CLASSROOM ASSESSMENT TECHNIQUES TO ASSESS CHINESE STUDENTS' SENSE OF DIVISION

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Abstract

This paper is about an explorative study of the use of classroom assessment techniques (CATs) by primary school mathematics teachers in China. Six female teachers and 216 third-grade students from two schools in Nanjing were involved. The focus was on assessing whole number arithmetic. Teachers' use of the CATs was investigated through lesson observations, feedback forms, interviews, and reports. In this paper we zoom in on one CAT in which students had to solve division problems without making use of the standard division algorithm, being the only procedure they had been taught. From the solutions teachers can infer whether their students really had deep understanding of the division operation. Only a few students could apply a solution strategy without using the standard algorithm. All teachers were initially unsure about what information they were supposed to find with this CAT and did not know how to deal with the results afterwards.

Key words: China, classroom assessment techniques, division, student work, textbook, whole number arithmetic

Introduction

Knowledge about students' learning is a sine qua non for educational decision making. Therefore, assessment – understood as the process in which students' responses to specially created or spontaneously occurring stimuli are used to draw inferences about their knowledge and skills (Popham, 2000) – plays a crucial role in education. Of the many different types of assessment that can be distinguished, formative assessment (Cizek, 2010) has the strongest link to teaching. It informs teachers directly about their students' learning processes so they can tailor their instruction to their students' needs. Formative assessment carried out by the teacher is often called "classroom assessment" (e.g., Mavrommatis, 1997). In fact, classroom assessment includes all teacher activities meant to collect information about their students' understanding of a particular topic.

To emphasize assessment's supporting role for teaching and learning, the Assessment Reform Group (1999) introduced the term "assessment for learning". This term was an eye-opener for many involved in assessing students' learning. In this new approach to assessment the focus shifts from a mainly test-based approach to one where assessment is more integrated with instruction and contains all kinds of informal assessment activities carried out by the teacher (Torrance, 2012).

From the moment that Black and Wiliam (1998) brought the power of classroom assessment to raise students' achievement to a larger audience, more and more research has been conducted on its practical applications. An example is the two-year project carried out with teachers in the United States (Leahy et al., 2005), in which eventually 50 "techniques" to improve teachers' classroom assessment practice were developed. Characteristic of these techniques is that they blur the divide between instruction and assessment and are low-tech, low-cost, and usually feasible for individual teachers to implement. Another characteristic of these "classroom assessment techniques" (CATs), as we will call them hereafter, is that these are often well-known activities done by teachers that are now deployed in a new way with a specific assessment focus.

Inspired by the work of Wiliam and colleagues (Leahy et al., 2005; Wiliam 2011), a project aiming to improve classroom assessment was started in the Netherlands. Recently, two consecutive small-scale studies were conducted in Dutch primary schools to investigate the feasibility and effectiveness of the CATs in mathematics education (Veldhuis and Van den Heuvel-Panhuizen, 2014). In these studies, CATs were used to help teachers quickly find information about students' abilities in whole number arithmetic and provide indications for further instruction. Two different formats were used: whole-classroom response systems directly informing the teacher and worksheets that the teacher has to check after the lesson. Results from the studies showed that using CATs had a positive effect on students' learning. The mathematics achievement of students who were in classes where CATs were used improved considerably more than that of students from a national norm sample. Moreover, teachers and students reported enjoying the CATs and finding them useful. The present paper reports on an explorative study similar to the Dutch studies, but carried out in China.

In China, which has a long history of examination-oriented education, an assessment reform in basic education was kicked off by the Ministry of Education as part of the New Curriculum Reform in 2001. Therefore, during the last decade much attention has been given to putting assessment into the hands of teachers and help them perceive and practice the idea of assessment supporting teaching and learning (Zhang, 2009). However, despite this effort it was found that such assessment is only weakly relevant to teachers (Brown et al., 2011). Also, it seems that mathematics teachers in primary school in China tend to equate classroom assessment with their reactions to students' different responses, and they do not include questions to reveal students' thinking (Zhao, Van den Heuvel-Panhuizen and Veldhuis, in preparation).

The present study explored whether Chinese primary school teachers' classroom assessment practice in mathematics education can be improved by applying CATs. Similar to the Dutch project the focus was on CATs to be used in teaching whole number arithmetic in the second semester of Grade 3; in particular the focus was on the standard algorithm of division. For this mathematical topic a package of CATs meant for two and a half weeks of teaching was designed by the authors of this paper. The package was tried out in February–March 2014. In this paper only part of the study is discussed.

