ICME 13
Hamburg 2016

Reflections from inside on the Netherlands Didactic Tradition in Mathematics Education

Abstracts

Reflections from inside on the Netherlands didactic tradition in mathematics education

The Netherlands contribution to the ICME 13 Thematic Afternoon on 'European Didactic Traditions' is prepared by a committee consisting of

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Colophon

Reflections from inside on the Netherlands didactic tradition in mathematics education

Faculty of Science, Utrecht University
Printed by: All Print, Utrecht
Editorial assistant: Nathalie Kuijpers
Cover Design: Liza Koppen

Introduction

As part of the Netherlands strand of the ICME 13 Thematic Afternoon on ‘European Didactic Traditions’ two volumes are being prepared to be published within the Springer book series with volumes coming out of ICME 13. This booklet contains the abstracts of the chapters to be included in these two volumes. In the volume titled **Reflections from inside on the Netherlands didactic tradition** in mathematics education 30 authors, who have all their background in the Netherlands, reflect in 17 chapters on their experiences and views on teaching mathematics, educating mathematics teachers, being involved in curriculum development and test design, and doing research in mathematics education. The volume shows the Netherlands didactic tradition in mathematics education from the different perspectives of authors from various affiliations. Although many chapters address aspects of the domain-specific instruction theory of Realistic Mathematics Education (RME) and particular conceptualizations of it, the scope of the volume is broader than only RME. Among other things, the volume also tells the story of what in the Netherlands is thought to be important to teach students, how this has changed over the years and resulted in different types of textbooks. The volume does not have boundaries between school levels and covers mathematics education for students of all ages. Similarly, describing the tradition means involving the whole range of current, past, and future developments in mathematics education, though all in an exemplary way. Some chapters zoom in on particular mathematical competences such as thinking skills, on mathematical subdomains such as calculus and geometry, or on issues like the role of contexts and common sense, and the challenges of the use of digital tools, but the relation with mathematicians is discussed as well. A special place in the volume is taken by he Freudenthal Institute and its predessessors that were the successive epicenters of the driving force of the reform movement on mathematics education in the Netherlands.

Marja van den Heuvel-Panhuizen
Editor

Utrecht, July 2016

Mathematics for teams – Developing thinking skills in mathematics education

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Mathematics is more than just basic skills. Mathematical thinking should be an important aspect of mathematics education. In the Netherlands higher-order thinking skills like mathematical problem solving, reasoning, modelling and communicating mathematics have been part of the examination program since 1989. To assess these skills in an authentic and open way, the Mathematics A-lympiad, a contest for teams in upper secondary school, was designed. As a result of its success, similar activities have been created for lower secondary and for primary school. The Mathematics A-lympiad assignments fulfil specific requirements, such as being accessible for all students, eliciting mathematical thinking and providing opportunity for different strategies and solutions. In our chapter we will discuss how these assignments can be an instrument to implement these higher-order thinking skills in regular mathematics education as well.

Task contexts in Dutch mathematics education

Pauline Vos

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International comparative studies show that the Dutch use many mathematics tasks with real-life connections in their mathematics education. This phenomenon can already be observed in the earliest Dutch mathematics textbooks, which consisted entirely of tasks with real-life contexts. These tasks reflect an emphasis on usefulness which can be traced back to traits of Calvinism in the Dutch culture. In my chapter characteristics of contexts in recent mathematics tasks in the Netherlands are studied. Underlying frame is the notion of usefulness as a perception of students on future practices outside school. I distinguish: bare tasks (without contexts), tasks with mathematical contexts (e.g. pattern problems), dressed-up tasks (pseudo-realistic tasks with silly questions that hide a mathematical question), tasks with realistic contexts and questions that are meaningful within the context, and tasks with authentic contexts. I analyzed a mathematics textbook chapter and a sample of recent examination papers, which both show that Dutch mathematics education contains many links to real life, presented verbally and visually. The contexts are drawn from a wide spectrum of areas, reflecting that mathematics can be found anywhere in society. Task contexts often come from recreational or professional practices, demonstrating to students the usefulness of mathematics in their future lives beyond school.

Mathematics and common sense – The Dutch school

Rijkje Dekker

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The old illusion in Dutch mathematics education was that the teacher could lead the students into a completely new world, ignoring all their prior knowledge and common sense. Nowadays, in many Dutch mathematics lessons, the teachers encourage their students to use their common sense. This is the result of a silent revolution in mathematics education. In my chapter I will offer a collage of the work of several mathematics educators, who have helped to put the common sense of students in the middle of Dutch mathematics education. We will meet students from age 6 till 16 working with whole numbers, fractions, geometry and exponential functions and we will discover how their common sense plays a crucial role in the development of their mathematical knowledge.

