



# Visual working memory and number sense: Testing the double deficit hypothesis in mathematics

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**Background.** Evidence exists that there are two main underlying cognitive factors in mathematical difficulties: working memory and number sense. It is suggested that real math difficulties appear when both working memory and number sense are weak, here referred to as the double deficit (DD) hypothesis.

**Aims.** The aim of this study was to test the DD hypothesis within a longitudinal time span of 2 years.

**Sample.** A total of 670 children participated. The mean age was 4.96 years at the start of the study and 7.02 years at the end of the study.

**Methods.** At the end of the first year of kindergarten, both visual–spatial working memory and number sense were measured by two different tasks. At the end of first grade, mathematical performance was measured with two tasks, one for math facts and one for math problems.

**Results.** Multiple regressions revealed that both visual working memory and symbolic number sense are predictors of mathematical performance in first grade. Symbolic number sense appears to be the strongest predictor for both math areas (math facts and math problems). Non-symbolic number sense only predicts performance in math problems. Multivariate analyses of variance showed that a combination of visual working memory and number sense deficits (NSDs) leads to the lowest performance on mathematics.

**Conclusions.** Our DD hypothesis was confirmed. Both visual working memory and symbolic number sense in kindergarten are related to mathematical performance 2 years later, and a combination of visual working memory and NSDs leads to low performance in mathematical performance.

Math difficulties have long-term negative consequences on children's school careers (e.g., Fletcher & Vaughn, 2009). Recent studies support multiple deficit neuropsychological models of mathematical difficulties and suggest that the difficulties occur due to neuropsychological weaknesses (Willcutt *et al.*, 2013). The double deficit (DD) hypothesis (Wolf & Bowers, 1999) has been used as a view on these neuropsychological weaknesses. However, studies focusing on double weaknesses regarding mathematical difficulties are scarce. Nevertheless, evidence exists that there are two main underlying cognitive factors in mathematical difficulties: working memory and number sense (e.g., Dehaene & Cohen, 1997; Geary, 2010; Passolunghi & Lanfranchi, 2012). However, few

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studies have examined these underlying factors in one model. Therefore, the present study aimed to test the DD hypothesis in mathematics within a longitudinal design.

### **Existing theories on early mathematical development**

Based on common theories on underlying factors in numerical and mathematical development, empirical studies have shown that there are, in addition to several other factors such as (oral) language and non-verbal reasoning, two main underlying cognitive factors in mathematical difficulties: working memory and number sense (e.g., Dehaene & Cohen, 1997; Geary, 2010; Passolunghi & Lanfranchi, 2012). Von Aster and Shalev (2007), for example, describe a four-step developmental model that enables predictions of possible neuropsychological dysfunctions for developmental dyscalculia. It postulates that the (inherited) core-system representation of cardinal magnitude (i.e., non-symbolic number sense; step 1) provides the basic meaning of number. They state the first step as a necessary precondition for children to learn to associate a perceived number of objects or events with spoken or, later, written and Arabic symbols (i.e., symbolic number sense). The process of linguistic (step 2) and Arabic (step 3) symbolization constitutes in turn a precondition for the development of a mental number line (i.e., ordinality; step 4). The authors state that this spatially oriented number line develops during elementary school and requires additional cognitive components including working memory. Recently, Geary (2013) distinguished in a comparable manner between mechanisms that facilitate children's early numeracy learning. These mechanisms may include an inherent sense of magnitude (i.e., non-symbolic number sense), fluent mapping of basic mathematical symbols onto this intuitive number sense (i.e., symbolic number sense), and the ability to explicitly operate on these symbols and understand the logical relations among them. Besides, Geary (2013) emphasizes the ability to maintain effortful attentional control, as measured by working memory tasks, as differentiating factor between children with and without problems in formal mathematics. LeFevre *et al.* (2010) also propose a model about longitudinal predictors of mathematics in which quantitative skills (i.e., number sense) and spatial attention (i.e., visual working memory) are, besides linguistic pathways, included as forms of knowledge that relate differentially to componential mathematical knowledge. Fuchs *et al.* (2012) likewise revealed linguistics (operationalized as oral language) as contributor to pre-algebraic knowledge, next to symbolic calculations and non-verbal reasoning.

### **Visual working memory and number sense**

Working memory, or the ability to store and manipulate information during a task (Baddeley, 1986), is considered important for early mathematical performance because incoming information must be stored (i.e., intermediate steps) and manipulated (i.e., used in calculations) during the dissolving of mathematical tasks (e.g., Passolunghi, Cargnelutti, & Pastore, 2014; Swanson, 2011; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). This idea is confirmed by several longitudinal studies (for a review, see Raghobar, Barnes, & Hecht, 2010). Both visual and verbal working memory types were repeatedly related to math achievement and development (e.g., Andersson & Lyxell, 2007; Krajewski & Schneider, 2009). In the present study, visual working memory was included, because this has consistently been found to be associated with mathematical performance (St Clair-Thompson & Gathercole, 2006). A possible explanation for the importance of visual working memory is that visual working memory might serve as a workspace for spatially

ordering numerical information while performing mathematical tasks. In young children especially, numbers and quantities are related to spatial information, because numerical information is represented spatially. Manipulating numbers thus requires a good functioning spatial workspace (Kolkman, Kroesbergen, & Leseman, 2014).

