



**ICME 13**

**Hamburg 2016**

**Reflections from abroad  
on the Netherlands  
Didactic Tradition  
in Mathematics Education**

*Abstracts*



**Reflections from abroad on the Netherlands  
didactic tradition in mathematics education**

The Netherlands contribution to the ICME 13 Thematic Afternoon on 'European Didactic Traditions' is prepared by a committee consisting of

Marja van den Heuvel-Panhuizen

Paul Drijvers

Michiel Doorman

Marc van Zanten

## **Colophon**

Reflections from abroad on the Netherlands didactic tradition in mathematics education

© 2016 – Freudenthal Institute

Faculty of Science, Utrecht University

Printed by: All Print, Utrecht

Editorial assistant: Nathalie Kuijpers

Cover Design: Liza Koppen

## Introduction

As part of the Netherlands strand of the ICME 13 Thematic Afternoon on ‘European Didactic Traditions’ two volumes are being prepared to be published within the Springer book series with volumes coming out of ICME 13. This booklet contains the abstracts of the chapters to be included in these two volumes. In the volume titled **Reflections from abroad on the Netherlands Didactic Tradition in Mathematics Education** 45 authors from abroad reflect in 19 chapters on their experiences with mathematics education in their countries that was inspired by ideas and materials based on the domain-specific instruction theory of Realistic Mathematics Education (RME) that was developed in the Netherlands. The chapters contain examples from 16 countries situated on 5 continents. For some authors their acquaintance with the Dutch approach started with meeting Freudenthal and others from the Freudenthal Institute or its predecessors that caught their interest. Others became involved in a joint project with the Dutch. Most chapters zoom in on particular principles of RME, how RME deals with particular content and how tasks and lessons are developed and how research in mathematics education is carried out. The authors discuss what aspects of RME appealed to them and explain how RME has influenced their thinking on mathematics education, the projects they are working on, and that RME has sometimes even altered aspects of their countries’ tradition in teaching and learning mathematics. Consequently, it will not be a surprise that the chapters in this volume express much appreciation for RME. Yet, in addition to their approval, the authors also articulate the challenges of RME. It is apparent that a particular approach to mathematics education cannot simply be transplanted to another country. This knowledge is not new, but what is new is that the chapters show how a ‘local’ approach to mathematics education – which in fact RME is – has turned out in other countries. The authors have elucidated how they have adapted RME to their circumstances and their view on mathematics education. By showing how others have used RME and made their own interpretations of it, a mirror is held up to RME, which in turn also benefits its further development. The chapters make it clear that looking at RME from abroad and from the perspective of other cultural contexts can put a brighter spotlight on the essence of RME than only reflections and deliberations from inside.

Marja van den Heuvel-Panhuizen  
Editor

Utrecht, July 2016

# **From tinkering to systemic innovation – The role of teachers in the guided redesign of mathematics education in the United States**

David C. Webb <sup>\*</sup>, Frederick A. Peck <sup>#</sup>

<sup>\*</sup> *University of Colorado, Boulder, USA*

<sup>#</sup> *University of Montana, USA*



The history of Realistic Mathematics Education (RME) in the United States has positioned teachers at the center of innovation from its early years to present day. From the first proof-of-concept study at a high school in Milwaukee to current professional development opportunities offered at innovation centers, the application and spread of RME is best characterized as a teacher-centered approach to principled reconsideration of how students learn mathematics. Such reconsideration of beliefs and conceptions is often motivated when teachers re-experience mathematics through the lens of progressive formalization and related didactic approaches. Through a series of cases that articulate teacher interpretation and application of RME in US classrooms, we highlight how teacher participation has led to systemic innovation. These efforts, while inspired and supported by professional development and curricula, have been inspired and sustained by teachers who provide colleagues proof of concept in local contexts.

