A Theory of Scalar Mesons

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We discuss the effect of the instanton induced, six-fermion effective Lagrangian on the decays of the lightest scalar mesons in the diquark–antidiquark picture. This addition allows for a remarkably good description of light scalar meson decays. The same effective Lagrangian produces a mixing of the lightest scalars with the positive parity $q\bar{q}$ states. Comparing with previous work where the $q\bar{q}$ mesons are identified with the nonet at 1200-1700 MeV, we find that the mixing required to fit the mass spectrum is in good agreement with the instanton coupling obtained from light scalar decays. A coherent picture of scalar mesons as a mixture of tetraquark states (dominating in the lightest mesons) and heavy $q\bar{q}$ states (dominating in the heavier mesons) emerges.
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1 Introduction

We study in this paper the strong decays of the lightest scalar mesons: σ, κ, f_0, a_0 and the relations of the light scalars to the scalar mesons observed in the 1-2 GeV range.

Recent experimental and theoretical evidence for the existence of σ and κ [1–4] (see also [5]) indicates that light scalars make a full SU(3) flavor nonet. Their mass spectrum, with the peculiar inversion of the κ and f_0 or a_0 mass ordering, speaks however against the naive $q\bar{q}$ picture. The most natural explanation for such complete multiplet with inverted mass spectrum is that these mesons are diquark–antidiquark bound states. The $K\bar{K}$ molecular constitution [6], advocated to explain the degeneracy of f_0/a_0 with the $K\bar{K}$ threshold, would lead most likely to incomplete multiplets.

The picture where the light scalar mesons are diquark–antidiquark states bound by color forces has been discussed by several authors [7–9]. In this picture the diquarks, which we will indicate with $[q_1q_2]$, are in color $\bar{\mathbf{3}}$, spin $S = 0$ and flavor $\bar{\mathbf{3}}$, and antidiquarks in the conjugate representations. Diquark–antidiquark bound states (tetraquarks, for short) naturally reproduce the SU(3) nonet structure with the correct mass ordering, as indicated by the explicit quark composition:

$$
\begin{aligned}
\sigma^{[0]} &= [ud][\bar{u}\bar{d}] \\
\kappa &= [su][\bar{u}\bar{d}];\ [sd][\bar{u}\bar{d}]\ (+\text{ conjugate doublet}) \\
f_0^{[0]} &= \frac{[su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]}{\sqrt{2}} \\
a_0 &= [su][\bar{s}\bar{d}];\ \frac{[su][\bar{s}\bar{u}] - [sd][\bar{s}\bar{d}]}{\sqrt{2}};\ [sd][\bar{s}\bar{u}]
\end{aligned}
\qquad (1)
$$

While the mass spectrum of the light scalar mesons is well understood in terms of diquark–antidiquark bound states, the overall picture of scalar mesons is still affected by two drawbacks. First, the strong decays into two pseudoscalar mesons ($S \to PP$) have so far escaped a satisfactory theoretical understanding in the quark rearrangement picture. In particular, the $f_0 \to \pi\pi$ coupling is too small compared to experiments (according to the ideal-mixing decomposition in (1) it should vanish) and the $a_0 \to \eta\pi$ coupling largely exceeds its experimental value. Second, the identification of the $q\bar{q}$ scalar sates is an open issue. The latter must exist, as indicated by the well identified nonets corresponding to[1] orbital angular momentum $L = 1$ and quantum numbers $J^{PC} = 1^{++}, 1^{+-}, 2^{++}$ [10]: all predicted states are unambiguously identified but for $J^{PC} = 0^{++}$.

In this paper we show that these two main problems are solved by the instanton induced effective six-fermion Lagrangian [11], the same effective interaction which solves the problem of the $\eta - \eta'$ masses [12,13]. Such Lagrangian has two important effects in scalar mesons dynamics: (i) it generates a mixing between tetraquarks and $q\bar{q}$ states, (ii) it provides an additional amplitude which brings the strong decays of the light scalars in good agreement with data. Former studies of instanton-induced effects in scalar meson dynamics can be found in the literature [14], but these two effects have not been discussed before.

