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COLLECTIVE ACTION AND THE COMMON ENEMY EFFECT

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How is collective defence by players affected when they face a threat from an intelligent attacker rather than a natural threat? This paper analyses this question using a game-theoretic model. Facing an intelligent attacker has an effect if players move first and visibly set their defence strategies, thereby exposing any players who do not defend, and if the attacker is, moreover, not able to commit to a random attack. Depending on the parameters of the game, the presence of an intelligent attacker either increases the probability that players jointly defend (where such joint defence either does or does not constitute a utilitarian optimum), or decreases the probability that players jointly defend (even though joint defence is a utilitarian optimum).

Keywords: Common enemy effect; Defence games

JEL Codes: D74, H41, C72

1. INTRODUCTION

A diverse literature across many disciplines has studied what may be called the *common enemy effect*: the presence of a common enemy, either in the form of a potential natural disaster or in the form of an intelligent enemy, increases the probability that agents act collectively. In psychology and sociology, the presence of such a common enemy is argued to change agents' preferences, e.g. increasing their feelings of solidarity.¹ Our paper instead provides a game-theoretic rationale for the common enemy effect, where the agents' preferences are stable, but where the presence of a common enemy changes their incentives. We specifically focus on the differences between facing a common enemy in the form of a natural threat and in the form of an intelligent enemy.

A first effect of facing an intelligent attacker rather than a natural threat identified in the analysis is the *defensive efficiency effect*. Intuitively, consider two adjacent countries which each have a nuclear plant, and either face the threat of an accident at this plant or the threat

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¹See e.g. Simmel (1955), Coser (1956) (the so-called *in-group out-group hypothesis*), and McLaughlin and Pearlman (2012) in sociology. Other related references include Heider (1946) and Antal, Krapivsky, and Redner (2006) in social psychology, and Levy (1989) and Sirin (2011) in political science.

of a terrorist attack on the plant. Each country can take protective measures, but if even one unprotected nuclear plant is hit, both countries suffer equally. Assume that the only difference between the two types of threats is that terrorists attack any unprotected nuclear plant, whereas nature attacks a random plant. Then, if defensive costs are too high to give the individual country incentives to take protective measures when facing a natural threat, the fact of facing a terrorist threat rather than a natural threat increases the probability that the countries coordinate on taking protective measures. When facing a terrorist threat, each country may take protective measures, just because the other country is doing so: if one country stops taking measures unilaterally, and if this is visible to terrorists, the country knows that it will automatically be targeted.

A basic version of our model where the defensive efficiency effect is at work is the following. Two defenders can benefit from a public good with value V to both of them. Each individual defender can take defensive measures at a cost C . In the absence of a threat (Figure 1(a)), joint non-defence trivially is both the unique equilibrium and the utilitarian optimum (in the sense that this strategy profile maximises the sum of the defenders' payoffs). The *game against the attacker* (Figure 1(b)) is a sequential move game which proceeds as follows. First, the defenders simultaneously decide whether they individually defend or not. Next, the intelligent attacker (she) observes the defenders' decisions, and decides whether or not to attack a single defender (he). If she attacks a non-defending defender, the value of the public good is reduced to zero to both defenders, in any other case the value of the public good is fully maintained to both defenders. Joint non-defence is an equilibrium, because a defender who defends unilaterally cannot make any difference, as the other non-defending defender will be attacked anyway. But assuming that $V > C$, joint defence is an equilibrium as well, because a defender who unilaterally stops defending, is automatically attacked. Joint defence is also the utilitarian optimum.

The fact that defenders face an intelligent attacker (Figure 1(b)), rather than no threat, induces them to jointly defend. To what extent is this effect due specifically to the fact that the intelligent attacker is a strategic player? To answer this question, we also analyse the game against nature (Figure 1(c)), where the only difference with the game against the attacker is that nature randomly attacks a single defender. The payoffs when both defenders defend and when neither defends, continue to be the same. However, if only one defender defends, nature with probability $\frac{1}{2}$ still attacks the defending attacker. Thus, if one defender defends, both defenders with probability $\frac{1}{2}$ obtain V , and with probability $\frac{1}{2}$ obtain zero. It is clear now that if $V/2 < C < V$, the only equilibrium of the game against nature is one where no defender defends – even though joint defence continues to be the utilitarian optimum. Thus, in this case the presence of an intelligent attacker (rather than of nature) indeed induces play of the utilitarian optimum.

	Defend	Don't
Defend	$V - C, V - C$	$V - C, V$
Don't	$V, V - C$	V, V

(a). Game without a threat

	Defend	Don't
Defend	$V - C, V - C$	$-C, 0$
Don't	$0, -C$	$0, 0$

(b). Game against the attacker

	Defend	Don't
Defend	$V - C, V - C$	$V/2 - C, V/2$
Don't	$V/2, V/2 - C$	$0, 0$

(c). Game against nature

FIGURE 1 Games with a public good

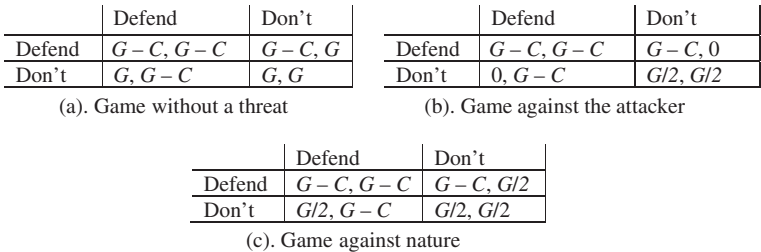


FIGURE 2 Games with a private good

Yet, for a smaller defence cost, the presence of an intelligent attacker rather than nature, leads to a *non-defensive inefficiency effect*. In the nuclear example, if defensive costs are so low that individual countries prefer to take unilateral protective measures when facing the threat of an accident, the fact of instead facing a terrorist threat, *decreases* the probability that the countries coordinate on taking defensive measures. When facing the threat of a terrorist attack, it is possible that no country takes protective measures, because no other country is doing so: if one country takes measures unilaterally, this will not make any difference anyway, as terrorists will target any country that did not take measures.²

In Figure 1, consider a smaller defence cost, such that $C < V/2 < V$. While this does not make any difference for the games in Figures 1(a) and (b), the game against nature (Figure 1(c)) now has a unique equilibrium where all defenders defend, which is also the utilitarian optimum. While such a joint-defence equilibrium continues to exist in the game against the attacker, an equilibrium now also exists where no defender defends. Therefore, when the defence cost is low, the presence of an intelligent attacker, rather than of a natural threat, makes it possible that defenders get locked into an equilibrium where nobody defends.

Finally, for yet other parameters, the presence of an intelligent attacker, rather than nature, has a *defensive inefficiency effect*: while it is the utilitarian optimum not to defend, the presence of an intelligent attacker may make players defend. For instance, it may be that in a neighbourhood with a low crime rate, it is not optimal for everyone to install costly burglar alarms. But if everyone else already has a burglar alarm (assuming that thieves can observe who does and does not have an alarm), the individual household has no choice but to install such an alarm as well.

This effect occurs in a variant of the game in Figure 1 where defence concerns each defender's own private good, rather than a public good. Let each defender obtain value G from his own private good. When the defender is attacked and does not defend, the value of his private good is reduced to zero; if the defender is attacked and defends (at cost C), the value of his private good is maintained, even when the defender is attacked. Importantly, the defender always maintains the value of his private good when not attacked. This leads to Figures 2(a)–(c) for the respective games. We assume here that $C > G/2$, so that joint non-defence is the utilitarian optimum.³ Joint non-defence is now the only equilibrium

²It is now the presence of a natural threat rather than of an intelligent attacker which induces joint defence. For sociological literature that suggests that natural threats increase cohesion, see the papers cited in Carroll et al. (2005). Furthermore, this may help explain Kriesberg's point (1973, 249) that the presence of a common enemy decreases rather than increases cohesion (for a recent overview of similar arguments, see McLauchlin and Pearlman, 2012).

³If instead $G/2 > C$, joint defence is the utilitarian optimum and the unique equilibrium in both the game against nature and in the game against the intelligent attacker.

	Efficiency	Inefficiency
Defensive	<i>Proposition 2</i>	<i>Proposition 3</i>
Non-defensive	-----	<i>Proposition 1</i>

FIGURE 3 Three intelligent attacker effects

in the game without a threat, as well as in the game against nature (note that in the latter game, the individual defender's payoff is not affected by the other defender's strategy). Yet, in the game against the attacker, there is also a joint defence equilibrium, even though this is not the utilitarian optimum. The fact that everyone is defending, makes it costly for the individual defender to deviate, as he is then automatically attacked.