# Searching for alternatives for New Math in Belgian primary schools – Influence of the Dutch model of Realistic Mathematics Education

Dirk De Bock, Wim Van Dooren, Lieven Verschaffel

*KU Leuven, Belgium*



We sketch the turbulent history of primary mathematics education in Belgium during the last (half) century. The outline starts with traditional mathematics in the period before and shortly after World War II, an approach that is often, but partly unjustly, labelled as ‘mechanistic’. Then we focus on the rise of New Math or ‘modern mathematics’ in the 1970s. We briefly discuss its roots and describe how this structural approach, which basically followed the development at the secondary level, was implemented in Belgian primary schools. By the early 1980s, New Math was being strongly criticized, which paved the way for its fall during the 1990s. This leads us to the current curricula that are strongly inspired by the Dutch model of Realistic Mathematics Education (RME), while maintaining valuable elements of the strong Belgian tradition in developing students’ mental and written calculation skills and even some (minor) New Math accents. We describe in some detail the influence of RME on the different mathematical domains in these curricula, as well as some new challenges that arise on the horizon.

# The impact of Hans Freudenthal and the Freudenthal Institute on the project ‘Mathe 2000’

Erich Ch. Wittmann

*Institute for Development and Research in Mathematics Education (IEEM),  
TU Dortmund University, Germany*



In the preface of the volume ‘Five years IOWO’, published in 1976, Hans Freudenthal stated:

IOWO is not a research institute; its members do not regard themselves as researchers but as producers of instruction, as engineers in the educational field, as curriculum developers. Engineering needs background research and can produce research as fall-out. Though both of them will be visible in the present account, its nucleus is our productive work, represented by a few specimens, and embodies our views on mathematics as a human activity and on curriculum development as a classroom activity, guided by curriculum developers, in close contact with all those interested in mathematics education.

It is not only this general view of developmental research that has been guiding us when we founded ‘Mathe 2000’ ten years later. Freudenthal’s didactical analyses of mathematical contents and processes as well as empirical studies at the FI, conducted along these lines, in particular by Adri Treffers and Marja van den Heuvel-Panhuizen, have been equally inspiring for our work in which we tried to achieve a balance between mathematical structure and applications and in which we also tried to give the practice of skills a proper place.

# Reflections on Realistic Mathematics Education in South Africa

Cyril Julie <sup>\*</sup>, Faaiz Gierdien <sup>#</sup>

<sup>\*</sup> *University of the Western Cape, Cape Town, South Africa*

<sup>#</sup> *University of Stellenbosch, Stellenbosch, South Africa*



The project Realistic Mathematics Education in South Africa (REMESA) was introduced in South Africa during a period when curriculum changes were introduced to fit the educational ideals of the ‘new’ South Africa. During the project’s implementation there were two curriculum changes. In our chapter we reflect upon the appropriation by practicing teachers of two modules developed during these two curriculum dispensations. Three modules – ‘Vision geometry’, ‘Global graphs; Functions, tables and graphs’ and ‘The exponential function’ – were developed by a team comprising staff from the Freudenthal Institute and the Mathematics Education sector of the University of the Western Cape. The modules were implemented in classrooms. Teachers viewed the module ‘Vision geometry’ with skepticism whilst the module ‘Global graphs’ was more readily accepted. The appropriation was thus differential. In current school mathematics policy documents and learning materials, the major ideas of the module ‘Vision geometry’ are virtually invisible. The ideas from the module ‘Global graphs’ are more visible. This can be ascribed to the prominence of graphical representations in South African school mathematics curricula. The two instances point in the direction that the proximity of innovative approaches to the operative curriculum plays an important role with respect to teachers’ adoption of the resources for their practice.



# **Learning to look at the world through mathematical spectacles – A personal tribute to Realistic Mathematics Education**

Abraham Arcavi

*Weizmann Institute of Science, Israel*



As professionals of mathematics education, we seldom offer personal accounts of our own learning and development. Although such subjective experiences may be idiosyncratic and hardly generalizable, a brief ‘racconto’ of what and how one came to know may be useful – firstly, to those from whom we learned (maybe what we learned from them is not what they intended to teach us, and this is worth explicating), secondly, to those whom we teach (for them to know who we are and some of the sources of our learning), and, thirdly, to some colleagues willing to start conversations and to share experiences. In my essay, I subjectively describe aspects of the inspiration generated by the wise, applicable and effective principles of mathematics instruction that Realistic Mathematics Education has offered to us all, influencing the approaches to teaching and learning and doing in mathematics education.