Materials and Methods

Six female third-grade mathematics teachers (age *M* = 32; *SD* = 7.23 years) and 216 students from two primary schools in Nanjing tried out the CATs. Both schools are located in an urban district. School I, of which two teachers (Teachers A and B) and 60 students participated, has an average reputation. School II, of which four teachers (Teachers C, D, E and F) and 156 students took part, has a good reputation for its quality of education and the facilities in this school are better than those in School I. All teachers used the 苏教版 Textbook published by Jiangsu Education Publishing House.

Because the CATs should be integrated in the teachers' teaching practice there had to be a close fit between the CATs and the mathematics content provided by the textbook. This implied that we could not simply take over the CATs we had developed for the Dutch project. In the Chinese textbook, students start to learn multiplication and division in the first semester of Grade 2. After learning the basic knowledge and skills of multiplication and division (the multiplication tables), students already learn the algorithms for multiplication and division near the end of the first semester of Grade 2. This means students become familiar with the standard digit-based algorithmic vertical notation of multiplication and division from a very early age on. The teaching/learning trajectory of these algorithms consists of problems with a progressively increasing number of digits. At the beginning of the second semester in Grade 3 the students have arrived at solving division problems in which three-digit numbers have to be divided by one-digit numbers (see Tab. 1 for the content of the Chapter 1 on division that is dealt with in Grade 3 in February, 2014).

Lesson	Type	Topic	Example problems
1	New	Quotient is a three-digit number	600÷3 = 200 986÷2 = 493
2	New	Quotient is a two-digit number	312÷4 = 78
3	Repetition of Lesson 1 and 2		
4	New	0 in dividend (and quotient)	0÷3 = 0 306÷3 = 102
5	New	0 only in quotient	432÷4 = 108
6	New	Two-step division problem	There are two bookshelves with four layers. When there are 224 books in total, how many books are on one layer?
7	Repetition of Lesson 4, 5 and 6		
8	Repetition of the whole chapter		

Tab.1: Lesson plan for teaching the standard algorithm for division of three-digit numbers divided by one-digit numbers in Chapter 1, second semester of Grade 3

When designing the CATs for this chapter, two requirements were taken into account. The CATs should be linked to the lesson objectives and they should provide teachers with information about their students' learning to help them to reach a deeper understanding than just knowing whether or not students have answered a problem correctly. In total, for this chapter 13 CATs were developed.

The CAT of our focus in this paper is *Solving division problems without standard algorithm*. This CAT was planned for Lesson 8, when the students have had extensive practice in using the standard algorithm. Normally, at this stage, most students are able to carry out the algorithm and solve division problems without making mistakes. However, this does not automatically mean that they have a deep understanding of the division operation. It is also possible that students just apply the procedure in a mindless, mechanistic way. To make decisions for further instruction teachers need to know how stable students' understanding is. When students only have superficial knowledge, they might get in trouble when they have to use the division algorithm with, for example, decimal numbers.

The main idea behind this CAT is to reveal whether students have a clue on how to solve a division problem without using the standard algorithm. Therefore teachers provided the students with a worksheet containing four division problems – 468÷2=, 594÷6=, 480÷3=, and 816÷4= – presented in horizontal number sentences.

To help the teachers understand the purpose and procedure of the CATs, four meetings were organized in which the CATs were discussed. To collect data about the use of the CATs, all of one teacher's lessons in which she used the CATs were observed and recorded, and after each lesson she was interviewed. The other five teachers were observed and video recorded for at least one lesson per week. In the end, all teachers wrote a short report about whether and why they liked or disliked the CATs.

Results

According to the teachers' reports, the students were given at most 10 minutes for solving the four division problems. When the worksheets were handed to the teachers, they quickly scanned students' solutions and their first conclusion was that the majority of the students answered most of the division problems correctly and that most students gave an explanation for how they solved them.

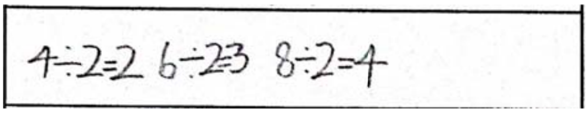
As the teachers reported, 468÷2 was not difficult for the students. But instead of solving the problem without using the algorithm, as was demanded, more than half the students did in fact use the standard algorithm. While the students' notations in horizontal number expressions suggest that they did carry out a number of sub-divisions based on splitting the dividend, what they really did was a step-by-step processing of digits, which is similar to an algorithmic approach (see Tab. 2a).