Although number sense is defined in different ways, there is overall agreement that number sense refers to an intuitive understanding of numbers, their magnitude, relationships, and how they are affected by operations (e.g., Gersten, Jordan, & Flojo, 2005). An important distinction in number sense is that between non-symbolic number sense (i.e., comparing magnitudes) and symbolic number sense (i.e., counting and Arabic symbols; cf. Desoete, Ceulemans, De Weerd, & Pieters, 2012; Kolkman, Kroesbergen, & Leseman, 2013). Non-symbolic number sense, or the understanding and ability to discriminate quantities (such as dot patterns), is thought to be based on a cognitive system dedicated to processing quantity information (Dehaene, 2001). Deficits in such a system are often related to serious problems in learning mathematics (Von Aster & Shalev, 2007). However, for the process of learning mathematics, it may be even more important to link such non-verbal representations to the corresponding verbal representations, such as in counting objects (Kolkman *et al.*, 2013). Therefore, the distinction between non-symbolic and symbolic numerical skills has recently been the subject of numerous studies on mathematical learning (e.g., Toll, Van Viersen, Kroesbergen, & Van Luit, 2015). Although it is not yet clear whether the non-symbolic skills, or the symbolic skills of number sense are most closely related to math learning difficulties (e.g., Sasanguie, De Smedt, Defever, & Reynvoet, 2011), there is no doubt that deficits in number sense form the basis of the majority of math learning difficulties (e.g., Mazzocco & Thompson, 2005) because manipulating numbers and the quantities they represent is the very basis of mathematics.

### **The DD hypothesis**

In research on developmental dyslexia, the DD hypothesis (Wolf & Bowers, 1999) has been used as a conceptualization in identifying children with reading impairment. Heikkilä, Torppa, Aro, Närhi, and Ahonen (2015) have extended the DD view from reading disabilities to math difficulties. It turns out that deficits in rapid automatized naming and phonological awareness were primarily linked to reading disabilities but not to math problems. Other recent studies suggest that both difficulties co-occur due to shared neuropsychological weaknesses in working memory, processing speed, and verbal comprehension (Willcutt *et al.*, 2013). As past research on mathematical performance has pointed to two main underlying cognitive factors – working memory (e.g., Rotzer *et al.*, 2009) and number sense (e.g., Landerl, Bevan, & Butterworth, 2004) – the DD hypothesis is better adapted to math achievement by selecting other predictors than those used in dyslexia research (Dehaene & Cohen, 1997). Träff (2013) examined the relative contributions of number sense and general cognitive abilities (i.e., working memory) in 10- to 13-year-olds and demonstrated, in accordance with the developmental model of mathematical learning (Fuchs *et al.*, 2010), that both constructs underlie different aspects of mathematical performance. Kroesbergen and Van Dijk (2015) studied the DD hypothesis in mathematics with both number sense and working memory in 154 primary school children, and their results confirm this hypothesis. They showed that both visual-spatial working memory and number sense have an almost equally important role in mathematical development. In their study, children with a DD scored the lowest. As this study (Kroesbergen & Van Dijk, 2015) is the first and only study that explicitly investigated the DD hypothesis for mathematics, replication of the results is necessary. Investigating

the DD in younger children would be valuable, as research especially showed the highest correlation between number sense and mathematical skills in kindergarten and first grade (e.g., Jordan, Glutting, & Ramineni, 2010). Whereas former studies are based on a one-time point, it would be useful to test the DD hypothesis in mathematical achievement from a longitudinal perspective as this would make it possible to learn more about correlations between early predictors and later mathematical performance. Because working memory is also related to number sense (Toll & Van Luit, 2014a), one could argue that early working memory deficits (WMDs) can cause both difficulties in number sense and math deficits later on, which then create a seemingly causal effect between number sense and mathematical performance. As the study focuses on the time period in which children are introduced to formal mathematics, a longitudinal study can yield more information about these relations. Hence, the aim of this study was testing the DD hypothesis within a time span of 2 years.

