

$$\max_{u,v} \left(\frac{\sum_{o=1}^O u_o \times y_{o0}}{\sum_{i=1}^I v_i \times k_{i0}} \right)$$

$$\text{subject to: } \frac{\sum_{o=1}^O u_o \times y_{on}}{\sum_{i=1}^I v_i \times k_{in}} \leq 1 \quad n = 1, \dots, N$$

Where

- y_{o0} = quantity of output “o” for DMU₀
- u_o = weight attached to output o, $u_o > 0, o = 1, \dots, O$
- k_{i0} = quantity of input “i” for DMU₀
- v_i = weight attached to input i, $v_i > 0, i = 1, \dots, I$

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The equation is solved for each DMU iteratively (for $n=1, 2, \dots, N$); therefore, the weights that maximize the efficiency of one DMU might differ from the weights that maximize the efficiency of another DMU [17, 18]. Theoretically, these weights can assume any non-negative value, while the resulting technical efficiency scores vary only within a scale of 0 to 1, subject to the constraint that all other DMUs also have efficiencies between 0 and 1.

However, the ratio formulation expressed above leads to an infinite number of solutions, because if (u^*, v^*) is a solution, then $(\alpha u^*, \alpha v^*)$ is another solution [15, 17, 19, 20]. To avoid this problem, one can impose an additional constraint by setting either the denominator or the numerator of the ratio to be equal to 1 (for example, $v'x_j = 1$), which translates the problem to one of either maximising weighted output subjected to weighted input being equal to 1 or of minimising weighted input subjected to weighted output being equal to 1 [15, 21]. This would lead to the multiplier form of the equation as expressed as follows [15, 19, 20]:

$$\max_{\mu, v} (\mu' y_j),$$

Subject to:

$$v'x_j = 1$$

$$\mu' y_j - v'x_j \leq 0, j = 1, 2, \dots, J$$

$$\mu, v \geq 0$$

This maximization problem can also be expressed as an equivalent minimization problem [15, 19].

Technically, a DEA-based efficiency analysis can take either an input- or output-orientation. In an input-orientation, the primary objective is to minimize inputs, while in an output-orientation the goal is to attain the highest possible output with the given amounts of inputs. In our case, an output-oriented DEA model was deemed more appropriate on the premise that district health teams have essentially a fixed set of inputs to work with at any given time [3,5,6]. In other words, the district health system stewards would have more leverage in controlling outputs through innovative programming rather than raising additional resources.

As performance and institutional capacity are expected to vary across districts [4], a variable-returns-to-scale (VRS) approach was also considered more relevant to the study setting. This approach allows for economies and diseconomies of scale, rather than imposing the laws of direct proportionality in input-output relationships as espoused in the constant-returns to scale (CRS) [16-19]. A VRS model also offers the advantage of decomposing Overall Technical