Theories of Order in Carnap's *Aufbau*

Paul Ziche

From Lines of Influence Towards Unified Discourses: "Construction Theories" and "Theories of Order" in the *Aufbau*

Rudolf Carnap's *Logischer Aufbau der Welt* is packed with references to what seem to be not only multiple, but extremely divergent philosophical traditions. A prime example is § 3, very early in Carnap's book, and devoted to the "method" (in the German original, Carnap uses an even broader term, "Der Weg") of Carnap's search for a "Konstitutionssystem", a "constructional system of concepts".[1] Without further discussion, this paragraph employs several terms in parallel in order to characterize the very aim of Carnap's project. What the title of this paragraph announces, namely an "analysis of reality" via a "theory of relations", is re-phrased as the task of solving "all problems of the pure theory of ordering/reinen Ordnungslehre". In the same paragraph, Carnap offers yet another description of his project: what he aims at is the development of a "construction theory/Konstitutionssystem". In this paragraph, thus, great importance is given to the notion of *order*, and a quick look at other early texts of Carnap confirms the relevance of this concept. According to his *Physikalische Begriffsbildung*, it is the task of science to "collect and order" insights with the aim of constructing (in the German text, Carnap is here already using the term "*Aufbau*") a comprehensive ordering, a "Gesamtordnung" of what we perceive (Carnap 1926, 1, 5). In *Der Raum*, it is ideas from projective geometry

P. Ziche (✉)
Department of Philosophy, University of Utrecht,
Janskerkhof 13A, 3512BL Utrecht, The Netherlands
e-mail: p.g.ziche@uu.nl

that make Carnap understand space as an "Ordnungsgefüge", as an ordered structure (see below, section "Order between mathematics, metaphysics, and innovative philosophico-scientific projects"). In the *Aufbau* itself, the motive of order remains prominently present. A constructional system intends to present a "uniform *ordering of concepts*" (Carnap 1928, § 157) that takes place on the level of structures, not of content or of "materially new insights". The fact that the basis of these constructions needs to be sought in relations can likewise be summarized in the notion of "Ordnungssetzungen/initial ordering concepts" (Carnap 1928, § 75).

When relating his constructional theory to other philosophical positions in the appendix to § 3 of the *Aufbau*, Carnap refers to three broad fields: "logistics", "applied theory of relations", and "construction theory/Konstitutionstheorie". For the first two, the reference author is clear; in both cases, Bertrand Russell is the most important point of departure. The third field, however, opens up the horizon of references in a rather striking fashion; that the title of this paragraph is adopted for Carnap's entire project in the *Aufbau* makes this only more remarkable. Carnap here refers to the philosopher/psychologist Theodor Ziehen[2] with his *Erkenntnistheorie auf physiologischer und physikalischer Grundlage*, to Hans Driesch, perhaps the most ambitious theorist of order with his transformation of philosophy into an *Ordnungslehre*, but also known as vitalist metaphysician of biology and author on parapsychology, and to Walter Dubislav, member of the "Berlin Group" of scientific philosophers and – together with Karl Wilhelm Clauberg[3] – author of a *Systematisches Wörterbuch der Philosophie*.[4] Carnap also draws connections to Edmund Husserl and Alexius Meinong, and, somewhat "more remotely", to the "classificatory systems of concepts" of the scientist-philosopher Wilhelm Ostwald, of the psychologists-philosophers Wilhelm Wundt and Oswald Külpe and of the theologian *plus* philosopher Paul Tillich.[5]

This selection of authors is more than just remarkable. What connects them? If there are connections, how strong are they? The first point to notice is that Carnap

---

[2] Ueberweg's history of philosophy in the edition from 1906 only briefly names Ziehen as one of the philosophical authors dealing with the status of psychology and with psychology's role within epistemology; as far as established movements are concerned, he is related to the "immanence philosophy" of which Wilhelm Schuppe was the main protagonist (Heinze 1906, 375–6). The 12th edition from 1923 emphasizes his role in the development of "empiriocriticism" beyond its original formulation in the works of Avenarius and Mach, and stresses the positivist elements in Ziehen's philosophy (Oesterreich 1923, 401–407). On Ziehen, see the website maintained by A. Herbst and the paper by Th. Mormann in this volume.

[3] While Dubislav is given due attention – with respect to his logical and methodological ideas – in Milkov and Peckhaus 2013, Part IV, his *Wörterbuch* and his cooperation with Clauberg are hardly discussed in the literature. Clauberg, a medical doctor, is sometimes wrongly identified with the NS-doctor Carl Clauberg. The Kant-Gesellschaft names him in 1920 as a new member with the profession of being a "Medizinalpraktikant". In 1929 he became Privatdozent, in 1935 Professor for medicine in Berlin.

[4] On Driesch, Ziehen and Dubislav see in more detail section "Getting closer to Carnap: Ziehen, Driesch, Dubislav".

[5] The inclusion of the theologian Tillich may seem surprising; however, Tillich was indeed well known in this period for his ideas on ordering the sciences (Ziche 2004).

himself introduces some caveats. The projects of Ziehen, Driesch and Dubislav are seen as "independent" from each other (§ 3), and Carnap distances himself from their "systems": "We will indicate agreements between our system and the just-mentioned systems on the few occasions when they occur, but our approach is, on the whole, quite different from those others because of the methodological tools which we shall employ" (§ 3). With respect to Ostwald, Wundt, Külpe and Tillich, Carnap himself explains that they remain "remote" from his own project because they lack a derivation of "concepts from one another".

The methodological challenge posed by these references is easy to summarize: what can we learn about Carnap's project when we start from the assumption that there are indeed important conceptual links that can support the joint occurrence of these authors and texts in the *Aufbau*? Carnap's somewhat restrained statements as to the importance of these authors for his own project become balanced by the intricate web of cross-references, implicit and explicit, between these authors, and by the fact that the notion of "order", one of the core concepts that Carnap himself adopts for characterizing his goals in the *Aufbau*, in all of these authors.

The recent literature on Carnap has emphasized throughout that a straightforward logical-empiricist interpretation of the *Aufbau* is highly problematic, and has devoted much work to reconstructing numerous lines of influence in the *Aufbau*. This has resulted in a considerable number of not really compatible readings of this text. Three methodological trends stand out: the singling out of individual lines of influence, either in terms of single authors or of particular movements in philosophy[6]; reading the *Aufbau* in a "reconciliatory" way, based upon viewing the text as drawing upon highly diverse contexts in an ecletic fashion (Mormann, this volume); or, which is in a way a variation of the second option, an interpretation of the text as consciously bringing together diverse traditions and thus – as is argued in Michael Friedman's *Parting of the Ways* (Friedman 2000) – as questioning deeply engrained tensions between philosophical traditions.[7]

I propose a different approach. Taking seriously the idea that one can indeed establish a common and broadly shared conceptual core of "theories of order", I suggest that we should distance ourselves as far as possible from thinking in terms of (more or less) clearly circumscribed forms of philosophy. This implies that we should free ourselves from feeling too greatly surprised when seeing Carnap in peaceful and fruitful interaction with apparently divergent movements. The surprise we are inclined to feel when looking at his references to those different movements is precisely a case of singling out some strands from a continuous discourse that

---

[6] For an analysis that is very strongly focussed on one single author – Husserl – see Haddock 2008.