### **Present study**

The present study was based on the general idea that both visual working memory and number sense have been found to be involved as explanatory factors in mathematics and that, as a consequence, real math difficulties appear when both working memory and number sense are weak (known as the DD hypothesis). Applying the DD hypothesis to mathematical learning problems from a developmental perspective can provide useful insight into the cognitive factors involved in the development of mathematical skills. For the purpose of testing the DD hypothesis within a longitudinal time span of 2 years, groups of children were classified based on their performance (visual working memory and number sense) in the first year of kindergarten<sup>1</sup> at 5 years of age, and the mathematical proficiency of those groups in first grade at 7 years of age was compared. The main research questions, including the linked hypotheses, in this study are as follows:

- Question A: How are visual working memory and number sense at the start of kindergarten related to mathematical performance 2 years later? Hypothesis A: Both visual working memory and number sense are predictors for mathematical performance in the first grade.
- Question B: Can difficulties in visual working memory and/or difficulties in number sense at the start of kindergarten explain the individual differences between children in their mathematical performance in first grade? Hypothesis B1: Children with a weakness in either visual working memory or number sense have lower mathematical abilities than children without such a weakness. Hypothesis B2: Children with a weakness in both visual working memory and number sense have lower mathematical abilities than children with a weakness in only one of them.

## **Method**

### **Participants**

A total of 670 children, 337 boys (50.3%) and 333 girls (49.7%), participated in this study. Parental consent was obtained for all children. The children attended 23

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<sup>1</sup> In the Netherlands, children begin attending kindergarten when they reach the age of four. They attend, on average, 2 years of kindergarten before moving to first grade in September of the year in which they turn 6 years of age.

different Dutch primary schools. The schools represented different areas of the Netherlands. Data from two time points – end of the first year (out of two) of kindergarten and end of first grade (covering a time span of 2 years) – were filtered to answer the formulated research questions. The mean age of the children was 4.96 years ( $SD = 3.69$  months) at the first time point and 7.02 years ( $SD = 8.17$  months) at the second time point. The Raven's Coloured Progressive Matrices (RCPM; Raven, 1962), a non-verbal test for reasoning ability, was administered halfway through first grade. As the norms of this test are outdated and unreliable due to the Flynn effect, only raw scores are presented. The average raw score was 26.37 ( $SD = 4.32$ ;  $N = 670$ ; range: 14–36).

### **Procedure**

At the end of the first year of kindergarten, the children were screened on visual working memory and number sense. The two tests designed to measure visual working memory and the non-symbolic number sense test were administered on the computer. The symbolic number sense test was a pencil-and-paper test. All four tasks were administered by trained graduate students with degrees in education or psychology and were conducted in a fixed order. The children were tested individually in a quiet room within their school for 30 min.

Two years later, at the end of first grade, the children's mathematical performance was examined with two different tests. The math fluency test was administered individually by trained graduate students outside of the classroom. The math problems test was group administered by the teacher. In primary schools in the Netherlands, such assessment happens twice a year to monitor children's development.

### **Instruments**

#### *Visual-spatial working memory*

Visual working memory ability was measured with two computer-based tasks (Dot Matrix and Odd One Out) from the Automated Working Memory Assessment (AWMA; Alloway, 2007) which was translated and voice-recorded into Dutch. The AWMA has a stable construct validity and good diagnostic validity for children with low working memory skills. For children aged four and a half and 11 and a half years, the test-retest reliability of the two tasks was .81 and .74, respectively (Alloway, Gathercole, Kirkwood, & Elliott, 2008). Both working memory tasks started with a short practice session, and were automatically terminated when a child gave three incorrect answers within a set of items of the same length. Dot Matrix is a simple span measure, while Odd One Out measures complex span. In Dot Matrix, children were presented with a  $4 \times 4$  matrix on the computer screen. A red dot appeared shortly thereafter in one of the boxes and children had to point to the correct box. The test started with a block of one dot, building up to a block with a sequence of seven dots presented across the matrix. In Odd One Out, children were asked to point out the odd shape in a row of three geometrical shapes, and to remember the location of this shape. Then, three new shapes appeared. At the end of each trial, three empty boxes appeared, and the children were asked to point to the consecutive locations of the odd shapes. The test started with a set of one trial, building up to a block with a sequence of seven trials within a set. The scores on both tasks could range from 0 to 28. The scores on the two

tasks ( $r = .37, p < .01$ ) were standardized and merged (mean) into a single visual working memory score.

#### *Number sense*

Number sense was measured with a non-symbolic and a symbolic task. In the non-symbolic task, Dot Comparison, children were asked to compare two arrays with dots and had to indicate the array with the greatest number of dots. The dots varied not only in number but also in size (e.g., Gebuis, Cohen Kadosh, De Haan, & Henik, 2009). The total score was the number of items solved correctly from a total of 30. A further description of the task, as well as test reliabilities, can be found in Kolkman *et al.* (2013). The symbolic task consisted of twenty items from the Early Numeracy Test-Revised (ENT-R; Van Luit & Van de Rijt, 2009). The ENT-R is a standardized pencil-and-paper test for children between the ages of 4 and 7 years. The twenty items of version B were part of four components (using numerals, synchronized and shortened counting, resultative counting, and general understanding of numbers). The total of correct answers (0–20) was the final score. As the correlation between the two number sense scores appeared to be small ( $r = .25, p < .01$ ), no composite score was calculated, and the tasks were added as separate variables in subsequent analyses.

