



Global monopoles can change Universe's topology



Anja Marunović*, Tomislav Prokopec

Institute for Theoretical Physics, Spinoza Institute and EMMEΦ, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands

ARTICLE INFO

Article history:

Received 7 April 2015

Received in revised form 11 March 2016

Accepted 12 March 2016

Available online 16 March 2016

Editor: M. Trodden

ABSTRACT

If the Universe undergoes a phase transition, at which global monopoles are created or destroyed, topology of its spatial sections can change. More specifically, by making use of Myers' theorem, we show that, after a transition in which global monopoles form, spatial sections of a spatially flat, infinite Universe becomes finite and closed. This implies that global monopoles can change the topology of Universe's spatial sections (from infinite and open to finite and closed). Global monopoles cannot alter the topology of the space-time manifold.

© 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The question of global properties (topology) of our Universe is a fascinating one, and it has been attracting attention for a long time. Yet only as-of-recently the data have been good enough to put meaningful observational constraints on the Universe's topology. While Einstein's equations uniquely specify local properties of space-time (characterized by the metric tensor), they fail to determine its global (topological) properties. Friedmann, Robertson and Walker (FRW) were first who observed that the most general solution corresponding to spatially homogeneous Universe with constant curvature κ of its spatial sections is the following FLRW metric (L stands for Lemaître),

$$ds^2 = -c^2 dt^2 + \frac{a^2(t) dr^2}{1 - \kappa r^2} + a^2(t) r^2 [d\theta^2 + \sin^2(\theta) d\phi^2], \quad (1)$$

where c is the speed of light and $0 \leq r < \infty$, $0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$ are spherical coordinates. Recent cosmic microwave background and large scale structure observations tell us that, at large scales the metric (1) describes quite accurately our Universe. When κ in (1) is

1. negative ($\kappa < 0$), then spatial sections of the Universe are hyperbolic,
2. zero ($\kappa = 0$), then spatial sections are flat;
3. positive ($\kappa > 0$), then the spatial sections are positively curved and they are locally homeomorphic to the geometry of the three dimensional sphere.

Older literature typically assumes that $\kappa \leq 0$ implies infinite spatial sections, while when $\kappa > 0$, spatial sections are compact. While the latter statement is correct, recent advancements in our understanding of (topology of) three dimensional manifolds tell us that we must be much more careful when drawing conclusions from the observational fact that the metric describing our observable Universe is well approximated by the FLRW metric (1). Namely, various boundary conditions could be imposed on the Universe's spatial sections [1], giving as a result a large number of possible three dimensional manifolds, only one of which corresponds to that of our Universe.

Let us now briefly recall the relevant observational facts. The first observational evidence that supports that we live in a (nearly) flat universe ($\kappa \approx 0$) was presented in 2000 by the balloon experiments Boomerang [2] and Maxima [3]. A recent bound on κ [4] is obtained when observations of BAOs (Baryon Acoustic Oscillations) are combined with the Planck data [5] and the polarization data from the WMAP satellite (WP),

$$\Omega_\kappa = -0.003 \pm 0.003, \quad \Omega_\kappa = -\frac{\kappa c^2}{H_0^2}, \quad (2)$$

where $H_0 \simeq 68$ km/s/Mpc (when the BAO data are dropped, one obtains $0.006 > \Omega_\kappa > -0.086$ [5]). Eq. (2) implies a large lower bound on the curvature radius of spatial sections, $R_c = 1/\sqrt{|\kappa|} \geq 60$ Gpc. The bound (2) implies the following robust conclusion: “our [observable] Universe is spatially flat to an accuracy of better than a percent” (cited from page 42 of Ref. [5]).

Even if the Universe is spatially flat, it can be made finite by imposing suitable periodic boundary conditions; the precise nature of periodic conditions determines Universe's global topology [1]. Although different scenarios have been considered in literature (good reviews are given in Refs. [1,6–10]), so far no evidence has been

* Corresponding author.

E-mail addresses: a.marunovic@uu.nl (A. Marunović), t.prokopec@uu.nl (T. Prokopec).

found that would favor any of the proposed models. For example, extensive mining of the CMB data has been performed [11–13] in order to find pairs of circles, which are a telltale signature for non-trivial large-scale topology of the Universe, but so far no convincing signature has been found.