[7] For an overview, see again Mormann's text in this volume. For recent examples of this broader approach, see Carus 2007; Awodey and Klein 2004; Köchy 2010. – A clear example of an approach that – despite its considerable breadth – narrows down the field, is to be found in Coffa 1991. See f.i. p. 1: "Within the field of epistemology one may discern three major currents of thought in the nineteenth century: positivism, Kantianism, and what I propose to call the semantic tradition." – Gereon Wolters' (1994, 2004) discussion of various styles of philosophizing would deserve more attention in this context, though Wolters, too, is focussing quite strongly on tensions between the various styles.

does not provide clear joints at which to carve it. What requires explanation, then, is not so much how Carnap succeeded in bringing together those movements, or what he took from each of them individually, but rather why we think them as being so diverse and irreconcilable. This question cannot be answered here, but will be raised again at the end of this paper. I'll repeatedly refer to one instance in particular – the role of psychology and the alleged tension between psychology and logic or logically inspired philosophy – where the confrontations that we are wont to expect were, to a surprising extent, absent in the discourses that Carnap latches onto.

This historiographical stance makes the strong assumption that the juxtaposition of the authors in Carnap's lists of references has a function beyond being a strategy for a broad legitimation through referring to as many reference authors as possible.[8] What this approach promises to give us is more insight into the dynamics of the philosophical discourse around 1900: a discourse that refuses to come neatly packaged and which precisely for this reason requires its participants to search for philosophically ambitious concepts that could function within this entire field, and that could hold it together. "Order" is a prime candidate here.

"Order" between Mathematics, Metaphysics, and Innovative Philosophico-Scientific Projects

"Order" is a term with a strong theological-metaphysical tradition. Around 1900, it is a term discussed by numerous thinkers: authors as diverse as Russell, Driesch, Cassirer, Ziehen, Whitehead, and many others ascribe a prominent role to this notion. Carnap's usage of the term "order" does indeed – and that will be the claim of the following discussions – refer to this broader discourse. However, widespread though discourse about "order" was around 1900, these theories were never really established in a unified form. While "order" was a highly prominent concept in this period, the history of theories of order does not amount to a genuine success story. Let's start the reconstruction of discourses about order with some significant interactions, in the name of "order", between mathematicians/logicians and psychologists.

"Order" was a key concept in nineteenth century innovations in mathematics and logic, strongly related to innovative and philosophically important issues in algebra, in the axiomatization of geometry and in set theory.[9] In fact, it arises in a number of

---

[8] The equally interesting and intricate issue of Carnap's "reference politics" – raised by Mormann in his paper in this volume – is, thus, consciously left out of consideration here.

[9] In this paper, I shall focus on projective geometry as an important inspiration form mathematics. However, the highly general conceptualization given to theories of order by the various authors discussing this concept might make it possible to also discuss the way how the relationship between these sub-fields of mathematics was perceived in this period (and then, interestingly, both within mathematics proper – see, for instance, the broad range of theories that Whitehead includes in his treatise on algebra, Whitehead 1898, and outside of mathematics proper).

the key mathematical discourses in this period. One of the most prominent of these concerns *projective geometry* (see already Nagel 1939). Projective geometry studies strong generalizations of existing geometry, characterized by those properties that remain invariant under projections. As is shown in a number of beautiful duality theorems in projective geometry, statements about the relationship between lines and points can be translated into dual statements about point-line relationships; lines and points, in those theorems, become interchangeable. Metric properties do not matter here; projective geometry investigates what – in Staudt's classic from 1847 (Staudt 1847) – has been called "Lage", "situation". Situations are characterized by relational properties. The most important example of a relational notion that gains prominence in debates about projective geometry is that of "betweenness": what does it mean, for example, for a point to lie between two other points? This question became important in an investigation of the Euclidean axiomatics for geometry that fails to incorporate a notion of betweenness. Put differently, projective geometry does not talk about the traditional objects of geometry such as points or lines, but about the relations that hold between these objects.

In *Der Raum* (Carnap 1922), Carnap is very clearly working in this tradition. The formal aspects of space need to be investigated on the basis of a thoroughgoing generalization of traditional geometry that leads from talking about points, lines and planes to a "*pure theory of relations*" or "*theory of order/Ordnungslehre*" (the very term employed by Hans Driesch as the name for his foundational project in philosophy), clearly inspired by projective geometry (e.g. Carnap 1922, 18–21; see the extensive discussion in Mormann 2003; on a more general level, see Nagel 1939). What a formal analysis of space has to study, are "Ordnungsgefüge" (Carnap 1922, 14), ordered structures. As Thomas Mormann reminds us, Carnap's usage of the term "Ordnungsgefüge" in *Der Raum* is itself not part of the jargon of mathematics, although mathematical issues clearly are essential for Carnap's early analysis of space (Mormann 2003). While this term – again a term that is clearly influenced by Driesch's writings – is not a term proper to mathematics, it is a term indigenous in a broader discourse within which mathematics in general and projective geometry in particular can be given a place.

The thinker who perhaps best exemplifies the requirement to think in terms of "order" on all levels, from the highest abstractions in mathematics and logic down to the level of his everyday life, is the mathematician *Moritz Pasch* (on Pasch see Schlimm 2010). An order fanatic also in daily life, he set out to repair the omissions in Euclidean axiomatics, thereby giving the axioms for "betweenness" a central role; he explicitly generalized mathematics from talking about geometrical objects (in terms of what he calls "Stoffwörter", substance terms) to a relations-based mathematics (in which the "Fügemittel", the connectives, are what determines the objects; Pasch 1926, 261), and he pays close attention to logical issues such as implicit definitions. Pasch also exemplifies the complex network in which these theories unfold. In editing his collected papers, he was supported by his Giessen colleague, the Gestalt psychologist *Kurt Koffka*, and Pasch not only published in

Hans Vaihinger's *Annalen*, but explicitly viewed his approach to mathematics as supportive of Vaihinger's "as-if-philosophy" (e.g. Pasch 1921).[10]

Some general strands emerge already in these preliminary remarks on the notion of "order" as it was discussed around 1900. The driving motivation behind the innovations in mathematics appears to have been the search for a more general (if possible, for the most general) approach in mathematics, in the philosophy of mathematics, and in logic. Thinking in terms of relations, not in terms of fixed first elements, seemed to be the most promising way to achieve this generality. This implies that the basic elements within a generalized mathematics (and, analogously, in generalizations in other fields) need not be atomistically simple; complex elements are possible. The link to psychology is remarkably strong, and it is two-sided. Not only does, for instance, Pasch display no contact fear in his cooperation with Koffka. Psychologists in this period, as part of their search for a methodological basis for their field, and – perhaps even more importantly – for allies in the conflicts between various forms of science, were particularly quick to pick up innovations at the borderline between fundamental mathematics and logic (Ziche 2002; 2008 chap VI.5).

It is precisely these ideas that can be traced throughout in discussions about "order" around 1900. *Bertrand Russell* devotes a brief manuscript note from 1898 to this concept (Russell 1990). He is strongly influenced in this text by the Italian mathematician Mario Pieri and by Pieri's ideas concerning projective geometry, and Russell also refers to Staudt. Russell is interested in what projective geometry can teach us about the basic concepts of geometry in particular, and of mathematics in general. In his note, he combines more clearly geometric notions with a discussion of what minimally determines a series and what is required for establishing an unequivocal sequential order in closed or open series. The motive of generalization, the step away from objects with certain qualities individually ascribed to these objects and towards relational analyses, is crucial for his approach. Russell discusses generalized sets of axioms,[11] and "order" is explicitly treated as a concept of the highest possible generality in mathematics: "Order is something which all series have in common" (Russell 1990, 355). Russell is clearly aware that there might be a psychological perspective on these issues (when three points on a circle are close to each other, it seems natural – in a very basic Gestalt-like argument – to say that they have an order, but this does not work when they form an equilateral triangle (Russell 1990, 353)), but he clearly recommends a formal analysis as being more general than the psychological approach. In the *Principles of Mathematics*, Russell takes up all these problems in a chapter devoted to "The meaning of order" (Russell

---

[10] While this may appear surprising in the light of the pessimistic and Nietzschean aspects of Vaihinger's project to reveal that our thinking is everywhere, in philosophy, mathematics, and the sciences, based upon „fictions", there is also a strongly mathematical strand in his arguments; Vaihinger has been particularly interested in exploring his theory's basis in, and implications for, topics in mathematics.