#### *Mathematical performance*

Mathematical performance was measured with the same tests as used in Kroesbergen and Van Dijk (2015) – a math facts and math problems test. Children's ability to reproduce math facts was tested with the first two columns of the Speeded Number Facts Test (Tempo Test Rekenen; De Vos, 1992). This is a numerical facility test which requires children to solve as many addition and subtraction fact problems (e.g.,  $2 + 4 = ?$ ) as possible within 1 min. The psychometric value has been demonstrated on a sample of 10,059 children (Ghesquière & Ruijsenaars, 1994). Children's ability to solve math problems was tested with the standardized national math test of Cito (Janssen, Scheltens, & Kraemer, 2005). This 'Cito' test is a national Dutch test with good psychometric properties that are commonly used in Dutch schools to monitor the progress of primary school children. For each grade, there are two level-appropriate tests – one is administered in January, the other in June (at the halfway point and end of the school year). The problems are presented in a booklet. Each problem comes with a picture that sometimes, but not always, contains necessary information. The test is administered groupwise by the teacher, who reads the problems out loud. An example of such problems is: 'The sign says: 10 chocolates for 3 euros only. How many euros do you have to pay for 20 chocolates?' The test contains 50 items that are administered on two separate days. Raw scores are converted into competence scores that increase throughout primary school, enabling a comparison of the results of different tests. Reliability of the first grade test has been computed on  $\alpha = .92$  (Janssen *et al.*, 2005).

#### **Data analyses**

Due to illness, absence, or technical issues, no scores on either the RCPM, the two working memory, or the two number sense tasks were available for 37 children (5.5%). Little's (1988) MCAR test indicated data were missing completely at random,  $\chi^2(33) = 32.35, p = .50$ . This means that the missing values pattern was non-informative,

meaning that the children who were absent during the tests did not introduce significant bias. Therefore, expectation maximization was used as a multiple imputation technique to manage the missing data (e.g., Schafer, 1997).

Prior to the main analyses, outlier analyses were conducted. No univariate outliers were found on math facts or math problems. Six outliers (four high scores and two low scores) for the simple span working memory measure, six outliers (all low scores) found for the non-symbolic number sense test, and eight outliers (all high scores) for the symbolic number sense test were reduced to a maximum of three standard deviations above or below the mean.

After providing the descriptive statistics, two steps in analyses were discerned. In the first step, correlation and multiple regression analyses were conducted to examine the relation between visual working memory, non-symbolic number sense, symbolic number sense, and mathematics. The multiple regression models were carried out using a mixed-model design with a multilevel approach in which a correction was made for the nested structure of the data (children nested into schools). All variables were included as standardized variables to enable a direct comparison of their predictive value. The variance accounted for by the predictors can be calculated by one minus the ratio of the residual variance ( $\sigma^2$ ) of the model and the residual variance of the intercept only model.

Prior to the second step, a group classification was performed resulting in four main groups. The four main groups are a no-deficit (ND) group, a WMD group, a number sense deficit group (NSD), and a DD group. As number sense is operationalized into non-symbolic and symbolic number sense, the NSD and DD group can be further divided into three subgroups: a group consisting of children with weaknesses in both non-symbolic and symbolic number sense and two groups with a weakness in either non-symbolic or symbolic number sense. It should be noted that at the age of five, relatively few children were found with a WM deficit only. Whereas a math performance comparison of the main groups can provide answers for the second research question, including the subgroups as well provides the opportunity to get a deeper insight in the role of non-symbolic and symbolic number sense in mathematical performance. Hence, analyses of variance were carried out twice to test group differences on math performance (math facts and math problems). Again, a mixed-model design with a multilevel approach in which a correction was made for the nested structure of the data (children nested into schools) was used. In the first analysis, the differences between the four main groups were examined; in the second analysis, the subgroups were examined additionally. In all the steps above, a correction was made for age within the analyses. IQ score was not selected as a covariate in the main analyses, but in alternative analyses reported in footnotes 2–7, because the children's IQ score was assessed one and a half years later (halfway through first grade) than their visual working memory and number sense skills.

## Results

### *Descriptive statistics, correlations, and regressions*

In Table 1, the descriptive statistics on the five variables are presented. In Table 2, bivariate and partial correlations between the separate tasks, corrected for age, are presented. Significant correlations were found between visual working memory and the two math tasks and between (non-)symbolic number sense and the two math tasks.

**Table 1.** Descriptive statistics for working memory, (non-)symbolic number sense, and mathematics

Task	N	M	SD	Min	Max
Visual WM <sup>a</sup>	670	-0.00	0.82	-2.17	2.78
Non-symbolic NS	670	24.27	4.25	12	30
Symbolic NS	670	6.18	3.01	0	15
Math facts	670	22.11	7.24	3	43
Math problems	670	45.43	15.34	0	88

Note. <sup>a</sup>Combined standardized score. WM = working memory, NS = number sense.

