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Spatially varying coefficient models in real estate: Eigenvector spatial filtering and alternative approaches

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ABSTRACT

Real estate policies in urban areas require the recognition of spatial heterogeneity in housing prices to account for local settings. In response to the growing number of spatially varying coefficient models in housing applications, this study evaluated four models in terms of their spatial patterns of local parameter estimates, multicollinearity between local coefficients, and their predictive accuracy, utilizing housing data for the metropolitan area of Vienna (Austria). The comparison covered the spatial expansion method (SEM), moving window regression (MWR), geographically weighted regression (GWR), and genetic algorithm-based eigenvector spatial filtering (ESF), an approach that had not previously been employed in real estate research. The results highlight the following strengths and limitations of each method: 1) In contrast to SEM, MWR, and GWR, ESF depicts more localized patterns of the parameter estimates and does not smooth local particularities. 2) ESF is less affected by multicollinearity between the local parameter estimates than MWR, GWR, and SEM. 3) Even though the in-sample explanatory power and prediction accuracy of ESF is superior compared to the competitors, repeated sampling indicates a limited out-of-sample fit and prediction accuracy, suggesting over-fitting tendencies. 4) The application of ESF is less intuitive than MWR and GWR, which are available off-the-shelf.

1. Introduction

Interest in hedonic models that consider the spatial heterogeneity of pricing effects to explore real estate markets in urban areas has grown rapidly (Helbich, Brunauer, Hagenauer, & Leitner, 2013; Lu, Charlton, Harris, & Fotheringham, 2014). Conventional global hedonic models assume a unitary housing market across space that can be modeled through a single price function being representative throughout a city (Bitter, Mulligan, & Dall'erba, 2007). Such models are increasingly questioned due to their unrealistic simplification of housing markets (McMillen & Redfearn, 2010). As a consequence, local hedonic models emerged as an alternative to explore spatially varying housing prices. Even though spatially varying pricing effects are congruent with urban economic theory (Redfearn, 2009) referring to “micro-market effects” (Sunding & Swoboda, 2010, p. 558), and emerging where local legislation and policy regulation are effective (Helbich, Brunauer, Vaz, & Nijkamp, 2014), their incorporation in hedonic models constitutes a methodological challenge. However, neglecting spatial heterogeneity might have serious consequences for model estimation, such as biased regression coefficients, resulting in inappropriate conclusions (LeSage & Pace, 2009; Páez, Fei, & Farber, 2008). No less important, since policy strategies rely on such models, it is critical for decision makers to have models that have the highest fit (Ahn, Byun, Oh, & Kim, 2012; Bourassa, Cantoni, & Hoesli, 2010) and that inform them properly about local housing market conditions, for example through visualizations of spatially varying marginal prices (Ali, Partridge, & Olfert, 2007). Such models also reduce the risk for mortgage lenders and appraisal agencies by obviating loan losses and erroneous real estate assessments.

Despite these appealing methodological and practical advantages of localized models (e.g. Fotheringham, Charlton, & Brunsdon, 2002; Griffith, 2008) in real estate applications, there is still disagreement over which local hedonic approach is superior (Ahn et al., 2012). In this regard, comparative studies are helpful to contrast the merits of different modeling techniques, particularly in light of the increase in the number of applications and the proliferation of new approaches (Páez et al., 2008). Until now, simulation experiments based on artificially generated data with known properties have dominated the comparative analysis literature (e.g., Páez, Farber, & Wheeler, 2011). Even though such investigations greatly improve our knowledge of the advantages and limitations of specific hedonic models, without linking them to more complex real-world case studies, simulation studies cannot entirely uncover their practical relevance. Consequently, empirical model assessments complementing simulations are essential. As model competition outcomes are data-dependent and might cause contradictory results, Bourassa et al. (2010) recommend that empirical comparisons utilize a single dataset.

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Therefore, the principal objective of this study was to address the model performance of four spatially varying coefficient models using a housing dataset for the metropolitan area of Vienna, Austria. As opposed to Farber and Yeates (2006) and Bitter et al. (2007), this study compared SEM, MWR, and GWR by applying a more rigorous out-of-sample accuracy assessment, resulting in less optimistic results than when using the R^2 as performance measure. There are three reasons for selecting these models: their performance is good, they have remarkable recognition in urban housing studies, and they support an enhanced understanding of local market conditions (e.g. Helbich et al., 2014; Kestens, Theriault, & Des Rosiers, 2006; Osland, 2010; Sunding & Swoboda, 2010). The second innovation was the introduction of ESF to model geographically varying relationships and to test the predictive performance of this approach relative to SEM, MWR, and GWR. It is this model, which had not previously been utilized in the context of hedonic modeling, that makes this study not only of interest for urban analysis, but also of practical relevance to urban policymaking. Finally, as ESF grounds on stepwise variable selection procedures which only test a limited number of variable combinations (i.e., the interaction terms between the eigenvectors and housing predictors), a genetic algorithm-based approach had been proposed as alternative.

2. Spatial hedonic price analyses

The theoretical foundation of hedonic modeling is motivated by Lancaster's (1966) theory of consumer utility, which argues that it is not the good itself that generates utility, but the good's specific characteristics. Grounded in this notion, Rosen (1974) developed hedonic pricing theory, which explains that a house price is the sum of its utility-bearing characteristics. Housing is thus considered a heterogeneous good consisting of non-separable structural and neighborhood features (Malpezzi, 2003). Each of these characteristics has its individual implicit price. Because property is fixed in space, a household implicitly chooses a bundle of different goods by selecting a specific house, seeking to maximize its utility. Hence, a household's purchasing decision theoretically reflects an optimal configuration of housing attributes and their paid transaction price (Sheppard, 1997).

Hedonic analysis provides a well-established approach to deconstruct a total house price, and to determine corresponding marginal prices (Malpezzi, 2003). A hedonic equation and its associated unknown parameters are estimated through non-spatial and spatial econometric regression or geostatistical approaches (e.g., Anselin & Arribas-Bel, 2013; Kuntz & Helbich, 2014). Besides the specification of the functional form (Helbich, Jochem, Mücke, & Höfle, 2013), spatial effects subsuming spatial autocorrelation (SAC) and spatial heterogeneity, challenge model estimation (Dubin, 1998). Spatial effects are deduced from the durability and spatial fixation of properties, questioning the validity of non-spatial regression (McMillen & Redfearn, 2010). Accordingly, by assuming spatial equilibrium between supply and demand, one global regression model is assumed to be valid for an entire market, and the estimated parameters are constant across space. Once a dwelling is constructed, it becomes immovable, and supply becomes inelastic (Schnare & Struyk, 1976). These supply inelasticities are coupled with a differentiation in demand emerging from dissimilar households (e.g., due to income variation, diverse socioeconomic characteristics), which value housing properties differently (Quigley, 1985). Both issues cause local supply–demand imbalance (Bitter et al., 2007) and challenge unitary housing markets. Therefore, functional disequilibrium and housing market segmentations are rational (Goodman & Thibodeau, 2003; Kestens et al., 2006), causing distinct patterns of price differentials that manifest as spatially heterogeneous marginal prices (Palm, 1978). Consequently, if this assumption of market segmentation is accepted, but not appropriately modeled, the hedonic coefficients are biased and models have a loss of explanatory power (Bitter et al., 2007; Bourassa et al., 2010; Helbich et al., 2014; Schnare & Struyk, 1976), while local price variations remain hidden.