[11] Take an example: from statements of the form "Any three not collinear planes determine a point", he comes to the more general one that "All space consists of a collection of points, and three qualities may be found, which are all possessed, though in different magnitudes, by different points" (Russell 1990, 346–7).

1937, 207–217; projective geometry is given a separate treatment, Russell 1937, 381–392). Again, Russell here intends to answer an extremely general question, namely: "What *is* order?" (Russell 1937, 207), and he answers this question in terms of the fundamental role of the relation of "betweenness" in the production of series.

With *Hans Driesch* (see section "Getting closer to Carnap: Ziehen, Driesch, Dubislav"") and *Wilhelm Ostwald*, two further authors from Carnap's list share Russell's conviction that progress towards a more general and therefore more fundamental conception of science, logic and epistemology requires us to take account of these innovations in mathematics, and they both embarked upon large-scale projects to bring about this generalizing step themselves. In this context both explicitly claimed – controversially, to be sure, but not absurdly, when one considers the context – that they were the genuine discoverers of modern logic, by virtue of their theories being even more general and fundamental than those of logicians working in a more mathematical tradition (Ziche 2011). In both cases, they, too, proclaim a theory of order as providing the basis for all forms of reasoning in science and logic.

In the case of Ostwald – chemist, Nobel Prize laureate, proponent of a monistic world view, and innovator of the philosophy of nature as a philosophical subdiscipline (on Ostwald, see Görs et al. 2005, on Carnap and Ostwald see Carus 2006, 66–69, who remains strongly focussed on Ostwald as a positivist) –, the theory of order lies at the very foundation of his (pyramidal) ordering of the sciences. The science of order is more general than traditional mathematics, and also more general than the monistic foundation that Ostwald gave to the natural and cultural sciences in terms of his "energetics", i.e. the idea that the law of the conservation of energy is the most general basis for analyzing natural (and cultural) processes. The formation of scientific concepts, on the basis of an ordering of impressions – stated in a way that is closely analogous to what Carnap discusses in *Physikalische Begriffsbildung* – is one of the topics that a theory of order has to analyze (see, e.g., Ostwald 1914, 106–123). For Ostwald, "order" is the key concept within any ordering of the sciences, not only in the elementary sense of arranging these sciences in an ordered system, but also in the sense of analyzing the most general methodological procedures that any science has to follow.[12] A particularly striking example of the generalizing attitude associated with the notion of "order" in the context of a classification of the sciences can be found in *Paul Oppenheim*'s (the wonderfully efficient co-author of classical papers together with Hempel, Kemenyi, Putnam, and others) texts from the 1920s. Under the title of a *Natural order of the sciences*, he presents an ordering scheme with a formal structure (based on a coordinate system that also allows for intricate coordinate transformations) within which all sciences can be placed onto one and the same horizontal level. Oppenheim's ordering scheme is multi-polar. He does not assume a clear directedness from simple to complex, and does not work along dichotomies such as natural sciences vs. humanities or metaphysics vs. empirical sciences. All these fields get a place within his ordering system that Oppenheim himself can, precisely because of its horizontal inclusiveness,

---

[12] Given the importance of the issue of concept formation in Ostwald, I cannot accept Carnap's negative verdict concerning the lack of derivative relations between concepts in Ostwald.

describe as being "tolerant" (Oppenheim 1926, 1928; on Oppenheim, see Müller and Ziche 2013).

Two further examples may suffice to illustrate how richly textured the field of theories of order is. From Ostwald, the self-proclaimed logician and the widely acclaimed philosopher of nature, a line can be drawn to *Alfred North Whitehead*. Whitehead discusses projective geometry (under the title "descriptive geometry") at length in his *Treatise on Universal Algebra* (Whitehead 1898, 214–228; see also Gandon 2004), and "order" is also one of the key concepts in Whitehead's *Process and Reality* from 1929 (Whitehead 1978). The "order of nature" develops in an organic fashion, starting from a "'given'" that is itself a complex "concrescence of objectivations" (Whitehead 1978, 83). "Order" is a generic term; there always needs to be some specific order, not merely "order" in a vague and general sense (Whitehead 1978, 83). Whitehead also emphasizes another facet of the ideal of order: it strongly appeals to our feelings and emotions; a "lure for feeling" arises from "the enjoyment of this ideal" (Whitehead 1978, 85).

As a final example, look at *Ernst Cassirer* whose importance for Carnap's *Aufbau*-project has been discussed in a number of important publications (Sauer 1985; Friedman 2000; Richardson 1992, on Cassirer's reception of formal logic see Heis 2010, on his philosophy of mathematics Mormann 2008). Cassirer's *Substanzbegriff und Funktionsbegriff* from 1910 is referred to explicitly in the *Aufbau* in the context of relational descriptions of structures (§§12, 64, 75). However, the issue of "order" ranges even broader in Cassirer's writings. In his essay review on "Erkenntnistheorie nebst den Grenzfragen der Logik" from 1913, Cassirer places the recent advances in epistemology and logic in an existential dimension. In all areas of knowledge, the "consciousness of the general connection" has become "alive" (Cassirer 1913, 1). An insight into this general connectedness can serve as an antidote against omnipresent "Zersplitterung", fragmentation, that can be repaired via the resources of logic, more precisely via a theory of those concepts that allow us to study the realm of knowledge, "das Ganze der Erkenntnis" (Cassirer 1913, 13), in its systematic unity. Those most fundamental and most general concepts need to be "ultimate notions of form", "letzte Formbegriffe", that express the possible types of relations between contents. The "object" dissolves in a texture of relations that is held together by highest rules and principles – and it is this texture that Cassirer explicitly calls "order" (Cassirer 1913, 13–15, 53). His paradigm examples are taken from the mathematical problems of serial order and from an understanding of structured manifolds. This is as much an issue in mathematics *as* in the physiology and psychology of perception; a "concept" is nothing but the conscious perfection ("bewußte Vervollkommnung und Durchbildung") of this structure.[13] Again, innovations in fundamental mathematics, the search for

---

[13] Cassirer uses the notion of "order" also to discuss the difference between idealism and realism (Cassirer 1913, 53): while idealism views the "cognized order/erkannte Ordnung" as that what is ultimately objective, the realist has to relate objectivity to absolute substances. Külpe's realistic philosophy (Külpe 1912–1923) is important for Cassirer here.

ultimately general concepts, and psychological aspects – both in individual psychology and in the analysis of the spirit of an entire epoch – come together.

A sequel to this article from 1927 takes up this psychological dimension in its very title: "Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie". The term "order" continues to be crucial here (Cassirer1927, 65, here as "Zuordnung"), and Cassirer indeed studies affirmatively the possible forms of interaction between psychology and a foundational theory of logic, motivated by mathematical innovations. What he aims at is an overarching unity of problems, a "übergreifende Problemeinheit" that covers both logic and psychology, thereby showing that he has no contact fear at all with respect to psychology.[14] In this text, it is the notion of meaning, "Bedeutung", that has to provide this uinty. The web of references spun by Cassirer is comparable in complexity and range with what we encounter in Carnap: Russell, Ziehen, Külpe, Schlick are all referred to in his paper (Cassirer 1927, 50–1, 68).