The two effects induced by the effective instanton Lagrangian are closely connected. The tetraquark–$q\bar{q}$ mixing makes it possible to identify the heavy scalars around 1.5 GeV as predominantly $q\bar{q}$ states with a non-negligible tetraquark component. The latter is essential to explain the anomalous mass spectrum of such mesons, as originally proposed in [8]. On the other hand, integrating out the heavy $q\bar{q}$ components, the tetraquark–$q\bar{q}$ mixing manifests itself into the non-standard $S \to PP$ decay amplitude for the light scalar mesons which improves substantially the agreement with data. These two independent phenomena lead to obtain two independent phenomenological determinations of the non-perturbative parameter which controls the matrix elements of the instanton Lagrangian in the scalar sector. The two determinations turn out to agree, reinforcing the overall consistency of the picture.

The paper is organized as follows: in Section 2 we illustrate the two main effects of the instanton interaction in the scalar sector, constructing the corresponding effective Lagrangians in terms of meson fields. In Section 3 we present a numerical analysis of $S \to PP$ decays in this scheme, demonstrating the relevant phenomenological role of the instanton contribution; as a further cross-check, we also show that $S \to PP$ decays are badly described under a pure $q\bar{q}$ picture of the light scalar mesons, with or without instantons. The results are summarized in the Conclusions.

2 Instanton effects in scalar meson dynamics

QCD instantons produce an effective interaction which reduces the $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ global symmetry of the quark model in the chiral limit to $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ times baryon number. The effect can be described by the following effective Lagrangian [12] (see also [15]):

$$\mathcal{L}_{\mathrm{I}} \propto \mathrm{Det}(Q_{LR}) \ , \qquad (Q_{LR})^{ij} = \bar{q}_L^i q_R^j \ , \tag{2}$$

where i and j denote flavor indices (summation over color indices is understood).

With three light quark flavors, $\mathcal{L}_{\mathrm{I}}$ is proportional to the product of three quark and three antiquark fields, antisymmetrised in flavor and color, and it includes a term of the type

$$\mathrm{Tr}(J^{[4q]} J^{[2q]}) \ , \tag{3}$$

where

$$J_{ij}^{[4q]} = [\bar{q}\bar{q}]_i [qq]_j \ , \qquad J_{ij}^{[2q]} = \bar{q}_j q_i \ , \tag{4}$$

[1] The fact that the $J^{PC} = 0^{++}$ components of the $q\bar{q}$ states must have $L = 1$ can most easily be understood as follows. The operator $\bar{q}(0)q(0)$ can be written as $\bar{q}_L q_R + \bar{q}_R q_L$. Now $\bar{q}_L$ and q_R both have helicity R but opposite momentum, hence opposite spin. Taking the momenta in the z-direction, we find that the spin structure of the fields here is $(1,2) + (2,1)$, so $S_z^{\mathrm{tot}} = 0$. This is to be compared with the pion case, where γ_5 flips the relative sign: $\bar{q}_L q_R - \bar{q}_R q_L = (1,2) - (2,1)$. The latter is clearly the mode with total spin $S = 0$, since it can be written as $\epsilon_{ij} u_i v_j$, but in the scalar case, it is the $S_z = 0$ component of the state with total spin $S = 1$. To obtain total angular momentum $J = 0$ we therefore need orbital angular momentum $L = 1$. The gamma algebra then shows that such $L = 1$ contribution of the wave functions to $\bar{q}(0)q(0)$ originates from the relativistic terms that mix the large components of the Dirac spinors with the small ones.

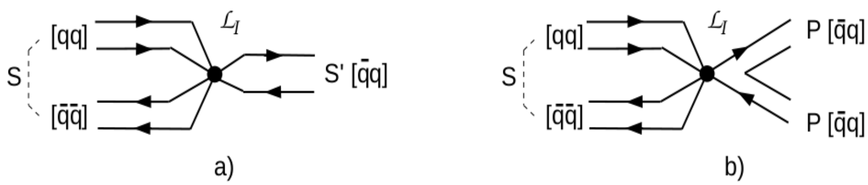

Figure 1: The two main effects of the instanton Lagrangian in the scalar sector: (a) the tetraquark-$q\bar{q}$ mixing; (b) the Zweig-rule violating $S \to PP$ amplitude.

and

$$[qq]_{i\alpha} = \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\bar{q}_c^{j\beta}\gamma_5 q^{k\gamma}. \tag{5}$$

$[qq]_{i\alpha}$ is the spin-0 diquark operator, latin indices indicate flavor, greek indices stand for color and $\bar{q}_c$ is the charge-conjugate of the quark field. As shown in Fig. 1, this term can lead to a mixing between tetraquark and $q\bar{q}$ scalar states (Fig. 1a), and an effective S(tetraquark)$\to PP$ coupling (Fig. 1b) which allows the ideally-mixed f_0 state in (1) to decay into two pions.