3. Modeling spatial variation: a review

Spatially varying coefficient models emerged to circumvent the limitations of using spatial regimes in global models, for example, that discrete market boundaries are known in advance and homogeneity within each region is present (Anselin & Arribas-Bel, 2013). Since spatial regimes were not relevant to the present study, the subsequent sections deal only with SEM, MWR, and GWR.

3.1. Spatial expansion method

A classic approach to model spatial structural instability is Cassetti's (1972, 1997) SEM (see Section 4.2), a precursor of GWR. Here, global coefficients are parameterized by polynomials, where covariates are expanded by spatially explicit variables within an ordinary least squares (OLS) framework (Fotheringham, Charlton, & Brunsdon, 1998). However, Pace, Barry, and Sirmans (1998) showed that a polynomial expansion is too imprecise to model spatial variation effectively. While polynomials have appealing usage, they lack robustness and tend to over-smooth local variation, and higher-order polynomials induce multicollinearity. Nevertheless, SEM has received attention in the real estate context from Can (1992), Kestens et al. (2006), and Bitter et al. (2007). For instance, Can (1992) interacted a small set of structural housing variables with neighborhood quality to model spatial drifts. Complementing Can (1992), Fik, Ling, and Mulligan (2003) utilized a fully interactive model that includes higher-order polynomials. Due to numerous interaction terms, Fik et al. (2003) had to limit the number of structural characteristics. Because such a reductionistic model is affected by omitted variables, its estimates are most likely biased. Although SEM is an improvement over global models (Pavlov, 2000), it is criticized for its inability to capture spatial trends other than those that are non-complex and broad, simultaneously discarding valuable local variation. In contrast to Pavlov (2000), who relaxed the parametric assumption of SEM by using non-parametric functions of spatial coordinates, Fotheringham et al. (2002) promoted moving window approaches.

3.2. Moving window and geographically weighted regression

Both MWR and GWR (Fotheringham et al., 2002) circumvent the modeling inflexibility problems of SEM. GWR extends MWR through additional distance-based weightings (see Section 4.3). A benefit of MWR and GWR is that marginal prices are allowed to vary smoothly across space by setting regional dummies or polynomial expansions aside. From a theoretical viewpoint, Bitter et al. (2007) argue that, by restricting the number of sales per local regression, GWR partly mimics appraisers' sales comparisons and price adjustment processes. Despite these appealing properties, GWR is under debate. For example, Wheeler and Tiefelsdorf (2005) and Griffith (2008) referred to multicollinearity problems amongst GWR estimates. While weak correlation affects the ability to interpret model output, strong dependencies make a reliable separation of individual variable effects hardly possible (Wheeler & Tiefelsdorf, 2005). Páez et al. (2011) noted that GWR itself artificially introduces multicollinearity, even if the input covariates are uncorrelated, while Jetz, Rahbek, and Lichstein (2005) reported sign reversals that can be traced back to multicollinearity, causing a local omitting variable bias. However, model calibration, which is based on predictive performance, remains unaffected (Brunsdon, Charlton, & Harris, 2012). Others, including Wheeler (2009) and Vidaurre, Bielza, and Larrañaga (2012), have proposed integrating ridge and lasso regression into GWR to alleviate collinearity complications (Ahn et al., 2012). However, these extensions have not found resonance in real estate. Fotheringham et al. (2002) examined the calibration procedures of hedonic GWR models and concluded that an adaptive bandwidth reduces the volatility of the regression coefficients compared to a fixed one. Closely related to bandwidths are complications with extreme coefficients. Cho, Lambert, Kim, and Jung (2009) showed that fixed bandwidths are more prone to extreme coefficients than adaptive ones, especially in areas with spatially sparse data. However, Páez et al. (2011) noted that these restrictions rest on simulation studies with insufficient sample sizes (e.g. Wheeler, 2009; Wheeler & Tiefelsdorf, 2005).

Several hedonic studies emphasize the appealing empirical performance of GWR. For example, Kestens et al. (2006) and Bitter et al. (2007) challenged SEM and GWR. As expected, both SEM and GWR out-perform global models, while GWR is superior to SEM in terms of prediction accuracy and explanatory power. Kestens et al. (2006) conveyed similar results, additionally stressing that SEM has the ability to distinguish between non-spatial and spatial heterogeneity, which is not possible with GWR. However, SEM results in over-generalized patterns. Comparing GWR and MWR, Páez et al. (2008) found similar prediction power for GWR and MWR, although the results differed in terms of prediction error. Not unexpectedly, Farber and Yeates (2006) and Osland (2010) reported a better GWR performance compared to OLS. Farber and Yeates (2006) also measured GWR against MWR. Again, GWR was more precise. Contradicting the findings of Osland (2010) and Helbich et al. (2014), Gao, Asami, and Chung (2006) reported no significant improvement using GWR, and concluded that OLS is sufficient. They argued that the spatial extent of their study site – one district in Tokyo – was too small to show price heterogeneities.

In conclusion, this literature review showed that only a limited number of techniques are currently utilized in housing studies. Little empirical consensus exists about which model performs best to analyze spatially varying relationships, which supports the need for further research. The application of ESF (Griffith, 2008) had not previously been considered in real estate research and its potential remained unknown.

4. Methods

4.1. Eigenvector spatial filtering

The principal aim of using ESF is to avoid SAC-based regression misspecification. The topology-based approach (Griffith, 2000, 2012) has several advantages compared to other filtering techniques (Getis, 1990; Griffith & Peres-Neto, 2006). For example, Getis's (1990) approach is restricted to: a) positive SAC, b) the variables must have a natural and positive origin, and c) each variable must be filtered separately. In contrast, Griffith's (2008) approach is not limited in this respect, and, more importantly, it can be extended to model geographically varying relationships.

Initially, topology-based ESF rests upon de Jong, Sprenger, and van Veen (1984), who pioneered the relationship between eigenvalues and the Moran's I coefficient (MC, Cliff & Ord, 1973). In accordance with Griffith (2000), ESF applies eigenvector decomposition in order to extract a set of EVs from a given contiguity matrix (Getis, 2009; Patuelli, Schanne, Griffith, & Nijkamp, 2012), which also emerges in the numerator of the MC statistic. This matrix is defined as follows:

$$\left(\boldsymbol{I}-\frac{\mathbf{1}\mathbf{1}^T}{N}\right)\boldsymbol{C}\left(\boldsymbol{I}-\frac{\mathbf{1}\mathbf{1}^T}{N}\right) \tag{1}$$

where $\mathbf{I}$ represents the $N\times N$ identity matrix having 1 s in the main diagonal and 0 s elsewhere, $\mathbf{1}$ is a $N\times 1$ vectors of 1 s, $\boldsymbol{C}$ gives the topological spatial arrangement of N spatial units, and T denotes the matrix transpose. These resulting EVs have the appealing properties of being mutually uncorrelated and orthogonal, each mimicking a certain degree of latent SAC, representing global to local patterns (Tiefelsdorf & Boots, 1995; Tiefelsdorf & Griffith, 2007). EV1 contains numerical values resulting in the largest possible MC, whereas EV2 expresses the set of values having the largest obtainable MC by any possible set of EVs that are orthogonal and uncorrelated with EV1. This decomposition continues for the remaining N EVs, through the highest possible negative SAC (Griffith, 2000).