These ideas remain alive in later texts by Cassirer. A particularly interesting example is a manuscript from 1940 on "basic phenomena", "Basisphänomene" (Cassirer 1995, 111–195; on the problematic dating, see Möckel 2005, 294). According to Cassirer's critical analysis of the Vienna circle's positivism, basic phenomena cannot be arrived at in a formal procedure (Cassirer 1995, 118). They need to be experienced. In this context, he embeds his analysis into an extremely broad historical panorama, including both Goethe – as a witness for the idea of experientially accessible basic phenomena – and another author from classical German philosophy, Friedrich Heinrich Jacobi (together with Hume, Fries/Nelson, Dilthey), as supporting the idea of a direct access to a realm of basic phenomena. Again, psychology is a partner in his arguments; not, however, in the form of a mechanistic psychology of associations, but as a descriptive psychology à la Dilthey, Husserl, Natorp, Hönigswald (Cassirer 1995, 138sqq.), or in the form of an experimental psychology of thought as developed by Oswald Külpe and his colleagues at Würzburg (Cassirer 1995, 141; on Külpe and his school see section "Conceptual trends: Complex elements and abstract content").

Many of these ideas are taken up, in changing contexts, in later years. Even if the theory of order did not live up to become a generally acknowledged success story, we still find a chapter on "Ordering" in Nelson Goodman's *Ways of Worldmaking* (Goodman 1978, 12–15; see also Carnap 1963, 19, on Goodman's being the first author to propose an improved system along the lines of the *Aufbau*). Even if Goodman's discussion of order may seem to stand in a thoroughly changed philosophical and logical context, we can still find the key ideas of the order-discourse in the 1920ies in later texts. The Dutch logician E.W. Beth addresses the issue of order

---

[14]Cassirer 1913, 36–43, gives an extensive discussion of Ziehen's ideas concerning epistemology. Interesting for discussions about psychologism – see below, sections "Some Carnapian implications" and "Whence demarcation? – Concluding remarks" – is Cassirer's statement (Cassirer 1913, 44) that Richard Hönigswald inverts Carl Stumpf's claim that only what is psychologically adequate can find a place within logic, in a particularly illuminating fashion: psychology must not fail to conform to the "logical notion of truth".

in 1960 in a paper on "Ordnung in der Logik". In this text, he rejects, in precisely the terms that we already encountered in early Carnap, the reproach that mathematical logic itself does not present us with an "ordered structure/geordnetes Gefüge" (Beth 1962, 161) due to the co-existence of various theories in the foundations of logic (intuitionst, classical, multi-valued, modal). Beth in particular intends to criticize this thesis, and he, too, requires us to advance the level of generality on which we have to deal with philosophical issues. He introduces a category of "abstract philosophy", dealing with, in an open list of issues, general-particular, necessity, set, category, infinite-finite, and many more, thereby emphasizing that many discussions of these issues in the classical texts of philosophy bear close resemblance with modern mathematical logic.[15]

Conceptual Trends: Complex Elements and Abstract Content

For Carnap's constructional system, just as for all the authors mentioned so far, the basic elements of the system need not be simple in the sense of being ultimate atomistic objects or representations. The other key idea that all the theorists of order share is the conviction that we need to analyse reality in terms of *relations*, not of substances or ultimate atoms. Finally, the possibility to have *experiences* of these complex ultimate states is addressed by all the authors mentioned so far. This turns out to be the core of the theories of order as they were discussed around 1900: we can and should indeed begin with complex elements that can be experienced in their very character of being first foundations. In the perspective of many of the authors in the 1920ies, it was precisely the generality of the theories that could be built from these assumptions that made it possible to neglect further technicalities (for instance the questions how 'structures' might relate to 'relations of order'). Carnap's *Aufbau*-theory of "Elementarerlebnisse" that need to be thought of in an anti-atomistic fashion and that are unanalysable in the atomistic sense but that nevertheless allow for a further study via quasi-analysis, as presented with explicit reference to discussion in psychology in §§ 67–9 of the *Aufbau*, on the one hand fits this context, while Carnap can also be read as exploring how far we can get in further exploring these notions in a logic-inspired framework.

Via their experiential dimension, the theories as presented so far become related to two further discourses, namely that of psychology (or more precisely the discourse at the intersection between philosophy and psychology, without being intended to by psychologistic at least), and to the idea that there is a direct experience of logical certainty. The conviction that we not only need, but also can experience complex basic states binds together virtually all of the protagonists of the debate about order.

In which sense and to which extent this experiential aspect is taken up in the *Aufbau* should be an issue for further discussion. Carnap himself refers to authors

---

[15] The metaphysical and theological dimensions also remain present, see e.g. Schmidt 1956, who at the same time emphasizes the relevance of logic.

working in this line. A particularly strong statement to the effect that basic logical or epistemological notions have an experiential dimension can be found in the writings of *Johannes Volkelt*. In his small 1922-booklet on *Gefühlsgewißheit*, Volkelt discusses the possibilities and limitations of experiencing intuitive certainty. It is in fields such as metaphysics, aesthetics or "the science of values/ Wertwissenschaft" that intuitive certainty is particularly valuable. But also in these fields, intuition needs to be employed under the continuous control of thinking. However, we encounter important aspects of intuitive certainty also in the realm of logic. There is, according to Volkelt, a distinctive "feeling for the logical", a "*Gefühl für das Logische*" which is – in agreement with the anti-atomistic tendencies that have already been stated – not a feeling for individual facts, but for their connection (Volkelt 1922, 24).[16] In the work Carnap refers to in the *Aufbau,* Volkelt's *Gewissheit und Wahrheit* from 1918, Volkelt discusses the issue of intuitive certainty (Volkelt 1918, 538–558; he, too, refers to Jacobi in this context, p. 544) in the context of an argument directed at finding, yet again, the most general form for philosophical theorizing. Volkelt argues for epistemology as occupying this prominent place, and for epistemology's thus being even more general than a theory of order (Volkelt 1918, 108). The fundamental statements of epistemology are explicitly called "neutral", they are not based upon an emphatic concept of subjectivity, and they should be seen as being pre-logical (Volkelt 1918, 59–64).

At this point it is worthwhile to make a brief excursion and discuss another author on Carnap's list, *Oswald Külpe*. Best known in philosophical circles in his time for his project in realistic philosophy (Külpe 1912–1923), he was also hugely influential as an experimental psychologist of thought, and it is in this function that Cassirer discusses him. Külpe and his group in Würzburg, in which the first generation of Gestalt psychologists got their training, claimed to have established experimentally that human thought is not built up from simple atomistic elements; irreducible thought states can be vividly experienced as giving direction to our thought even if the telos of these directed thoughts itself is not represented (for an overview, see Ziche 1999). These states are devoid of concrete content, where "concrete" is understood in the sense of consisting of simple sense experience. Still, those states have a distinctive phenomenal character in their being directed towards some cognitive goal, i.e. they themselves are characterized by being strongly relational. Külpe's and his school's work is important in its implications for the relationship between psychology and philosophy. In distancing itself from associationist and empiricist theories of the genesis of mental content, their thought psychology could even claim to experimentally test philosophical theories such as Husserlian phenomenology without being troubled by anxieties concerning a psychologistic fallacy (Ziche 1998).

---

[16] Again, the historical contextualization is broad and complex; let me only point out some features: again, Volkelt explicitly refers – as Cassirer also did, see above – to Jacobi, but places him in a long list also containing mystics (Paracelsus, Guyau). Fichte, Driesch, and Husserl are also among the authors he refers to (Volkelt 1922, 11, 14).