2.1 The tetraquark–$q\bar{q}$ mixing

The quantum numbers of $J_{ij}^{[4q]}$ and $J_{ij}^{[2q]}$ match those of the tetraquark and $q\bar{q}$ scalar states S_{ij} and S'_{ij}

$$S = \begin{bmatrix} \frac{a^0}{\sqrt{2}} + \frac{f_0^{[0]}}{\sqrt{2}} & a^+ & \kappa^+ \\ a^- & -\frac{a^0}{\sqrt{2}} + \frac{f_0^{[0]}}{\sqrt{2}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \sigma^{[0]} \end{bmatrix} \quad (S = [qq][\bar{q}\bar{q}]) \ , \tag{6}$$

$$S' = \begin{bmatrix} \frac{a'^0}{\sqrt{2}} + \frac{\sigma'^{[0]}}{\sqrt{2}} & a'^+ & \kappa'^+ \\ a'^- & -\frac{a'^0}{\sqrt{2}} + \frac{\sigma'^{[0]}}{\sqrt{2}} & \kappa'^0 \\ \kappa'^- & \bar{\kappa}'^0 & f_0'^{[0]} \end{bmatrix} \quad (S' = q\bar{q}) \ , \tag{7}$$

where the the neutral isoscalar states are not necessarily mass eigenstates. The instanton Lagrangian thus generate a mixing term

$$\mathcal{L}_{\rm mix} = \gamma \ {\rm Tr}(S \cdot S') \ . \tag{8}$$

A mixing of this form was introduced in [8] and is essential for a consistent identification of the scalar mesons around 1.5 GeV in terms of $q\bar{q}$ states, with a possible addition of a scalar glueball [17]. Indeed, the well identified $I = 1$ and $I = 1/2$ states around 1.5 GeV, $a_0(1450)$ and $K^*(1430)$, also show a reversed mass ordering, although smaller than in the case of the light states. The anomaly can be explained as a contamination of the inverse hierarchy of the light tetraquark states via $\mathcal{L}_{\rm mix}$. The coefficient γ was determined phenomenologically from a fit to the mass spectrum [8] to be:

$$|\gamma| \simeq 0.6 \ {\rm GeV}^2 \ . \tag{9}$$

With this value and the observed masses, the bare masses of the lightest $q\bar{q}$ scalars turn out to be slightly above 1 GeV. The result goes well with the estimate obtained in [13] from a consistent description of pseudoscalar states (including the η') and scalar $q\bar{q}$ states within a linear sigma model. The bare $q\bar{q}$ masses agree also with the natural ordering of P-wave states, that predicts 0^{++} masses to be smaller than 1^{++} and 2^{++} masses [8, 18].

It was observed in [18] that the value of the effective coupling in (9) is much larger than what expected by usual QCD interactions for such Zweig-rule violating effect. To obtain this mixing by usual QCD interactions, it is necessary to annihilate completely quarks and antiquarks in the initial state and to produce from vacuum those of the final state. A strongly suppressed transition [18], which instead is provided almost for free by the instanton Lagrangian.

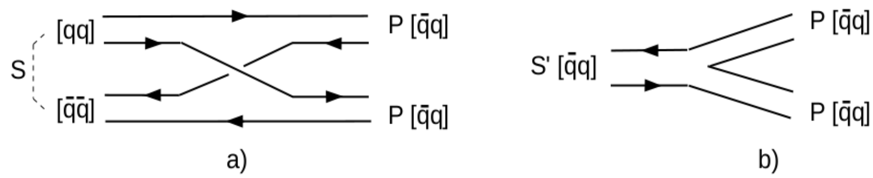

Figure 2: Leading quark-flavor diagrams for the decays into two pseudoscalar mesons of tetraquark (a) and $q\bar{q}$ (b) scalar mesons.