Due to missing degrees of freedom and a preference for more parsimonious models, the full set of N EVs must be reduced to a smaller set of so-called candidate EVs. This reduction ensures the elimination of EVs that represent trivial amounts or the wrong nature of SAC. For that purpose, Tiefelsdorf and Griffith (2007) proposed that $MC/MC_{\max} > 0.25$, where $MC_{\max}$ is the largest positive MC value. This approach depicts only those EVs that have at least about 5% redundant information. In other words, only relevant map patterns are selected. Subsequently, only the candidate EVs significantly related to a response variable, conditionally on the "real" covariates, are identified through selection algorithms (e.g., stepwise selection, shrinkage and selection methods; Seya, Murakami, Tsutsumi, & Yamagata, 2014), which yield the final set of EVs.

Rather than using the final EVs to correct for SAC on a global level, Griffith (2008, p. 2761) extended the basic linear model by means of interaction terms between the selected EVs and the predictors to model spatially varying coefficients in the following manner:

$$\widehat{\mathbf{Y}}\approx\left(\beta_0\mathbf{1}+\sum_{k_0=1}^{K_0}\boldsymbol{E}_{k_0}\beta_{k_0}\right)+\sum_{p=1}^{P}\left(\beta_p\mathbf{1}+\sum_{k_p=1}^{K_p}\boldsymbol{E}_{k_p}\beta_{k_p}\right)\bullet\boldsymbol{X}_p+\boldsymbol{\varepsilon} \tag{2}$$

where $\widehat{\mathbf{Y}}$ is the $n\times 1$ vector of prices, $\boldsymbol{X}_p$ is a $n\times 1$ vector of independent variable p $(p=1,2,3,\dots,P)$, $\boldsymbol{E}_{k_p}$ is the k_p EV $(k=1,2,3,\dots,K)$ that describes the variable p, $\beta_0, \beta_{k_0}, \beta_{k_p}$ are estimated regression coefficients, and $\boldsymbol{\varepsilon}$ is an independent and identically distributed error term. Note that • denotes the element-wise matrix multiplication and the interaction terms are given by $\boldsymbol{E}_{k_p}\bullet\boldsymbol{X}_p$. The parameters are estimated by means of OLS. The first part of the equation represents the spatially varying intercept, and the second part represents the spatially varying coefficients. After rearranging, the regression coefficients constitute the global impact, while the individual EVs mimic local modifiers of these global effects across space:

$$\boldsymbol{Y}=\beta_0\mathbf{1}+\sum_{p=1}^{P}\boldsymbol{X}_p\bullet\mathbf{1}\beta_p+\sum_{k=1}^{K}\boldsymbol{E}_k\beta_{E_k}+\sum_{p=1}^{P}\sum_{k=1}^{K}\boldsymbol{X}_p\bullet\boldsymbol{E}_k\beta_{pE_k}+\boldsymbol{\varepsilon}. \tag{3}$$

Two appealing ESF properties are that the coefficients vary around the global value β, and that multicollinearity amongst coefficients can be easily ascertained in terms of common EVs. In practice, the outlined procedure is challenging due to a large set of covariates and interaction terms, eventually larger than the available number of degrees of freedom. Griffith (2008) originally proposed forward variable selection to find significant interactions, but this procedure is computationally slow (Seya et al., 2014) and only investigates iteratively a rather small number of variable combinations, posing the danger of an inappropriately selected set of variables. Because the model possibilities are 2^k, where k denotes the number of predictors, testing all possible models to determine the optimal combination computationally is rarely feasible (Alberto, Beamonte, Gargallo, Mateo, & Salvador, 2010). Additionally, simplified models are easier to interpret.

In order to identify the most relevant interactions[1] in a parsimonious manner, the application of an evolutionary computing strategy seems promising. Stochastic search strategies, such as genetic algorithms (GA) (Goldberg, 1989; Reggiani, Nijkamp, & Sabella, 2001) imitating natural evolution, are effectively capable of selecting an optimal subset of covariates (e.g. Ahn et al., 2012; Alberto et al., 2010). Nevertheless, these approaches have so far been virtually ignored by real estate

[1] Note that housing variables are always included as covariates.

economists. Following Scrucca (2013), a GA produces a population of subsets (chromosomes), each including a randomly selected set of predictors and thus representing a potential solution. Based on evolutionary principles, new populations are generated. Each chromosome represents a potential variable set. Variables are encoded as a binary string of 1's and 0's, where 1 means the presence and 0 the absence of a predictor (gene). The length of a chromosome is given by the number of variables. Commonly applied genetic principles are selection, crossover, and mutation (Reggiani et al., 2001). The selection operator allows only the fittest offspring to reproduce and pass on its genetic properties. Population diversity is introduced by means of crossover and mutation. The former produces offspring by combining different parts of chromosomes, while the latter randomly modifies the values of genes of a chromosome. The efficiency and fitness of a chromosome is evaluated on the basis of a cost function. In this case, the Akaike information criterion (AIC) is used for evaluation purposes, which considers the model fit and penalizes less parsimonious models. The evolutionary process evolves until the algorithm is terminated, either due to a maximum number of iterations having been performed, or the fitness function not improving for a number of generations. Because less fit offspring are extinct, the GA is likely to find a near-optimal subset of predictors (Hagenauer & Helbich, 2012).

Finally, in order to obtain the final and mappable coefficients, all ESF model parts with common attributes are collected and then factored out in order to determine its spatially varying coefficient (Griffith, 2008).

4.2. Spatial expansion method

The SEM expands global regression coefficients by means of aspatial attributes and/or spatial coordinates (Cassetti, 1972, 1997). Utilizing the geographic context for parameter expansion allows the modeling of a spatial drift (Fotheringham et al., 1998), that is, covariates are interacted with locational attributes (Fik et al., 2003). The spatial drift is represented by polynomials of a certain degree of the spatial coordinates (u_i, v_i) of location i. Formally, the following simple model serves as the initial model:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{4}$$

where y_i represents the response, β the estimated coefficients, x_i a covariate, and ε_i an error with $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2 \boldsymbol{I})$. For illustration, β_1 is expanded linearly by the coordinates (u_i, v_i) as:

$$\beta_{1i} = \gamma_0 + \gamma_1 u_i + \gamma_2 v_i. \tag{5}$$

Substituting the expanded parameter β_1 in Eq. (4) results in the terminal SEM, which can be estimated by OLS:

$$y_i = \beta_0 + (\gamma_0 + \gamma_1 u_i + \gamma_2 v_i) x_i + \varepsilon_i. \tag{6}$$

The complexity of the modeled spatial drift depends on the selected order of the polynomials. Lower-order (e.g., second-order) polynomials are common (Kestens et al., 2006). The selection of an appropriate expansion is key; however, this assumes that a priori knowledge of the actual spatial pattern present which is rarely the case.