The above considerations make an implicit assumption that spatial curvature of the Universe is given and that it cannot be changed throughout the history of our Universe. In this letter we argue that this assumption ought to be relaxed, and we propose a dynamical mechanism:

formation of global monopoles at an early universe phase transition,

by which the (average, measured) spatial curvature of the Universe can change in the sense that it will become positive if it starts slightly negative or zero. Strictly speaking this is true provided the Universe was before the transition non-compact, i.e. it was created with no periodic boundary conditions imposed on it.

This claim will leave many readers with a queasy feeling since, when κ changes from $\kappa \leq 0$ to $\kappa > 0$, spatial sections could change from infinite (hyperbolic or parabolic) to finite (elliptic), thus changing the topology of spatial sections. One should keep in mind that all this happens at *space-like* hyper-surfaces of constant time, and hence it is not in contradiction with any laws of causality. And yet it does leave us with an uncomfortable feeling that ‘somewhere there’ distant spatial sections of the Universe are *reconnecting*, thereby changing them from infinite to finite and periodic. This will be the case provided all spatial dimensions are equally affected, which is the case in the mechanism considered in this letter. Even though not directly observable today, this process can have direct consequences for our future. Indeed, when an observer in that reconnected Universe sends a (light) signal, it will eventually arrive from the opposite direction. Furthermore, the future of a spatially finite (compact) universe can change from infinite and uneventful to finite and singular (namely, if cosmological constant is zero such a universe will end up in a Big Crunch singularity). Because of all of these reasons, a tacit consensus has emerged that no topology change is possible in our Universe (albeit strictly speaking measurements constrain the Universe’s spatial topology only after recombination). We argue in this work that this consensus needs to be reassessed.

In fact, the idea that the curvature of spatial sections could change can be traced back to the work of Krasinski [14] based on Stephani’s exact solution [15] to Einstein’s equations. Even though Krasinski has argued that the curvature of spatial sections could dynamically change, he has not offered any mechanism by which such a change could occur [16]. In this letter we provide such a dynamical mechanism.

A particularly instructive case to consider is the maximally symmetric de Sitter space, whose geometry can be clearly visualized from its five dimensional (flat, Minkowskian) embedding (see Fig. 1),

$$dS^2 = -dT^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2, \quad (3)$$

$$R_H^2 = T^2 - R^2, \quad R^2 = X_1^2 + X_2^2 + X_3^2 + X_4^2.$$

Thus de Sitter space is geometrically a four dimensional hyperboloid \mathbb{H}^4 , and its symmetry is the five dimensional Lorentz group, $SO(1, 4)$, which has – just like the Poincaré group of the symmetries of Minkowski space – 10 symmetry generators. This means that de Sitter space also has 10 global symmetries (Killing vectors). Common coordinates on de Sitter space (3) are those of constant curvature of its spatial sections, and they include: (a) closed (global) coordinates have $\kappa > 0$; (b) flat (Euclidean) coordinates (Poincaré patch) have $\kappa = 0$ and (c) open coordinates (hyperbolic

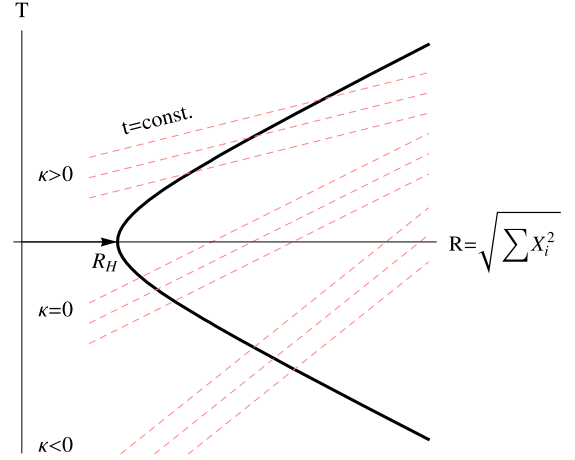


Fig. 1. Hypersurfaces of constant time of de Sitter \mathbb{H}^4 with a time dependent κ .