In summarizing the conceptual trends discussed so far, we started with the idea of *complex elements*. A precisely analogous characterization can be given of the generalized theories in fundamental mathematics: a more fundamental theory in mathematics can be said to be richer in content than the more derivative ones, the kind of abstraction involved in arriving at these generalized theories may be called a *contentful abstraction.* In ordering the sciences or in admitting forms of psychology that may enter into a constructive dialogue with logic, these projects display a high degree of tolerance, but of a tolerance that is at the same time embedded into a project for arriving at the very foundation of scientificity and that we may thus dub a form of *rigorous tolerance*. With the idea of tolerance, we see a particularly clear example of a term that originates in an integrative discourse aimed at bringing together the various sciences, and that then acquires a far more technical meaning in, for instance, Carnap's own syntactical and semantical considerations. The same holds for notions such as "relation" or "structure", "function" and other related concepts that all function both as technical terms within the technical contexts of logic and the philosophy of mathematics *and* as generalizing terms in broader debates. However, we should be aware that many participants in the debates that are reconstructed here would not accept the idea that a stronger focus on logical precision would get us outside the broadly shared conceptual framework.

Getting Closer to Carnap: Ziehen, Driesch, Dubislav

How are these ideas reflected and worked out in the authors Carnap refers to more affirmatively in § 3 of the *Aufbau*? Let's start with *Theodor Ziehen*, an author who occupies in many respects an intermediary position. Ziehen, medical doctor, psychiatrist, private philosopher, explicitly deplores the lack of attention devoted to the notion of "order" in the context of mathematics, more specifically within set theory (Ziehen 1917, 25). The only author he refers to here is Driesch. This lack of a clear analysis of the mathematical/logical meaning of "order" is the more problematic because, according to Ziehen, this notion is as fundamental for set theory as that of cardinality. Many of the then current suggestions for a definition of the notion of "order" are dismissed by Ziehen as being tautologies or mere stipulations. Ziehen himself, however, does not give a definition either; the most specific hints one gets from his text are the insistence on similarity as essential for talking about order, and a reference to the well-ordering theorem (Ziehen 1917, 27–30) – and it is clear, yet again, that Ziehen views an answer to the question as to the relationship between logic and set theory as essential for understanding which kind of theory is the most fundamental in mathematics.

Ziehen's enormous epistemological treatise from 1913, *Erkenntnistheorie auf psychophysiologischer und physikalischer Grundlage* – the Ziehen-text Carnap refers to in the *Aufbau* –, clearly participates in all the discourses discussed so far. I shall highlight three issues: the search for generalized concepts and theory forms; the role of intuitive certainty; the notion of reduction.

1. One of the first features that strikes the reader are the numerous neologisms that Ziehen suggests as replacements for traditional epistemological notions. All traditional conceptions in epistemology are bound to induce misleading prejudices (Ziehen 1913, 1–2). In place of "sensations", "Empfindungen" (a term he still employs in his psychological writings), he suggests a new term, "Gignomene", appropriated by Ziehen into German in a consciously awkward way as "Werdnisse". The "Gignomene" provide the "general factual basis/allgemeinen Tatbestand" for all forms of epistemology, and it is epistemology's task to "classify" and "order" these ultimate elements (Ziehen 1913, 3).[17] Again, the sources Ziehen refers to are numerous and include, among others, Jacobi – again! – and Jacobi's critique of Kant (Ziehen 1913, 2), Vaihinger, the British empiricists, and Avenarius who engaged in similarly neologistic projects (Ziehen 1913, 9).

   These most general notions, however, do not yet solve the problem of also providing the "most general representation" that we can entertain (Ziehen 1913, 499). What these notions do not provide, namely, is a most general property. Ziehen's argument departs from the conviction that all our representations need to have content and therefore cannot be completely general (Ziehen 1913, 44, 499). It is here that Ziehen occupies a transitory position: the most general elements of epistemology are not themselves able to support a most general account of what can be known; genuine unity and universality are possible, but only in the form of a "Weltbild", not in the sense of a general property. This "Weltbild" is itself the result of an ordering process consisting of collecting the "Gignomene" as completely as possible, and of then classifying them (Ziehen 1913, 516).

2. The step towards increasingly more general forms of epistemology has a surprising epistemological implication: It distances the entire project from the search for certainty. In the preface, Ziehen goes as far as to state that an epistemology in the sense of a "'theory of certainty'" does not exist (Ziehen 1913, V). His argument for this strong claim derives from the idea that any attempt to establish objective certainty refers us back to forms of subjective certainty, more specifically to the "feeling of certainty/Gewißheitsgefühl" and to consistency in making associations (Ziehen 1913, 497). What an epistemology should strive at, if certainty is unattainable, has already been stated: the ordering of the "Gignomene" (Ziehen 1913, 498). Hardly anywhere can we find a stronger statement as to the promises that could be attached to the project of ordering: the result of the process towards ever more general forms of epistemology can replace the traditional goals of any form of epistemology.

3. The status of the "Gignomene" at the basis of all epistemology raises difficult questions for any attempt at a reductionist analysis of, e.g., sensations. Ziehen deems it possible to further analyze the "Gignomene" into different constituents without assuming yet another layer of elements. Thomas Mormann has worked

---

[17] Ziehen introduces quite a number of other novel concepts; one of the most interesting ones is that of a "Koinade" (Ziehen 1913, 15–6), which stands for a clearly demarcated complex of sensations. – In the supplements to Ziehen 1913, he refers, among other authors, to Hermann Grassmann, thereby making explicit his indebtedness to new foundational discourses in mathematics.

out in detail how Carnap's notion of quasi-analysis can be understood as pursuing a comparable project (Mormann, this volume).

The arguments start getting repetitive when we turn towards the most outspoken order-theorist of this period, *Hans Driesch*, from whose writings, as has already been stated, Carnap derives important terms.[18] Driesch himself gives an interesting and diverse pedigree for his own *Ordnungslehre*: Meinong and his pupils; Russell, as far as logic in a narrow sense – restricted to the applicability of mathematics – is concerned; the Neo-Kantians Nicolai Hartmann and Johannes Rehmke; Henri Bergson, but with him there are only occasional points of contact (Driesch 1923, 10).

"Ordnungslehre" is Driesch's term for "logic" (Driesch 1923, 2). In introducing this term, his claims clearly go beyond just a change of labels. A theory of order, as conceived by Driesch, is far more general than traditional logic; it comprises what "'logic', 'ethics', 'aesthetics' are, taken together", if only one frees logic from epistemology which is an alien ingredient as far as logic is concerned (Driesch 1923, 4).[19] In particular, in its very generality, it goes beyond the psychologistic dualism between the normative and the descriptive (Driesch 1923, 4–5). At the same time, the "Ordnungslehre" replaces traditional epistemology by a more general form of philosophical theory. A theory of order is not about cognition, and would remain intact even if the solipsistic stance were to hold. Truth issues remain undiscussed; correctness in thinking is warranted via an "immediate knowledge of correctness", a "unmittelbares Richtigkeitswissen" (Driesch 1923, 5), which is stated in terms reminiscent of Külpe's school in psychology. In the highly general sense of being founded upon self-reflection ("Selbstbesinnung") with the possibility of getting access to this "consciousness of being directed" as the basis for correct thinking, also logic has a psychological basis. Psychology and logic both come to mutually support each other; in its function of being a "reflection on one's own self/ Selbstbesinnung", psychology comes before logic, while logic is prior if understood as "science of laws/Gesetzeswissenschaft" (Driesch 1923, 6sqq.). The issue of truth or falsity is eclipsed by the fact that a theory of order is so general as to go beyond issues of cognition and of certainty. Thus, the epistemological debate that first gave rise to the psychologism controversy is of no relevance for his account (Driesch 1923, 8).[20]

---

[18] On the role of Driesch for *Der Raum* see also Stone 2006, with a good account of the status of Driesch in his time. Driesch has produced an autobiography which informs in detail about his career (Driesch 1951).