4.3. Moving window and geographically weighted regression

Fotheringham et al. (2002) popularized GWR modeling by extending local regression to the spatial domain. By considering a subset of the input data, GWR estimates a series of weighted least squares regressions and facilitates continuously changing price functions. Formally, the GWR specification can be written as:

$$y_{(u_i,v_i)} = \beta_{0(u_i,v_i)} + \sum_{k=1}^{K} \beta_{k(u_i,v_i)} x_k + \varepsilon_{(u_i,v_i)} \tag{7}$$

where y is the response variable, x_k are the k^{th} predictors, (u_i, v_i) are the coordinates of the ith point, $\beta_{k(u_i,v_i)}$ is a continuous function on the location i, and ε represents an error term with $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2 \boldsymbol{I})$. The estimation of $\hat{\boldsymbol{\beta}}$ at location i is done by a locally weighted OLS estimator:

$$\hat{\boldsymbol{\beta}}_{(u_i,v_i)} = \left(\boldsymbol{X}^T \boldsymbol{W}_{(u_i,v_i)} \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{W}_{(u_i,v_i)} \boldsymbol{y} \tag{8}$$

where $\boldsymbol{W}$ represents an $n \times n$ diagonal spatial weight matrix, which has distance-dependent weights $w_{(u_i,v_i)n}$ as diagonal elements, and 0 as non-diagonal elements. The simplest weighting function is the discontinuous box-car kernel, also termed as the MWR. Here, a point is considered in a local regression if the distance d between the points i and j is less than a threshold value $b(w_{ij} = 1)$; otherwise, it is excluded $(w_{ij} = 0)$:

$$w_{ij} = \begin{cases} 1 & \text{if } |d_{ij}| < b \\ 0 & \text{if } |d_{ij}| > b \end{cases}. \tag{9}$$

The box-car kernel neglects distance decay effects. Therefore, many studies apply continuous functions (e.g., Redfearn, 2009), like the commonly used Gaussian kernel, where the weights w_{ij} decline with increasing distance:

$$w_{ij} = exp\left(-0.5\left(\frac{d_{ij}}{b}\right)^2\right) \tag{10}$$

where d_{ij} is the Euclidean distance between point i and j, and b represents the kernel's bandwidth. Regardless of the kernel type, the bandwidth is crucially important (Fotheringham et al., 2002). If the selected bandwidth is too small, only a small number of observations are considered in each local regression, resulting in unstable fits and large variances. In contrast, an overly large bandwidth over-smooths and induces a bias by masking local characteristics, and the estimates shrink to their global counterparts. To achieve a bias-variance tradeoff, bandwidth optimization strategies are preferred to an ad hoc selected bandwidth (Fotheringham et al., 2002; Páez et al., 2011). McMillen and Redfearn (2010) showed that an adaptive bandwidth is appropriate for housing studies, particularly when dwellings are spatially non-uniformly distributed. To determine an ideal number of nearest-neighbor points for each local regression, optimizing the cross-validated prediction error yields robust results (e.g., Fotheringham et al., 2002). However, McMillen and Redfearn (2010) speculated that the optimal bandwidth might be larger than the one identified by cross-validation (CV) optimization.

5. Study area and data

To address the research questions in an empirical context, the metropolitan area of Vienna, Austria, was selected as the study area. Its specific house price pattern – namely high house prices in the Wienerwald area, local price hot spots in the north-west and the south of Vienna, and decaying prices toward the eastern and western areas – makes this area ideal for investigating local price variations. Georeferenced owner-occupied, single-family home data were provided by the UniCredit Bank Austria AG for the years 2007 to 2009. Each house has eight attributes describing the physical structure, including the logged transaction price recorded in euros serving as a response variable. Due to skewed distributions, the covariates total floor area and total plot area were also transformed to their logs. Additionally, the proportion of academics at the administrative level of enumeration districts is attached to each individual house serving as socio-economic proxy variable. This dataset is published by Statistics Austria. Enumeration districts are the smallest available administrative units in Austria and have the advantage, compared to the municipality level, that most houses under investigation are nested within a unique

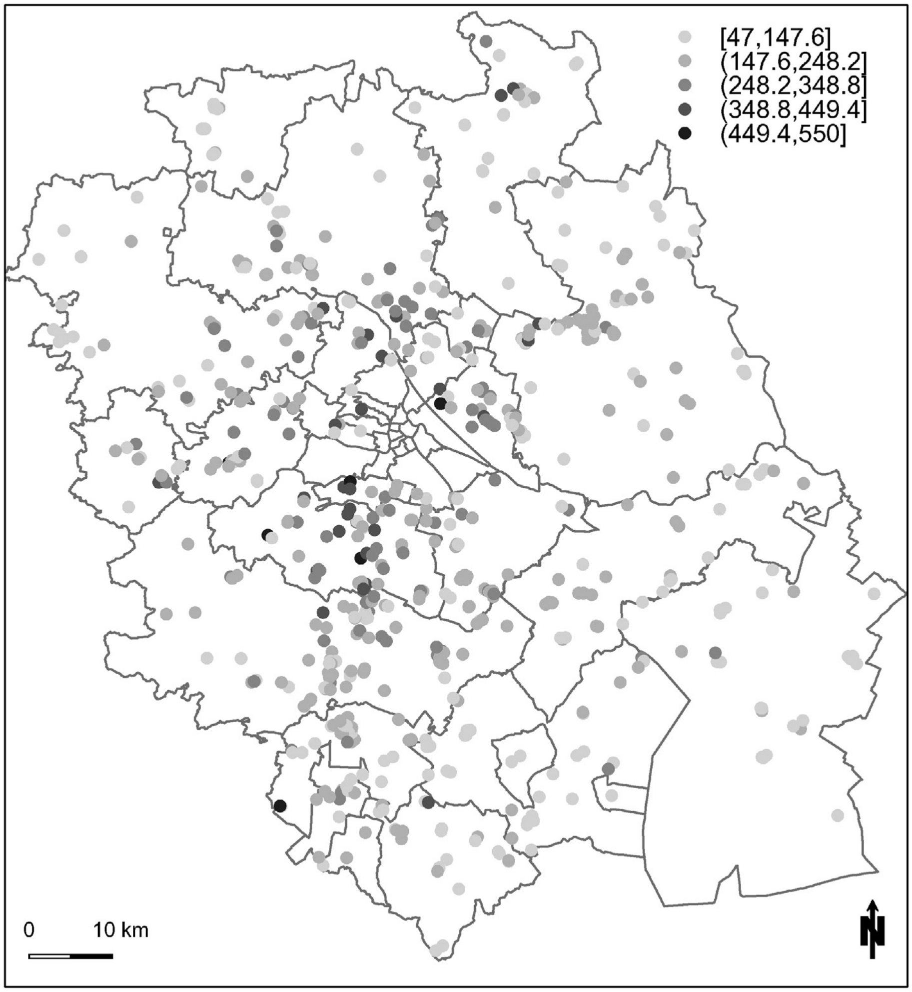

Fig. 1. Study area and locations of single-family homes (prices in €1000, gray lines delimit districts).

spatial unit preventing a more complex modeling design (i.e., multilevel modeling). After screening the data for missing values, 648 houses remained in the dataset. While Fig. 1 gives an impression of the spatial distribution of the houses and their transaction prices, Table 1 provides descriptive statistics of the variables.