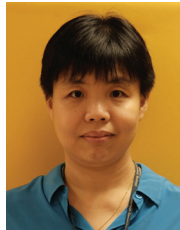
# Graphing linear equations – A comparison of the opportunity-to-learn in textbooks using the Singapore and the Dutch approach to teaching equations

Berinderjeet Kaur <sup>\*</sup>, Wong Lai Fong <sup>#</sup>, Simmi Naresh Govindani <sup>^</sup>

<sup>\*</sup> *National Institute of Education, Singapore,*

<sup>#</sup> *Anderson Secondary School, Singapore*

<sup>^</sup> *Yishun Secondary School, Singapore*



Textbooks may be considered as proxies of the opportunity-to-learn (OTL) students get. We examine the OTL afforded by two textbooks, one using the Singapore approach and the other the Dutch approach for graphing linear equations. Both textbooks provide opportunities for students to connect the mathematical concepts to meaningful real-life situations, practice questions for self-assessment, and reflect on their learning. However, the approaches presented in the two textbooks are different. As for the activities, the Dutch approach textbook has the same context for all the activities which are related. In the Singapore approach textbook the activities are self-contained and can be carried out independently from each other. Furthermore, there is no one context that cuts through all the activities. With regards to the complexity of demand for student performance the classroom activities, practice questions and prompts for reflection in the Dutch-approach textbook provide students with more scope for reasoning and communication and promote the development of the disciplinarity orientation of mathematics. Lastly from reflections of two lead teachers teaching in Singapore schools and using the Singapore-approach textbook it is apparent that they see merit in the Dutch-approach textbook.

# **Low achievers in mathematics – Ideas from the Netherlands for developing a competence-oriented view**

Petra Scherer

*University of Duisburg-Essen, Germany*



Although in Germany a competence-oriented view on teaching and learning mathematics has been one of the guiding principles for primary mathematics since the early 1990s, it was not appreciated for low achievers or for students in special education. Research in special education mostly focused on diagnosis with regard to deficiencies and the usual teaching practice in special education could be characterized by learning step-by-step in a rather mechanistic and reproductive way. Influenced by research papers and encouraging classroom experiences with low-achieving students in the Netherlands, my research focused on the question to what extent competence-oriented diagnosis followed by an inquiry-based learning approach would be appropriate also for children with special needs, or especially for them. Instead of underestimating these students' abilities, it seemed necessary to give them the opportunity to show what they are capable of, e.g. by using more open problems that show the ideas students have in mind. In several projects and studies referring to different mathematical topics it could be shown that even low achievers benefited from an open approach and were able to choose individual strategies, make use of structures and relations, find patterns and show creative and effective work.

# From the bottom up – Reinventing Realistic Mathematics Education in Southern Argentina

Betina Zolkower <sup>\*</sup>, Ana María Bressan <sup>#</sup>, Silvia Pérez <sup>^</sup>, María Fernanda Gallego <sup>^</sup>

<sup>\*</sup> *Brooklyn College, City University of New York, USA*

<sup>#</sup> *Ministerio de Educación, Río Negro, Argentina*

<sup>^</sup> *Instituto de Formación Docente Continua, San Carlos de Bariloche, Río Negro*



Our chapter focuses on the Grupo Patagónico de Didáctica de la Matemática (GPDM), a collective of about twenty teachers and teacher educators in a Southern Argentinean town united by a commonly shared interest in making mathematics meaningful, relevant, and accessible to all students. Through collaborative processes of design, try outs, reflective discussion, revision, new try outs, and so on, in spiral movements that interconnect their own mathematizing with that of their students, since the year 2000 GPDM participants have been learning about, adapting, implementing, and contributing to Realistic Mathematics Education (RME). The chapter is organized as follows. First we outline the state of mathematics education in Argentina in the 1990s. Next, we describe how the GPDM was formed, how participants learned about and implemented RME, and how the group's sphere of influence in grades K-12 as well as in pre-service and in-service mathematics teacher education expanded from the local to the regional, national, and international level. We close with a reflection on what we have learned in this creative appropriation process. Throughout the chapter, a selection of annotated vignettes illustrates the manner in which the legacy of Hans Freudenthal materialized and continues to materialize in Argentinean classrooms.