[19] The theory of order also determines the fundamental categories of metaphysics. See Driesch 1922, III: "Theory of order, or logic, that can also be called – in the broadest sense of the term – theory of experience or theory of science"/"Ordnungslehre oder Logik, die auch, im weitesten Sinne des Wortes, Erfahrungslehre oder Wissenschaftslehre heißen kann", provides the basic structure of the "Wirklichkeitslehre", the theory of reality.

[20] This point is made very forcefully in Driesch 1913 (this text is almost entirely structured along the Würzburg School's ideas on experimental thought psychology): when taking the theory of order as a foundation, it becomes clear that many researchers, "logicians" as well as "psychologists", work in the same direction, and Driesch consequently emphasizes the surprising agreement

Again, it is the notion of order that supports this going beyond traditional notions of certainty, and towards a generalized form of philosophy that transcends the divide between logic, epistemology, and psychology. Driesch defines "knowledge" via the experience of order: "I know when I consciously have something ordered before me" ("Ich weiss, wenn ich bewußt Geordnetes mir gegenüber habe"; Driesch 1923, 1). This experience of order also yields the definiens for "science/Wissenschaft" as that kind of "knowledge that is consciously present as being complete and organically structured/das bewußt vollständige und gefügehaft gegliederte Wissen" (Driesch 1923, 16). Order can and needs to be experienced ("geschaut"), in the form of syntheses that are not produced consciously by the person who has the experience. Again, it is important to be aware of the degree of generality Driesch is operating upon here: we may no longer think in terms of a contrast between the conceptual and other forms of experience (Driesch 1923, 29–30, with reference to Vokelt's "Gefühlsgewißheit"). Various forms of elements can all be perceived as being "ultimately ordered", with the "Tone der ordnungshaften Endgültigkeit" (Driesch 1923, 320): both intuitive (green, red, c-flat, warm, sweet, pain) and abstract ("this", "such"; "related"; "different"; "because",…) qualities can be perceived in this way. None of those elementary qualities is simple, we always encounter "complexes" (Driesch 1923, 321). Külpe's Würzburg School and Gestalt psychology form an important historical context for these arguments (Driesch 1923, 341).

In Carnap's list of order theoreticists in § 3, the author whose contribution to a theory of order is most difficult to place is probably *Walter Dubislav*. Dubislav gives an extensive discussion of a theory of structures in his *Die Definition* from 1931 where he devotes an entire chapter to a "structural theory/Strukturtheorie" of the coordination of signs and objects via definitions (Dubislav 1931, 96–106). The text referred to by Carnap, Dubislav/Clauberg's *Systematisches Wörterbuch der Philosophie*, likewise departs from a theory of definitions, and intends to "systematically" present the key concepts of philosophy, following the techniques used to define concepts within axiomatic definitions, and at the same to analytically describe existing usage without falling into dialleles, i.e. giving circular definitions. "Definition" itself is defined (not in an introduction, but in the lemma "definition" itself; in fact, the introduction continuously refers to the lemmata in the dictionary that thereby comes to include its own theory) as a reduction of a complex symbol to primitive symbols (Clauberg and Dubislav 1923, 117). The dictionary is primarily organized in terms of "definition chains/Kettendefinitionen", where a "Kettendefinition" is defined in order-theoretic terms as "an ordered set whose elements are definitions" (Clauberg and Dubislav 1923, 117). Ordered sets themselves become defined via axioms (lemmata "Ordnung" and "Menge, geordnete", the axioms on p. 293). What Dubislav and Clauberg aim at, is an ordered set of definitions that can order and arrange all fundamental concepts in philosophy in a way that (a) stands up to the demands required by a theory of definition, and (b) does justice to the existing usage of these terms. The impetus towards order is

---

in their views (Driesch 1913, V–VI). What he intends to present under the title of a "logic" is a concept that *both* logicians and psychologists claim as theirs.

spelled out in the broad discussion of the classification of the sciences (Clauberg and Dubislav 1923, 538–543) which is supported by extensive diagrams. A rather striking feature of their definitional project is the absence of a list of primitive concepts; they apparently see their project as completed once they succeeded in arranging existing definitions into a clearly ordered whole.

Some Carnapian Implications

It has already been shown that the notion of "order" does indeed pervade the *Aufbau*, and that Carnap uses it repeatedly for characterizing the *Aufbau*'s project as a whole. Strong evidence for the importance of this notion can be derived from Carnap's discussion of "problems of essence" in §§ 158sqq. Thinking in terms of order helps solve, or dissolve, traditional metaphysical problems. "Order forms" are what lies at the basis of the traditional distinction between types of objects; order is imposed upon "the one, unified domain of elements which are propertyless and merely connected through relations" (§ 162). Consequently, there are infinitely many forms of order, and traditional dualistic modes of thinking – mind-body dualism, in particular – dissolve when faced with the far more flexible and integrative notion of being an object that follows from conceiving of object types as order forms. Terms from the order theorists discussed so far are taken up and modified by Carnap in this context; an example is Carnap's discussion of "Parallelverläufe" in § 168 which can be compared to Ziehen's notion of "Parallelveränderungen" (Ziehen 1913, 25, 28).

This kind of anti-dualist argument provides a basis for the various forms of tolerance and neutrality that Carnap propagates. In the present context, the neutrality with respect to different theories of how we should understand reality is particularly remarkable: "*Construction theory represents the neutral foundation which they* ["the so-called epistemological schools of realism, idealism, and phenomenalism"] *have in common*" (§ 178).[21] In the same terms that were used by the order theorists for describing the role of the notion of "order" as the most fundamental concepts of all, Carnap envisages the status of his construction theory as the fundamental theory lying behind all epistemological stances. The differences between these stances does, then, in fact not lie on the level of epistemology, but derives from metaphysical tenets that are added upon the basic structure only at a later moment. This is stronger than just the claim for metaphysical abstinence or anti-metaphysics; Carnap argues from the intrinsic structure of the project of finding the ultimate common core of philosophical theories. Similar arguments and concepts return in a number of key passages in the *Aufbau*. Carnap frames his argument for the availability of various "system forms" (§§ 59–60) in terms of different "orders of concepts" or "of

---

[21] Similar ideas are voiced in Ernst Mach's *Analyse der Empfindungen* that is addressed in the *Aufbau* a number of times (also in § 3 where Carnap explicitates his indebtedness to theories of order). For Mach, whether a (in itself neutral) element becomes a senssation or a physical object, depends on how it enters into functional dependencies to other elements (Mach 1922, 13).

construction" (§ 59) that all have their individual merits and difficulties. Within an order-theoretic framework, it is not required that we unequivocally ascribe to one of these system forms an ultimate or exclusive correctness – this would get us away from epistemology and into metaphysics. As in a number of the theories presented here, the questions of truth and certainty fall outside the constructional framework.

Of particular interest in the current context is the position Carnap himself takes as regards psychology. In § 151 of the *Aufbau*, he emphasizes strongly that his construction theory must not be understood as being psychologistic. Interestingly, Carnap clearly sees the need to refer explicitly to the potential danger of being read as a psychologist.[22] The anti-psychologistic argument that he employs is again derived from order theory: what finally repudiates the charge of psychologism, is the differentiation of different spheres of objects; construction theory leads to higher-level objects, and thus does not support any form of reduction in terms of lower-level objects (or states), as claimed by psychologists. The same holds for values (§152).

