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Abstract
In the two-player electronic mail game (EMG), as is well-known, the probability of collective action is lower the more confirmations and re-confirmations are made available to players. In the multi-player EMG, however, as we show players may coordinate on equilibria where they require only few of the available confirmations from each other to act. In this case, increasing the number of available may either create equilibria with positive probability of collective action when none existed before, or may increase the probability of collective action, if equilibria with positive probability of collective action already existed for fewer available confirmations.

Keywords: Multi-Player Electronic Mail Game, Collective Action, Common Knowledge

JEL classification: D82, D85, D71

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1. Introduction

Consider Rubinstein's (1989) *electronic mail game* (henceforth EMG) where an informed and an uninformed player may or may not have an opportunity to benefit from collective action. The informed player finds out that the opportunity arises, and sends a message to the uninformed player. If this message arrives with certainty, the players achieve common knowledge about the opportunity. If the message gets lost with small probability, however, the informed player does not know whether the uninformed player knows about the opportunity. A remedy would now seem that the uninformed player automatically confirms receipt of the informed player's message. Still, if such a confirmation gets lost with small probability as well, then the uninformed player who receives the informed player's message and sends a confirmation, does not know whether the informed player knows that the uninformed player knows about the opportunity. From this perspective, it would seem useful that an automatic communication protocol sends confirmations and confirmations of confirmations back and forth indefinitely, in a process that only ends when a message gets lost, or when a deadline is reached.

Yet as shown by Rubinstein (1989), the players now only achieve collective action if all available confirmations are received, which in case there is no deadline on communication means that players never act, as a message gets lost sooner or later. While being able to communicate longer allows players to validate higher-order statements of the type "I know that you know that I know that… you know that there is an opportunity", and would seem to bring them closer to common knowledge, paradoxically exchanging messages for a longer time makes players less likely to act, and they are best off if at most one confirmation is sent.

The purpose of this paper is to show that this paradox need not exist for the *multi-player* EMG, in that in this variant of the EMG longer communication may increase the probability of collective action. The intuition is the following. In the multi-player EMG, the automatic communication protocol does not just allow for one string of confirmations and re-confirmations, but for multiple such strings, which moreover grow exponentially in number as the deadline is extended. If players coordinate on requiring the receipt of messages in only a few of such strings, when the deadline is extended by one additional stage, it continues to be true that one more round of confirmations is required, which as such decreases the probability of collective action. Yet, this may be more than compensated by the fact that the larger number of available message strings creates more back-up communication channels for the players, thus countering the effect of noise. This explains why allowing players of the multi-player EMG to validate additional higher-order statements may make them more likely to act.

Two authors treat variants of the multi-player EMG, but obtain results that confirm those of the two-player EMG.[1] Morris (2002a, 2002b) treats a multi-player EMG where a threshold of players needs to jointly act to achieve collective action, and where at each communication stage a new sample of players smaller than the threshold is randomly picked. Either all players in this sample receive a confirmation from the sample of players taken at the previous stage, or none of them does. Corroborating Rubinstein (1989)'s result, Morris shows that if the communication process continues indefinitely, the players do not achieve collective action. Coles (2009) presents a multi-player EMG with a single informed player where uninformed players do not send confirmations to one another, and where the informed player only sends confirmations to the uninformed players when having received confirmations from all uninformed players. His results also confirm Rubinstein's. The reason for the difference between our results and those of Morris and of Coles lies in the restrictions that these authors put on their communication protocols, such that not every possible level and type of higher-order knowledge is produced. These restrictions eliminate the effect of having multiple alternative communication channels that plays a key role in our paper.[2]

The paper is structured as follows. Section 2 introduces our multi-player EMG, and Section 3 characterizes the set of pure-strategy Bayesian Nash equilibria of this game where collective action takes place with positive probability. Section 4 identifies circumstances in which extending the deadline is beneficial to players. We end with a discussion in Section 5.