sections) have $\kappa < 0$. Krasinski has, however, pointed out that there are also de Sitter coordinates in which κ changes in time. Both cases, when κ changes from negative to positive, and v.v. are possible. An example of the metric when $\kappa(t)$ changes from negative to positive can be easily inferred from [14],

$$ds^2 = -\frac{c^2(r/r_0)^4}{[1 + ctr^2/r_0^3]^2[(Hr_0/c)^2 - ct/r_0]} dt^2 + \frac{1}{[1 + ctr^2/r_0^3]^2} [dr^2 + r^2 d\theta^2 + \sin^2(\theta) d\phi^2], \quad (4)$$

where r_0 , c , H are constants. That this is a de Sitter space can be checked, for example, by evaluating the Riemann tensor. One finds

$$R_{\mu\nu\alpha\beta} = (R/12)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}), \quad (5)$$

where $R = 12H^2/c^2 \equiv 12/R_H^2$ is the Ricci curvature scalar, $H = \text{const.}$ is the Hubble parameter and $R_H = c/H$ is the Hubble radius. Relation (5) holds uniquely for maximally symmetric spaces such as de Sitter space. The curvature of spatial sections of de Sitter in (4) can be inferred from the Riemann curvature of spatial sections,

$${}^{(3)}R_{ijkl} = \frac{{}^{(3)}R}{6}(g_{ik}g_{jl} - g_{il}g_{jk}), \quad {}^{(3)}R = \frac{24ct}{r_0^3} \equiv \frac{6\kappa(t)}{a^2(t)}, \quad (6)$$

from which we infer,

$$\kappa(t) = \frac{4cta^2(t)}{r_0^2}, \quad \text{with } a(t) = e^{Ht}, \quad (7)$$

which means that $\kappa < 0$ for $t < 0$, $\kappa = 0$ for $t = 0$, and $\kappa > 0$ for $t > 0$. Note that topology of spatial sections changes at $t = 0$. For $t < 0$ the sections are three dimensional hyperboloids, with a time dependent (physical) throat radius $r_c(t) = r_0^{3/2}/\sqrt{-ct}$, for $t = 0$ they are paraboloids and for $t > 0$ they are three-spheres with a (time-dependent) radius, $r_c = r_0^{3/2}/\sqrt{ct}$ (see Fig. 1). Consequently, topology of spatial sections changes at $t = 0$, as can be seen in Fig. 1 [17]. A similar (albeit inhomogeneous) construction is possible on FLRW space-times. While this shows that there are observers for which topology of spatial sections of an expanding space-time changes, it does not tell us how to realize such a change, and whether such a change is possible in a realistic setting. This is what we address next.

2. The model

The action for gravity we take to be the Einstein–Hilbert (EH) action,

$$S_{EH} = \frac{c^4}{16\pi G_N} \int d^4x \sqrt{-g} R, \quad (8)$$

where G_N is the Newton constant, R is the Ricci curvature scalar, and g is the determinant of the metric tensor $g_{\mu\nu}$. In our model the EH action is supplemented by the action that governs the dynamics of global monopoles,

$$S_\phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi^a) (\partial_\nu \phi^a) - V(\phi^a) \right), \quad (9)$$

with the Higgs type of $O(3)$ symmetric potential

$$V(\phi^a) = \frac{\mu^2}{2} \phi^a \phi^a + \frac{\lambda_\phi}{4} (\phi^a \phi^a)^2 + \frac{\mu^4}{4\lambda_\phi}, \quad (10)$$

where repeated indices a indicate a summation over $a = 1, 2, 3$. μ is a mass parameter and λ_ϕ is a self-coupling. The scalar field $\vec{\phi} = (\phi^a)$ ($a = 1, 2, 3$) consists of 3 real components, such that the action (9) is $O(3)$ -symmetric. When $\mu^2 < 0$ the vacuum exhibits a field condensate, $\phi^a \phi^a \equiv \phi_0^2 = -\mu^2/\lambda_\phi$, which spontaneously breaks the $O(3)$ symmetry of the action to an $O(2)$, such that the resulting vacuum manifold \mathcal{M} has a symmetry of the two dimensional sphere, $\mathcal{M} = O(3)/O(2) \sim S^2$. The two excitations along the two orthogonal directions of S^2 are the two massless Goldstone bosons, while the excitation orthogonal to \mathcal{M} is massive, $m^2(\phi_0) = 2\lambda_\phi \phi_0^2 = -2\mu^2$. The potential (10) is chosen such that the energy density of the (classical) vacuum is $V(\phi_0) = 0$.