# Realistic Mathematics Education in China

Xiaotian Sun, Wei He

*Department of Mathematics and Mathematics Education, Minzu University of China, Beijing*



In our chapter, we start with a historical review of how Professor Hans Freudenthal and Realistic Mathematics Education (RME) became known in China, and how the academic exchange between Chinese scholars in the field of mathematics education and researchers at the Freudenthal Institute initiated and continued later on. Then we discuss the positive impact of RME. Specifically, we cite some living examples for how the theoretical and empirical research substances related to RME-influenced mathematics curriculum development in China. These examples include the fields of curricular policy making, textbook design and classroom teaching.

# The enrichment of Belgian secondary school mathematics with elements of the Dutch model of Realistic Mathematics Education since the 1980s

Dirk De Bock, Dirk Janssens, Johan Deprez

*KU Leuven, Belgium*



In their search for alternatives for the failed New Math movement of the 1960s and 1970s, Belgian mathematics educators looked with great interest to the Dutch model of Realistic Mathematics Education (RME), developed by Hans Freudenthal (1905-1990) and his team at the University of Utrecht. In our chapter we primarily focus on how, from the mid-1980s until the mid-1990s, valuable elements of that model were integrated in Belgian secondary school mathematics. At that time, the influence of Dutch mathematics education on Belgian curricula was quite substantial, but some form of collaboration between the communities of mathematics teachers in both countries already existed since the early 1950s. However, from the 1950s until the 1970s, school mathematics in both countries evolved largely independent of each other. In Belgium, the structural New Math approach, with Georges Papy (1920-2011) as its main representative, became dominant in school mathematics, while the modernization of school mathematics in the Netherlands was strongly inspired by Freudenthal's RME model emphasizing the role of applications and modelling.

# Echoes and influences of Realistic Mathematics Education in Portugal

João Pedro da Ponte <sup>\*</sup>, Joana Brocardo <sup>#</sup>

<sup>\*</sup> *Instituto de Educação, Universidade de Lisboa, Portugal*

<sup>#</sup> *Escola Superior de Educação, Instituto Politécnico de Setúbal, Portugal*



Our chapter traces the connections between Realistic Mathematics Education (RME) and Portuguese developments in mathematics education in terms of research studies and curriculum development. The basis for this work is a literature review of papers and other documents, with special attention to the period 2005-2015, and research studies organized by mathematical topic. Although there is no research group in Portugal that is perfectly aligned with RME principles and curriculum materials, noticeable influences may be seen in the frequent references made in some research groups to key RME ideas, notably the importance of students working from tasks in meaningful contexts, the role of representations and models to support students' thinking, and the levels of students' mathematical activity. This is most noticeable in conceptual frameworks for developmental research studies in the area of number and in the use of realistic contexts in task design, and it is also apparent in the official 2007 Portuguese curriculum document.

# Supporting the mathematical learning processes by means of maths conferences and maths language tools

Christoph Selter, Daniel Walter

*Institute for Development and Research in Mathematics Education (IEM),  
TU Dortmund University, Germany*



In recent decades, the instructional theory of Realistic Mathematics Education (RME) has exerted a powerful influence on mathematics education around the world. In this context, the idea of progressive mathematization has gained international acceptance. In our chapter we illustrate the way in which we benefited from the idea of organizing the teaching and learning of mathematics in keeping with this guiding principle. After some personal memories of the first author, we start by describing what we consider to be the central elements of the principle of progressive mathematization. This is followed by our description of two measures, the ‘maths conferences’ and ‘maths language tools’, for rendering the learning and teaching concepts entailed by the principle of progressive mathematization even more expedient and fruitful. The contribution concludes with an explanation of how we understand the term ‘realistic’ in RME and how we use it in maths conferences and maths language tools.