Emphasizing the difference with psychologism becomes particularly pressing when one considers that the techniques Carnap himself uses in his quasi-analysis are indeed related to theories in psychology. Via the shared interest in a general, mathematics-based theory of what a science is or has to be, even the more clearly logical techniques (such as definition by abstraction or implicit definitions) do not stand unrelated to psychology. The strength of Carnap's remarks in favour of Gestalt theory in the *Aufbau* has recently been questioned (Mormann 2003, 18–9). In § 67 (which is not directly addressed by Mormann; see also Carnap 1963, 16 on the influence of Gestalt theory on his project), Carnap states very clearly that what is ultimately given are "experiences in their totality and undivided unity". These cannot be analyzed as being built up from discrete elements; all we can state is that they are relationally positioned in a "stream of experience". Two points need to be made here: First, Carnap himself does not reckon Gestalt theory among the (usual) psychological theories. He is talking about "Gestalttheorie", not about Gestalt psychology, and views this theory as being more general than psychological theories: it certainly has effects within psychology, but is also of great relevance for areas other than psychology (§ 67; in the same paragraph, he is also referring to Driesch). Second, the method of quasi-analysis occasionally uses vocabulary that is clearly influenced by the Würzburg school of thought psychology. This is particularly clear in § 71 when Carnap discusses the example of hearing a chord: what we perceive are not "constituents", but "different directions in which we can proceed from it to other chords". Taken together with the earlier discussion of order theories presented here, there is no need to take apart a more logical (Frege/Hilbert/Russell) and a more psychological approach to these methodological issues. This, however, only helps to make the question as to how precisely these authors relate to each other even more urgent.

---

[22] There is a real issue here; Martin Kusch's discussion of the intricacies of the psychologism-debate has Carnap pretty strongly on the side of the anti-psychologists, but also gives evidence that Carnap was indeed charged with being a psychologist (Kusch 1995, 7), though only much later.

Whence Demarcation? – Concluding Remarks

The conclusion from looking at the closely-knit network of related theories and arguments, centering around the notion of "order", clearly cannot be that one can or should gloss over the differences between these theories in a light-handed fashion. What the historical field-studies can clarify, however, is the type of question that should be asked in order to structure these debates. This involves *both* appreciating those ideas and notions that were intended to hold an extremely broad and internally diverse discourse together, *and* getting a clearer view of the specificities of individual positions and of the dynamics that lead to the formation of groups or movements that come to oppose each other. The crucial unifying factor in these arguments has been the search for a form of theory that is more general than existing theories (as regards the foundations of mathematics, as regards forms of philosophy, as regards the different forms of scientific disciplines), and that therefore lies beyond the tensions that exist between these theories. Carnap clearly participates in this discourse, and the claims as to "tolerance" and "neutrality", so important for the *Aufbau*'s project, mark this commitment to unifying discourses. At the same time, tolerance and neutrality have to be pursued on the basis of, and with the aim of, consistently adopting scientific ways of reasoning.

This only makes the question as to where, why, and when the various discourses start to part company – to take up Friedman's metaphor for the philosophical dynamics of this period – more urgent. Take the problem of direct experience: the extremely broad usage of this term, and its mystical connotations, is definitely no longer universally shared; but where are the points of rupture? Take the psychologism-issue: is it indeed sensible to think in terms of broad tolerance in the light of Husserl's and Frege's anti-psychologistic invectives? Hasn't anti-psychologism been crucial for establishing modern logic and modern philosophy of mathematics? What the discussions in the 1920s can and should teach us, is that we rather invert the burden of proof: how did it come that anti-psychologism (or particular brands of logical empiricism, neopositivism…) could acquire the status of becoming the unquestioned basis for twentieth century scientific philosophy? Issues in the reception and self-reception of Carnap's thinking in the 1920s become important here: to which extent does his autobiography (Carnap 1963) contribute to a more streamlined picture? To which extent is Quine's (speculative) suggestion adequate that Carnap started as a "single-minded phenomenalist" who was then pressed by Neurath towards a more physicalist stance (Quine 1994)?

These questions cannot be answered here. What the study of the 1920s-debates minimally teaches is that many of the apparently clear disjunctions between schools, forms of science or forms of philosophy only start being introduced later; and, more ambitiously, that at the very least authors from numerous fields worked hard to establish a scientifically (of course: under a suitably generalized notion of science) informed type of philosophy that could transcend those disjunctions.

Bibliography

Awodey S, Klein C (2004) Carnap brought home. The view from Jena. Open Court, Chicago/La Salle

Beth EW (1962) Ordnung in der Logik. In: Kuhn H, Wiedmann F (eds) Das Problem der Ordnung, Meisenheim a. Anton Hain, Glan, pp 161–176

Carnap R (1922) Der Raum. Ein Beitrag zur Wissenschaftslehre. Reuther & Reichard, Berlin

Carnap R (1926) Physikalische Begriffsbildung. G. Braun, Karlsruhe

Carnap R (1928) Der logische Aufbau der Welt. Weltkreis, Berlin-Schlachtensee

Carnap R (1963) Intellectual autobiography. In: Schilpp PA (ed) The philosophy of Rudolf Carnap. Open Court, LaSalle, pp 1–84

Carnap R (1967) The logical structure of the world and pseudoproblems in philosophy (trans: George, RA), Routledge/Kegan Paul, London

Carus AW (2007) Carnap and twentieth-century thought. Explication as enlightenment. Cambridge et al., CUP

Cassirer E (1913) Erkenntnistheorie nebst den Grenzfragen der Logik. Jahrbücher der Philosophie 1:1–59

Cassirer E (1927) Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie. Jahrbücher der Philosophie 3:31–92

Cassirer E (1995) Nachgelassene Manuskripte und Texte. Vol 1: Zur Metaphysik der symbolischen Formen, John Michael Krois (ed), Meiner, Hamburg

Clauberg KW, Dubislav W (1923) Systematisches Wörterbuch der Philosophie. Meiner, Leipzig

Coffa JA (1991) The semantic tradition form Kant to Carnap. To the Vienna Station, Wessels L (ed). CUP, Cambridge

Driesch H (1913) Die Logik als Aufgabe. J.C.B. Mohr (Paul Siebeck), Tübingen

Driesch H (1922) Wirklichkeitslehre. Ein metaphysischer Versuch. Emmanuel Reinicke, Leipzig

Driesch H (1923) Ordnungslehre. Ein System des nicht-metaphysischen Teils der Philosophie, New ed., Diederichs, Jena

Driesch H (1951) Lebenserinnerungen. Aufzeichnungen eines Forschers und Denkers in entscheidender Zeit. Ernst Reinhardt, München/Basel

Dubislav W (1931) Die Definition. Meiner, Leipzig

Friedman M (2000) A parting of the ways. Carnap, Cassirer, and Heidegger. Open Court, Chicago/LaSalle

Gandon S (2004) Russell et l'*Universal Algebra* de Whitehead: La géométrie projective entre ordre et incidence (1898–1903). Revue d'histoire des mathématiques 10:187–256

Goodman N (1978) Ways of worldmaking. Harvester Press, Hassocks

Görs B, Psarros N, Ziche P (eds) (2005) Wilhelm Ostwald at the crossroads between chemistry, philosophy and media culture. Leipziger Universitätsverlag, Leipzig