It is well known that the action (9)–(10) permits non-vacuum classical solutions known as global monopoles [19]. Global monopoles have a non-vanishing vacuum energy, but they do not decay as they are stabilized by topology. The simplest such solution of the equations of motion is a hedgehog-like spherically symmetric solution of the form,

$$\vec{\phi}(t, \vec{r}) = \phi(r) (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)^T, \quad (11)$$

where θ , φ and r are spherical coordinates. One can show [20] that the topological charge (also known as the winding number) of that solution is unity,

$$Q[\phi^a] = \frac{1}{8\pi} \int dS^{ij} \frac{\epsilon^{abc} \phi^a \partial_i \phi^b \partial_j \phi^c}{[\phi^e \phi^e]^{3/2}} = 1, \quad dS^{ij} = dx^i \wedge dx^j, \quad (12)$$

and that it is stable under small field perturbations.

Global monopoles are generically created at a phase transition by the Kibble mechanism [19] (at least of the order one per Hubble volume) if the effective field mass matrix, $[m_{\text{eff}}^2(\phi^a = 0)]^{ab} = \partial^2 V_{\text{eff}} / \partial \phi^a \partial \phi^b$ changes from having all positive eigenvalues to at least one negative eigenvalue [21,22], which can be realized in e.g. a hot Big Bang. Global monopoles with charge $Q = 1$ and $Q = -1$ (which can be obtained by interchanging any two coordinates in Eq. (11)) are equally likely to form, and the monopoles of opposite charge will strongly attract each other (by a force independent of distance) and efficiently annihilate [26], such that a network of global monopoles will eventually reach a scaling solution [27] with about four monopoles per Hubble volume at any given time.

One important and defining property of global monopoles is their *solid deficit angle* [23] (see also the Appendix), which extends to the particle horizon associated with the monopole creation event.

3. Global and local properties of space-time

As shown in the Appendix, at sufficiently large distances, the metric of a monopole can be approximated by a FLRW metric with a solid deficit angle. This deficit angle generates a spatial Ricci tensor that breaks spatial homogeneity of the FLRW space-time and decays as ${}^{(3)}R_r^r \simeq 4\Delta/[m_\phi^2 a^4 r^4]$, ${}^{(3)}R_\theta^\theta = {}^{(3)}R_\varphi^\varphi \simeq \Delta/[a^2(t)r^2]$, where Δ is the deficit angle and m_ϕ the monopole mass. In the limit of a large number of monopoles per Hubble volume all with $\Delta \ll 1$ and if in the absence of monopoles the spatial curvature is zero, a local observer that is sufficiently far from any individual monopole will observe a metric that can be approximated by a FLRW metric with a positive spatial curvature. Hence, an observer in such a universe will tend to conclude that the Universe is spatially compact and finite.

In this section we show that a (spatially flat) cosmological space filled with randomly distributed global monopoles must have spatial sections that are closed, and therefore its spatial geometry is that of a three dimensional sphere. This conclusion is reached based on the well known *Myers' theorem* [24], which states that for any Riemannian manifold whose Ricci curvature R_i^j is positive and limited from below as,

$$\|R_i^j\| > R_{\min} > 0 \quad (13)$$

the distance function $d(x; x')$ is limited from above by $d(x; x') < \pi/\sqrt{R_{\min}}$ in any number of (spatial) dimensions. This then implies that the manifold is *globally closed* and its radius of curvature is limited from above as $r_c \leq \pi/\sqrt{R_{\min}}$. This powerful theorem relates local properties of Riemannian manifolds to their global properties. In particular, when applied to global monopoles, that, at asymptotically large distances, the minimum component of the Ricci is (see Eq. (22) in the Appendix),

$$\|{}^{(3)}R_i^j\| = {}^{(3)}R_r^r \simeq \frac{4\Delta}{m_\phi^2 a^4 r^4}. \quad (14)$$