Figure 3 summarises the three possible effects that we identify, with reference to the corresponding propositions in Section 4. This reveals that a non-defensive efficiency effect does not exist. Such an effect would say that players defend in the game against nature, while this is not the utilitarian optimum. If defenders value only the private good (Figure 2), this is impossible: the individual defender's payoff is not affected by the decisions of other defenders, so that equilibrium and utilitarian optimum necessarily coincide. If defenders value only the public good (Figure 1), it is also impossible, because if the individual defender prefers to defend unilaterally in the game against nature ($V > C/2$), it is certainly in the defenders' joint interests of defenders to defend ($V > C$).

The paper is structured as follows. In Section 2, we treat related literature. In Section 3, we introduce a more general model with n defenders of which Figures 1 and 2 are special cases, where defenders both value the public good and their own private good, the defence cost varies, and defenders who are attacked and do not defend may still benefit from the defensive efforts of non-attacked defenders (in which case one defender's defensive efforts becomes a substitute for the other defender's defensive efforts). This allows us to precisely characterise, in three consecutive propositions in Section 4, which of the three effects in Figure 3 applies as a function of the parameters of the model.

In Section 5, we treat two variants of our model, namely a model where the attacker is able to commit to an attacking strategy, and a model where defender and attackers set their strategies simultaneously. This reveals that the assumptions in Section 3 that defenders visibly move first in setting their defensive strategies, and that the attacker cannot commit to a random attacking strategy, are crucial for the results in Section 4. We end with a discussion in Section 6.

2. RELATED LITERATURE

We here treat disparate game-theoretic papers based on intuitions somewhat similar to the intuition underlying our defensive efficiency effect, but which translate these intuitions into models that differ both from each other and from our model. Most of these models may best be understood with respect to the simple two-player game in Figure 4.

	<i>Contribute</i>	<i>Don't</i>
<i>Contribute</i>	R, R	S, T
<i>Don't</i>	T, S	P, P

FIGURE 4 Two-player game with $R > P$, and $R - T > S - P$

In Mesterton-Gibbons and Dugatkin's (1992) model of *by-product mutualism*, the game in Figure 4 is explicitly treated, where it is assumed that $R > P$ and $(R - T) > (S - P)$, so that joint contribution is the utilitarian optimum. A common enemy is added to the game (where no distinction is made between a natural threat and an intelligent attacker), and is assumed to affect the adversity faced by the two players, where an increase in adversity is assumed to increase both $(R - T)$ and $(S - P)$. For a small degree of adversity, both $(R - T)$ and $(S - P)$ are negative. The game then becomes a *prisoner's dilemma*, and joint failure to contribute is the unique equilibrium. As one increases the degree of adversity, since by assumption $(R - T) > (S - P)$, $(R - T)$ first becomes positive, while $(S - P)$ remains negative. In this case, we obtain a *stag hunt*, where both joint contribution and joint failure to contribute are equilibria. As one increases the degree of adversity further, both $(S - P)$ and $(R - T)$ become positive, and the unique equilibrium is one where both players contribute. As the degree of adversity is increased, joint contribution thus becomes more likely. In our model, we look inside the black box of how the payoffs in Figure 1 are generated. This leads us to make a clear distinction between the adversity of facing a natural threat, and the adversity of facing an intelligent attacker, and also leads us to identify two effects on top of the defensive efficiency effect (see Figure 3), not identified in the literature.⁴

In Münster and Staal (2011), contributing is interpreted as investing in production, whereas not contributing means investing in appropriating what is produced by others. At Stage 2 of their model, separate groups of players play the game in Figure 4, with the strategies interpreted in the specified way. At Stage 1, as the product of a collective decision process, each group decides how much to invest in trying to appropriate the production of other groups. As players face a budget constraint determining how much effort they can put in appropriation, collectively deciding to invest in across-group appropriation at Stage 1, will leave fewer resources for individually investing in within-group appropriation at Stage 2. Thus, the fact that there are competing groups, stops players from investing in within-group conflict. On top of the different context, a key difference with our model is that Münster and Staal's argument relies on players being able to make decisions collectively, whereas our argument is strictly based on individual decision-making.

Both Bornstein, Gneezy, and Nagel (2002) and Riechmann and Weimann (2008) run an experiment on the minimum effort coordination game due to Van Huyck, Battalio, and Beil (1990). For $(P - S) > (R - T) > 0 > (S - P)$, the game in Figure 1 can be interpreted as a basic version of this game, where C represents high effort, and D represents low effort. One then obtains a stag hunt game, with (C, C) as the payoff dominant but risk dominated equilibrium, and (D, D) as the risk dominant but payoff dominated equilibrium. In both experiments, in the control treatment, individual groups play the minimum effort coordination game. In competition treatments, pairs of groups play this game 'against each other', in the sense that the payoffs in the group which achieves a lower minimal effort than the other group, are lowered. Both papers find that in the competition treatment, participants are more likely to coordinate on the (C, C) equilibrium. Our paper differs in that we look at the effect of the presence of an intelligent attacker rather than of a natural threat, whereas the

⁴In terms of the framework of Mesterton-Gibbons and Dugatkin (1992), in Figure 1(a): $R - T = -C$ and $S - P = -C$; in Figure 1(b): $R - T = V - C$, $S - P = -C$; in Figure 1(c): $R - T = V/2 - C$, $S - P = V/2 - C$. In Figure 2(a): $R - T = -C$, $S - P = -C$; in Figure 2(b): $R - T = G - C$, $S - P = G/2 - C$; in Figure 2(c): $R - T = G/2 - C$, $S - P = G/2 - C$. It is thus not systematically true that $R - T > S - P$ in our model. Furthermore, it is not systematically true that, comparing the game without a threat to either of the games with a threat, adversity as defined by the authors is systematically increased (e.g. in both Figure 1(a) and (b), $S - P = -C$). Finally, adversity does not change in a systematic way when comparing the game against the attacker to the game against nature.

experiments look at the effect of group competition rather than no group competition. Further, the effect we look at changes the set of equilibria, whereas in the two papers mentioned, the effect of competition concerns only equilibrium selection.

Hoyer (2012) models a network formation game, of which Figure 1 may be considered as the simplest possible case. Action *C* is now interpreted as agreeing to share information, and action *D* as not agreeing. Benefits of information sharing only arise when a pair of players jointly decides to share information, where such a joint decision is represented as a *link* between players. In the *n*-player version of this game, players not only benefit from their own links, but also from the links of their neighbours, the links of the neighbours of their neighbours, and so on. For high linking costs, in the absence of an intelligent attacker, players do not form links. However, if the players face an attacker who removes links, equilibria exist where they do form links. This is because if an individual player now defects the intelligent attacker removes a link such that players lose a lot of information (as the attacker can then cut the network in two). Key differences with the current paper are that (1) in Hoyer (2012) pairs of players can agree to play a strategy profile that is to their mutual interest (cooperative game theory), whereas in the current paper, players cannot make such agreements (non-cooperative game theory); (2) Hoyer only considers the effect of an intelligent attacker compared to the absence of a threat, whereas here we focus on the effect of facing an intelligent attacker rather than a natural threat.

In Hugh-Jones and Zultan (2013), in each stage of a repeated game, an attacker decides which of several groups to attack, where attacking a group means attacking a random player within this group. At the same time, a random player among the non-attacked players in the targeted group can decide whether or not to help the attacked player. Groups differ according to the willingness of their members to help attacked players, where the attacker does not observe group types. It is in the collective interest of individuals in the same group to maintain a reputation for being helpful. As reputation has a weakest-link characteristic, where deviation of a single player causes immediate damage to the group's reputation, maintaining a reputation is also in the individual's interest. The defence game we model differs in the sense that our players cannot come to the rescue of other players, but can only decide whether or not to defend their own position. Also, in our model, there is only one group with a single and known type of defenders, such that group reputation is not relevant.

Finally, in Kovenock and Roberson (2012) two players *A* and *B* separately play a two-player Colonel Blotto game against player *C*. In each game, the two players allocate their respective endowments over *n* battlefields. In each battlefield, the player allocating more resources wins, and players are better off the more battlefields they win. If player *A* has more resources than player *B*, then the fact that *C* is a common enemy may create an equilibrium where player *A* transfers some of his endowment to player *B*. In this sense, the presence of a common enemy creates other-regarding behaviour. Aside from the quite different context of the game, a key difference in our game is that we do not allow for transfers between players.