2. Multi-player electronic mail game

The multi-player EMG is played by a set of players $\{1, 2, \ldots, I\}$. We use symbols i, j, k, l to refer to generic players. In the game, the following compound lottery takes place from stages 0 to z. At stage 0, Nature with probability p chooses state r (= there is an opportunity for

[1] The following papers modify the EMG in other ways than introducing multiple players. Dulleck (2007) shows that boundedly rational players with imperfect recall can still coordinate on requiring few messages; Strzalecki (2010) argues the same by applying a level-k reasoning model. Dimitri (2004) shows that when messages from different players get lost with different probabilities, coordinated action can still occur. Binmore and Samuelson (2001) and De Jaegher (2008) investigate the effect of communication being voluntary instead of automatic.

[2] Recently, Coles and Shorrer (2012) treat a variant of Coles (2009), where again uninformed players do not communicate with each other, and where the event of a message getting lost does not occur independently from other messages getting lost. As shown by the authors, because of this correlation, players are able to coordinate on equilibria where only a few messages are required.

collective action), and with probability $(1-p)$ chooses state q (= there is no opportunity). In state q, no player receives messages. In state r, at stage 1, Nature (n) independently with probability $(1-\varepsilon)$ lets each player i receive a *message string* $n \to i$ (denoting a message from Nature directed to player i), and with probability ε does not let this player receive a message string. We typically assume ε to be small. When any player i receives message string $n \to i$ at stage 1, Nature at stage 2 independently with probability $(1-\varepsilon)$ lets each player $j \neq i$ receive message string $n \to i \to j$ (denoting that j knows that i knows that state r occurs), and with probability ε does not let j receive this message. When any player j receives $n \to i \to j$ at stage 2, Nature at stage 3 independently with probability $(1-\varepsilon)$ lets each player $k \neq j$ (including $k = i$) receive message string $n \to i \to j \to k$ (denoting that k knows that j knows that i knows that state r occurs), and with probability ε does not let k receive this message string. In this manner, each individual received message string, denoted as m, continues to be forwarded until it gets lost, or until the ultimate communication stage z has been reached.

At stage $(z+1)$, the players simultaneously choose an action from the same action set {Q, R}, where R means acting ("revolting") and Q means not acting ("quitting"). The players' payoffs are summarized in Table 1. Each player obtains payoff 0 when doing Q, whatever the state, and whatever other players do. Each player incurs loss L when doing R in state q whatever other players do, and when doing R in state r when not all other players do R as well. Each player obtains benefit H when doing R in state r when *all* other players do R as well. We assume that $L > H > 0$, and typically consider L to be large.[3]

	State q: Prob. $(1-p)$	State r: Prob. p	
		One or more others play Q	All others play R
Action Q	0	0	0
Action R	$-L$	$-L$	H

Table 1 Payoffs of individual player as function of states and actions; $L > H > 0$, $p < 1/2$.

In order to describe the information structure and to define strategies and equilibria of the multi-player EMG, it is useful to see this game as a standard simultaneous moves game with incomplete information. In such a game, typically a simple lottery (Ω, π) consisting of a set of states of the word Ω and a probability distribution π, determines which state of the world

[3] The version of the EMG treated here is a multi-player version of the two-player EMG of Morris and Shin (1997), which without loss of generality, differs slightly from Rubinstein's (1989) original game.

occurs (where typical state of the world ω occurs with probability $\pi(\omega)$). To reduce the compound lottery in the multi-player EMG to a simple lottery, consider the directed graph referred to as the *maximal communication tree,* an example of which is found in Figure 1 for the case $I = 3$, $z = 2$. The root node with label n has a link, represented as $n \to i$, to each of I nodes with labels 1 to I, at distance 1 from the root node. Each such node with label i has a link $i \to j$ to each of $(I - 1)$ nodes with labels $j \neq i$ at distance 2 from the root node. From each such node with label j again departs a link $j \to k$ to each of the $(I - 1)$ nodes with labels $k \neq j$, at distance 3 form the root node. And so on, where the maximal communication tree expands until terminal nodes at distance z from the root node are reached, so that for a distance t, with $1 \leq t \leq z$, it has $I(I-1)^{t-1}$ nodes at distance t from the root node. Note that to each path $n \to \ldots i \to j$ contained in the maximal communication tree starting at the root node, corresponds a possible message string.