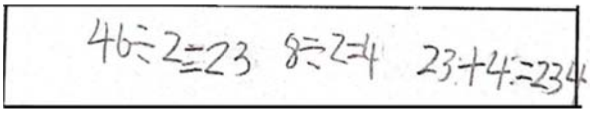
Although both students whose work is shown in Tab. 2a came to the correct answer, one might wonder whether they really have insight in the division operation. In contrast to this way of working, a solution that gives a better guarantee for having insight is using the number values of the dividend by splitting 468 into 400, 60, and 8, making three divisions, and adding the results. This solution is shown in Tab. 2b.

However, the real proof of having a good understanding of the division operation was delivered by 594÷6.

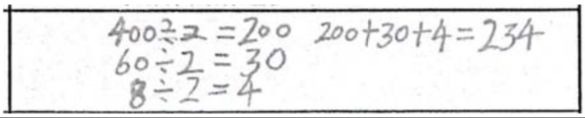
468÷2

a Digit-based horizontal notations of sub-divisions





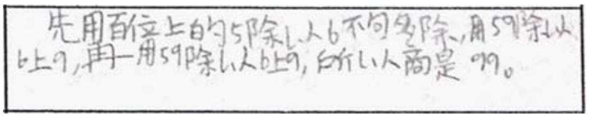
b Splitting up the dividend and horizontal notation of sub-divisions



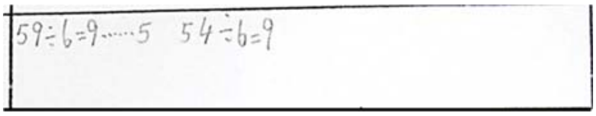
Tab. 2: Two types of student solutions for 468÷2

594÷6

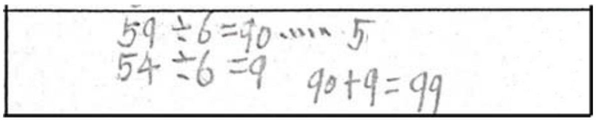
a Verbal description of division algorithm*



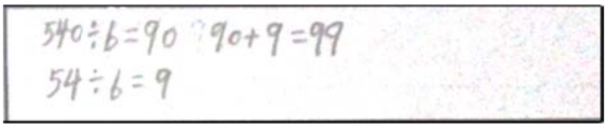
b Horizontal notation of division algorithm (digit-based)

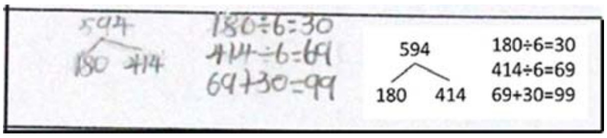


c Horizontal notation of division algorithm (with number value)

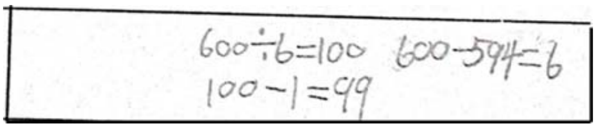


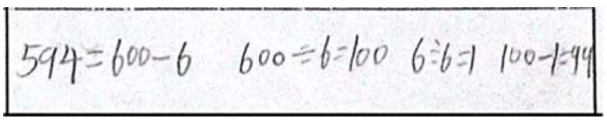
d Using a whole-number-based splitting strategy





e Using a smart strategy





* Translation: Firstly, I used 5 in the hundreds place divided by 6, which was not enough. Then I used 59 divided by 6. I wrote down 9. In addition, I used 59 to be divided by 6, which equals to 9. So the quotient is 99.

Tab. 3: Five types of student solutions for 594÷6

According to all the teachers 594÷6 was very difficult for their students. To solve this division almost all students stuck to the standard algorithm, either by describing it in words (see Tab. 3a) or by writing down the algorithm in a horizontal digit-based way (see Tab. 3b). Yet, while still using a digit-based approach some students were also aware of the number value of the digits (see

Tab. 3c), indicating that they have a notion of what is going on when you have to divide a number. Notwithstanding this, their solution was still based on the standard algorithm. Teachers A and B discovered that many of their students solved the problems in this way, which they considered as "mixing up different strategies and notations". Facing these –what they called– "seemingly right but wrong expressions", they felt that they did not know how to explain to their students what they did wrong.