3. THE MODEL

Our *game against the attacker* involves a finite number *n* of defenders and one attacker. At Stage 1, simultaneously with all other defenders, each defender *i* decides whether to set $x_i = 1$ (=defend) or $x_i = 0$ (=do not defend). The attacker observes the defenders' defensive decisions. At Stage 2, the attacker can either not attack (set $y_i = 0$ for every defender *i*), or attack one particular defender *j* (set $y_j = 1$ for one defender *j*, and set $y_k = 0$ for all

defenders $k \neq j$). The *game against nature* is identical to the game against the attacker, except that at Stage 2 nature moves instead of the attacker, where nature always decides to randomly attack a single defender, independently of defenders' strategies.

The *individual defensive success function* $g_i = g(x_i, y_i)$ denotes whether defender i succeeds in holding off an attack ($g_i = 1$), or not ($g_i = 0$). We assume that $g(0, 0) = g(1, 0) = g(1, 1) = 1$, and $g(0, 1) = 0$, meaning that a defender i is only *not* successful in holding off an attack when he is not defending ($x_i = 0$) and is attacked ($y_i = 1$). Each individual defender i obtains payoff $f(g_1, g_2, \dots, g_n)\alpha V + (1 - \alpha)g_i G - Cx_i$, where V is the value of the public good, G is the value of the private good, α is the weight each defender attaches to the public good, C is the defence cost, and $f(g_1, g_2, \dots, g_n)$ is the *common defensive success function*. The latter denotes the defenders' *collective* success in avoiding an attack on the public good. Given that only a single defender can be attacked, this function can only take on two values, namely $f(1^n)$ (where 1^n denotes that n inputs are 1), or $f(0, 1^{n-1})$ (where 1^{n-1} denotes that $(n-1)$ inputs are 1, and 0 that and one input is 0). Note that the value obtained by a defender i from his own private good is only affected by his own defensive success, and not by that of other defenders.

In the game against the attacker, the payoff of the attacker is $-f(g_1, g_2, \dots, g_n)\alpha nV - (1 - \alpha) \sum_{i=1}^n g_i G - K \sum_{i=1}^n y_i$. She thus incurs a cost equal to minus the sum of the benefits which the defenders obtain, where K is the cost of attacking an individual defender.⁵ Given that the attacker can at most attack a single defender, $\sum_{i=1}^n g_i G$ either takes on a value of G or of $(n-1)G$, whereas $K \sum_{i=1}^n y_i$ takes on a value of K or 0.

We assume that the production function $f(\cdot)$ takes on the CES form (cf. Cornes 1993; Sandler 2006), adjusted to take into account that the standard CES function is not defined for zero input, such that $f(g_1, g_2, \dots, g_n) = \left\{ \left[\left(\frac{1}{n} \right) \sum_{i=1}^n (g_i + 1)^\pi \right]^{1/\pi} - 1 \right\}$. Since by assumption only one single g_i can take on a value of zero, this function takes on two possible values, namely $f(1^n, \pi)V = \left\{ \left[\left(\frac{1}{n} \right) \sum_{i=1}^n 2^\pi \right]^{1/\pi} - 1 \right\} V = V$, and $f(0, 1^{n-1}, \pi)V = \left\{ \left[\frac{1}{n} + \frac{n-1}{n} 2^\pi \right]^{1/\pi} - 1 \right\} V$.

When π equals negative infinity, $f(0, 1^{n-1}, \pi)$ equals 0, and the individual defenders' defensive successes g constitute perfect complements in defending the public good,⁶ meaning that when one non-defending defender is attacked, the entire public good is destroyed. In this case, there is a perfect negative externality of unsuccessful defence, as may be the case in the nuclear example given in the introduction. When $\pi = 1$, $f(0, 1^{n-1}, \pi) = (n-1)/n$, and individual successes g constitute perfect substitutes in defending the public good: when $(n-1)$ out of n defenders are successfully defending, a portion $(n-1)/n$ of the public good is preserved. The negative externality of a successful attack on non-defending defenders is such here that only a part of the public good proportional to the number of successfully attacked defenders (namely one), is destroyed. For instance, if the defenders are countries benefiting from the public good of a common market, successful attack on a single country may diminish the benefit of the common market by a factor proportional to the size of the country.

⁵Note that the attacker's payoff does not equal minus the sum of the defenders' payoffs, as this would include their defensive costs as well. However, our game is a zero-sum game in the benefits.

⁶See Hirschleifer (1983) for this terminology. It should be noted that one can alternatively look at the extent to which the defenders' actual defensive efforts, rather than their defensive successes, are complements, substitutes, or have a best-shot nature in the defence of the public good. E.g., when defensive *successes* are *complements* ($f(0, 1^{n-1}, \pi) = 0$), and defenders play the game against nature, defensive *inputs* are perfect *substitutes*, as each defender's defensive effort increases the probability that V is obtained by $1/n$; if players play the game against the attacker, defensive inputs are also perfect complements, as each defender's effort is necessary to obtain V .

As π approaches positive infinity, $f(0, 1^{n-1}, \pi)$ approaches 1, and the individual successes g operate in a best shot common defensive success function, where the highest defensive success determines the extent to which the public good is preserved. For instance, let us assume that countries have connected electricity grids, and have overcapacity in their electricity production, so that if there is a disruption in production in one country, the other country can produce electricity for it. In this extreme case, the public good can only be destroyed if no country defends. In general, as we let $f(0, 1^{n-1}, \pi)$ range from 0 to 1, we let the common defensive success function range from one with perfect complements to one that takes on the best shot form. We consider in particular $0 \leq f(0, 1^{n-1}, \pi) < 1$.

We assume that $[1 - f(0, 1^{n-1}, \pi)]V > C$.⁷ In the game against the attacker, for defenders who value only the public good, when all other defenders defend, $[1 - f(0, 1^{n-1}, \pi)]V$ is the benefit to the individual defender of defending (yielding payoff V) rather than not defending (yielding payoff $f(0, 1^{n-1}, \pi)V$). In the game against nature, for defenders who value only the public good, $[1 - f(0, 1^{n-1}, \pi)]V$ is the *aggregate* benefit to all defenders, of *one* defender defending rather than not defending: with probability $1/n$, the individual defender is attacked, in which case all defenders obtain nV rather than $f(0, 1^{n-1}, \pi)nV$. The assumption $[1 - f(0, 1^{n-1}, \pi)]V > C$ therefore implies that in the game against the attacker, when defenders value only the public good, the individual defender prefers to defend when all other defenders defend; at the same time, it implies that defenders who value only the public good achieve a utilitarian optimum in the game against nature when all defend. In the game against nature, $[1 - f(0, 1^{n-1}, \pi)](V/n)$ measures the benefit to the individual defender who values only the public good, of defending rather than not defending. We consider both the case where this benefit does and does not exceed C .

We further assume that $G > C$. In the game against the attacker, for defenders who value only the private good, when all other defenders defend, G is the benefit to the individual defender of defending (yielding payoff G) rather than not defending. Our assumption $G > C$ thus implies that in the game against the attacker, if defenders value only the private good, the individual defender prefers to defend if all other defenders defend. Together with the above, this means that $\alpha[1 - f(0, 1^{n-1}, \pi)]V + (1 - \alpha)G > C$, so that defenders who value both the public and the private good prefer to defend in the game against the attacker if all other defenders defend.

At the same time, we assume that $(G/n) < C$. In the game against nature, when defenders value only the private good, (G/n) is at the same time the benefit to an individual defender, and the aggregate benefit to all defenders, of this defender defending rather than not defending: with probability $1/n$ this defender is targeted, in which case he obtains G rather than 0 (where G is then also the benefit to the sum of the defenders' payoffs). Our assumption $(G/n) < C$ thus implies that in the game against nature, if defenders value only the private good, joint non-defence is both the unique equilibrium and the utilitarian optimum. Yet, as we assume at the same time that $[1 - f(0, 1^{n-1}, \pi)]V > C$ and make no assumptions about the relation between $[1 - f(0, 1^{n-1}, \pi)](V/n)$ and C , when defenders value both the private good and the public good, in the game against nature, joint non-defence may or may not be the utilitarian optimum and may or may not be the unique equilibrium. Finally, we assume that $\alpha[1 - f(0, 1^{n-1}, \pi)]nV + (1 - \alpha)G > K$, so that it is a best response for the attacker to attack if no defender defends.