6. Results

To test the generalizability of the model, the dataset was divided into a training set and a test set utilizing random sampling (Hastie, Tibshirani, & Friedman, 2009). We used 80% and 90% of the data as the training set; the remaining data were used to explore the out-of-sample prediction performance. Because the spatial distribution of the randomly selected hold-out data matters for accuracy assessments (LeSage & Pace, 2004), this step was repeated 100 times. Exactly the same data partitions were used for all models. However, to get a better understanding of the model behaviors, here focus is on one randomly selected training dataset (80% in size). Section 6.4 presents the results for the 100 replications.

6.1. A non-stationary spatially filter model

Based on a binary 5-nearest neighbor weight matrix,[2] 116 out of 648 EVs with positive SAC beyond the threshold value of $MC/MC_{\max} > 0.25$ (Tiefelsdorf & Griffith, 2007) were extracted. To further reduce the candidate EVs while considering the 9 housing variables, house price was regressed onto them. As outlined in Section 4.1, a GA-based selection strategy with a binary decision variable and an AIC-based fitness function was performed. The parameters of the GA operators were defined as recommended by Scrucca (2013): initial population size = 50, crossover probability = 0.8, and mutation probability = 0.1. The maximum number of iterations was set to 10,000 runs. To avoid overfitting, the number of consecutive generations without improvements in the fittest value was set to 100. Because the number of EVs (116) is small, the GA converged after 254 iterations; 43 EVs are significantly related to price. The selected EVs[3] represent regional and global patterns. Overall, the pure spatial dimension of the house prices, represented by the depicted EVs, explains approximately 30% (adjusted R^2) of the price variance, emphasizing the importance of geography. Subsequently, the interaction terms for the 43 EVs and 9 housing characteristics render 387 covariates.

A similarly set-up GA was utilized for the second covariate selection including the housing covariates and their interaction terms with the EVs. Already after a few generations the AIC-based fitness value was significantly reduced, and the GA quickly converged after 1802 generations due to no marked improvements in the fitness function. Note that the GA selected a distinctively parsimonious model consisting of 168 covariates compared to the stepwise approach with 323 covariates. Depending on the magnitude of the interaction effects, two classes

[2] Preliminary robustness checks concerning different contiguity matrix specifications on model performance show only marginal impacts about model quality.

[3] The following EVs were selected: EV4-9, EV11-12, EV15-19, EV22, EV24, EV27-29, EV31, EV35, EV37, EV39, EV41, EV51, EV53-54, EV62, EV66, EV72-73, EV81, EV84, EV92, EV94, EV97, EV107, and EV109-115.

Table 1
Description of variables.

Abbrev.	Name	1st QT	Median	3rd QT	Cat. 0	Cat. 1
lnp	Log of transaction price (€)	11.695	12.044	12.346	–	–
lnareatot	Log of total floor area (square meters; except cellar)	4.605	4.786	5.019	–	–
lnareapl	Log of plot area (square meters)	6.004	6.398	6.679	–	–
age	Age of building at time of sale (years)	6.750	20.000	38.000	–	–
condh1	Condition of the house (0 = good, 1 = poor)	–	–	–	489	159
heat1	Quality of the heating system (0 = good, 1 = poor)	–	–	–	596	52
cellar1	Existence of a cellar (0 = no, 1 = yes)	–	–	–	416	232
garage1	Quality of the garage (0 = good, 1 = poor)	–	–	–	280	368
terr1	Existence of a terrace (0 = no, 1 = yes)	–	–	–	420	228
acad	Proportion of academics (%, 2001, enumeration district)	14.790	18.280	24.880	–	–

QT = quantile.

appear to be present. For example, while "lnareatot" has 18 local modifiers, referring to more regional and local EVs, "terr1" has only 12 EVs across all spatial scales. Overall, with an adjusted R^2 of 0.829, the model displays good performance. The F-test confirms model validity ($F = 14.750$; $p < 0.001$). Neither the MC nor the Breusch–Pagan (BP) tests depict any model anomalies, referring to a well-specified ESF model.

6.2. Spatial expansion method

Spatial expansion specifications with first- to third-order polynomial interactions were implemented. To reduce multicollinearity, the coordinates were centered as in Kestens et al. (2006) and non-significant interactions were removed via a stepwise procedure. The AIC supports the third-order polynomial expanded model, which is further discussed. The adjusted R^2 is 0.631. Even after centering the input variables, the SEM is still affected by strong multicollinearity as indicated by the variance inflation factors (>100). Testing the model assumptions, the BP test refers to homoscedasticity ($BP = 53.477$, $p = 0.379$), but the MC points to minor but significant spatial residual patterns ($MC = 0.082$, $p < 0.001$). A potential reason is that spatial heterogeneity is not appropriately modeled by means of a crude and inflexible spatial trend.

6.3. Moving-window and geographically weighted regression

A MWR was implemented with an adaptive box-car kernel. The bandwidth was optimized to 125 points, which resulted in a model with an AIC of 232. In addition, a GWR model was estimated with an adaptive Gaussian kernel function. CV suggests that 35 out of 648 points should be considered for each local model. As in Sunding and Swoboda (2010), by slightly increasing the bandwidth beyond the CV optimized value, McMillen and Redfearn's (2010) concern that the bandwidth is underestimated cannot be confirmed. Sensitivity analyses using an adaptive bisquare and tricube kernel yielded very similar results. GWR resulted in a lower AIC score (187) than MWR. However, both models are still facing spatial residual patterns. Whereas the MC for GWR showed reduced residual SAC ($MC = 0.062$, $p = 0.005$) compared to SEM, the one for MWR rose ($MC = 0.103$, $p < 0.001$).

6.4. Model comparison

Table 2 summarizes the estimated coefficients across the models. The median parameters are in accordance with the literature. One of the prime benefits of local models is the possibility to map the estimated coefficients in order to explore local relationships and marginal effects. To create more appealing visualizations, ordinary kriging is used for interpolation. For illustrative purposes, Fig. 2 visualizes the marginal price surfaces for each method for the variable floor area. Because ESF is not based on sliding windows during its calibration or a polynomial expansion, it apparently produces more localized results. However, the patterns of marginal prices of ESF, MWR, and GWR roughly resemble each other. For example, the models show lower prices in the eastern areas. In contrast to ESF, MWR, and GWR, SEM is, as expected, not able to capture spatial variations beyond large-scale trends around the core city.

Next, multicollinearity effects between the local coefficients were explored (Páez et al., 2011; Wheeler & Tiefelsdorf, 2005). However, as the true values of the parameters are unknown, we could not assess whether the found collinearity between the local parameters is intrinsically wrong.

Table 2
Estimated parameters.