A proviso concerns the hadronic matrix element of $J^{[2q]}$. In the fully non-relativistic approximation, one would have:

$$\langle 0|\bar{q}(0)q(0)|S'\rangle = \Psi(0) \ , \tag{10}$$

where $\Psi(0)$ is the non-relativistic wave-function in the origin, which vanishes for P-waves. However, for relativistic quark fields we get a non-vanishing result, proportional to $v = p/E$. For QCD, Coulomb like, bound states, $v \sim \alpha_S$, and the P-wave nature of S' results only in a mild suppression.

The instanton induced mixing could in principle be determined by the following matrix element

$$\langle S|\mathcal{L}_{\rm I}|S'\rangle \propto \langle S|{\rm Tr}(J^{[4q]}J^{[2q]})|S'\rangle \approx {\rm Tr}\left(\langle S|J^{[4q]}|0\rangle\langle 0|J^{[2q]}|S'\rangle\right) \ . \tag{11}$$

However, at present we do not have a reliable independent information on the matrix elements of $J^{[4q]}$ and $J^{[2q]}$ between scalar states and vacuum.

2.2 $S \to PP$ decays

The leading mechanisms describing the decays of tetraquark and $q\bar{q}$ scalar states into two pseudoscalar mesons are illustrated in Fig. 2. In the tetraquark case, the diagram in Fig. 2a (denoted as *quark rearrangement* in [9]) explains the affinity of f_0 and a_0 to $K\bar{K}$ channels. A non vanishing $f_0 \to \pi\pi$ amplitude appears if the possible mixing of $f_0^{[0]}$ and $\sigma^{[0]}$ in (1) is taken into account. However, as we illustrate in a quantitative way in the following section, the mixing alone does not lead to a good fit of all $S \to PP$ decays [8,9]. As already observed in [9], this problem indicates the presence of additional contributions to the $S \to PP$ amplitudes, generated by a different dynamical mechanism. This mechanism can be traced back to the instanton amplitude in Fig. 1b.

In constructing effective Lagrangians for the $S(S') \to PP$ decays an important role is played by chiral symmetry. In the chiral limit the octet components of the light pseudoscalar mesons

$$\Phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{bmatrix} \ , \tag{12}$$

can be identified with the Goldstone bosons of the spontaneous breaking of ${\rm SU}(3)_L \times {\rm SU}(3)_R$ into ${\rm SU}(3)_{L+R}$. Following the general formalism of Ref. [19] (see also [20]), the effective Lagrangian of lowest dimension allowed by chiral symmetry contributing to $S \to PP$ decays, with P restricted to the octet components, turns out to be composed by only two independent operators:

$$O_1(S) = \frac{F^2}{2}{\rm Tr}(Su_\mu u^\mu) \qquad \text{and} \qquad O_2(S) = \frac{F^2}{2}{\rm Tr}(S){\rm Tr}(u_\mu u^\mu) \ , \tag{13}$$

where

$$u_\mu = iu^\dagger \partial_\mu U u^\dagger = -(\sqrt{2}/F)\partial_\mu \Phi + \mathcal{O}(\Phi^2) \ , \qquad U = u^2 = e^{i\sqrt{2}\Phi/F} \ . \tag{14}$$

Being dictated only by chiral symmetry, an identical structure holds for the $S' \to PP$ transitions of the $q\bar{q}$ scalar states.

The relative strength of the effective couplings of these operators can be determined by the correspondence of their flavor structures with a given quark-flavor diagram. In the tetraquark case, the leading amplitude in Fig. 2a contributes to both O_1 and O_2, generating the combination $O_1 - \frac{1}{2}O_2$ [9]. Taking into account also contributions with the singlet pseudoscalar field, the effective operator generated is:

$$O_f(S) = O_1(S) - \frac{1}{2}O_2(S) + O_S(S) \ , \qquad O_S(S) = \frac{F^2}{2}\left[-\mathrm{Tr}(Su_\mu)\mathrm{Tr}(u^\mu) + \frac{1}{2}\mathrm{Tr}(S)\mathrm{Tr}(u_\mu)\mathrm{Tr}(u^\mu)\right] \ . \qquad (15)$$