In both the game against nature and the game against the attacker, a strategy of a defender i is the probability p_i of defending (i.e. setting $x_i = 1$). In the game against the attacker, a

⁷Note that this means that we only allow the CES production function to approach the best-shot case, meaning $f(0, 1^{n-1}, \pi) < 1$.

strategy of the attacker consists of a probability q_i of attacking each defender i (i.e. setting $y_i = 1$) as a function of any possible realisation of values (x_1, x_2, \dots, x_n) , where it must be the case that $\sum_{i=1}^n q_i \leq 1$. The equilibrium concept used is the *subgame perfect equilibrium* (Selten 1975): the attacker's equilibrium strategy must be such that each attacking decision that she makes as a function of a profile of defence decisions (x_1, x_2, \dots, x_n) , is a best response if this profile is realised. As the game against nature is a simultaneous move game between the defenders, the entire game is a subgame, and every Nash equilibrium of this game is also subgame perfect. We may thus apply the subgame perfect equilibrium concept in this case as well.

4. THREE INTELLIGENT ATTACKER EFFECTS

In the appendix, in three consecutive lemmata, we derive the pure-strategy subgame perfect equilibria,⁸ and the utilitarian optimum from the defenders' perspective of the game against nature and the game against the attacker, as a function of the parameters. For all parameters we consider, the game against the attacker is a coordination game, which always has both a joint defence and a joint non-defence equilibrium. Depending on the parameters, the game against nature instead either has a unique joint defence, or a unique joint non-defence equilibrium. Finally, depending on the parameters, the utilitarian optimum is either joint defence or joint non-defence, and coincides for both games, as for symmetric play by the defenders it does not matter whether they face nature or the attacker. Evidently, given the positive externality created by defending in the game against nature when defenders value the public good, if the individual defender is willing to unilaterally defend in the game against nature, joint defence is both the utilitarian optimum, and the unique equilibrium in the game against nature. However, if joint defence is the utilitarian optimum, it need not be the unique equilibrium in the game against nature.

The lemmata in the appendix allow us to immediately derive in three consecutive propositions the conditions for the three intelligent attacker effects described in Figure 3. The results can be most easily described by means of the values in Definition 1 below. C_L is the benefit to the *individual* defender of defending rather than not defending when no other defender defends in the game against nature. We further define values α_H, f_L and n_L for which defence cost C exactly equals the benefit C_L . In the same manner, C_H is the *joint* benefit to defenders of defending rather than not defending when no other defender defends. We further define values α_L, f_H and n_H for which defence cost C exactly equals the benefit C_L . Finally, for describing the results in the propositions, it is useful to define specific values of C_L and C_H : $C_L(\alpha = 1) = [1 - f(0, 1^{n-1}, \pi)](V/n)$, $C_H(\alpha = 1) = [1 - f(0, 1^{n-1}, \pi)]V$, $C_L[f(0, 1^{n-1}, \pi) = 0] = \alpha(V/n) + (1 - \alpha)(G/n)$, $C_H[f(0, 1^{n-1}, \pi) = 0] = \alpha V + (1 - \alpha)(G/n)$, $C_L(n = 2) = \alpha[1 - f(0, 1^{n-1}, \pi)](V/2) + (1 - \alpha)(G/2)$, and $C_H(n = 2) = \alpha[1 - f(0, 1^{n-1}, \pi)]V + (1 - \alpha)(G/2)$, $C_H(G = 0) = \alpha[1 - f(0, 1^{n-1}, \pi)]V$.

⁸The game against the attacker also has a mixed-strategy subgame perfect equilibrium. E.g., the games against the attacker in Figure 1(b) and (c) have a mixed-strategy subgame perfect equilibrium where each defender defends with respectively probability $p = C/V$, and $p = (2C - G)/G$. We ignore these equilibria because for the general model, they are tedious to compute, and do not change the results, which are driven only by the form of the unique equilibrium in the game against nature. If defenders do not defend (respectively defend) in the unique equilibrium of the game against nature, then even if players play the mixed-strategy equilibrium in the game against the attacker, there continues to be a defensive effect (respectively non-defensive effect), as defenders who play the mixed equilibrium defend more often (respectively less often) than in the game against nature.

Definition 1.

- $C_L = \alpha[1 - f(0, 1^{n-1}, \pi)](V/n) + (1 - \alpha)(G/n)$,
 $C_H = \alpha[1 - f(0, 1^{n-1}, \pi)]V + (1 - \alpha)(G/n)$;
- $\alpha_L = \frac{C - (G/n)}{[1 - f(0, 1^{n-1}, \pi)]V - (G/n)}$, $\alpha_H = \frac{C - (G/n)}{[1 - f(0, 1^{n-1}, \pi)](V/n) - (G/n)}$;
- $f_L = \frac{\alpha V + (1 - \alpha)G - nC}{\alpha V}$, $f_H = \frac{\alpha V + (1 - \alpha)(G/n) - C}{\alpha V}$;
- $n_L = \frac{\alpha V[1 - f(0, 1^{n-1}, \pi)] + (1 - \alpha)G}{C}$, $n_H = \frac{(1 - \alpha)G}{C - \alpha[1 - f(0, 1^{n-1}, \pi)]V}$;

Proposition 1 looks at the conditions for which the *non-defensive inefficiency effect* applies. An example of parameters meeting the conditions in Proposition 1 is found in Figure 1, where $\alpha = 1$, $f(0, 1^{n-1}, \pi) = 0$, $n = 2$, and $V/2 > C$. In general, as we have assumed that joint non-defence (respectively defence) is the utilitarian optimum when only the private good (respectively the public good) receives weight, a necessary condition for the effect to apply is that the public good receives positive weight. In terms of the defence cost, *ceteris paribus* (henceforth c.p.) the non-defensive inefficiency effect applies when the defence cost is low enough to make joint defence the only equilibrium in the game against nature, in which case joint defence is also necessarily the utilitarian optimum (provided that the condition that the defence cost is lower than the benefit of unilaterally defending in the game against nature, is not already valid whenever our assumption $C < G$ is valid).

The condition for the non-defensive inefficiency effect to apply can also be stated, c.p., as a function of the other parameters. In terms of α , given our assumption that joint non-defence is the only equilibrium in the game against nature when only the private good receives weight ($C > (G/n)$), the effect can never apply if joint non-defence is also the only equilibrium when only the public good receives weight ($C \geq C_L(\alpha = 1)$). If instead $C < C_L(\alpha = 1)$, the effect applies if the public good receives sufficient weight. In terms of $f(0, 1^{n-1}, \pi)$, the effect can never apply if unilateral defence is not a strict best response in the game against nature even when individual successes in defending the public good are perfect complements, i.e. $C \geq C_L[f(0, 1^{n-1}, \pi) = 0]$ (as the benefit of defending unilaterally in the game against nature is then as high as possible). If instead $C < C_L[f(0, 1^{n-1}, \pi) = 0]$, the effect applies if the complementarity of the individual successes in defending the public good is sufficiently high. Finally, in terms of n , the effect can never apply if unilateral defence is not a strict best response in the game against nature even when there are only two defenders, i.e. $C \geq C_L(n = 2)$ (as the benefit of defending unilaterally in the game against nature is then as high as possible). If instead $C < C_L(n = 2)$, the effect applies if the number of defenders is sufficiently low.

Proposition 1. The *non-defensive inefficiency effect* applies, c.p., for:

- (i) C such that $(G/n) < C < \min \{C_L, G\}$ (conditional on $\alpha > 0$);
- (ii) α such that $\alpha_H < \alpha \leq 1$ (conditional on $C < C_L(\alpha = 1)$);
- (iii) $f(0, 1^{n-1}, \pi)$ such that $0 \leq f(0, 1^{n-1}, \pi) < f_L$ (conditional on $C < C_L[f(0, 1^{n-1}, \pi) = 0]$);
- (iv) n is such that $2 \leq n < n_L$ (conditional on $C < C_L(n = 2)$).

Proof. For this proposition to apply, Lemma 3(ii) should be valid, along with Lemma 1(ii). It is now easy to check that all the conditions in Lemma 3(ii) are more restrictive. Therefore, the proposition applies under the conditions of Lemma 3(ii).