ESF	1st QT	Median	3rd QT
lnareatot	0.207	0.338	0.518
lnareapl	−0.118	0.036	0.191
age	−0.013	−0.009	−0.005
condh1	−0.264	−0.163	−0.042
heat1	−0.177	−0.020	0.189
cellar1	−0.030	0.095	0.178
garage1	−0.224	−0.104	0.012
terr1	0.012	0.100	0.187
acad	0.005	0.014	0.026
MWR			
lnareatot	0.172	0.296	0.465
lnareapl	0.019	0.053	0.146
age	−0.010	−0.008	−0.007
condh1	−0.110	−0.080	−0.044
heat1	−0.176	−0.129	−0.075
cellar1	0.049	0.097	0.139
garage1	−0.111	−0.088	−0.059
terr1	0.020	0.054	0.095
acad	0.013	0.016	0.018
GWR			
lnareatot	0.150	0.289	0.440
lnareapl	0.033	0.065	0.143
age	−0.010	−0.009	−0.007
condh1	−0.117	−0.087	−0.043
heat1	−0.174	−0.117	−0.071
cellar1	0.050	0.098	0.151
garage1	−0.120	−0.087	−0.057
terr1	0.022	0.047	0.086
acad	0.015	0.017	0.018
SEM			
lnareatot	0.150	0.283	0.401
lnareapl	0.061	0.105	0.176
age	−0.010	−0.008	−0.007
condh1	n.a.	−0.095	n.a.
heat1	−0.204	−0.122	0.024
cellar1	0.034	0.090	0.192
garage1	−0.134	−0.081	−0.050
terr1	−0.001	0.060	0.118
acad	n.a.	0.012	n.a.

Note that not all GWR covariates (e.g. age, condh1) show significant spatial variability of the parameters (Leung, Mei, & Zhang, 2000). Thus, a mixed-GWR (Fotheringham et al., 2002) might be an extension. Where the stepwise algorithm had not selected a related interaction term, "n.a." refers to SEM parameters not expanded by the coordinates and the estimated global parameter is reported.

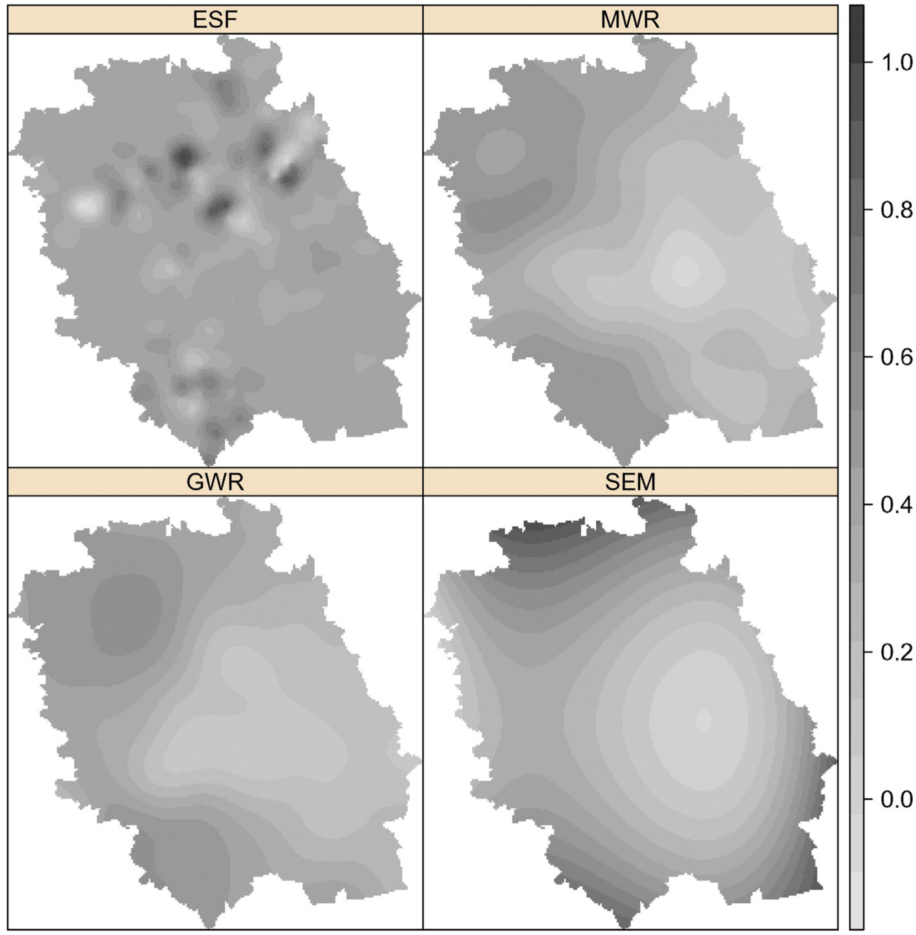

Fig. 2. Spatially varying coefficients for total floor area.

Due to a lack of diagnostic tools (e.g., Wheeler, 2009) for ESF and SEM, pair-wise Spearman correlation matrices were computed between the local parameter estimates. The correlation plots in Fig. 3 indicate that dependencies are reduced using ESF compared to the other models. The results for SEM must be interpreted with care due to strong multicollinearity. With the exception of ESF, all models produce at least one pair of local coefficients that are highly correlated ($\rho > 0.7$). The remaining collinearity in the ESF approach results from those EVs spanning simultaneously multiple interaction terms. In particular, SEM features extreme correlations between several coefficients, which is not unexpected and already highlighted in the literature (e.g. Fotheringham et al., 2002; Pace et al., 1998). Strong multicollinearity effects are less pronounced in MWR and GWR than in the SEM model.

Fig. 4 summarizes the in-sample prediction accuracy. The ESF model scatters predictions more closely around the 1:1 line. Compared to ESF, GWR tends to underestimate average priced houses and lean toward occasional large errors in the medium price range. Table 3 reports the Spearman's ρ correlation coefficients between the observed and the in-sample predictions, and confirms with ρs of 0.924 and 0.904 the suitability of ESF compared to its competitors. Furthermore, the root mean square error (RMSE) based on leave-one-out cross-validation (LOOCV; Hastie et al., 2009), which estimated a model for each $n-1$ sample and used the put aside data for accuracy testing, showed lower LOOCV errors for ESF than for MWR, GWR, and SEM, all indicating rather comparable LOOCV errors.

While in-sample accuracy assessments are overly optimistic, the predictive performance had also been evaluated by means of hold-out samples of 10% and 20% of the entire data. The out-of-sample results counter the in-sample ones (Table 3). Independent of the hold-out sample size, MWR, GWR, and SEM perform significantly better than ESF as indicated by the median RMSEs and median Spearman's ρs. Even though only two sample partition sizes were tested, it seems that ESF performs more accurately when the test data are small (i.e., 10%), whereas the competitors show only minor differences.

7. Discussion and conclusions

There is growing interest in urban analyses and policymaking to model house price variations locally (e.g. Helbich et al., 2014; Redfearn, 2009; Sunding & Swoboda, 2010). However, this increasing attention is challenged by a lack of consensus on how to model local variation of housing prices appropriately, as well as by divergent and contradictory empirical results across different models. This provided the impetus for the present study, which compared four spatially varying coefficient models in terms of a) their spatial patterns of the estimated parameters, b) multicollinearity effects between the local coefficients, and c) their predictive accuracy using data for the Vienna region. Hedonic models were estimated by means of SEM, MWR, and GWR and compared to a model that had not previously been employed in real estate research, namely ESF. The key findings can be summarized as follows.