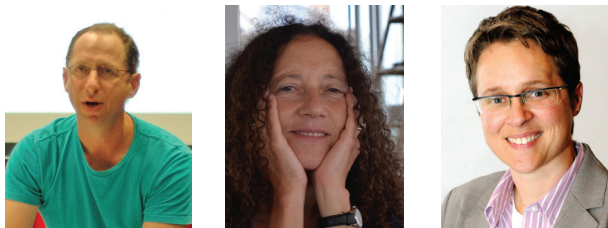


## Reinventing RME at Berkeley – Emergence and development of a course for pre-service teachers

Dor Abrahamson <sup>\*</sup>, Betina Zolkower <sup>#</sup>, Elisa Stone <sup>\*</sup>

<sup>\*</sup> *University of California, Berkeley, USA*

<sup>#</sup> *City University of New York, USA*



A central principle of Realistic Mathematics Education (RME) is that learners experience guided opportunities to reconstruct cultural practices and artifacts in the course of attempting to solve engaging problems using emerging resources as structuring tools. The same principle, we submit, plays out at the meta level, across ages, geography, and functions, where instructors experience opportunities to reinvent RME as they adapt its principles to satisfy specific design constraints and local needs. Our chapter recounts a collaborative effort to create at the Graduate School of Education of the University of California, Berkeley, graduate and undergraduate courses for pre-service mathematics teachers that incorporate tenets of RME while accommodating prescribed and emerging constraints of local contexts, such as stipulation of federal funding as well as the collective histories and prior schooling experiences of pre-service teachers, most of whom are encountering this didactical approach for the first time.

# The influence of Dutch didactics on Korean mathematics education

Kyeong-Hwa Lee <sup>\*</sup>, YeongOk Chong <sup>#</sup>, GwiSoo Na <sup>^</sup>, JinHyeong Park <sup>+</sup>

<sup>\*</sup> *Seoul National University, Korea*

<sup>#</sup> *Gyeongin National University of Education, Korea*

<sup>^</sup> *Cheongju National University of Education, Korea*

<sup>+</sup> *Myongji University, Korea*



Dutch didactics – in Korean mathematics education society often referred to as Realistic Mathematics Education (RME) – has become one of the major perspectives on mathematics education which have been widely discussed and applied by Korean mathematics educators and mathematics teachers to reform Korean mathematics education over the past 30 years. Our chapter briefly depicts how RME has been introduced in both theoretical and practical viewpoints through doctoral and master theses as well as through journal articles and curriculum documents in Korea. It turns out that RME has provided integral and meaningful issues to be constantly discussed among Korean mathematics educators since its introduction in the 1980s. In conclusion, RME has contributed largely to activating and reshaping Korean mathematics education in multiple ways although several barriers to overcome or perspectives to modify have emerged due to Korea’s different social and educational backgrounds. Parts of these barriers as well as recognized benefits come to the fore through feedback and reflections from the teachers and students who experienced RME in Korean contexts, as described at the end of our chapter.

# The influence of Realistic Mathematics Education outside the Netherlands – The case of Puerto Rico

Omar Hernández-Rodríguez <sup>\*</sup>, Jorge López-Fernández <sup>\*</sup>, Ana Helvia Quintero-Rivera <sup>\*</sup>, Aileen Velázquez-Estrella <sup>#</sup>

<sup>\*</sup> *University of Puerto Rico*

<sup>#</sup> *Carmen D. Ortiz School*



In our chapter we describe the genesis and evolution of Realistic Mathematics Education (RME) in Puerto Rico, and analyze the aspects that allowed or deferred its influence on local mathematics education. RME was introduced in Puerto Rico thanks to a group of mathematics professors at the University of Puerto Rico, Río Piedras Campus, who collaborated, first with staff from Wisconsin University and later more closely with a team of designers from the Freudenthal Institute. This was the beginning of a collaboration that lasted several years and accounted for the design and development of quality educational materials adapted to the Puerto Rican reality. The initial goal was to develop a curriculum for the elementary level, but it soon developed into a more comprehensive project that included training for teachers and developers, implementation efforts, and research initiatives that was called ‘Las matemáticas en contexto en Puerto Rico’ (MeC-PR). RME in Puerto Rico went through interconnected, and sometimes overlapping, stages of design, training, implementation, and research. All of them left their mark in different areas such as educational practices, official documents, and research practices.

# The impact of Dutch mathematics education on Danish mathematics education

Mogens Niss

*INM (Department of Science and Environment), Roskilde University, Denmark*



Hans Freudenthal – in his capacity as a mathematician as well as a very articulate and thoughtful mathematics educator, as an international ‘politician’ of mathematics education, as the founder of ‘Educational Studies in Mathematics’, as a prolific writer, as an organizer of meetings and conferences – exerted quite an influence on Danish mathematics education from the late 1960s onwards. The Dutch mathematics education tradition thus founded always received close attention from the Danish mathematics education community. In my chapter I outline and discuss the nature of this influence and I attempt provide an explanation of why this tradition has resonated so well with implicit and explicit movements in Denmark.