Next, Proposition 2 looks at the conditions under which the *defensive efficiency effect* applies. An example of parameters meeting the conditions in Proposition 2 is found in Figure 1, for $\alpha = 1$, $f(0, 1^{n-1}, \pi) = 0$, $n = 2$, $V/2 < C$. In general, as this effect requires that joint defence is the utilitarian optimum, a necessary condition for the effect to apply is again that the public good receives positive weight. Moreover, the value of the private good needs to exceed the benefit of unilaterally defending in the game against nature, as otherwise our assumption that the value of the private good exceeds the defence cost automatically implies that joint defence is the unique equilibrium in the game against nature.

In terms of the defence cost, c.p. the defensive efficiency effect applies if the defence cost is high enough to make joint non-defence the only equilibrium in the game against nature, and low enough to make joint defence the utilitarian optimum (provided that the condition that the benefit of unilaterally defending in the game against nature is smaller than the defence cost, is not already valid whenever our assumption $(G/n) < C$ is valid, and provided that the condition that the joint benefit of defending is larger than the defence cost, is not already valid whenever our assumption $C < G$ is valid).

In terms of α , if the benefit of unilaterally defending in the game against nature is weakly smaller than the defence cost even if only the public good receives all weight ($C \geq C_L(\alpha = 1)$), the effect applies as long as the public good receives sufficient weight to make joint defence the utilitarian optimum. If instead the benefit of unilaterally defending in the game against nature exceeds the defence cost if only the public good receives weight ($C < C_L(\alpha = 1)$), the effect applies for intermediate weights attached to the public good (sufficiently low so that unilaterally defending is not a best response in the game against nature, and sufficiently high so that the joint benefit of defence exceeds the defence cost).

In terms of $f(0, 1^{n-1}, \pi)$, consider the case where individual successes in defending the public good are perfect complements ($f(0, 1^{n-1}, \pi) = 0$). If in this case the defence cost weakly exceeds the benefit of unilaterally defending in the game against nature, but does not exceed the joint benefit of defending (so that the conditions for the effect to exist are valid), then the effect applies in general for sufficiently high complementarity of individual successes (ensuring that joint defence is the utilitarian optimum). If instead, when individual successes are perfect complements, the benefit of unilaterally defending in the game against nature exceeds the defence cost (thus violating the condition for the effect to exist), the effect applies for intermediate degrees of complementarity of individual successes (sufficiently high to make joint defence the utilitarian optimum, and sufficiently low to make joint non-defence the only equilibrium in the game against nature).

Furthermore, in terms of n , if the defence cost does not exceed the benefit of unilaterally defending in the game against nature for two defenders (i.e. $C \leq C_L(n = 2)$), the effect applies only if the number of defenders is not too low to make joint defence an equilibrium in the game against nature. If the defence cost does exceed the benefit of unilaterally defending in the game against nature for two defenders (i.e. $C > C_L(n = 2)$), this is certainly true for a higher number of defenders as well, and the number of defenders does not need to be further restricted from below to make the effect apply. If the defence cost weakly exceeds the joint benefit of defending even if there are only two defenders (i.e. $C_H(n = 2) \leq C$), this will certainly be true for higher numbers of defenders as well, and the effect cannot apply. If the joint benefit of defending exceeds the defence cost for two defenders (i.e. $C_H(n = 2) > C$), two cases apply. Note that the weighted joint benefit of defending the private good (i.e. $(1 - \alpha)(G/n)$) approaches zero as the number of defenders approaches infinity. If the weighted joint benefit of defending the public good weakly exceeds the defence cost (i.e. $C_H(G = 0) \geq C$), the number of defenders does not need to be restricted from above to make the effect apply. If the weighted joint benefit of defending the public good does not exceed

the defence cost (i.e. $C_H(G=0) < C$), the effect only applies only if the number of defenders is not too high.

Proposition 2. The *defensive efficiency effect* applies, c.p., for:

- (i) C such that $\max \{C_L, (G/n)\} < C < \min \{G, C_H\}$ (conditional on $\alpha > 0$, $C_L < G$).
- (ii) α such that $\alpha_L < \alpha < \alpha_H$ for $C < C_L(\alpha = 1)$, and $\alpha_L < \alpha \leq 1$ for $C \geq C_L(\alpha = 1)$;
- (iii) $f(0, 1^{n-1}, \pi)$ such that $f_L < f(0, 1^{n-1}, \pi) < f_H$ for $C < C_L[f(0, 1^{n-1}, \pi) = 0]$, and $0 \leq f(0, 1^{n-1}, \pi) < f_H$ for $C_L[f(0, 1^{n-1}, \pi) = 0] \leq C < C_H[f(0, 1^{n-1}, \pi) = 0]$;
- (iv) n such that $n_L < n < n^*$ if $C \leq C_L(n = 2)$, and $2 \leq n < n^*$ if $C_L(n = 2) < C < C_H(n = 2)$ (where $n^* = n_H$ if $C_H(G = 0) < C$, and $n^* = \infty$ if $C_H(G = 0) \geq C$).

Proof. The results are obtained simply by combining the necessary and sufficient conditions for Lemma 3(i) and Lemma 1(ii) to apply.

Finally, Proposition 3 looks at the conditions for which the *defensive inefficiency effect* applies. An example of parameters meeting the conditions in Proposition 1 is found in Figure 2, for $\alpha = 0$, $n = 2$. In general, as we have assumed that joint non-defence (respectively joint defence) is the utilitarian optimum if only the private good (respectively the public good) receives weight, a necessary condition for the effect to apply is that the private good receives non-zero weight. Moreover, the value of the private good needs to exceed the benefit of joint defence, as otherwise our assumption that the value of the private good exceeds the defence cost automatically implies that joint defence is the utilitarian optimum, contradicting the effect. In terms of the defence cost, c.p. the defensive inefficiency effect now occurs if the defence cost is sufficiently high.

In terms of α , as the effect automatically applies if only the private good receives weight (see Figure 2), the effect applies in general as long as the private good receives enough weight. In terms of $f(0, 1^{n-1}, \pi)$, if the defence cost weakly exceeds the joint benefit of defending even when individual successes in defending the public good are perfect complements (i.e. $C \geq C_H[f(0, 1^{n-1}, \pi) = 0]$), this will be true for any lower level of complementarity as well. If this is not the case (i.e. $C < C_H[f(0, 1^{n-1}, \pi) = 0]$), so that for perfect complements joint defence is the utilitarian optimum, the effect exists if individual success in defending the public good are to a sufficient degree substitutes. Finally, in terms of n , if the defence cost weakly exceeds the joint benefit of defending even when there are only two defenders (i.e. $C_H(n = 2) \leq C$), the effect applies for any number of defenders. If this is not the case (i.e. $C_H(n = 2) > C$), the effect exists if the number of defenders is sufficiently high, conditional on the weighted joint benefit of defending the public good being smaller than the defence cost ($C > C_H(G = 0)$), so that as the number of defenders is increased, eventually the joint benefit of defending is smaller than the defence cost.

Proposition 3. The *defensive inefficiency effect* applies, c.p., for:

- (i) C such that $C_H < C < \min \{G, [1 - f(0, 1^{n-1}, \pi)]V\}$ (conditional on $\alpha < 1$, $C_H < G$);
- (ii) α such that $0 \leq \alpha < \alpha_L$;
- (iii) any $0 \leq f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$ if $C \geq C_H[f(0, 1^{n-1}, \pi) = 0]$; $f(0, 1^{n-1}, \pi)$ such that $f_H < f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$ if $C < C_H[f(0, 1^{n-1}, \pi) = 0]$, or;
- (iv) any $n \geq 2$ if $C_H(n = 2) < C$, and n such that $n > n_H$ if $C_H(n = 2) \geq C$ (conditional on $C > C_H(G = 0)$).

Proof. For this proposition to apply, Lemma 3(i) should be valid, along with Lemma 1(i). It is now easy to check that all the conditions in Lemma 1(i) are more restrictive. It follows that the proposition applies under the conditions of Lemma 1(i).

5. EXTENSIONS⁹

5.1. Commitment to random attacks

In Section 4, we have looked at a refinement of the Nash equilibrium, in the form of the subgame perfect equilibrium. We start this section by pointing out that it need not be the case that all Nash equilibria of the game are also subgame perfect. Consider the parameters under which the defensive efficiency effect applies. Then a Nash equilibrium exists for the game against the attacker where the attacker randomly attacks a single defender as soon as any of the defenders defend, and does not attack otherwise. In this equilibrium defenders do not defend. Effectively, the attacker then threatens to behave as nature as soon as any of the defenders defend. Yet, such a threat is not credible, as the attacker should in fact attack only any remaining non-defending defender, as soon as any defender defends. For this reason, the described Nash equilibrium is not subgame perfect.