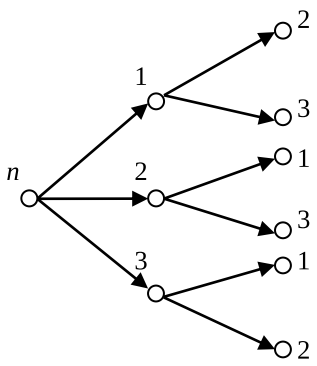

Figure 1. Maximal communication tree when I = 3, z = 2.

Define as a *tree g* any connected subgraph of the maximal communication tree, that has as its root the node with label n. Denote by G the set of all trees contained in the maximal communication tree, where we assume this to include the empty tree. E.g., in the simple case in Figure 1, the number of trees contained in the maximal communication tree equals $\sum_{x=0}^{3}\binom{3}{x}\sum_{y=0}^{(I-1)x}\binom{(I-1)x}{y} = 125$.[4] To *every* such tree g now corresponds a state of the world where state r occurs and all message strings contained in g arrive, where in short we refer to

[4] This includes the empty tree; for each of the 3 cases with a single node at distance 1 from the root node, 4 different trees with either 0, 1 or 2 nodes at distance 2; for each of the 3 cases with 2 nodes at distance 1, 16 different trees with either 0, 1, 2, 3 or 4 nodes at distance 2; for the single case with exactly 3 nodes at distance 1, 64 different trees with either 0, 1, 2, 3, 4, 5 or 6 nodes at distance 2.

this as state of the world g. In the same manner, we use symbol G not only for the set of all trees, but also for the set of all states of the world where state r occurs. Furthermore, as already noted, whenever state q occurs, the set of received message strings is automatically empty, where in short we refer to this as state of the world q. We thereby obtain that in the reduced-form simple lottery, Ω is the set (q, G), $\pi(q)$ equals $(1-p)$, and for any g in G, $\pi(g)$ equals $p(1-\varepsilon)^{x_1}\varepsilon^{I-x_1}(1-\varepsilon)^{x_2}\varepsilon^{(I-1)x_1-x_2}\ldots(1-\varepsilon)^{x_z}\varepsilon^{(I-1)x_{z-1}-x_z}$, where x_t denotes the number of message strings arriving at stage t, with $0 \le x_1 \le I$ and with $x_t \le (I-1)x_{t-1}$ for $2 \le t \le z$.

Furthermore, it is standard in a game with incomplete information that a signal map assigns to each state of the world a signal for each player. In the multi-player EMG, we denote by μ_i player i's signal map, and by $\mathfrak{M}_i$ the set of signals he can observe. Concretely, the signal map μ_i: $(q, G) \rightarrow \mathfrak{M}_i$ assigns to state of the world q the empty message string set ϕ received by i, and to each state of the world g in G message string set $\mu_i(g)$, consisting of each message string that i receives in g. The set of signals $\mathfrak{M}_i$, with typical element M_i, is therefore a collection[5] of message string sets containing a $\mu_i(g)$ for each g in G. One and the same signal may be received for several states of the world. E.g. in Figure 1, denote $m_1 = n \rightarrow 2 \rightarrow 1$, m_1' $= n \rightarrow 3 \rightarrow 1$, $m_2 = n \rightarrow 3 \rightarrow 2$ and $m_3 = n \rightarrow 2 \rightarrow 3$. Consider $g = \{m_1, m_1', m_2\}$. Then $\mu_1(g)$ $= \{m_1, m_1'\}$. Yet, 1 also receives signal $\{m_1, m_1'\}$ in states of the world $\{m_1, m_1'\}$, $\{m_1, m_1', m_3\}$, and $\{m_1, m_1', m_2, m_3\}$.