For math facts, the correlations vary between .15 (small; non-symbolic number sense) and .48 (medium; symbolic number sense). For math problems, the correlations vary between .24 (small; non-symbolic number sense) and .50 (large; symbolic number sense).<sup>2</sup>

In Table 3, the results of two mixed-model analyses are presented. In these models, a correction was made for the nested structure of the data (level 1: individual students; level 2: schools). In the first analysis, math facts were the dependent variable, whereas in the second analysis, math problems were the dependent variable. To test the relative effect of visual working memory and (non-)symbolic number sense, the three predictors were entered together. Age was also included as predictor of math performance.<sup>3</sup> In the first analysis, it appeared that age, visual working memory, and symbolic number sense were significant predictors of children's scores on math facts. Non-symbolic number sense did not predict children's scores on math facts. The variance in math facts accounted for by the four predictors is 24% (1-0.67/0.89). In the second analysis, the four variables were each a significant predictor of children's scores on math problems. The variance in math problems accounted for by the four predictors was 28% (1-0.65/0.90). In both analyses, symbolic number sense appeared to have more explaining value (Math facts:  $B = .40$ ; Math problems:  $B = .42$ ) than non-symbolic number sense (Math facts:  $B = .02$ ; Math problems:  $B = .11$ ) or visual working memory (Math facts:  $B = .15$ ; Math problems:  $B = .12$ ).

### Multivariate analyses of variance

In favour of the second research question, a classification was made based on the combined score for visual working memory, along with the non-symbolic and symbolic number sense score at the first time point. For each selection measure, a division was made between the 25% lowest scoring children and the 75% children scoring above the 25th percentile. The cut-off criterion of 25% was used firstly to create appropriate group sizes, and secondly because this criterion is often used to indicate low-achieving children or those at risk or having difficulties in mathematics (e.g., Stock, Desoete, & Roeyers, 2010; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011). The division was made as close

<sup>2</sup> Partial correlations corrected for both age and IQ score reveal similar results. For math facts, they vary between .10 (small; non-symbolic number sense) and .43 (medium; symbolic number sense). For math problems, they vary between .18 (small; visual working memory and non-symbolic number sense) and .41 (medium; symbolic number sense).

<sup>3</sup> The alternative analyses (correcting for age and IQ score) reveal a similar outcome with regard to the four predictors in both analyses and reveal IQ score as a significant predictor for math facts (Est. = .11, SE = 0.03,  $t(657.99) = 3.12$ ,  $p < .01$ ) and for math problems (Est. = .31, SE = 0.03,  $t(656.85) = 9.44$ ,  $p < .01$ ).



**Table 2.** Correlations between age, working memory, (non-)symbolic number sense, and mathematics

	Visual WM	Number sense		Basic math	
		Non-symbolic	Symbolic	Math facts	Math problems
Age	.18**	.10*	.24**	.07	.07
Visual WM		.21**	.44**	.32**	.29**
NS Non-symbolic	.19**		.25**	.15**	.24**
NS Symbolic	.41**	.24**		.48**	.50**
Math facts	.31**	.15**	.48**		.54**
Math problems	.28**	.24**	.50**	.53**	

Note. WM = working memory, NS = number sense. Bivariate correlations above diagonal; partial correlations corrected for age below diagonal. \* $p < .05$ ; \*\* $p < .01$ .

**Table 3.** Results of mixed-model analyses for math facts and math problems

	Est.	SE	df	t	p	$\sigma^2$	Wald Z
Math facts						0.67	17.98**
Intercept	.01	0.07	21.54	0.09	.93		2.47*
Age	-.07	0.03	667.67	-2.09	.04		
Visual WM <sup>a</sup>	.15	0.04	667.38	4.00	.00		
Non-symbolic NS	.02	0.03	662.49	0.69	.49		
Symbolic NS	.40	0.04	667.75	10.70	.00		
Math problems						0.65	17.94**
Intercept	-.05	0.07	19.06	-0.68	.51		2.42*
Age	-.06	0.03	665.37	-1.78	.08		
Visual WM <sup>a</sup>	.12	0.04	665.04	3.36	.00		
Non-symbolic NS	.11	0.03	659.52	3.39	.00		
Symbolic NS	.42	0.04	665.75	11.35	.00		

Note. All variables are standardized. WM = working memory, NS = number sense. <sup>a</sup>Combined standardized score. \* $p < .05$ ; \*\* $p < .01$ , two-sided tested.

as possible to a 25–75% division (visual WM: 25.2–74.8%; non-symbolic NS: 26.6–73.4%; symbolic NS: 30.4–69.6%). The children were classified into four different groups: a ND group with 300 children (44.8%) thrice belonging to the children scoring above the 25th percentile, a WMD group consisting of 69 children (10.3%) with a visual working memory score below the 25th percentile, a NSD group consisting of 201 children (30.0%) with scores below the 25th percentile on either or both of the number sense tasks, and a DD group with 100 children (14.9%) belonging to the low-scoring children (<25th percentile) on visual working memory and at least one of the number sense tasks. Within the NSD and the DD group, three subgroups can be distinguished: children with weaknesses in both non-symbolic and symbolic number sense (scoring below the 25th percentile on both tasks), children with a weakness in non-symbolic number sense only (scoring below the 25th percentile on that task), and children with a weakness in symbolic number sense only (scoring below the 25th percentile on that task).