Yet, if instead the attacker has a commitment device available that allows her to commit to a random attack, the attacker will do so. This can be modelled in a simple manner. We add a stage 0 to the game introduced in Section 3, where the attacker can either visibly choose to play the game against the defenders in the role of an intelligent attacker, or to play the game against the defenders in the role of nature. In the former case, the game proceeds exactly as described for the game against the attacker in Section 3, and the attacker either obtains $-[anf(0, 1^{n-1}, \pi)V + (1-\alpha)(n-1)G] - K$ if in equilibrium no defender defends, or $-anV - (1-\alpha)nG$ if all defenders defend. In the latter case, everything proceeds as the game against nature, with the addition that the attacker always incurs attacking cost K , as a random defender is always attacked. In this case, the attacker again obtains $-[anf(0, 1^{n-1}, \pi)V + (1-\alpha)(n-1)G] - K$. As it is valid by assumption that $[1-f(0, 1^{n-1}, \pi)]anV + (1-\alpha)G > K$, as soon as there is positive probability that the defensive equilibrium is played in the game against the attacker, the attacker prefers to commit to a random attacking strategy by choosing to play the game against the defenders in the role of nature.

As an illustration, terrorists seem often to resolve to random attacks. Arce M. and Sandler (2005) argue that this is done in order to maximise terror, by making everyone feel at risk at every possible time and location. In our model, a rationale for random attacks can be provided without appealing to such an argument. As a further illustration, consider the case where the defenders are firms in a price cartel (a public good to defenders), and where defensive measures take the form of precautions to keep the cartel secret. If taking precautions is expensive, a competition authority which targets firms that are not taking such precautions, may inadvertently encourage cartel formation. If, however, the competition authority

⁹It is straightforward to construct an extension justifying what is in political science referred to as *diversionary foreign policy*, or as the *scapegoat hypothesis* (see Levy 1989; Sirin 2011). In a variant of the game with incomplete information, defenders do not know whether they play the game against nature or the game against the attacker. The government, which seeks to achieve the utilitarian optimum, knows which game is played, and can send a cheap talk signal (Crawford and Sobel 1982) of this at stage 0. If the (commonly known) probability that the game against the attacker is played is sufficiently high, the government always sends the same signal to the defenders, and defenders play as in the game against the attacker. Thus, by not informing defenders when they are facing a game against nature, the government induces joint action.

strategically delegates (cf. Vickers 1985) to employees performing random audits, the individual firm gets an incentive to free-ride on taking precautions, and the cartel breaks down.

On the other hand, if taking precautions is cheap, it may be that a random attack always induces all players to take precautions, whereas targeted attacks create the possibility of lock-in, where the individual firm does not take precautions because no other firm is taking precautions. In this case, the competition authority need not commit to an attacking strategy, since it is clear that it will target the weakest link. Finally, if the defenders are tax evading firms (where tax evasion is a private good), and if defensive measures take the form of hiding tax evasion, it is again better for the tax authority to commit to random audits. Specific about this case is that, if firms are better off if none of them takes defensive measures, the tax authority's commitment also makes the firms better off.

5.2. Simultaneous moves game

In the game against the attacker introduced in Section 3, defenders move first by setting their defensive strategies, which are then observed by the attacker, who sets her attacking strategy. How essential is it for the intelligent attacker effects that the defender moves first? What if instead defenders and attacker set all their strategies simultaneously? In equilibrium, players then continue to play mutual best responses. This means that in equilibrium, it continues to be the case that the intelligent attacker targets any weakest link. Yet, in any equilibrium where defence takes place with positive probability, defenders are able to deviate and stop defending, without this having consequences. For this reason, a joint defence equilibrium no longer exists in the simultaneous moves game. In such a candidate equilibrium, the attacker does not attack, so that defenders have no reason to defend.

Consider furthermore a candidate equilibrium where no defender defends. As the attacker is unable to observe the defenders' defensive decisions, the incentives for the individual defender to unilaterally defend are the same as in the game against nature. Thus, for the parameters treated in Proposition 1, where joint non-defence is not an equilibrium in the game against nature, in the simultaneous moves game joint non-defence is not an equilibrium either. In this case, the only equilibrium is a mixed Nash equilibrium where defenders randomise on how often to defend in such a way that the attacker remains indifferent, and the attacker randomises on how often to attack in such a way that the defenders remain indifferent. The non-defensive inefficiency effect is reinforced here, as it is necessarily the case that defence is less likely to take place in the game against the attacker than in the game against nature.

For the parameters treated in Propositions 2 and 3, joint non-defence is the unique equilibrium in the game against nature. This is also the unique equilibrium in the game against the attacker for the simultaneous moves game. Contrary to what is required for a mixed equilibrium, just as in the game against nature, if no other defender defends, the individual defender does not have any incentive to unilaterally defend. It follows that in these cases, the results of the sequential move game do not extend to the simultaneous moves game, so that the defensive efficiency effect and the defensive inefficiency effect no longer apply.

Proposition 4. Consider the simultaneous moves game against the attacker. Then:

- (i) A pure-strategy Nash equilibrium where all defenders defend and where the attacker does not attack, *never* exists.
- (ii) A pure-strategy Nash equilibrium where no defender defends, and where the attacker attacks a random defender, exists if $C \geq C_L$.

- (iii) A mixed Nash equilibrium where defenders defend with positive probability exists if $C < C_L$. In this equilibrium, the attacker attacks with probability C/C_L , and each defender does not defend with probability $K/(nC_H)$.

Proof. For the proof see the Appendix 1.

6. DISCUSSION

In this paper, we used a game-theoretic model to investigate the effect of the presence of a common threat on joint defence by a group of defenders (in short, the common enemy effect). We have in particular focussed on the effect of defenders facing an intelligent attacker (who performs targeted attacks), rather than a natural threat (which performs random attacks), or in short: the intelligent attacker effect. Necessary conditions for intelligent attacker effects to apply, are the following: (1) defenders move first, and visibly set their defence decisions, thus visibly exposing weakest links; (2) the attacker cannot credibly commit to not attacking weakest links. Under these conditions, the presence of an intelligent attacker rather than nature, increases the probability that defenders defend against the attacker, and benefit from such joint defence, if some further conditions are met. Defence should concern to a sufficient extent a public good, such that joint defence is in defenders' joint interest, but the weight of the public good should not be so high as to make unilateral defence a best response when facing a natural threat. The defence cost should be sufficiently high such that it is not an equilibrium to defend when facing a natural threat, but sufficiently low such that joint defence is still in defenders' joint interests. The number of defenders should be sufficiently high such that the individual defender does not find unilateral defence worthwhile when facing a natural threat, but the number of defenders should not be so high that joint defence stops being in the interest of the defenders as a group. Finally, the individual defenders' successes in defending the public good should not be so complementary that unilateral defence becomes worthwhile when facing a natural threat, but not so substitutable that joint defence stops being in the defenders' interests as a group.

If instead the public good receives a lot of weight, the defence cost are very low, the number of defenders is very low, or individual successes in defending the public good are to a high degree complementary, it is a best response to unilaterally defend in the game against nature, and in general joint defence is also the utilitarian optimum. In this case, the fact that the attacker is intelligent discourages rather than encourages joint defence, and creates inefficiency. When facing an intelligent attacker, defenders may get locked into not defending, as unilateral defence is not productive in the presence of an intelligent attacker.

Finally, if instead the public good receives little weight, the defence cost is very high, the number of defenders is very high, or individual successes in defending the public good are to a low degree complementary, in the utilitarian optimum no defence takes place, and this is also the unique equilibrium in the game against nature. In this case, the fact that the attacker is intelligent encourages joint defence, but this joint defence is not in the defenders' interests as a group. Defenders may now get locked into a suboptimal joint defence equilibrium, where no defender wants to unilaterally deviate, because any deviating defender is automatically attacked.