The instanton induced coupling in Fig. 1b has a completely different flavor structure. From the chiral realization of the currents in (3),

$$\begin{aligned} \langle PP|J^{[2q]}|0\rangle &= (u_\mu u^\mu)_{\mathcal{O}(\Phi^2)} + \ldots \\ \langle PP|\mathrm{Tr}(J^{[4q]}J^{[2q]})|S\rangle &= \mathrm{Tr}(Su_\mu u^\mu)_{\mathcal{O}(\Phi^2)} + \ldots \propto \mathrm{Tr}(S\partial_\mu\Phi\partial^\mu\Phi) + \ldots \end{aligned} \qquad (16)$$

where dots denote higher-order terms in Φ and in the chiral expansion. It follows that instanton effects are encoded only by $O_1(S)$. Taking into account both the leading quark-rearrangement diagram and the instanton contribution, decays of scalar tetraquark states into pseudoscalar mesons should be described by the effective Lagrangian

$$\mathcal{L}_{\mathrm{decays}}(S) = c_f O_f(S) + c_I O_1(S) \qquad (17)$$

where we expect $|c_I| \ll |c_f|$ since that the instanton contribution is a subleading effect.

Similarly to the case of the S–S' mixing, we are not able to evaluate the hadronic matrix element of $J^{[4q]}$ in (16) from first principles, therefore we cannot predict the value of the effective instanton coupling c_I in (17). However, an interesting crosscheck of the normalization of the instanton effective Lagrangians is obtained under the hypothesis that the leading contribution to the amplitude in Fig. 1b is the S' pole term arising by the contraction of the diagrams in Fig. 1a and Fig. 2b.

The $S' \to PP$ decays of the heavy $q\bar{q}$ states have been analyzed in Ref. [8] and found to be reasonably well described by the effective Lagrangian

$$\mathcal{L}_{\mathrm{decays}}(S') = c'_f \ O_1(S') \ , \qquad (18)$$

corresponding to the chiral realization of the diagram in Fig. 2b. The value of the effective coupling is found to be $c'_f \approx 6.1$ GeV^{-1} ($c'_f = 2A'$ in the notation of Ref. [8]). Under the plausible hypothesis of S' pole dominance for the instanton-induced $S \to PP$ amplitude we thus expect

$$\left|c_I^{(S'-\mathrm{pole})}\right| = \frac{|\gamma| \ c'_f}{M^2_{S'} - M^2_S} \sim \frac{|\gamma| \ c'_f}{M^2_{S'}} \sim 0.016 \qquad (19)$$

where we have used $M_{S'} \sim 1.5$ GeV.

The above estimate should be taken cautiously. Non-pole terms are not expected to be totally negligible, and the poor experimental information about the heavy scalar meson decays implies a sizable uncertainty in the value of c'_f. Nonetheless it is very encouraging that the value in (19) is consistent with the phenomenological value of c_I that we shall obtain in the next section from the phenomenology of light scalar decays with the effective Lagrangian (17).

3 Phenomenological analysis of $S \to PP$ decays

In this section we analyze the decays of the light scalars into two pseudoscalar mesons using the effective Lagrangian in (17). As discussed before, such Lagrangian with $|c_I| \ll |c_f|$ corresponds to the tetraquark hypothesis for the light scalar mesons, taking into account instanton effects.

The $g_{SP_1P_2}$ couplings, defined by

$$\mathcal{A}\left(S \to P_1(p_1)P_2(p_2)\right) \ = \ g_{SP_1P_2} \ p_1^\mu p_{2\mu} \ = \ g_{SP_1P_2} \ \frac{1}{2}(M_S^2 - M_{P_1}^2 - M_{P_2}^2) \ , \qquad (20)$$

−	σ	κ	f_0	a_0
M(MeV)	441	800	965	999

Table 1: Numerical values used for the light scalar masses (see Ref. [1–4, 22])

are reported in Table 2. The decay rates are then given by:

$$\Gamma(S \to P_1 P_2) = |\mathcal{A}(S \to P_1 P_2)|^2 \frac{p^*}{8\pi M_S^2} ,$$

where p^* is the decay momentum.

For comparison, we also attempt a fit of light scalar meson decays with the effective Lagrangian

$$\mathcal{L}_{\rm eff}(S') = c'_f O_1(S') + c'_I O_f(S') \tag{21}$$

where, contrary to what advocated so far, we identify the $q\bar{q}$ scalar field in (7) with the nonet of light scalar mesons. In this scheme the leading operator is $O_1(S')$ while instanton effects contribute to $O_f(S')$ (the corresponding $g_{SP_1P_2}$ are reported in the third line of Table 2). The comparison shows that, beside the problems with the mass spectrum, the $q\bar{q}$ hypothesis for the light scalar mesons has also serious difficulties in fitting $S \to PP$ data.