Assume that the monopole–monopole correlation function is that of randomly (Poisson) distributed monopoles with an average distance squared, $\langle a^2(\vec{x} - \vec{x}')^2 \rangle \simeq a^2 \sigma^2$. Let us consider a sphere of radius $ar \sim a\sigma N^{1/3}$, in which the average number of monopoles is, $N \sim (ar/\sigma)^3 \sim 1/\Delta$, which is the number needed to close the space (here we neglect factors of order unity, such as the volume of the 3-dimensional unit sphere, $\mathcal{V}_0(S^3) = 2\pi^2$). The Poisson distribution then implies that the maximum distance between two monopoles is $ar_{\max} \sim a\sigma [\ln(\Delta^{-1})]^{1/3}$. Now, according to Myers' theorem, a spatial section of the Universe filled with monopoles is a closed inhomogeneous manifold with a (comoving) radius of curvature, $r_c \lesssim m_\phi a \sigma^2 \Delta^{-1/2} [\ln(\Delta^{-1})]^{2/3}$. While this represents an upper bound, the actual Universe's curvature radius will be smaller, probably of the order $r_c \sim \sigma/\sqrt{\Delta}$, where a dilute monopole gas is assumed, $ma\sigma \gg 1$.

To conclude, due to their deficit angle, formation of global monopoles can have an impact on global (topological) properties of the (spatial sections of the) Universe. In particular, a (spatially flat) universe filled with global monopoles will have a closed geometry and, in the limit of many monopoles where each has a small deficit angle, it will closely resemble a closed universe with a constant positive spatial curvature κ . We have thus shown that, if before a phase transition at which global monopoles form, spatial sections of the Universe are flat or slightly negatively curved and therefore can be (in the absence of non-trivial topology) infinite, after the phase transition the Universe will have on average a positive curvature and its spatial sections will be homeomorphic to a three-sphere and hence compact.

Even though the Universe filled with global monopoles resembles a FLRW universe with $\kappa > 0$, it is an inhomogeneous universe with an uncertain future (at the moment it is not clear to us whether the Universe will end up in a Big Crunch or it will expand forever). Furthermore, an observer residing sufficiently close to a monopole, on top of the usual Hubble flow will feel a repulsive gravitational force that points away from the monopole core (see e.g. [25]). By studying physical effects of this force a local observer will be able to distinguish between a homogeneous universe and a universe filled with global monopoles. Next, while on a clump of matter photons and particles get deflected by an angle that depends on their velocity, sufficiently far from a global monopole the deflection angle will be equal for (relativistic and nonrelativistic) massive particles and photons, *i.e.* it will be independent on particle's speed.

Furthermore, measuring spatial curvature on the largest observable scales (such as it is done by modern CMB observations) can provide an upper limit on the number of monopoles in our horizon. Since CMB observatories such as the Planck and WMAP satellites measure κ on the Hubble scale, they can be reinterpreted as the upper limit on the total solid deficit angle in our Hubble volume generated by global monopoles,

$$0 < \sum_{i \in \text{Hubble shell thickness } \sigma} \Delta_i < -(\Omega_\kappa)_{\text{max}} \sim 0.01, \quad (15)$$

where $-(\Omega_\kappa)_{\text{max}}$ represents the upper limit on $-\Omega_\kappa = \kappa c^2/H_0^2$ allowed by the observations and $H_0 \simeq 68$ km/s/Mpc is the Hubble parameter today.

4. Discussion

In this letter we show that formation of global monopoles in the early Universe can lead to a change in (average) spatial curvature κ of the Universe such that, if the Universe starts with an average $\kappa = 0$, after the phase transition, $\kappa > 0$.

An important question is whether our model is consistent with all of the current observations. As mentioned above, global monopoles eventually reach a scaling solution with about 4 monopoles per Hubble volume. Each global monopole will generate a perturbation in matter density that corresponds to a gravitational potential of the order $\psi_N \sim \Delta$, implying that Δ must be smaller than about 10^{-5} (which is the amplitude of a typical potential generated by inflationary perturbations). A mild breakdown of spatial homogeneity and isotropy is consistent with CMB observations, and may be related with some of the CMB anomalies, examples being the observed tantalizing hints for non-Gaussianities [28,29].

Furthermore, above we found that a typical spatial curvature generated by monopoles is of the order, $\kappa \sim \Delta/\sigma^2 \sim (N_M^{2/3} \Delta)(aH)^2$, which increases (decreases) in accelerating (decelerating) space-times, where N_M is the number of monopoles in the observational volume ($N_M \sim 4$ if the observational volume equals the Hubble volume). This then implies that the spatial curvature induced by global monopoles in our Hubble volume is of the order, $\kappa \sim \Delta$, which is about 2 orders of magnitude too small to be observable. A way out is to have N copies of scalar fields, each with an $O(3)$ symmetric potential and leading to formation of global monopoles. In this case each of the corresponding global monopole networks would be today in the scaling solution, implying that $\kappa \sim (4N)^{2/3} \Delta$ per Hubble volume. Thus one could get $\kappa \sim 10^{-3}$ (which is observable) with $N \sim 250$ copies, where we took $\Delta \sim 10^{-5}$. Such a large number of scalar fields can occur *i.e.* in some string compactification models.