If player i receives signal M_i, he learns that the true state of the world lies in the set of pre-images $\mu_i^{-1}(M_i)$ of M_i. The posterior probability that a state of the world ω in $\mu_i^{-1}(M_i)$ occurs then equals $\pi(\omega)/\pi(\mu_i^{-1}(M_i))$, where $\pi(\mu_i^{-1}(M_i))$ is the sum of the probabilities of all the states of the world in the set $\mu_i^{-1}(M_i)$. The collection of sets of states of the world obtained by assigning a set of pre-images to each M_i in $\mathfrak{M}_i$, is player i's *information partition*. A strategy for player i is a map α_i: $\mathfrak{M}_i \rightarrow \{Q, R\}$. A strategy profile $(\alpha_1, \alpha_2, \ldots, \alpha_I)$ is a pure-strategy Bayesian Nash equilibrium if for each player i, for each signal M_i, $\alpha_i(M_i)$ is such that for the action $A_i \neq \alpha_i(M_i)$,

$$\sum_{\omega \in \mu_i^{-1}(M_i))} [\pi(\omega) / \pi(\mu_i^{-1}(M_i))]\, u_i\big(\alpha_i(M_i), \alpha_{-i}(\mu_{-i}(\omega)), \omega\big) \ge$$
$$\sum_{\omega \in \mu_i^{-1}(M_i))} [\pi(\omega) / \pi(\mu_i^{-1}(M_i))]\, u_i\big(A_i, \alpha_{-i}(\mu_{-i}(\omega)), \omega\big)$$

[5] "Collection", and later on "family" are used as synonyms of sets, to avoid the expression "set of sets".

where α_{-i} is the profile of strategies of all other players, where μ_{-i} refers to each other player's signaling map, and where u_i is player i's payoff as a function of his own action, the actions of others, and the state of the world. We call a *collective-action equilibrium*[6], any pure-strategy Bayesian Nash equilibrium where it occurs with positive probability that all players act in state r.

We focus on collective-action equilibria where equilibrium strategies meet the following restriction:

(R1) The strategy α_i of every player i is such that: if for a $M_i \in \mathfrak{M}_i$ we have $\alpha_i(M_i) = R$, then for any $M_i' \in \mathfrak{M}_i$ with $M_i' \supset M_i$, we also have $\alpha_i(M_i') = R$; if for a $M_i \in \mathfrak{M}_i$ we have $\alpha_i(M_i) = Q$, then for any $M_i'' \in \mathfrak{M}_i$ with $M_i'' \subset M_i$, we also have $\alpha_i(M_i'') = Q$.

(R1) means excluding inefficient equilibria where players coordinate on doing Q conditional on the receipt of particular message strings. For small noise, such equilibria are obviously inefficient, as collective action is then unlikely to take place. Focusing on equilibria meeting (R1), a candidate equilibrium strategy of any player i can now be described in a more concise manner. *First*, to describe a strategy α_i it suffices to characterize the collection of all message string sets leading i to play R, where we call an individual message string set in this collection a *sufficient set*; all non-included message string sets then necessarily lead the player to play Q. Formally, for a given strategy α_i, define the set $\mathfrak{M}_i^R = \{ M_i \in \mathfrak{M}_i : \alpha_i(M_i) = R \}$. Then any element of $\mathfrak{M}_i^R$, with typical element denoted as S_i, is a sufficient set for i. *Second*, by (R1), if i acts when receiving sufficient set S_i, he also acts when receiving any $S_i' \in \mathfrak{M}_i^R$ such that $S_i' \supset S_i$. For a given set S_i' leading i to act, we may therefore succinctly describe part of i's strategy by focusing only on the minimal sufficient subset of message strings, denoted $S_i^{\min}$, leading i to act (with $S_i^{\min} \subset S_i'$). Any superset of $S_i^{\min}$ then necessarily leads i to act as well. Formally, a *minimal sufficient set* is any $S_i^{\min} \in \mathfrak{M}_i^R$ such that there does not exist $S_i'' \in \mathfrak{M}_i^R$ with $S_i'' \subset S_i^{\min}$. Player i's strategy is now fully described by a collection of minimal sufficient sets $\mathbb{S}_i^{\min} = \{ S_i^{\min}, \ldots, S_i'^{\min}, S_i''^{\min}, \ldots \}$, and any collective-action equilibrium