We end by noting that our model does not take into account any psychological or cognitive effects that the presence of an intelligent attacker may have. For instance, once an intelligent attacker is present, the defenders may incur cognitive costs by the fact that they now

need to put themselves in the shoes of the intelligent attacker – whereas when facing a natural threat they do not face such costs. In this respect, we consider our model as a benchmark, in that we precisely predict in which circumstances game-theoretic rationales for intelligent attacker effects should apply. A question for future research is then to investigate whether effects exist which cannot be explained with such a rationalist approach, and if so under what circumstances.

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APPENDIX 1. LEMMATA AND PROOF OF PROPOSITION 3

Lemma 1. In both the *game against nature* and the *game against an attacker*, the sum of the defenders' payoffs is maximised when

(i) No defender defends, c.p., for:

- C such that $C_H < C < \min \{G, [1 - f(0, 1^{n-1}, \pi)]V\}$ (conditional on $C_H < G, \alpha < 1$);
- α such that $0 \leq \alpha < \alpha_L$;
- any $0 \leq f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$ if $C \geq C_H[f(0, 1^{n-1}, \pi) = 0]$; $f(0, 1^{n-1}, \pi)$ such that $f_H < f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$ if $C < C_H[f(0, 1^{n-1}, \pi) = 0]$, or;
- any $n \geq 2$ if $C > C_H(n=2)$; n such that $n > n_H$ if $C \leq C_H(n=2)$ (conditional on $C_H(G=0) < C$).

(ii) All defenders defend, c.p., for:

- C such that $(G/n) < C < \min \{C_H, G\}$ (conditional on $\alpha > 0$);
- α such that $\alpha_L < \alpha \leq 1$;
- $f(0, 1^{n-1}, \pi)$ such that $0 \leq f(0, 1^{n-1}, \pi) < f_H$ (conditional on $C < C_H[f(0, 1^{n-1}, \pi) = 0]$), or;
- n such that $2 \leq n < n^*$ (conditional on $C < C_H(n=2)$), where $n^* = n_H$ if $C_H(G=0) < C$, and $n^* = \infty$ if $C_H(G=0) \geq C$.

Proof. Let us look at the problem of maximising the sum of the defenders' payoffs in the game against nature, where p_i denotes the probability that the individual defender defends.

$$\max_{p_i} \sum_i \left\{ \begin{aligned} & p_i [\alpha(V/n) + (1-\alpha)G - C] + \\ & (1-p_i) [\alpha f(0, 1^{n-1}, \pi)(V/n) + (1-\alpha)[(n-1)/n]G] + \\ & \alpha \sum_{j \neq i} \frac{p_j V + (1-p_j)f(0, 1^{n-1}, \pi)V}{n} \end{aligned} \right\}$$

The first derivative of this expression with respect to a single p_i equals:

$$\begin{aligned} & [\alpha(V/n) + (1-\alpha)G - C] - \{\alpha f(0, 1^{n-1}, \pi)(V/n) + (1-\alpha)[(n-1)/n]G\} \\ & + \alpha(n-1) \frac{V - f(0, 1^{n-1}, \pi)V}{n} = \alpha[1 - f(0, 1^{n-1}, \pi)]V + (1-\alpha)(G/n) - C = C_H - C \end{aligned} \quad (1)$$

Next, let us look at the problem of maximising the sum of the defenders' payoffs in the game against the attacker:

$$\max_{p_i} [\alpha nV + (1-\alpha)nG] \prod_{i=1}^n p_i + [\alpha n f(0, 1^{n-1}, \pi)V + (1-\alpha)(n-1)G] \left[1 - \prod_{i=1}^n p_i \right] - C \sum_{i=1}^n p_i$$

The first derivative of this expression with respect to a single p_i equals:

$$[\alpha nV + (1 - \alpha)nG] \prod_{j \neq i} p_j - [\alpha n f(0, 1^{n-1}, \pi)V + (1 - \alpha)(n - 1)G] \prod_{j \neq i} p_j - C \quad (2)$$

It follows from (2) that for small $\prod_{j \neq i} p_j$ the sum of defenders' payoffs decreases in an individual p_i , and for large $\prod_{j \neq i} p_j$, the sum increases in p_i (as follows from our assumptions that $[1 - f(0, 1^{n-1}, \pi)]V > C, G > C$). Therefore, each individual p_i is either set equal to 0 or 1. Also, the sum decreases in p_i as soon as even one defender $j \neq i$ sets $p_j = 0$. So, a maximum where some defenders never defend, and others always defend, does not exist. It follows that in the optimum, either all defend or none defend. It is now easy to check that the difference in total defender payoff of having all defenders defending, rather than none of them, is again $C_H - C$. It follows that the utilitarian optimum is always identical in the game against nature and the game against the attacker.

By (1), in terms of the defence cost, all defenders should defend if $(G/n) < C < C_H$, where $(G/n) < C$ is valid by assumption. The condition in the proposition takes into account the possibility that $G < C_H$. As $(G/n) < C_H$ is impossible for $\alpha = 0$, a necessary condition for all defenders to defend is that $\alpha > 0$, with no further necessary conditions imposed, as $[1 - f(0, 1^{n-1}, \pi)]V > (G/n)$ follows from our assumptions that $[1 - f(0, 1^{n-1}, \pi)]V > C, C > (G/n)$.

In terms of α , by (1) all defenders should defend if $\alpha_L < \alpha \leq 1$. As our assumptions imply that $0 < \alpha_L < 1$, no further parameter restrictions need to be imposed for $\alpha_L < \alpha \leq 1$ to be possible. In terms of $f(0, 1^{n-1}, \pi)$, all defenders should defend if $0 \leq f(0, 1^{n-1}, \pi) < f_H$. A necessary condition for $f(0, 1^{n-1}, \pi)$ meeting this condition to exist is that $f_H > 0$, which is the case if $C < C_H[f(0, 1^{n-1}, \pi) = 0]$. Finally, if $C > C_H(G = 0)$, in terms of n , all defenders should defend if $2 \leq n < n_H = \frac{(1-\alpha)G}{C-\alpha[1-f(0,1^{n-1},\pi)]V}$. In this case, a necessary condition for such n to exist is that $C < C_H(n = 2)$. If $C \leq C_H(G = 0)$, all defenders defend for any n .

By (1), in terms of the defence cost, no defender should defend if $C_H < C < \min \{G, [1 - f(0, 1^{n-1}, \pi)]V\}$, where the right part of the condition is imposed by modelling assumptions, noting that we did not make any assumptions about the relation between G and $[1 - f(0, 1^{n-1}, \pi)]V$. Note that $C_H \geq (G/n)$ given that by assumption, $[1 - f(0, 1^{n-1}, \pi)]V > (G/n)$. Furthermore, given our assumption that $[1 - f(0, 1^{n-1}, \pi)]V > C$, there are no levels of C that meet the condition if $\alpha = 1$. If $G > [1 - f(0, 1^{n-1}, \pi)]V$, given that our assumptions imply that $[1 - f(0, 1^{n-1}, \pi)]V > (G/n)$, no further necessary condition needs to be imposed. If $G \leq [1 - f(0, 1^{n-1}, \pi)]V$, a further necessary condition is that $C_H < G$.

In terms of α , no defender should defend if $0 \leq \alpha < \alpha_L$. As our assumptions imply that $0 < \alpha_L < 1$, no further parameter restrictions are needed for $0 \leq \alpha < \alpha_L$ to be possible. Whenever $\alpha > 0$, in terms of $f(0, 1^{n-1}, \pi)$, two cases exist. For $C < C_H[f(0, 1^{n-1}, \pi) = 0]$ (meaning that $f_H > 0$), no defender should defend if $f_H < f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$, where the right part of this condition follows from our assumption that $[1 - f(0, 1^{n-1}, \pi)]V > C$, and where it is easy to check that our assumptions imply that $f_H < \frac{V-C}{V}$. For $C \geq C_H[f(0, 1^{n-1}, \pi) = 0]$ (meaning that $f_H \leq 0$), no defender should defend for any $f(0, 1^{n-1}, \pi)$ (i.e., for $0 \leq f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$). Finally, in terms of n , two cases exist. For $C < C_H(n = 2)$ (so that $n_H < 2$), no defender should defend for any n . For $C \geq C_H(n = 2)$ (so that $n_H \geq 2$), no defender should defend for $n > n_H$, where the latter is only possible conditional on $C_H(G = 0) < C$.

Lemma 2. In the *game against the attacker*, both a subgame perfect equilibrium exists where all defenders defend and the attacker does not attack, and a subgame perfect equilibrium where no player defends and the attacker attacks any defender.