In this letter we focus mainly on the case when the very early Universe is non-compact and when its spatial sections have a

constant negative or vanishing spatial curvature and argue that a phase transition at which global monopoles form can change the perceived spatial curvature to positive and that the spatial sections may thus become compact. We are very much aware of that this model of the Universe may be too simplistic and that the true geometry of the spatial sections of the early Universe may be much more complex. Firstly, the Universe's spatial sections could be inhomogeneous and could be made up of piecewise connected simple three dimensional Thurston geometries [30] that are used to classify three dimensional manifolds. The eight Thurston's classes of geometries are: (1) hyperbolic (H^3), (2) Euclidean (E^3), (3) spherical (S^3), (4) $H^2 \times R$, (5) $S^2 \times R$, (6) $SU(2, R)$, (7) NIL geometry, and (8) Solv geometry. This letter is not the place to address in depth the properties of any of these geometries (see Ref. [1] for an incomplete account of these geometries; a more complete, but more mathematical, discussion can be found in Refs. [30,31]). One should note, however, that spatial slices can correspond to the covering spaces of these geometries (in which case no periodic boundary conditions are imposed) or periodic conditions are imposed on them, which would make the Universe's spatial sections compact, and complicate the discussion of what happens when global monopoles form. This letter focuses primarily on the simple case in which a non-compact H^3 spatial geometry changes via E^3 to S^3 (assuming no periodic boundary conditions). If however the space is hyperbolic and compact (that can be achieved by imposing suitable periodic conditions on a compact fundamental domain) formation of topological defect will in general *not* change the topology of the Universe. This is so because compact H^3 or H^2 topologies have as the fundamental domain a compact space that cannot be smoothly changed to that of a flat or positively curved fundamental domain. To illustrate the point, consider $H^2 \times R$. The fundamental domains that correspond to a compact subspace H^2 comprise polygons P_{4g} , where $g \geq 2$ denotes the genus of the manifold. It is now clear that these domains cannot be smoothly changed to P_4 or P_6 , which correspond to the fundamental domains of flat two dimensional spaces. Finally, in the case when the topology of the Universe belongs to one of the last 3 Thurston classes ($SL(2, R)$, NIL or Solv), since they do not have the positively curved counterparts, we expect that a local observer will not be able to infer a change of topology from a non-compact one to a compact one. It is clear that further study is needed to fully understand the dynamics of topology of the spatial sections of our Universe.

Acknowledgements

This work is part of the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW). A.M. is funded by NEWFELPRO, an International Fellowship Mobility Programme for Experienced Researchers in Croatia and by the D-ITP. The authors thank Jose Maciel Natario for a very useful comment.

Appendix A

The energy-momentum tensor of a global monopole is given by

$$T_{\mu\nu}^\phi = -\frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = (\partial_\mu \phi^a)(\partial_\nu \phi^a) + g_{\mu\nu} \mathcal{L}_\phi. \quad (16)$$

Let us assume an AB metric of the form,

$$ds^2 = -c^2 dt^2 + A^2(t, r) dr^2 + B^2(t, r) r^2 [d\theta^2 + \sin^2(\theta) d\phi^2]. \quad (17)$$

In presence of a global monopole (11) the asymptotic behavior of the A and B functions in (17) is [20],

$$A^2 \xrightarrow{r \rightarrow \infty} \frac{a^2(t)[1 + f_0/r^2]}{1 - \Delta}, \quad B^2 \xrightarrow{r \rightarrow \infty} a^2(t) \left(1 + \frac{g_0}{r^2}\right) \quad (18)$$