# Two decades of Realistic Mathematics Education in Indonesia – From ICMI Shanghai to ICME Hamburg

Zulkardi <sup>\*</sup>, Ratu Ilma Indra Putri <sup>\*</sup>, Ariyadi Wijaya <sup>#</sup>

<sup>\*</sup> *Sriwijaya University, Indonesia*

<sup>#</sup> *Jogyakarta State University, Indonesia*



In our chapter we report on the process of adaptation of Realistic Mathematics Education (RME), a didactic approach founded by Freudenthal in the Netherlands. In the Indonesian context, RME is called ‘Pendidikan Matematika Realistik Indonesia’ (PMRI). First, the chapter starts with how RME came to Indonesia. It was Sembiring from the Institut Teknologi Bandung who saw Jan de Lange, the director of the Freudenthal Institute, Utrecht University, presenting a keynote at the ICMI conference in Shanghai in 1994. Then the story continues with the decision of the Indonesian government to send six doctoral candidates to the Netherlands to learn about RME. The chapter also explains the process and results from the development and implementation of RME through a Dutch-Indonesian project called ‘Do-PMRI’ (Dissemination of PMRI). Moreover, the chapter describes examples of implementation strategies such as developing a master program on RME, designing learning materials using RME theory and the development of a national contest of mathematics literacy using context-based mathematics tasks similar to those employed in the PISA test. The chapter ends with a discussion of two new initiatives at Sriwijaya University, namely the development of a Center of Excellence of PMRI and the establishment of a doctoral program on PMRI.

# **Anchoring contexts and mathematising – Reflections from India**

Usha Menon

*India*



The tradition of Realistic Mathematics Education (RME) has focused on the role of paradigmatic contexts that support model formation which go on to function as tools to mathematically deal with other situations. This aspect of the role of contexts has had an extensive elaboration in the Indian context where the affective dimensions of the context have come to play an important role. In addition, contexts have been designed considering the nature of the mathematical concept at hand, as well as linkages with representations and procedures. In my chapter I present the evolution of a trajectory for multiplication focusing on the idea of multiplication and on two-digit multiplication problems in particular. In both cases story contexts with choices to be made by the main character in the story play a crucial role. The ‘times’ idea is introduced through a context, in which the character gets a prize for making the least number of presses to take out the required biscuits. This incorporates the use of the two-color bead string called ‘Ganitmala’ leading to jumps on the empty number line. It engages children and also gives meaning for the multiplication symbol. This approach replaces the earlier context based on arrays and is seen to be more effective. In the case of two-digit multiplication, the affective dimension supports children to use the double splitting strategy.

# Implementing Realistic Mathematics Education in England and the Cayman Islands – Dealing with clashing educational ideologies

Paul Dickinson <sup>\*</sup>, Frank Eade <sup>#</sup>, Steve Gough <sup>\*</sup>, Sue Hough <sup>\*</sup>,  
Yvette Solomon <sup>\*</sup>

<sup>\*</sup> *Manchester Metropolitan University, UK*

<sup>#</sup> *Ministry of Education, Cayman Islands*



In our chapter, we discuss the issue of implementing Realistic Mathematics Education (RME) in the English education system over a number of years and education sectors. We also consider the experience of one of us in the Cayman Islands, a British overseas territory with an education system that is influenced by British tradition, but is distant from many of its politically driven accountability pressures and measures. We illustrate first the challenges of developing an RME approach which is operable within the English system, highlighting the issues of student expectation, dominant didactic practices and assessment, all of which influenced what we were able to do. Second, we describe the outcomes of interventions in England at early secondary school level (age 12-14, Key Stage 3) and at General Certificate of Secondary Education (GCSE) level (normally age 15-16, Key Stage 4, but also available in post-16 education). Finally, Frank Eade describes his experience of building on our early work to develop an RME approach in the Cayman Islands. We conclude with a discussion of the lessons learned from these challenges; we argue that despite the problems we encountered there are reasons to remain optimistic about the potential of an RME approach in the English system.