Dutch mathematicians and mathematics education – A problematic relation

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Delft University of Technology, the Netherlands

Mathematics as a compulsory school topic was introduced in the Netherlands in the first decades of the 19th century. While in the beginning there was some involvement of Dutch academic mathematicians, later on their engagement with mathematics teaching was only marginal. That changed in the second half of the 20th century. Hans Freudenthal, professor of mathematics in Utrecht, became deeply involved in mathematics teaching. He became the first director of the IOWO, the institute that dominated Dutch mathematics teaching from the seventies on. In the sixties, under the influence of New Math, other mathematicians had already played a role in modernizing mathematics, but from the seventies on, their role became minimal again. In the first decade of the 21th century the dominance of the ideas of Realistic Mathematics Education drew protests from mathematics departments at several universities. This criticism induced fierce and often heated debates. At the moment, these discussions have calmed down and it seems that a new understanding between the worlds of school and university mathematics is growing.

Dutch didactical approaches in primary school mathematics as reflected in two centuries of textbooks – Procedural, conceptual and dual textbooks

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Our chapter contains an overview of the most important textbook series used in the Netherlands from 1800 to 2015. We distinguish five time periods, and for each period we highlight the textbook series that are most characteristic. Three categories are distinguished in describing the textbooks that were in fashion in the successive periods: procedural, conceptual, and dual textbooks; this third category has elements of the first two. For the procedural textbook series, which are also referred to as 'mechanistic', memorization of mathematical facts, automatization of operational procedures and recognizing types of problems are the primary interest. Application is only considered at the very end of the teaching trajectory, and then rarely. Smart, flexible (mental) calculations and estimating are not part of the program. The conceptual textbook series have an opposite approach. In learning mathematical facts and procedures, understanding is highly valued and applications are included from the start as the basis for this. Number sense, flexible (mental) calculation, and estimation are central, next to algorithmic calculation. Students can design their own problems, develop solution methods and work on their own level. As expected, using different textbook series with different content and teaching methods results in pursuing different goals in mathematics education, which in turn results in different learning outcomes, as has been shown by national evaluations of progress in educational achievement.

Sixteenth century reckoners versus twenty-first century problem solvers

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In the sixteenth century a new way of doing arithmetic came into use in the Netherlands. Written arithmetic with Hindu-Arabic numbers replaced the traditional arithmetic method using coins on a counting board. Manuscripts and books in the vernacular taught the new method to future merchants, moneychangers, bankers, bookkeepers, etcetera, who wanted to learn recipes to solve the arithmetical problems of their future profession. They wanted to know what to do in a limited amount of practical arithmetical and financial situations. Learning mathematics was not an issue. If they would come across a new mathematical problem they would not know what to do, but they probably never met new mathematical problems! Five centuries later we want to teach our students mathematical skills to survive in a computerized and globalized society. They still need knowledge of number relations and arithmetical rules, but have to learn to apply this knowledge flexibly and meaningfully to solve new problems (approximately), to mathematise situations, and to evaluate, interpret and check output of computers and calculators. The 21^{st} century needs problem solvers, but to acquire the skills of a good problem solver a firm knowledge base – comparable with that of the sixteenth century reckoner – is still necessary.

A short history of the mathematics curriculum in primary education in the Netherlands

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What in many countries is quite common, namely that the government to a certain degree prescribes what has to be taught in primary school mathematics education, is in the Netherlands only recently the case. From 1806 on, when in the first educational law it was established that all primary schools had to teach mathematics, it took until 1993 before the Dutch government launched a formal intended primary school mathematics curriculum. Yet this only consisted of so-called 'Core goals,' which were described globally. The restraint of the Dutch government in providing educational prescriptions arises from the constitutional freedom of education. In 2010 this situation changed somewhat when the so-called 'Reference standards,' which prescribe in more detail what has to be taught in school, were formulated. Parallel to this history of the intended curriculum, there is also the history of the potentially implemented curriculum as laid down in textbooks, which in the Netherlands are of decisive influence on the enacted curriculum. In our chapter we shed light on changes in the content and performance expectations of the Dutch primary school mathematics curriculum over the last century.