(f_0 and g_0 are constants) and that of $\phi(r)$ in (11) is,

$$\phi(0) = 0, \quad \phi(r \rightarrow \infty) = \phi_0 - \frac{1}{\lambda_\phi \phi_0 a^2 r^2}, \quad (19)$$

with $\phi_0^2 = -\mu^2/\lambda_\phi$, see Eqs. (9)–(11). With these in mind, one obtains, for non-vanishing components of the stress-energy tensor the following asymptotic form,

$$T_r^r \xrightarrow{r \rightarrow \infty} -\frac{\phi_0^2(1 - 2/m_\phi^2 a^2 r^2)^2}{a^2 r^2(1 + g_0/r^2)} - \frac{2\phi_0^2}{m_\phi^2 a^4 r^4},$$

$$T_\theta^\theta = T_\varphi^\varphi \xrightarrow{r \rightarrow \infty} -\frac{2\phi_0^2}{m_\phi^2 a^4 r^4}, \quad (20)$$

where $m_\phi^2 = 2\lambda_\phi \phi_0^2 \gg H^2$. On the other hand, the Einstein equations for the manifold's spatial sections are,

$${}^{(3)}R_i^j - \frac{{}^{(3)}R}{2} \delta_i^j \simeq \frac{8\pi G_N}{c^4} T_i^j \Rightarrow {}^{(3)}R = -\frac{16\pi G_N}{c^4} \sum_i T_i^i \quad (21)$$

Upon comparing these with (20) one obtains that the monopole contributes to the relevant components of the spatial Ricci curvature tensor as,

$${}^{(3)}R_r^r \simeq \frac{4\Delta}{m_\phi^2 a^4 r^4}, \quad {}^{(3)}R_\theta^\theta = {}^{(3)}R_\varphi^\varphi \simeq \frac{\Delta}{a^2 r^2}, \quad \Delta = \frac{8\pi G_N \phi_0^2}{c^4}, \quad (22)$$

where Δ denotes the solid deficit angle.

