of the strained spin liquid.

Discussion.—Our results raise a number of further issues. First, we note that the simultaneous application of strain and of a physical magnetic field has nontrivial effects, which can be analyzed perturbatively by projecting onto the flux-free sector [3]. We have found that the LLL of the strained Kitaev model is shifted to a finite energy, while otherwise the spectrum of the matter Majorana fermions

remains qualitatively unchanged. The excitations of the now fully gapped system obey non-Abelian statistics similar to the elementary excitations of Kitaev's B phase [3] subject to a magnetic field. Second, and perhaps most interesting, is the fate of the Landau-level spin liquid when interactions beyond the Kitaev model are included [22]. While this question is beyond the scope of this work, it is likely that the large degeneracy of the many-body ground state will be lifted in favor of potentially exotic states driven by interactions between the Majorana fermions, akin to the physics of the fractional quantum Hall effect.

Realizations of our system appear possible using strained thin films of honeycomb-lattice iridates, where it has been proposed that homogeneous pressure or strain may drive the material into a Kitaev spin-liquid regime. Landau levels can then be generated by using shapes resulting from triaxial strain, circular arcs [23], or nanobubbles [12]. It has to be kept in mind, however, that inhomogeneous strain may induce additional interactions as it deforms the nearly 90° Ir—O—Ir bonds. A second possibility is offered by advances in designing artificial molecular structures on surfaces, where the electronic structure of strained graphene has already been demonstrated [24]. A third option is by simulating the Hamiltonian (1) using ultracold atomic gases. Phase plates [25] or digital mirror devices [26] can be used to directly project the distorted lattice onto a 2D cloud of atoms, and proposals for realizing an artificial spin-orbit coupling have been made [27].

In summary, we have demonstrated how to engineer a novel state of matter, where the emergent degrees of freedom of a fractionalized spin liquid—charge-neutral Majorana fermions—display a Landau-level structure of excitation energies. We have shown that this Landau-level structure can be efficiently detected in measurements of dynamic spin correlations, thanks to a mechanism of probe-induced bound-state formation.

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