First, while all four models reveal intuitive coefficients, the comparison of the geographically varying marginal prices indicates that ESF results in more localized parameter surfaces. This is mainly due to an alternative operationalization of how spatial heterogeneity is modeled (Griffith, 2008). While ESF extracts EVs from a contiguity matrix and interacts them with covariates, GWR uses overlapping sliding windows while performing weighted regressions, apparently provoking overly smooth coefficient patterns. Of course, reducing the MWR/GWR bandwidth would lead to more local analysis, but this would no longer reflect the numerically optimized bandwidth. In comparison to ESF, MWR, and

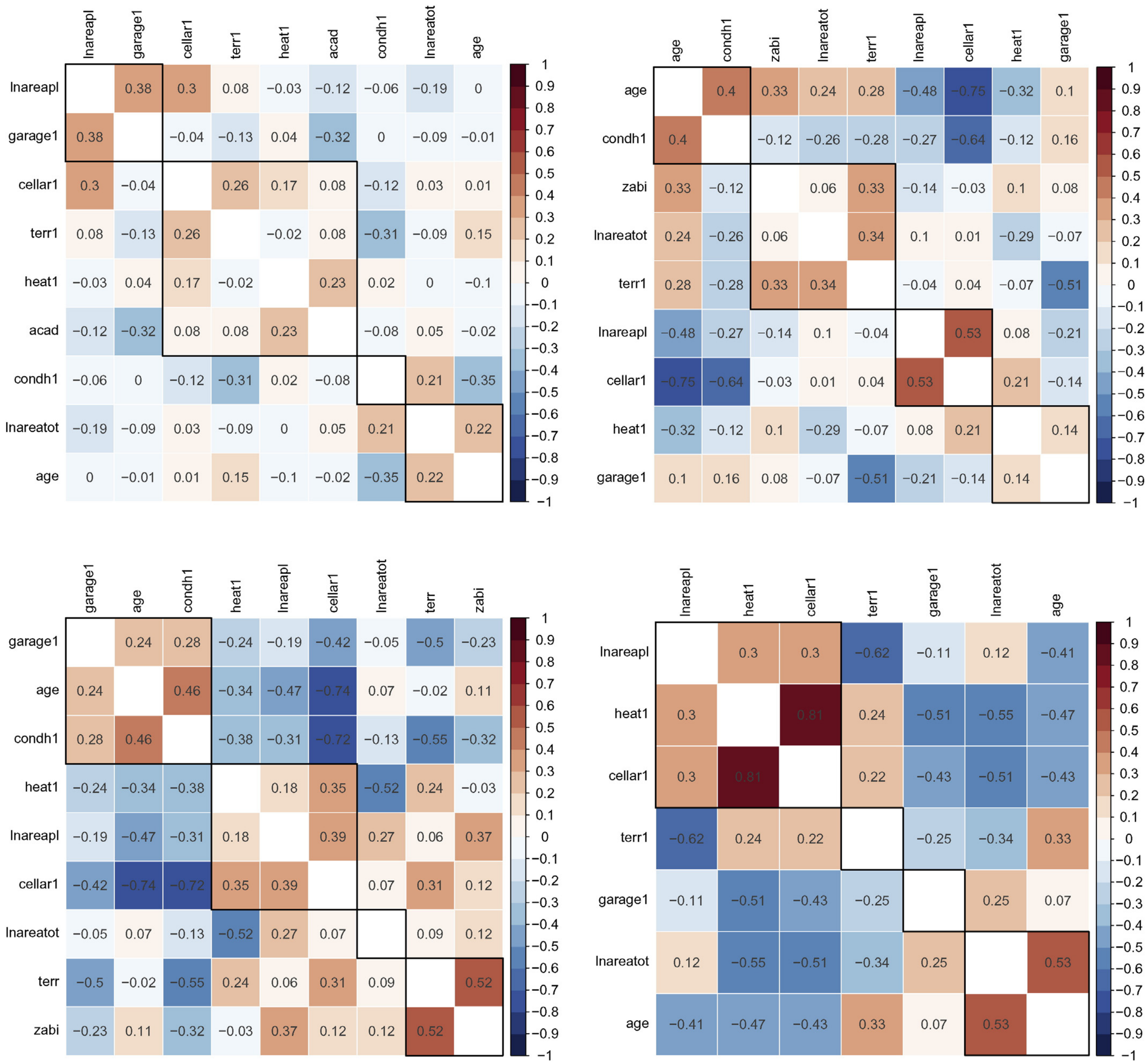

Fig. 3. Correlation matrices of the estimated parameters (ESF: top left, MWR: top right, GWR: bottom left, SEM: bottom right. The variables are ordered according to their correlations. Each rectangle represents a coefficient cluster based on hierarchical clustering. Note that in the SEM not all housing covariates are expanded by the coordinates).

GWR, SEM is not, as anticipated, capable of modeling marginal price variation across space appropriately, as indicated by residual SAC implying that the spatial variation is actually so complex and localized that it can be captured by lower-order polynomials (Bitter et al., 2007; Pace et al., 1998). While MWR and GWR have the availability of specific test statistics to determine whether a set of local parameter estimates exhibits significant spatial variation (Leung et al., 2000), all models allow quantifying the significance of spatial variation using the locally estimated parameters with a confidence interval around the equivalent global parameter.

Second, multicollinearity between coefficient pairs, a critique against GWR (e.g., Wheeler & Tiefelsdorf, 2005), is addressed. ESF gives insights how SAC inflates/deflates multicollinearity amongst the spatially varying coefficients. Common eigenvectors can inflate SAC, while unique eigenvectors can deflate it. Confirming Griffith (2008), the ESF-based coefficients seem to be less plagued by multicollinearity problems than those for GWR. However, these results are speculative: the true parameters are unknown and these values might also be correlated, in which case the estimated parameters are also collinear to some degree. Thus, our simple correlation analyses are premature, calling for simulation studies and the development of specific diagnostic tools for ESF as already available for GWR (Wheeler, 2009).