[6] The multi-player EMG always has a pure-strategy Bayesian Nash equilibrium where players never act, and has mixed equilibria, where upon the receipt of messages players randomize over acting or not. In all of the mentioned equilibria, each player's expected payoff is zero. Our focus on collective-action equilibria is based on the premise that players can coordinate on Pareto superior equilibria.

can be described as a profile of collections of minimal sufficient sets. The next section characterizes all collective-action equilibria.

3. Characterization of collective-action equilibria of the multi-player EMG

Intuitively, in any collective-action equilibrium, if player i receives at least all message strings in at least one individual minimal sufficient set $S_i^{\min}$, and finds this sufficient to act, it must be that player i expects that with high probability an event occurs such that all other players observe specific minimal sufficient sets of their own – where these players then also expect that with high probability the mentioned event occurs. This suggests that, just as each pure strategy can be described by a number of alternative minimal sufficient sets, each collective-action equilibrium can be characterized by a number of alternative events such that, if players expect any of these events to occur with high probability, they act. We call these *action-inducing events*. Theorem 1 shows that this intuition is indeed valid, and that action-inducing events correspond to specific trees in G which we call broom sets.

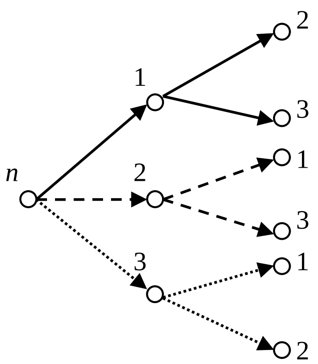

Figure 2. Maximal communication tree G, containing brooms b_1 (solid arrows), b_2 (dashed arrows), and b_3 (dotted arrows).

Formally, because of its form, we call a *broom b* any tree in G that contains a *single* path from the root node to a node with label i at distance $(z-1)$ from the root node, as well as each of the $(I-1)$ links of this node with label i to nodes with labels $j \neq i$ at distance z from the root note.[7] Denote by $\mathcal{B}$ the set of all brooms in G. Over any given set, denote by $\mathcal{P}^*[.]$ the

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[7] When z = 1 we consider the maximal communication tree as containing a single broom, namely the maximal communication tree itself.

power set excluding the empty set. Then a *broom set* B is any element of $\mathcal{P}^*[\mathcal{B}]$. In knowledge terms, broom set B corresponds to the event of *any* state of the world g occurring such that B is a subgraph of the tree g. E.g., in Figure 2, $\mathcal{B} = \{b_1, b_2, b_3\}$, and $\mathcal{P}^*[\mathcal{B}]$ consists of 7 broom sets, with $B_1 = \{b_1\}$, $B_2 = \{b_2\}$, $B_3 = \{b_3\}$, $B_4 = \{b_1, b_2\}$, $B_5 = \{b_2, b_3\}$, $B_6 = \{b_1, b_3\}$, $B_7 = \{b_1, b_2, b_3\}$. E.g. to B_4 corresponds the event where either of five states of the world occurs, where each time all message strings in b_1 and b_2 arrive, and additionally either no other message strings arrive; $n \to 3$ arrives; $n \to 3$ and $n \to 3 \to 1$ arrive; $n \to 3$ and $n \to 3 \to 2$ arrive; or $n \to 3$, $n \to 3 \to 1$ and $n \to 3 \to 2$ arrive.