Haddock GER (2008) The young Carnap's unknown master. Husserl's influence on Der Raum and Der Logische Aufbau der Welt. Ashgate, Aldershot/Burlington

Heinze M (red.) (1906) Friedrich Ueberwegs Grundriss der Geschichte der Philosophie, Part 4: Die Philosophie seit Beginn des neunzehnten Jahrhunderts, 10th edn., E.S. Mittler & Sohn, Berlin

Heis J (2010) "Critical philosophy begins at the very point where logistic leaves off": Cassirer's response to Frege and Russell. Perspect Sci 18:383–408

Herbst A. Website Theodor Ziehen: http://www.stork-herbst.de/sides/_thziehen.html

Köchy K (2010) Vielfalt der Wissenschaften bei Carnap, Lewin und Fleck. Zur Entwicklung eines pluralen Wissenschaftskonzepts. Berichte zur Wissenschaftsgeschichte 33:54–80

Külpe O (1912–23), Die Realisierung. Ein Beitrag zur Grundlegung der Realwissenschaften, 3 vols., Hirzel, Leipzig

Kusch M (1995) Psychologism. A case study in the sociology of philosophical knowledge. Routledge, London/New York

Mach E (1922) Die Analyse der Empfindungen und das Verhältnis des Physischen zum Psychischen, 9th edn. Gustav Fischer, Jena

Milkov N, Peckhaus V (eds) (2013) The Berlin group and the philosophy of logical empiricism. Springer, Dordrecht

Möckel C (2005) Das Urphänomen des Lebens. Ernst Cassirers Lebensbegriff. Meiner, Hamburg

Mormann T (2003) Synthetic geometry and *Aufbau*. In: Bonk T (ed) Language, truth and knowledge. Contributions to the philosophy of Rudolf Carnap. Kluwer, Dordrecht, pp 45–64

Mormann T (2008) Idealization in Cassirer's philosophy of mathematics. Philos Math 16:151–181

Müller T, Ziche P (2013) Oppenheim on order – the career of a logico-philosophical concept. In: Milkov N, Peckhaus V (eds) The Berlin group and the philosophy of logical empiricism. Springer, Dordrecht, pp 265–291

Nagel E (1939) The formation of modern conceptions of formal logic in the development of geometry. Osiris 7:142–223

Oesterreich TK (red.) (1923) Friedrich Ueberwegs Geschichte der Philosophie, Part 4: Die deutsche Philosophie des neunzehnten Jahrhunderts und der Gegenwart, E.S. Mittler & Sohn, Berlin

Oppenheim P (1926) Die natürliche Ordnung der Wissenschaften. Grundgesetze der vergleichenden Wissenschaftslehre. Gustav Fischer, Jena

Oppenheim P (1928) Die Denkfläche. Statische und dynamische Grundgesetze der wissenschaftlichen Begriffsbildung (Kant-Studien. Ergänzungshefte 62), Kurt Metzner, Berlin-Charlottenburg

Ostwald W (1914) Moderne Naturphilosophie. I. Die Ordnungswissenschaften. Akademische Verlagsgesellschaft, Leipzig

Pasch M (1921) Die Begründung der Mathematik und die implizite Definition. Ein Zusammenhang mit der Lehre vom Als-Ob. Annalen der Philosophie mit besonderer Rücksicht auf die Probleme der Als-Ob-Betrachtung 2:145–162

Pasch M (1926) Die axiomatische Methode in der neueren Mathematik. Annalen der Philosophie und philosophischen Kritik 5:241–274

Quine WV (1994) Comment. In: Salmon W, Wolters G (eds) Logic, language, and the structure of scientific theories. University of Pittsburgh Press/Universitätsverlag Konstanz, Pittsburgh/Konstanz, pp 345–351

Richardson AW (1992) Logical idealism and Carnap's construction of the World. Synthese 93:59–92

Russell B (1937) The principles of mathematics, 2nd edn. Allen & Unwin, London

Russell B (1990) Note on order [1898]. In: Griffin N, Lewis AC (eds) B. Russell, The collected papers, vol 2, Philosophical papers 1896–99. Unwin Hyman, London, pp 339–358

Sauer W (1985) Carnap's *Aufbau* in kantianischer Sicht. Grazer philosophische Studien 23:19–35

Schlimm D (2010) Pasch's philosophy of mathematics. Rev Symbol Log 3:93–118

Schmidt F (1956) Ordnungslehre. Reinhardt, München/Basel

Staudt GK Chr von (1847) Geometrie der Lage, Nürnberg, Friedrich Korn

Stone AD (2006) On the completion and generalization of intuitive space in *Der Raum*: Husserlian and Drieschian Elements, http://philsci-archive.pitt.edu/2896/1/Completion_and_Generalization.pdf

Volkelt J (1918) Gewissheit und Wahrheit. Untersuchung der Geltungsfragen als Grundlegung der Erkenntnistheorie. Beck, München

Volkelt J (1922) Die Gefühlsgewißheit. Eine erkenntnistheoretische Untersuchung. Beck, München

Whitehead AN (1898) A treatise on universal algebra with applications. CUP, Cambridge

Whitehead AN (1978) Process and reality. An essay in cosmology, Griffin DR, Sherburne DW (ed), Corrected edition. Free Press, New York

Wolters G (1994) Scientific philosophy: the other side. In: Salmon W, Wolters G (eds) Logic, language, and the structure of scientific theories. University of Pittsburgh Press/Universitätsverlag Konstanz, Pittsburgh/Konstanz, pp 3–19

Wolters G (2004) Styles in philosophy: the case of Carnap. In: Awodey S, Klein C (eds) Carnap brought home. The view from Jena. Open Court, Chicago/La Salle, pp 25–39

Ziche P (1998) "Reine Psychologie" und "Philosophie als strenge Wissenschaft" – Die Rezeption der Husserlschen Phänomenologie durch die 'Würzburger Schule'. In: Jahnke J, Fahrenberg J, Stegie R, Bauer E (eds) Psychologiegeschichte – Beziehungen zu Philosophie und Grenzgebieten. Profil, München/Wien, pp 347–366

Ziche P (ed) (1999) Introspektion. Texte zur Selbstwahrnehmung des Ichs. Springer, Wien/New York

Ziche P (2002) Wilhelm Thiery Preyers Psychomathematik: Die Mathematisierung der reinen Empfindungen. In: Dumont K, John M, van Zantwijk T (eds) Psychologie und Geschichte 10, H. 3–4: Empirische Psychologie und philosophischer Empirismus. Schlaglichter auf 250 Jahre Psychologiegeschichte. Festschrift Georg Eckardt, pp 283–295

Ziche P (2004) Orientierungssuche im logischen Raum der Wissenschaften. Paul Tillichs *System der Wissenschaften* und die Wissenschaftssystematik um 1900. In: Danz Chr (ed) Theologie als Religionsphilosophie. Studien zu den problemgeschichtlichen und systematischen Voraussetzungen der Theologie Paul Tillichs, Wien, Lit, pp 49–68

Ziche P (2008) Wissenschaftslandschaften um 1900. Philosophie, die Wissenschaften und der 'nicht-reduktive Szientismus', Chronos, Zürich

Ziche P (2011) Alternative claims to the discovery of modern logic: coincidences and diversification. In: François K, Löwe B, Müller T, van Kerkhove B (eds) Foundations of the formal sciences, vol VII, Bringing together philosophy and sociology of science. College Publications, London, pp 243–267

Ziehen T (1913) Erkenntnistheorie auf psychophysiologischer und physikalischer Grundlage. Gustav Fischer, Jena

Ziehen T (1917) Das Verhältnis der Logik zur Mengenlehre. Reuther & Richard, Berlin