Descriptive information pertaining to the four main groups, including the subgroups, is presented in Table 4. Preliminary analyses were performed to test whether the main

**Table 4.** Description and scores of four groups: number of children, gender, age, non-verbal iq score, visual working memory, (non-)symbolic number sense, and mathematics

Group	Gender				End year 1						End year 3							
	Boys		Girls		Age in months		Non-verbal IQ score		Visual WM		Non-symbolic NS		Symbolic NS		Math facts		Math problems	
	N	%	N	%	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
No deficit	300	56	144	48.0	60.25	3.45	27.55	3.90	0.45	0.65	26.66	1.87	7.98	2.70	24.14	7.43	50.66	14.45
WM deficit	69	38	55.1	31	44.9	59.04	3.55	26.30	4.16	-0.94	0.32	1.96	6.68	1.71	21.45	6.72	45.94	14.07
NS deficit	201	95	47.3	106	52.7	59.03	3.76	25.88	4.44	0.17	0.58	4.65	4.65	2.47	21.27	6.63	41.86	14.09
Both	41	18	43.9	23	56.1	58.95	3.19	24.68	4.34	0.02	0.59	19.05	2.90	3.05	18.37	6.88	32.10	12.99
Non-symbolic	74	35	47.3	39	52.7	60.05	3.79	26.55	4.36	0.35	0.68	18.05	2.99	7.20	23.77	7.08	47.54	12.87
Symbolic	86	42	48.8	44	51.2	58.19	3.81	25.89	4.49	0.09	0.44	26.21	1.95	3.20	20.51	5.30	41.63	13.10
Double deficit	100	48	48.0	52	52.0	58.23	3.87	23.89	4.15	-1.04	0.35	20.74	4.39	3.50	18.19	6.17	36.58	15.32
Both	40	16	40.0	24	60.0	58.35	4.17	23.15	3.64	-1.07	0.36	18.20	3.43	2.60	16.45	5.62	31.35	14.60
Non-symbolic	23	12	52.2	11	47.8	58.87	3.40	25.35	4.74	-0.98	0.41	18.09	3.01	5.91	20.26	6.40	44.30	14.14
Symbolic	37	20	54.1	17	45.9	57.70	3.84	23.78	4.17	-1.06	0.30	25.14	1.75	2.97	18.78	6.23	37.43	14.93

Note. WM = working memory, NS = number sense.

groups differ from each other on background measures (gender, age and IQ score). In addition to  $p$  values, effect sizes are reported using the partial eta-squared ( $\eta^2$ ). The critical values for the effect sizes ( $\eta^2$ ) are .01 for a small effect, .06 for a medium effect, and .14 for a large effect (Cohen, 1988). There was no association between gender and group classification for the main groups,  $\chi^2(3, N = 670) = 1.93, p = .59$ , but there were significant age differences,  $F(3, 666) = 9.83, p < .01, \eta^2 = .04$  [small effect], and significant differences in IQ score,  $F(3, 666) = 21.08, p < .01, \eta^2 = .09$  [medium effect], between the four groups. Children in the ND group are slightly older than the children in the NSD group (mean difference = 1.22,  $SE = 0.33, p < .01$ ) and children in the DD group (mean difference = 2.02,  $SE = 0.42, p < .01$ ). Children in the ND group have on average a slightly higher IQ score than the children in the NSD group (mean difference = 1.67,  $SE = 0.38, p < .01$ ) and children in the DD group (mean difference = 3.66,  $SE = 0.47, p < .01$ ). Age will be added as a covariate in the subsequent analyses, and IQ score will be added as covariate as well in the alternative analyses (reported in footnotes). In Table 4, furthermore, the scores of the four groups on visual working memory, (non-)symbolic number sense, and math performance are presented.

Two sets of multilevel analyses of variance were conducted. In the first set, the four main groups were included. In the second set, the subgroups were also included for a total of eight groups. The results in both sets are corrected for age (covariate). The results of the first set show significant differences between the four main groups, when corrected for age,<sup>4</sup> for math facts,  $F(3, 661.52) = 20.26, p < .01$  and math problems,  $F(3, 659.23) = 31.23, p < .01$ . The results of the *post-hoc* tests, with Bonferroni correction for experiment-rate errors, are presented in the upper part of Table 5.<sup>5</sup> The children in the ND group outperform the children in the other three groups on both measures. The children in the WMD group and the NSD group show similar performance on both measures, but both score better than the children in the DD group.