The set $\mathcal{P}^*\{\mathcal{P}^*[\mathcal{B}]\}$ lists all the families of broom sets in G. Define as a *Sperner family of broom sets* any element F of $\mathcal{P}^*\{\mathcal{P}^*[\mathcal{B}]\}$ such that no broom set in F is a subset of another broom set in F. Formally, a Sperner family of broom sets is any $F \in \mathcal{P}^*\{\mathcal{P}^*[\mathcal{B}]\}$ such that $\forall$ B, $B' \in F$, if $B \neq B'$, we have $B \not\subset B'$, $B \not\supset B'$. The set $\mathcal{P}^{*F}\{\mathcal{P}^*[\mathcal{B}]\}$ lists all Sperner families of broom sets in G. E.g. in Figure 2, $\mathcal{P}^{*F}\{\mathcal{P}^*[\mathcal{B}]\}$ consists of 18 Sperner families of broom sets, with $F_1 = \{B_1\}$, $F_2 = \{B_2\}$, $F_3 = \{B_3\}$, $F_4 = \{B_4\}$, $F_5 = \{B_5\}$, $F_6 = \{B_6\}$, $F_7 = \{B_7\}$, $F_8 = \{B_1, B_2\}$, $F_9 = \{B_2, B_3\}$, $F_{10} = \{B_1, B_3\}$, $F_{11} = \{B_1, B_2, B_3\}$, $F_{12} = \{B_1, B_5\}$, $F_{13} = \{B_2, B_6\}$, $F_{14} = \{B_3, B_4\}$, $F_{15} = \{B_4, B_5\}$, $F_{16} = \{B_4, B_6\}$, $F_{17} = \{B_5, B_6\}$, $F_{18} = \{B_4, B_5, B_6\}$. In knowledge terms, to each family of broom sets corresponds a set of alternative events.

Finally, for any Sperner family of broom sets $F = \{B, \ldots, B', B'', \ldots\} \in \mathcal{P}^{*F}\{\mathcal{P}^*[\mathcal{B}]\}$, define $\mathfrak{M}_i(F) = \{\mu_i(B), \ldots, \mu_i(B'), \mu_i(B''), \ldots\}$, where we recall that $\mu_i(g)$ is the set of all message strings that i receives in a tree g. $\mathfrak{M}_i(F)$ is thus a collection of message string sets that i receives, one for each broom set in the Sperner family of broom sets F. E.g. in Figure 2, for the family of broom sets $F_{15} = \{B_4, B_5\}$, $\mathfrak{M}_1(F_{15}) = \{\{n \to 1, n \to 2 \to 1\}, \{n \to 2 \to 1, n \to 3 \to 1\}\}$, $\mathfrak{M}_2(F_{15}) = \{\{n \to 1 \to 2, n \to 2\}, \{n \to 2, n \to 3 \to 2\}\}$, and $\mathfrak{M}_3(F_{15}) = \{\{n \to 1 \to 3, n \to 2 \to 3\}, \{n \to 2 \to 3, n \to 3\}\}$. Theorem 1 now shows that any profile of collections of minimal sufficient sets ($\mathbb{S}_1^{\min}, \mathbb{S}_2^{\min}, \ldots, \mathbb{S}_I^{\min}$) describing a collective-action equilibrium, corresponds to some Sperner family of broom sets $F = \{B, \ldots, B', B'', \ldots\}$, in that in this equilibrium for each player i, $\mathbb{S}_i^{\min} = \mathfrak{M}_i(F) = \{\mu_i(B), \ldots, \mu_i(B'), \mu_i(B''), \ldots\}$. Thus, in the case of Figure 2, there are 18 collective-action equilibria. E.g. to $F_{15} = \{B_4, B_5\}$ corresponds a collective-action equilibrium where $\mathbb{S}_1^{\min} = \mathfrak{M}_1(F_{15})$, $\mathbb{S}_2^{\min} = \mathfrak{M}_2(F_{15})$, and $\mathbb{S}_3^{\min} = \mathfrak{M}_3(F_{15})$. Put otherwise, a collective-action equilibrium exists where players act if and only if they either believe that with high probability action-inducing event B_4 occurs (= any state of the world g occurs such that both b_1 and b_2 are subgraphs of the tree g), or that action-inducing event B_5 occurs (= any state of the world g occurs such that both b_2 and b_3 are subgraphs of the tree g).[8]