The students who split the dividend in two or more whole numbers and divided them each and expressed the division in a horizontal notation (see Tab. 3d) really applied an alternative for the standard digit-based algorithm. However, a few students came up with rather far-fetched splits which made the teachers unsure about how to react to these solutions.

Despite this uncertainty, the teachers were quite sure that the student work that best revealed students' understanding of the division operation is the use of a smart solution, for example related to 600 to solve 594÷6 (see Tab. 3e). However, only about one or two students per class came up with such a solution. Teacher B was surprised that in her class two students, whom she considered as average (or even weak) students, now used such a smart strategy. In the teacher report Teacher B wrote:

"[This classroom assessment technique] expands students' thinking. They are supposed to command how to use the algorithm, but that should not be their only tool. They need to think about the features of particular division problems in order to calculate flexibly, rather than immediately think about the algorithm to solve all problems." (Teacher B, report)

All six teachers found it interesting to see their students' thinking. All of them, however, were also initially unsure about what information they were supposed to find, and three reported that even when they saw the students' responses they were still doubtful. In the interviews they also made it clear that they did not know how to deal with the results. As the main reasons they mentioned not being accustomed to asking students such questions or thinking about such questions themselves.

Conclusions

In this study we collected Chinese primary school mathematics teachers' first experiences with using CATs. Based on lesson observations, feedback forms, interviews, and teacher reports we can conclude that CATs can enrich Chinese teachers' assessment practice. All teachers agreed that the questions asked in the CATs were helpful to get to know more about students' learning, because the questions focus on students' mathematics understanding rather than on their skills or the accuracy of their calculations.

The CATs also gave teachers insight in what their students should learn and how to teach it. A first indication is that the teachers tried to redesign their instruction plan to assimilate the CATs into their lessons. Another sign is that some teachers taught their students to solve the CAT problems before using them in class; they probably wanted to prevent their students' bad performances in the CATs. Remarkably, some teachers even integrated parts of the characteristics of the CATs into their own teaching. This was, for example, illustrated by the fact that a teacher provided questions focusing on strategies rather than answers. In general, the use of CATs had a greater effect on instruction before class and in class, than after the class in which the CATs were used. In this sense, it looks more like teachers tried to merge the CATs with their previous instructional plan and use them as extra exercises than considering them as a turning point where instruction could be changed.

Notwithstanding, the teachers' reactions were overall positive; the teachers liked the CATs and considered them useful. So we think that CATs indeed can contribute to the improvement of Chinese teachers' assessment practice and consequently maybe also of their teaching practice in mathematics education.

Discussion

Of course, these conclusions should be taken with prudence. Only six teachers in only one grade and only one mathematical topic were involved. Further research is necessary to come to robust findings and generalisation. Moreover, despite the positive reactions, one might doubt whether the teachers really grasped the purpose of the CATs. In the case of the CAT *Solving division problems without standard algorithm*, it was remarkable that the first thing the teachers did was to check the correctness of the answers. This suggests that the teachers did not really see the CAT as a gateway to assess students' deep understanding of division. The fact that the teachers did not know how to react to their students' solutions, which they mentioned clearly in the interviews, can also be considered an indication of this.

What the students' solutions and the teachers' reactions also pointed at is how much mathematics education differs in different countries, even when it involves a rather straightforward topic such as division in the domain of whole number arithmetic. In this way the present study did not only give us information about whether classroom assessment practice in Chinese primary school mathematics classes can be improved by applying CATs, the study also brought another finding which we were not looking for at first to the fore. When starting this study, of course we were aware of the fact that students in China follow a teaching/learning trajectory for whole number arithmetic that starts with teaching students the digit-based algorithms from a very early age on. Therefore, we thought it would be helpful for the teachers to check whether their students can (still) solve division problems without using the standard algorithm. In the Netherlands, such a CAT would not be revealing for teachers because the Dutch trajectory for learning division is heavily grounded in whole-number-based calculation. So Dutch students will be quite able to not use the standard algorithm. When trying out this CAT in China, we did not realize that it would be virtually impossible for Chinese students to show their understanding of the division operation without using the fixed recipe of the digit-based algorithm.

Even more surprisingly, teachers also appeared to not comprehend the point of letting students try to use different solution strategies than the standard algorithm and had trouble in identifying whether they had used a different solution strategy or not.

In this way the CATs were not only an eye-opener for the teachers who were involved in our study, but also for us as researchers.

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