Proof. If all defenders defend, the individual defender does not want to deviate iff $\alpha V + (1 - \alpha)G - C > \alpha f(0, 1^{n-1}, \pi)V$. This is always true, as by assumption $[1 - f(0, 1^{n-1}, \pi)]V > C$ and $G > C$. If no defender defends, the individual defender does not want to deviate iff $\alpha f(0, 1^{n-1}, \pi)V + (1 - \alpha)[(n - 1)/n]G > \alpha f(0, 1^{n-1}, \pi)V + (1 - \alpha)G - C$ iff $C > (1 - \alpha)(G/n)$. This is always valid, as by assumption $C > (G/n)$.

Lemma 3. In the *game against the nature*, in the unique subgame perfect equilibrium

- (1) no defender defends, c.p., for:
 - (a) C such that $\max \{C_L, (G/n)\} < C < \min \{G, [1 - f(0, 1^{n-1}, \pi)]V\}$ (conditional on $C_L < G$);
 - (b) α such that $0 \leq \alpha < \alpha_H$ for $C < C_L(\alpha = 1)$, and α such that $0 \leq \alpha \leq 1$ for $C \geq C_L(\alpha = 1)$;
 - (c) (if $\alpha > 0$) $f(0, 1^{n-1}, \pi)$ such that $0 \leq f(0, 1^{n-1}, \pi) < (V - C)/V$ for $C \geq C_L[f(0, 1^{n-1}, \pi) = 0]$, and $f(0, 1^{n-1}, \pi)$ such that $f_L < f(0, 1^{n-1}, \pi) < \frac{V-C}{V}$ for $C < C_L[f(0, 1^{n-1}, \pi) = 0]$, or;
 - (d) any n such that $n \geq 2$ if $C > C_L(n = 2)$, and n such that $n > n_L$ if $C \leq C_L(n = 2)$.
- (2) all defenders defend, c.p., for:
 - (a) C such that $(G/n) < C < \min \{C_L, G\}$ (conditional on $\alpha > 0$);
 - (b) α such that $\alpha_H < \alpha \leq 1$ (conditional on $C < C_L(\alpha = 1)$);
 - (c) $f(0, 1^{n-1}, \pi)$ such that $0 \leq f(0, 1^{n-1}, \pi) < f_L$ (conditional on $C < C_L[f(0, 1^{n-1}, \pi) = 0]$), or;
 - (d) n such that $2 \leq n < n_L$ (conditional on $C < C_L(n = 2)$).

Proof.

(i) If no defender defends, the individual defender does not want to deviate iff:

$$\begin{aligned} & \alpha f(0, 1^{n-1}, \pi)V + (1 - \alpha)[(n - 1)/n]G > \\ & \alpha \{[(n - 1)/n]f(0, 1^{n-1}, \pi)V + (V/n)\} + (1 - \alpha)G - C \cdot \text{iff} \quad (3) \\ & C > \alpha[1 - f(0, 1^{n-1}, \pi)](V/n) + (1 - \alpha)(G/n) = C_L \end{aligned}$$

At the same time, by assumption, it must be valid that $C > (G/n)$, so that the lower bound on C is determined by whether C_L or (G/n) is largest. Also, by assumption it must be the case that $C < G$ and $C < [1 - f(0, 1^{n-1}, \pi)]V$, where the upper bound on C is determined by the smallest of the right-hand sides of the two latter expressions. It follows that, in terms of C , an equilibrium where no defender defends exists under the given condition. For $G \geq [1 - f(0, 1^{n-1}, \pi)]V$, this condition becomes $(G/n) < C < [1 - f(0, 1^{n-1}, \pi)]V$, which is valid under our modelling assumptions. For $G < [1 - f(0, 1^{n-1}, \pi)]V$ the condition becomes $C_L < C < G$, which is only possible if $C_L < G$.

In terms of α , if $C \geq C_L(\alpha = 1)$, given that by assumption $C > (G/n)$, by (3) no defender defends in equilibrium for any α . If instead $C < C_L(\alpha = 1)$, by (3) no defender defends in equilibrium if $0 \leq \alpha < \alpha_H$. In terms of $f(0, 1^{n-1}, \pi)$, by (3) no defender defends in equilibrium if $f(0, 1^{n-1}, \pi) > f_L$. If $C \geq C_L[f(0, 1^{n-1}, \pi) = 0]$, f_L is zero or negative, so that the condition does not further constrain $f(0, 1^{n-1}, \pi)$. If $C < C_L[f(0, 1^{n-1}, \pi) = 0]$, the condition

does constrain $f(0, 1^{n-1}, \pi)$. Finally, in terms of n , by (3) no defender defends if $n > n_L$. This condition does not further restrict n if $n_L < 2$, but does restrict n otherwise.

(ii) If all defenders defends, the individual defender does not want to deviate iff:

$$\alpha V + (1 - \alpha)G - C > \alpha\{(1/n)f(0, 1^{n-1}, \pi)V + [(n-1)/n]V\} + (1 - \alpha)[(n-1)/n]G \cdot \text{iff} \\ C < \alpha[1 - f(0, 1^{n-1}, \pi)](V/n) + (1 - \alpha)(G/n) = C_L \quad (4)$$

By assumption, a lower bound on C is (G/n) . It follows that a necessary condition for C to exist such that all defenders defend, is that $\alpha > 0$. Whether C_L constrains the cost levels for which defenders defend in equilibrium, depends on the relation between G and C_L .

In terms of α , by (4) all defenders defend in equilibrium if $\alpha_H < \alpha < 1$, where by $\alpha_H < 1$ one obtains that such ranges of α only exist conditional on $C < C_L(\alpha = 1)$. In terms of $f(0, 1^{n-1}, \pi)$, by (4) all defenders defend in equilibrium if $0 \leq f(0, 1^{n-1}, \pi) < f_L$, where by putting $f_L > 0$ one finds that such ranges of $f(0, 1^{n-1}, \pi)$ only exist conditional on $C < C_L[f(0, 1^{n-1}, \pi) = 0]$. Finally, in terms of n , by (4) all defenders defend in equilibrium if $2 \leq n < n_L$, where by putting $n_L > 2$ one finds that such ranges of n exist conditional on $C < C_L(n = 2)$. QED

Proof of Proposition 4.

- (i) If all defenders defend, it is a best response for the attacker not to attack. But given that the attacker does not attack, it is no longer a best response for the individual defender to defend.
- (ii) This follows from Equation (3) in the proof of Lemma 3, which now equally applies to the simultaneous move game against the attacker.
- (iii) In a mixed equilibrium of the simultaneous move game against the attacker, let defender j defend with probability p_j , and let the attacker attack any defender with probability q . Then the individual defender i is indifferent between defending and not defending if

$$q \left[\left(\frac{1}{n} + \sum_{j \neq i} \frac{p_j}{n} \right) [\alpha V + (1 - \alpha)G] + \left(1 - \frac{1}{n} - \sum_{j \neq i} \frac{p_j}{n} \right) [\alpha f(0, 1^{n-1}, \pi)V + (1 - \alpha)G] \right] \\ + (1 - q)[\alpha V + (1 - \alpha)G] - C \\ = q \left[\left(\sum_{j \neq i} \frac{p_j}{n} \right) [\alpha V + (1 - \alpha)G] + (1/n)\alpha f(0, 1^{n-1}, \pi)V \right] \\ + \left(1 - \frac{1}{n} - \sum_{j \neq i} \frac{p_j}{n} \right) [\alpha f(0, 1^{n-1}, \pi)V + (1 - \alpha)G] \right] V + (1 - q)[\alpha V + (1 - \alpha)G]$$

Iff

$$q = \frac{C}{\alpha[1 - f(0, 1^{n-1}, \pi)](V/n) + (1 - \alpha)(G/n)} = C/C_L$$

The attacker is indifferent between attacking and not attacking any defender i Iff:

$$\begin{aligned}
& p_i(-n[\alpha V + (1 - \alpha)G]) + (1 - p_i)(-\alpha n f(0, 1^{n-1}, \pi) V + (1 - \alpha)(n - 1)G) - K \\
& = (-n[\alpha V + (1 - \alpha)G]) \text{ Iff } (1 - p_i) = \frac{K}{\alpha[1 - (0, 1^{n-1}, \pi)]nV + (1 - \alpha)G} = K/(nC_H),
\end{aligned}$$

where this expression is smaller than 1 by our modelling assumptions.

QED