A socio-constructivist elaboration of Realistic Mathematics Education

Koeno Gravemeijer

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In my chapter I describe a socio-constructivist elaboration of Realistic Mathematics Education (RME) that emerged from my collaboration with Paul Cobb and Erna Yackel. It is argued that RME and socio-constructivism are compatible and complement each other. Socio-constructivism points to the critical role of the classroom culture, while RME offers a theory on supporting students in (re-)constructing mathematics. Furthermore, the role of symbols and models is discussed, which was considered problematic in constructivist circles, while being central in RME. The emergent modelling design heuristic is presented as a solution to this puzzle. Together, guided reinvention, didactical phenomenology, and emergent modelling, are combined to delineate RME as an instructional design theory. This is complemented by a discussion of pedagogical content tools as counter parts of the emergent modelling and guided reinvention design heuristics at the level of classroom instruction. Finally, research on student learning and enactment of RME in Dutch classrooms is discussed.

Mathematics & Didactics as a subject in primary school teacher education in the Netherlands

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From the 1970s, curricula of primary mathematics teacher education in the Netherlands drastically changed. This first occurred simultaneously with the changes in primary mathematics education. Teacher educators systematically discussed mathematics teacher education and implemented new content and new approaches in primary teacher education. Our chapter provides a chronological overview of how Dutch primary mathematics teacher education developed from the 70s until the present. We describe ideas about learning to teach mathematics and ideas about the relationship between the development of mathematical literacy and didactical proficiency of student teachers. Furthermore, the influence of national measures such as the introduction of nationwide tests for primary mathematics teacher education is discussed. The chapter ends with an impression of recent learning materials for student teachers and a reflection on new perspectives for integrating theory and practice, emphasizing the continuous search for a well-balanced way to interconnect mathematics and didactics.

Secondary mathematics teacher education in the Netherlands

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In our chapter, we discuss the education of mathematics teachers in the Netherlands. There are different routes for qualifying as a teacher. These routes target different student teacher populations, ranging from those who have just graduated from high school to those who have already pursued a career outside education or working teachers who want to qualify for teaching in higher grades. After discussing the complex structure this leads to, we focus on the aspects that these different routes have in common. We point out typical characteristics of Dutch school mathematics and discuss the aims and challenges in teacher education that result from this. We give examples of different methods used in Dutch teacher education, which we link to a particular model for designing vocational and professional learning environments. We end the chapter with a reflection on the current situation.

Digital tools in Dutch mathematics education – A dialectic relationship

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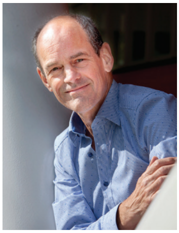

Nowadays, digital tools for mathematics education are sophisticated and widely available. These tools offer important opportunities, but also come with constraints. Some tools are hard to tailor by teachers, educational designers and researchers; their functionality has to be taken for granted. Other tools offer many possible educational applications, which require didactical choices. In both cases, one may experience a tension between a teacher's didactical goals and the tool's affordances. From the perspective of Realistic Mathematics Education, this challenge concerns both guided reinvention and didactical phenomenology. In my chapter, this dialectic relationship will be addressed through the description of two particular cases of using digital tools in Dutch mathematics education: the introduction of the graphing calculator, and the evolution of the online Digital Mathematics Environment. From these two case descriptions, it is concluded that students need to develop new techniques for using digital tools; techniques that interact with conceptual understanding. For teachers, it is important to be able to tailor the digital tool to their didactical intentions. From the perspective of Realistic Mathematics Education, I conclude that its match with using digital technology is not self-evident. Guided reinvention may be challenged by the rigid character of the tools, and the phenomena that form the point of departure of the learning of mathematics may change in a technology-rich classroom.

A short history of the development of calculus in Dutch secondary education

Martin Kindt

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Compared to neighboring countries, an effective nationwide introduction of calculus in secondary education took place rather late in the Netherlands, in 1958. This happened after much discussion between advocates of the teaching of calculus in school and their opponents in the first half of the 20^{th} century. After 1960 there has been an acceleration in curriculum changes. First the curriculum was influenced by the New Math movement. Then, in reaction to that, came the influence of Realistic Mathematics Education. This resulted in the introduction of new mathematics programs in upper secondary education. Actually, one might say: Every decade there was a revision phase of the curriculum.