The results of the second set show significant differences between the eight groups (including subgroups), when corrected for age,<sup>6</sup> for math facts,  $F(7, 660.06) = 11.82, p < .01$ , and math problems,  $F(7, 658.07) = 19.85, p < .01$ . The results of the *post-hoc* tests, with Bonferroni correction for experiment-rate errors, are presented in the lower part of Table 5.<sup>7</sup> To improve clarity, only the significant contrasts are shown. Four results are worth highlighting. First, the children in the ND group outperform the children in all other groups, except for the children with a weakness in non-symbolic number sense only (math facts and math problems) and children with a DD, but a weakness in non-symbolic number sense only (math problems). Second, the children in the WMD group only perform better than the DD group in which the children were weak on both non-symbolic and symbolic number sense (math facts and math problems) and the NSD group in which the children were weak on both non-symbolic and symbolic number sense (math problems). Third, the children with weaknesses in both non-symbolic and symbolic

<sup>4</sup> When corrected for IQ score as well, significant differences were revealed for math facts,  $F(3, 658.27) = 20.66, p < .01$ , and for math problems,  $F(3, 656.22) = 15.91, p < .01$ .

<sup>5</sup> The alternative analyses (corrected for age and IQ score) reveal different significant levels for (1) the ND group and WMD group on math problems and (2) the NSD group and DD group on math problems.

<sup>6</sup> When corrected for IQ score as well, significant differences were revealed for math facts,  $F(7, 650.71) = 7.80, p < .01$ , and for math problems,  $F(7, 650.58) = 11.85, p < .01$ .

<sup>7</sup> The alternative analyses (corrected for age and IQ score) reveal different significant levels for (1) the ND group and WMD group on both measures, (2) the ND group and DD non-symbolic group on math facts, (3) the WMD group and the DD both group on math facts, (4) the NSD non-symbolic group and the DD symbolic group on both measures, and (5) the NSD symbolic group and the DD both group on math facts.

**Table 5.** Post-hoc differences between the main groups on mathematics

	Math facts		Math problems	
	MD (SE)	<i>p</i>	MD (SE)	<i>p</i>
Contrasts of main groups				
ND versus WMD	2.86 (0.89)	<.01	6.32 (1.86)	<.01
ND versus NSD	2.53 (0.61)	<.01	8.58 (1.29)	<.01
ND versus DD	5.84 (0.78)	<.01	14.30 (1.63)	<.01
WMD versus NSD	-0.32 (0.93)	1.00	2.26 (1.95)	1.00
WMD versus DD	2.99 (1.04)	.03	7.98 (2.17)	<.01
NSD versus DD	3.31 (0.82)	<.01	5.72 (1.71)	<.01
Contrasts including subgroups				
ND versus WMD	2.92 (0.88)	.04	6.16 (1.81)	.02
ND versus NSD both	5.25 (1.09)	<.01	18.36 (2.26)	<.01
ND versus NSD symbolic	3.15 (0.82)	<.01	8.47 (1.68)	<.01
ND versus DD both	7.59 (1.11)	<.01	18.32 (2.29)	<.01
ND versus DD non-symbolic	4.63 (1.41)	0.03		(ns)
ND versus DD symbolic	4.88 (1.15)	<.01	14.01 (2.37)	<.01
WMD versus NSD both		(ns)	12.20 (2.69)	<.01
WMD versus DD both	4.78 (1.31)	<.01	12.17 (2.70)	<.01
NSD both versus NSD non-symbolic	-4.88 (1.27)	<.01	-14.86 (2.62)	<.01
NSD both versus NSD symbolic		(ns)	-9.89 (2.56)	<.01
NSD non-symbolic versus DD both	7.21 (1.30)	<.01	14.83 (2.68)	<.01
NSD non-symbolic versus DD symbolic	4.50 (1.33)	.02	10.52 (2.74)	<.01
NSD symbolic versus DD both	4.45 (1.25)	.01	9.86 (2.59)	<.01

Note. MD = mean difference, ND = no deficit, WMD = working memory deficit, NSD = number sense deficit, DD = double deficit, (ns) = not significant group difference.

number sense perform lower than children with a non-symbolic number sense weakness only (math facts and math problems) or a symbolic number sense weakness only (math problems). Fourth, the children in the non-symbolic DD group do not perform significantly better than the other DD groups, but do, in contrast to the symbolic DD and both DD groups, perform similar to all NSD groups.

## Discussion and conclusion

The aim of the present study was to test the DD hypothesis for mathematical achievement within a longitudinal time span of 2 years. The results provide answers for the two main research questions and enable decisions for the three formulated hypotheses.