3.1 Mass mixing of neutral states

The physical f_0 and σ states are in general a superposition of the ideally mixed states $\sigma^{[0]}$ and $f_0^{[0]}$ defined in Eqs. (6)–(7). Introducing a generic mixing between the mass eigenstates and the ideally mixed states

$$\begin{bmatrix} \sigma \\ f_0 \end{bmatrix} = \begin{bmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} \sigma^{[0]} \\ f_0^{[0]} \end{bmatrix} , \tag{22}$$

the value of the mixing angle ω is determined by the experimental values of the scalar masses (see [9, 21]). Fixing the scalar masses to the input values in Table 1 but for the poorly known κ mass, and letting the latter vary in the interval $[750, 800]$ MeV, we find that the deviations from the ideal mixing case are quite small. We find in particular $|\omega| < 5^\circ$, which we use as range to estimate the impact of a non-vanishing ω.

Another important ingredient to compute the physical amplitudes is the η–η' mixing. In the octet–singlet basis we define

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix} = U(\phi_{PS}) \begin{bmatrix} \eta_8 \\ \eta_0 \end{bmatrix}, \qquad U(\phi_{PS}) = \begin{bmatrix} \cos\phi_{PS} & -\sin\phi_{PS} \\ \sin\phi_{PS} & \cos\phi_{PS} \end{bmatrix} . \tag{23}$$

It is useful to consider also the mixing in the quark basis:

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix} = U(-\theta) \begin{bmatrix} \eta_q \\ \eta_s \end{bmatrix} , \tag{24}$$

where $\phi_{PS} + \theta + \tan^{-1}(\sqrt{2}) = 0$. From the analysis of the pseudoscalar meson masses, $\gamma\gamma$ decays of η and η' and $J/\psi \to \gamma\ \eta/\eta'$ [26], one obtains $\phi_{PS} \simeq -22^0$ ($\theta \simeq -33^0$).

The effect of $\eta-\eta'$ mixing in the $a_0 \to \eta\pi$ processes plays an important role in distinguishing the two hypotheses for the scalar mesons. In the four-quark case, the quark exchange amplitude (Fig. 2a) produces a pure η_s, while the instanton interaction (Fig. 1b) produces a pure η_q. Expressing the coupling to physical particles η and η' in terms of the octet-singlet mixing angle leads to:

$$g^{[4q]}_{a_0^+\pi^+\eta} = \sqrt{2} c_I \cos\theta - c_f \sin\theta = \frac{\sqrt{2}(c_f + c_I)\cos\phi_{PS} + (c_f - 2c_I)\sin\phi_{PS}}{\sqrt{3}} \tag{25}$$

In the $q\bar{q}$ hypothesis, where the role of two operators O_1 and O_f is exchanged, one finds

$$g^{[q\bar{q}]}_{a_0^+\pi^+\eta} = \frac{\sqrt{2}(c'_f + c'_I)\cos\phi_{PS} - (2c'_f - c'_I)\sin\phi_{PS}}{\sqrt{3}} \tag{26}$$

	$\sigma^{[0]}\pi^+\pi^-$	$f_0^{[0]}\pi^+\pi^-$	$f_0^{[0]}K^+K^-$	$\kappa^+K^0\pi^+$	$a_0\eta_q\pi$	$a_0\eta_s\pi$	$a_0^-K^-K^0$
$[qq][\bar{q}\bar{q}]$	$-c_f$	$\sqrt{2}c_I$	$\frac{1}{\sqrt{2}}(-c_f+c_I)$	c_f+c_I	$\sqrt{2}c_I$	$-c_f$	c_f+c_I
$q\bar{q}$	$\sqrt{2}c'_f$	$-c'_I$	c'_f	$c'_f+c'_I$	$\sqrt{2}c'_f$	$-c'_I$	$c'_f+c'_I$

Table 2: The $g_{SP_1P_2}$ couplings for tetraquark and $q\bar{q}$ scalar mesons, from the effective Lagrangians (17) and (21), respectively.