Third, ESF yielded appealing results regarding the in-sample model fits and in-sample predictive accuracies compared to SEM, MWR, and GWR. The results of the in-depth analysis of our sample match Griffith's (2008) work. ESF had approximately 25% higher goodness-of-fit values and 15–20% more accurate LOOCV predictions. Based on 100 randomly selected out-of-sample test datasets, however, these conclusions must be reversed. Whereas the out-of-sample predictions of SEM, MWR, and GWR are roughly comparable, ESF shows a pronounced prediction inaccuracy and a reduced fit. For example, ESF-based Spearman's ρ correlations using test data of 10% in size, yield a reduced fit of 25% compared to the competitors. However, as the MWR and GWR residuals show minor residual SAC and SEM is affected by pronounced multicollinearity, the results should be interpreted with care. As ESF is data-

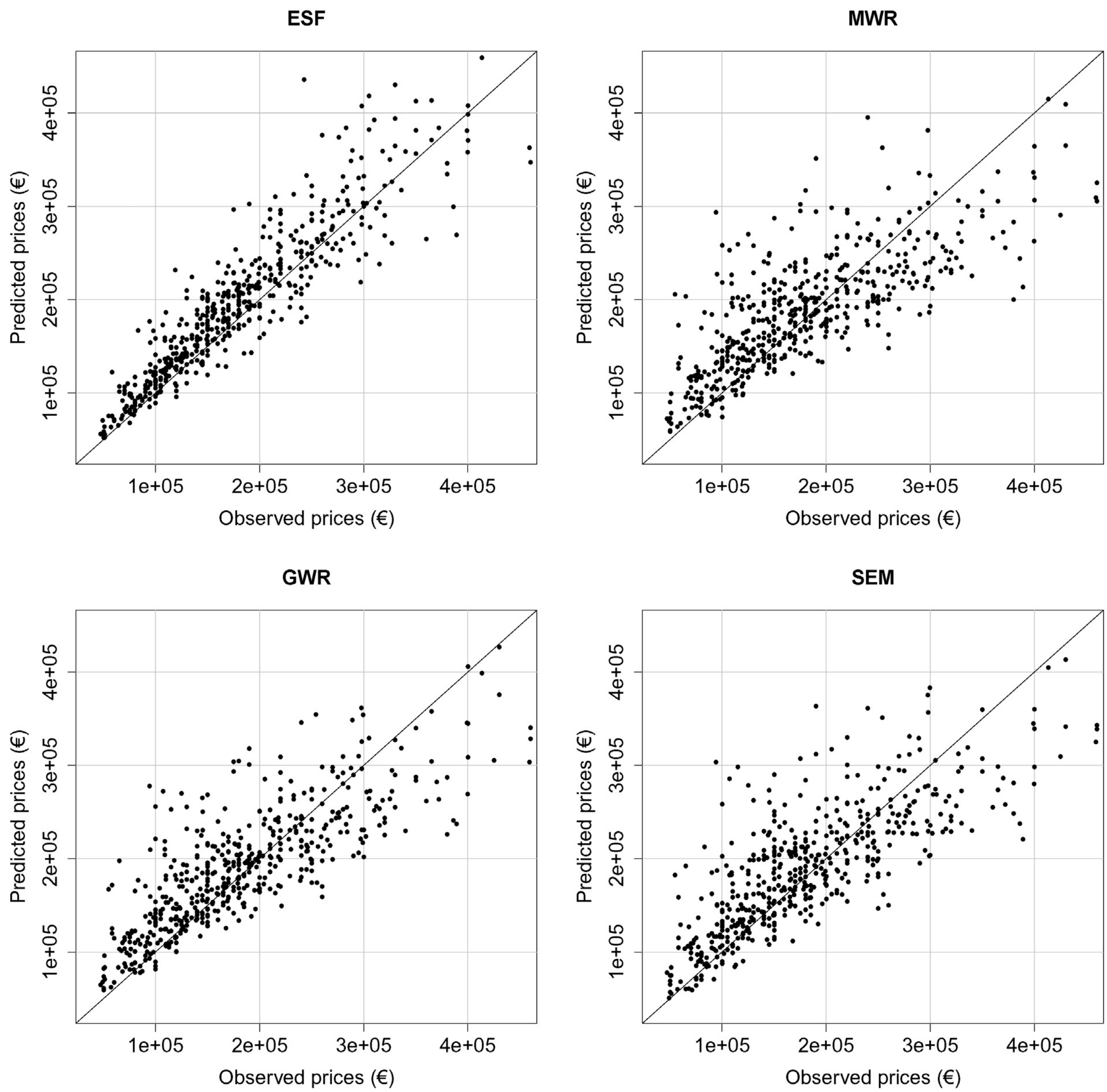

Fig. 4. In-sample prediction accuracies.

driven, the findings related to differences between the in-sample and the out-of-sample prediction accuracy suggest over-fitting. In contrast to MWR and GWR, ESF shows the tendency to perform more accurately when employing small-sized test data. Both issues call for attention in future studies.

Finally, we addressed practical issues affecting the application of the models. Because MWR and GWR are based on CV-based model calibration and ESF is grounded on eigenvector extraction from a neighborhood matrix, and involves a large set of interaction terms, both approaches are computationally costly. ESF is more of a constraint on small to medium-sized appraiser databases compared to MWR and GWR. To keep computations for larger housing datasets tractable, an alternative might be integrated nested Laplace approximation methods (Rue, Martino, & Chopin, 2009). While GWR is already implemented in current geographic information systems and is available as independent software, ESF presumes that the user has more in-depth knowledge of mathematics, which in turn requires coding skills (Chun & Griffith, 2013). In this respect, GWR is the more "user friendly" solution to modeling spatial heterogeneity (Lu, Harris, Charlton, & Brunsdon, 2014) and offers several extensions such as mixed-GWR where some covariates are expected to vary across space, while others are spatially constant (Fotheringham et al., 2002). A limitation of both MWR and GWR is that that they assume a linear functional form even though Helbich (2015) found non-linearities between some structural variables (e.g., age). To overcome this restriction, Helbich and Jokar Arsanjani (2015) made initial attempts linking ESF with non-linear generalized additive models (Wood, 2006). As advocated by Basile et al. (2014), (geo)additive hedonic models simultaneously considering spatial autocorrelation, spatial heterogeneity, and non-linearities might be a powerful complement.

To conclude, even though ESF is still a niche player in spatial analysis, it should be considered a valuable alternative method for real estate research that allows going beyond normal probability models, is highly capable of mapping local parameter estimates, while not facing multicollinearity between the local coefficients.

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Table 3
Results of the predictive accuracy assessment.

Size test data		In-sample Spearman's ρ			LOOCV RMSE		
		1st QT	Median	3rd QT	1st QT	Median	3rd QT
20% test data	ESF	0.919	0.924	0.930	0.260	0.276	0.333
	MWR	0.689	0.699	0.711	0.329	0.335	0.340
	GWR	0.697	0.706	0.716	0.327	0.332	0.336
	SEM	0.764	0.774	0.783	0.312	0.318	0.324
10% test data	ESF	0.899	0.904	0.910	0.265	0.272	0.297
	MWR	0.694	0.699	0.706	0.332	0.336	0.338
	GWR	0.702	0.707	0.713	0.327	0.331	0.334
	SEM	0.765	0.770	0.775	0.317	0.319	0.323
Size test data		Hold-out sample Spearman's ρ			Hold-out sample RMSE		
		1st QT	Median	3rd QT	1st QT	Median	3rd QT
20% test data	ESF	0.448	0.509	0.551	0.585	0.656	0.829
	MWR	0.659	0.687	0.730	0.325	0.341	0.361
	GWR	0.668	0.699	0.737	0.322	0.336	0.355
10% test data	SEM	0.638	0.684	0.725	0.331	0.351	0.371
	ESF	0.492	0.547	0.609	0.473	0.533	0.637
	MWR	0.643	0.689	0.740	0.309	0.332	0.362
	GWR	0.650	0.700	0.746	0.299	0.326	0.352
	SEM	0.620	0.687	0.729	0.321	0.346	0.371

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