Based on the results, hypothesis A – both visual working memory and number sense are predictors of mathematical performance in first grade – will be partly accepted. However, non-symbolic number sense showed no predictive relation to math facts. Symbolic number sense appears to be the strongest predictor for both math areas (math facts and math problems). With respect to question A, this means that both visual working memory and symbolic number sense at the start of kindergarten are related to mathematical performance 2 years later. Non-symbolic number sense at the start of kindergarten turned out to be related to the skill of mathematical problem solving, but not to the skill of retrieving mathematical facts at the end of first grade. So, whether

non-symbolic number sense is also a significant predictor, in addition to symbolic number sense and visual working memory, depends on the specific math area. Thus, the amount and type of contribution to mathematical achievement varies between the different aspects of arithmetic. This result is congruent with previous research (e.g., Fuchs *et al.*, 2010; Träff, 2013) which states that the development of different types of formal school mathematics depends on diverse constellations of numerical versus general cognitive abilities, and is congruent with studies that found a relation between more symbolic math skills and basic math performance in 6- to 8-year-old children (e.g., Sasanguie *et al.*, 2011). De Smedt, Noël, Gilmore, and Ansari (2013) provided an integrative review of the existing data that has addressed the question of how different components of numerical cognition (i.e., number sense) relate to mathematical skills and conclude that results are consistent across studies for the symbolic comparison skills, but rather contradictory for the non-symbolic comparison skills. The results of the present study, also, show contradictory results for the non-symbolic number sense measure, suggesting that those skills become less important as children are developing their mathematical symbolic skills during education (e.g., Kolkman *et al.*, 2013).

With respect to question B, two hypotheses were formulated, B1 and B2. Both hypotheses B will be accepted for both math areas (math facts and math problems). Children with a weakness in either visual working memory or (non-)symbolic number sense have lower mathematical abilities than children without such a weakness (Hypothesis B1). The ND group outperforms the other three groups on both math problems and math facts. Children with a weakness in both visual working memory and (non-)symbolic number sense have lower performance than children with a weakness in number sense or in visual working memory (Hypothesis B2). This means that, in line with the results of Kroesbergen and Van Dijk (2015), a combination of visual working memory and (non-)symbolic NSDs leads to low performance in mathematics and confirms the important role of both visual-spatial working memory and (non-)symbolic number sense in mathematical development. Problems with both visual working memory and (non-)symbolic number sense are sources of mathematical difficulties, and their combined presence leads to the lowest performance.

Because of the fact that number sense was split into non-symbolic and symbolic, a closer examination of the results became possible by classifying subgroups of children who fail in one or both aspects of number sense. This closer examination shows, among other things, that children with a weakness in non-symbolic number sense only, or in combination with a visual WMD, are less disadvantaged than their peers with symbolic number sense weaknesses or weaknesses in both non-symbolic and symbolic number sense. This means that math performance appeals more to symbolic number sense abilities than it does to non-symbolic comparison skills. This supports the results of studies which state that symbolic numerical skills are more important than non-symbolic skills as prerequisites of mathematical performance (Holloway & Ansari, 2009; Kolkman *et al.*, 2013; Toll & Van Luit, 2014a; Toll *et al.*, 2015).

The present study followed a large sample of children during a time span of 2 years. Despite these two advantages, there are three limitations that need to be taken into account when interpreting and generalizing the results. First, preliminary analyses revealed age and IQ score differences among the four groups. Therefore, in almost all analyses, a correction was made for age but not for IQ score. IQ score was not selected as a covariate in the main analyses, but in alternative analyses reported in footnotes, because the children's IQ score was assessed one and a half years later (halfway through first grade) than their visual working memory and number sense skills. This assessment was close to

the math measures at the end of first grade and was, because of the time frame and the high correlation between working memory and non-verbal IQ scores (Alloway & Passolunghi, 2011), likely to interfere with the predictive relation between the kindergarten measures and the outcome measures. The alternative analyses revealed similar results, but did indeed show, among other things, that the math performance of children with WMDs did not differ from children with NDs. Second, based on evidence from previous research (e.g., St Clair-Thompson & Gathercole, 2006) the focus of the present study was only on visual working memory. It would have made the study more comprehensive to also include verbal working memory, because verbal working memory has been found to have a unique association with mathematical skills in some studies (Jarvis & Gathercole, 2003), but not in others (St Clair-Thompson & Gathercole, 2006). Third, the variance accounted for in the regression models leaves room for other explaining variables that were not included in the present study. Therefore, next to verbal working memory, factors such as executive functions (Espy *et al.*, 2004) or language (Toll & Van Luit, 2014b) could also have been included for a multiple deficit instead of a DD approach.

To summarize, the present study contributes to the knowledge on visual working memory and (non-)symbolic number sense as long-term predictors of mathematical performance in early primary school. The results imply that both visual working memory and number sense, especially symbolic, could be useful in showing a possible reason for low math performance, and raise therewith implications for educational practices in kindergarten. Visual working memory and (symbolic) number sense are good predictive measures for already identifying children at risk for low performance in mathematics at the age of five. Besides including those skills in screening batteries in kindergarten, specific attention to the stimulation of those factors within the curriculum or through remedial programmes is advisable. Further research is necessary to investigate whether children with double weaknesses could be helped by stimulation of their early mathematical skills within an educational setting.

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