References

- [1] J.J. Levin, Topology and the cosmic microwave background, Phys. Rep. 365 (2002) 251, [http://dx.doi.org/10.1016/S0370-1573\(02\)00018-2](http://dx.doi.org/10.1016/S0370-1573(02)00018-2), arXiv:gr-qc/0108043.
- [2] P. de Bernardis, et al., Boomerang Collaboration, A flat universe from high resolution maps of the cosmic microwave background radiation, Nature 404 (2000) 955, arXiv:astro-ph/0004404.
- [3] A. Balbi, P. Ade, J. Bock, J. Borrill, A. Boscaleri, P. de Bernardis, P.G. Ferreira, S. Hanany, et al., Constraints on cosmological parameters from MAXIMA-1, Astrophys. J. 545 (2000) L1, arXiv:astro-ph/0005124; A. Balbi, P. Ade, J. Bock, J. Borrill, A. Boscaleri, P. de Bernardis, P.G. Ferreira, S. Hanany, et al., Constraints on cosmological parameters from MAXIMA-1, Astrophys. J. 558 (2001) L145 (Erratum).
- [4] A. Aubourg, S. Bailey, J.E. Bautista, F. Beutler, V. Bhardwaj, D. Bizyaev, M. Blanton, M. Blomqvist, et al., Cosmological implications of baryon acoustic oscillation (BAO) measurements, arXiv:1411.1074 [astro-ph.CO].
- [5] P.A.R. Ade, et al., Planck Collaboration, Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. (2014), arXiv:1303.5076 [astro-ph.CO].
- [6] P.A.R. Ade, et al., Planck Collaboration, Planck 2013 results. XXVI. Background geometry and topology of the Universe, arXiv:1303.5086 [astro-ph.CO].
- [7] P.A.R. Ade, et al., Planck Collaboration, Planck 2015 results. XVIII. Background geometry & topology, arXiv:1502.01593 [astro-ph.CO].
- [8] J.P. Uzan, R. Lehoucq, J.P. Luminet, New developments in the search for the topology of the universe, arXiv:gr-qc/0005128.
- [9] J.P. Luminet, B.F. Roukema, Topology of the universe: theory and observation, arXiv:astro-ph/9901364.
- [10] G.D. Starkman, Topology and cosmology, Class. Quantum Gravity 15 (1998) 2529.
- [11] P.M. Vaudrevange, G.D. Starkman, N.J. Cornish, D.N. Spergel, Constraints on the topology of the Universe: extension to general geometries, Phys. Rev. D 86 (2012) 083526, arXiv:1206.2939 [astro-ph.CO].
- [12] N.J. Cornish, D.N. Spergel, G.D. Starkman, E. Komatsu, Constraining the topology of the universe, Phys. Rev. Lett. 92 (2004) 201302, arXiv:astro-ph/0310233.
- [13] N.J. Cornish, D.N. Spergel, G.D. Starkman, Circles in the sky: finding topology with the microwave background radiation, Class. Quantum Gravity 15 (1998) 2657, arXiv:astro-ph/9801212.
- [14] A. Krasinski, On the global geometry of the Stephani Universe, Gen. Relativ. Gravit. 15 (1983) 673.
- [15] H. Stephani, Über Lösungen der Einsteinschen Feldgleichungen, die sich in einen fünfdimensionalen flachen Raum einbetten lassen, Commun. Math. Phys. 4 (1967) 137.
- [16] From Bergmann's work [18] it follows that a time-dependent change in κ can be realized by a space-like heat flow. It is, however, unclear whether such a flow can be realized in a realistic setting.
- [17] Here we use the notion of topology in a loose sense, to mean a change from an infinite space to a finite and compact space, whereby some topological invariants change. One can give to the notion of topology change a more rigorous meaning by making use of homotopy groups. For example, the homotopy groups of flat ($\kappa = 0$) spatial slices, which are homeomorphic to \mathbb{R}^3 , are trivial, $\pi_n(\mathbb{R}^3) = \{0\}$. When $\kappa > 0$, spatial slices are three spheres S^3 and some of its homotopy groups are nontrivial, $\pi_1(S^3) = \{0\}$, $\pi_2(S^3) = \{0\}$, $\pi_3(S^3) = \mathbb{Z}$, $\pi_4(S^3) = \mathbb{Z}_2$, etc. Finally, when spatial slices are three dimensional hyperboloids, \mathbb{H}^3 , its homotopy groups are trivial, $\pi_n(\mathbb{H}^3) = \{0\}$ ($n = 1, 2, 3, \dots$).
- [18] O. Bergmann, A cosmological solution of the Einstein equations with heat flow, Phys. Lett. A 82 (1981) 383.
- [19] T.W.B. Kibble, Topology of cosmic domains and strings, J. Phys. A 9 (1976) 1387.
- [20] A. Marunović, T. Prokopec, 2014, in preparation.
- [21] T. Prokopec, Symmetry breaking and the Goldstone theorem in de Sitter space, J. Cosmol. Astropart. Phys. 1212 (2012) 023, arXiv:1110.3187 [gr-qc].
- [22] G. Lazzari, T. Prokopec, Symmetry breaking in de Sitter: a stochastic effective theory approach, arXiv:1304.0404 [hep-th].
- [23] M. Barriola, A. Vilenkin, Gravitational field of a global monopole, Phys. Rev. Lett. 63 (1989) 341.
- [24] S.B. Myers, Riemannian manifolds with positive mean curvature, Duke Math. J. 8 (1941) 401.
- [25] A. Marunović, M. Murković, A novel black hole mimicker: a boson star and a global monopole nonminimally coupled to gravity, Class. Quantum Gravity 31 (2014) 045010, arXiv:1308.6489 [gr-qc].
- [26] R. Leese, T. Prokopec, Clustering of cosmological defects at the time of formation, Phys. Lett. B 260 (1991) 27.
- [27] D.P. Bennett, S.H. Rhie, Cosmological evolution of global monopoles and the origin of large scale structure, Phys. Rev. Lett. 65 (1990) 1709.
- [28] P.A.R. Ade, et al., Planck Collaboration, Planck 2013 results. XXIII. Isotropy and statistics of the CMB, Astron. Astrophys. 571 (2014) A23, arXiv:1303.5083 [astro-ph.CO].
- [29] P.A.R. Ade, et al., Planck Collaboration, Planck 2015 results. XX. Constraints on inflation, arXiv:1502.02114 [astro-ph.CO].
- [30] W.P. Thurston, Three dimensional manifolds, Kleinian groups and hyperbolic geometry, Bull. Am. Math. Soc. 6 (1982) 357, <http://dx.doi.org/10.1090/S0273-0979-1982-15003-0>.
- [31] Jeffrey Weeks, The Shape of Space, Marcel Dekker, New York, 1985, ISBN-13: 978-0824707095 and ISBN-10: 0824707095.