Processes	KLOE [23,25]		BES, Crystal Barrel, WA102	
$f_0 \to \pi^+\pi^-$	$1.43^{+0.03}_{-0.60}$	(1.3 ± 0.1)	2.32 ± 0.25 [27]	
$f_0 \to K^+K^-$	$3.76^{+1.16}_{-0.49}$	$\left(0.4^{+0.6}_{-0.3}\right)$	4.12 ± 0.55 [27]	
$a_0 \to \pi^0\eta$	2.8 ± 0.1	(2.2 ± 0.1)	2.3 ± 0.1 [28]	2.1 ± 0.23 [29]
$a_0 \to K^+K^-$	2.16 ± 0.04	(1.6 ± 0.1)	1.6 ± 0.3 [28]	

Table 3: Experimental data for the $S \to PP$ amplitudes in GeV. The number in brackets in the KLOE column refers to the parameterization of Ref. [30] without the σ pole (the absence of the σ contribution makes the $f_0 \to \pi^+\pi^-, K^+K^-$ results between brackets less reliable for the present analysis; similarly, we do not quote the $f_0 \to \pi^+\pi^-, K^+K^-$ results extracted from $\sigma(e^+e^- \to \pi^+\pi^-\gamma)$ [24] which suffer of a larger background). The numbers in the last column refer to other experiments where it has been possible to unambiguously extract the information on the partial amplitudes.

3.2 Numerical analysis

The results of the fit to $S \to PP$ amplitudes are reported in Table 4. We use the masses given in Table 1 and $\phi_{PS} = -22°$ [26]. We analyze both the two- and four-quark hypotheses, with or without the instanton contribution, using $\sigma \to \pi\pi$ and $f_0 \to \pi\pi$ as input channels. For simplicity, in Table 4 we compare the theoretical predictions with the KLOE data only. A comparison with other experimental results can simply be obtained by means of Table 3, where all the available experimental information is collected. The data for σ and κ decays are the results of the theoretical analyses in [3,4].

The values of the couplings derived in the four-quark hypothesis and $\omega = 0$ are:

$$c_f = 0.041\ \text{MeV}^{-1};\quad c_I = -0.0022\ \text{MeV}^{-1}\qquad (\omega = 0) \tag{27}$$

The negative sign of c_I is chosen to minimize the $a_0 \to \eta\pi$ rate. As shown by eq. (25), the negative interference between c_f and c_I increases for ϕ_{PS} more negative.

As an alternative strategy, we have performed a global fit, assigning conventionally a 10% error to the σ rate, 15% to κ, 30% to all others and $\phi_{PS} = -22°$ and searched for a best fit solution. The results of this fit are reported in the fourth column. The best-fit couplings are:

$$c_f = 0.020 \pm 0.002\ \text{MeV}^{-1},\quad c_I = -0.0025 \pm 0.0012\ \text{MeV}^{-1}\qquad (-5° \le \omega \le 5°) \tag{28}$$

Central values are for $\omega = 0$, errors are given by letting ω to vary in $\pm5°$ range (see sect. 3.1).

Some comments concerning the fit under the four-quark hypothesis are in order:

- There is a good overall consistency, the fit is stable and, as expected, $|c_I| \ll |c_f|$. The fitted value of $|c_I|$ is also perfectly consistent with the pole estimate presented in Eq. (19).
- The positive feature of the instanton contribution is that it provides a non-vanishing $f_0\pi\pi$ coupling independent from mixing and, at the same time, it improves the agreement with data on the 'clean' $a_0 \to \eta\pi$ channel.
- The relation between $\kappa \to K\pi$ and $a_0 \to KK$ is fixed by $SU(3)$ and does not depend on the value of the couplings. As a result it is impossible to fit simultaneously the central values of these two amplitudes without introducing symmetry breaking terms.