Ensuring usability – Reflections on a Dutch mathematics reform for students aged 12-16

Kees Hoogland

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In my chapter I look back at the implementation of W12-16, a major reform of mathematics education in lower and pre-vocational secondary education in the Netherlands. W12-16 stands for 'Wiskunde 12-16' [Mathematics 12-16] and involved the implementation of this reform for all students aged 12-16. The nationwide implementation of W12-16 started in 1990 and envisioned a major change in what and how mathematics was taught and learned. The content was widened from algebra and geometry to algebra, geometry and measurement, numeracy, and data processing and statistics. The learning trajectories and the instruction theory were based on the ideas of Realistic Mathematics Education: the primary processes used in the classroom were to be guided re-invention and problem solving. In the chapter's title, 'Ensuring usability' refers to the aim of the content being useful and understandable for all students, but also to the involvement of all relevant stakeholders in the implementation project, including teachers, students, parents, editors, curriculum and assessment developers, teacher educators, publishers, media and policy makers. In my chapter I reflect on the current state of affairs more than 20 years after the nationwide introduction. The main questions to be asked are: Have the goals been reached? Was the implementation successful?

Eighteenth century land surveying as a context for learning similar triangles and measurement

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This study investigates the value and applicability of the history of mathematics as a didactical tool for teaching mathematics. Recent literature has disclosed conceptual, cultural, and motivational arguments for including historical mathematical texts and methods in the mathematics curriculum. In our chapter we explore how these theoretical assumptions work out when designing historically-based instructional material and when using this material for teaching. The focus is on teaching measurement skills and the application of similar triangles to eight- and ninth-grade students. The profession of the Dutch land surveyor in the 18th century served as a historical context. Analyses of the data indicate that several aspects of this historical context are helpful for teaching these subjects. The practical activities along the 18th century lines appeared to have a positive effect on the students' motivation and on their conceptual understanding. The ninth graders reacted more positively to the historically inspired text than the eighth graders. The integration of historical elements, especially the need to read the old language, was generally not applauded, nor did we observe a positive cognitive effect. Yet, the practical activities inspired by the context appeared significantly effective and were judged positively by the students.

The emergence of meaningful geometry

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In 1968, when the Dutch education system was changed, the mathematics curriculum in Dutch secondary education was modified rigorously. This happened under influence of the New Math movement but also as an attempt to adapt the content of mathematics education to new developments in science and society. As a consequence, and also inspired by Klein's Erlanger Programme, geometry education moved into the world of linear algebra. However, this new geometry still very much resembled the Euclidean approach: focused on developing concepts with an axiomatic method and training students in logical deduction. The structure of the mathematical system remained the guideline for the intended learning process. As an alternative, influenced by the work of Fröbel and Ehrenfest, 'realistic' geometry education was developed in the Netherlands from the 1980s on. 'Grasping space', as Freudenthal called it, was considered as a key geometrical activity that supports the students' learning. Characteristic for this approach is the start from students' intuitive reasoning in three-dimensional contexts. Furthermore, the local organization of that reasoning was considered to guide the learning process, instead of the final structure of a mathematical system. In our chapter we discuss the changes in geometry education in the Netherlands during the past decades in interaction with issues of implementation. These changes reflect the emergence of meaningful and relevant geometry education.

Testing in mathematics education in the Netherlands

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Mathematics testing in the Netherlands focuses on informing schools, teachers, and students about student performance for both formative and summative purposes. The tests are used to monitor whether educational objectives have been achieved and whether content-specific standards have been mastered by the students. In our chapter, the content and objectives of the different national primary and secondary standardized tests are described. The focus is on the primary function of these tests, but their secondary function where tests are used for accountability is also discussed. In general, the tests are classified into four types: tests to adjust instruction; tests to evaluate proficiency and make decisions about students; tests to evaluate proficiency and make decisions about groups and schools; and tests to evaluate proficiency and make decisions about the quality of the educational system. We show that in practice these types often blend together, as test results are aggregated into group- and school-based indicators at the student level for school evaluation and accountability.

There is, probably, no need for such an institution – The Freudenthal Institute in the last two decades of the twentieth century

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In the seventies, IOWO became well-known in the mathematics education community. IOWO: An institute for the development of mathematics education, with flag bearer and source of inspiration Prof. Dr. Hans Freudenthal. For purely political reasons the government decided that there was no need for such an institution in the eighties, and that all collaborators should move to SLO, Netherlands Institute for Curriculum Development. Most people refused to accept this offer. Many letters were written by our international colleagues in order to let IOWO survive. The politicians found a very creative solution: five people were allowed to carry on within the university as researchers (only). In my chapter I describe how the remaining people took back what was 'stolen' from them. And within ten years the government found that a new very successful institute had been established. They were even 'proud' of this institute for its innovative ideas, and practical uses, based on developmental research.