Processes	$\mathcal{A}_{\rm th}([qq][\bar{q}\bar{q}])$			$\mathcal{A}_{\rm th}(q\bar{q})$		$\mathcal{A}_{\rm expt}$
	with inst.	no inst.	best fit	with inst.	no inst.	
$\sigma \to \pi^+\pi^-$	input	input	1.6	input	input	3.22 ± 0.04
$\kappa^+ \to K^0\pi^+$	7.3	7.7	3.3	6.0	5.5	5.2 ± 0.1
$f_0 \to \pi^+\pi^-$	input	[0–1.6]	1.6	input	[0–1.6]	1.4 ± 0.6
$f_0 \to K^+K^-$	6.7	6.4	3.5	6.4	6.4	3.8 ± 1.1
$a_0 \to \pi^0\eta$	6.7	7.6	2.7	12.4	11.8	2.8 ± 0.1
$a_0 \to K^+K^-$	4.9	5.2	2.2	4.1	3.7	2.16 ± 0.04

Table 4: Numerical results, amplitudes in GeV. Second and third columns: results obtained with a decay Lagrangian including or not including instanton effects, respectively. Fourth column: best fit with instanton effects included (see text). Fifth and sixth columns: predictions for a $q\bar{q}$ picture of the light scalars with and without instanton contributions. All results are obtained with a η–η' mixing angle $\phi_{PS} = -22°$. Second and fifth columns are computed with a scalar mixing angle $\omega = 0$. The $f_0 \to \pi\pi$ couplings in the third and sixth columns are computed with ω in the range $\pm 5°$ (see text). Data for σ and κ decays are from [3,4], the reported amplitudes correspond to: $\Gamma_{\rm tot}(\sigma) = 544 \pm 12$, $\Gamma_{\rm tot}(\kappa) = 557 \pm 24$.

- Being extracted from $\sigma(e^+e^- \to \pi\pi\gamma)$ data, the experimental values for the $f_0, a_0 \to KK$ couplings reported in Table 4 are subject to a sizable theoretical uncertainty (not shown in the table).
- A mixing angle ω in the $\pm 5°$ range (see sect. 3.1) affects, most significantly, the no-instanton case, leading to a non-vanishing $f_0 \to \pi\pi$ amplitude marginally consistent with data but it does not solve the $a_0 \to \eta\pi$ problem (Table 4, third column).

The fit under the $q\bar{q}$ hypothesis, reported in the fifth and sixth columns of Table 4 is much worse. Due to the exchange of f_0 and σ going from S to S', it remains true that there is no $f_0 \to \pi\pi$ in the absence of instantons and, similarly to the tetraquark case, we obtain $|c'_I| \ll |c'_f|$. However, the situation is drastically different for the $a_0 \to \pi^0\eta$ channel. In the $q\bar{q}$ case there is no sign conspiracy that produces the cancellation found before and the predicted amplitude is far from the observed one for any value of ϕ_{PS}, with or without instanton effects. This bad fit provides a further evidence, beside the inverse mass spectrum, against the $q\bar{q}$ hypothesis for the lightest scalar mesons.

4 Conclusions

The addition of the instanton-induced effective six-fermion Lagrangian lead us to a simple and satisfactory description of both the light scalar mesons below 1 GeV and the heavier scalar states around 1.5 GeV. The light mesons are predominantly tetraquark states (S), while the heavier ones are predominantly $q\bar{q}$ states (S'). Instantons induce a mixing between the two sets of states, which explains the puzzling mass spectrum of the heavier mesons. Moreover, integrating out the heavy states, instanton effects manifest themselves in the dynamics of the lightest states, generating the Zweing-rule violating amplitude which is necessary for a consistent description of the strong decays of the light scalar mesons.

The phenomenological determinations of the S–S' mixing and of the Zweig-rule violating $S \to PP$ amplitude suggest that the dynamics of the scalar mesons could be described, to a good extent, by a simple effective Lagrangian of the type

$$\mathcal{L}_{\rm eff,all} = {\rm Tr}\left(S\mathcal{M}_S^2 S\right) + {\rm Tr}\left(S'\mathcal{M}_{S'}^2 S'\right) + \gamma\ {\rm Tr}\left(SS'\right) + c_f O_f(S) + c'_f O_1(S') \tag{29}$$

where $\mathcal{M}_{S^{(\prime)}}^2 = a^{(\prime)} + b^{(\prime)}\lambda_8$ are appropriate mass matrices reflecting the inverse and normal mass ordering of tetraquark and $q\bar{q}$ states, and γ is the coupling encoding the instanton contribution.

Note Added After this work was submitted, an independent analysis of the instanton-induced mixing between two and four quark scalar mesons has been presented in [31].

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