Theorem 1. In the multi-player EMG, there exists a $\varepsilon^* < 1$ such that for any ε in $(0, \varepsilon^*]$, the following applies:

(i) Consider *any* Sperner family of broom sets $F = \{B, \ldots, B', B'', \ldots\} \in \mathcal{P}^{*F}\{\mathcal{P}^*[\mathcal{B}]\}$. Then if each $j \neq i$ considers $\mathfrak{M}_j(F) = \{\mu_j(B), \ldots, \mu_j(B'), \mu_j(B''), \ldots\}$ as a collection of alternative minimal sufficient sets, then it is a best response for i to consider $\mathfrak{M}_i(F) = \{\mu_i(B), \ldots, \mu_i(B'), \mu_i(B''), \ldots\}$ as a collection of alternative minimal sufficient sets.

(ii) Conversely, consider *any* collective-action equilibrium. Then a Sperner family of broom sets $F \in \mathcal{P}^{*F}\{\mathcal{P}^*[\mathcal{B}]\}$ can be found such that the profile $(\mathfrak{M}_1(F), \mathfrak{M}_2(F), \ldots, \mathfrak{M}_I(F))$ describes this equilibrium, in that it constitutes the profile of collections of minimal sufficient sets corresponding to this equilibrium.

Theorem 1 can be understood by means of the following intuitions. *First*, just as in Rubinstein's (1989) two-player EMG, if the message string set M_i received by player i does not contain message strings received later than at stage $(z-2)$, this can never be sufficient to i for acting. If M_i would be sufficient to i anyway, given that the loss of acting in the wrong circumstances is large, only confirmations of the message strings in M_i would be sufficient to $j \neq i$. But, given that i is now able to receive confirmations of these confirmations received by j, M_i cannot be sufficient itself to i. It follows that players can only consider penultimate or ultimate message strings received as sufficient for acting. This explains why action-inducing events consist of (pen)ultimate message strings.

Second, as by assumption every message string received by a player is forwarded to each other player, a *single* (pen)ultimate message string received may be sufficient to act. If player i receives a penultimate message string m, and believes that receipt of the ultimate confirmation of this message string is sufficient to all other players, then for sufficiently small noise, m is sufficient to i, as by assumption i is not able to receive confirmations of the ultimate confirmations. At the same time, if player j receives an ultimate message string m' which is a confirmation of m, and believes that m is sufficient to i and that a confirmation of

[8] Note that if a player receives all available message strings, he believes that with high probability both the events corresponding to B_4 and to B_5 occur.

m is sufficient to each $k \neq i$, j, then for sufficiently small noise, m' is sufficient to j. This explains why any action-inducing event may correspond to a single broom, containing a single penultimate message string, and all confirmations of this message string.

Third, while in equilibrium, a single (pen)ultimate message string received may suffice to a player for acting, players may lock each other into requiring multiple such message strings. Simply, if only a non-singleton set M_i of (pen)ultimate message strings is sufficient to i, then for each penultimate message string m in M_i, only a confirmation of m will be sufficient to other players. Similarly, for each ultimate message string m' in M_i, only the message string m'' of which m' is a confirmation, will be sufficient to a player j; furthermore, only the confirmations of m'' will be sufficient to a player k, with $k \neq i, j$. This again justifies why only all message strings in M_i are sufficient to i in the first place, and explains why any action-inducing event may correspond to a broom set rather than to a broom.

Fourth, as soon as message strings are sent at more than one stage, given that there are now multiple penultimate message strings, players can also coordinate on finding multiple alternative message string sets sufficient. There may therefore be several alternative action-inducing events.

As is easy to see, the Pareto-worst collective-action equilibrium is one where each player only acts when receiving each potentially available message string. In this case, the Sperner family of broom sets describing the equilibrium consists of one single broom set, namely the broom set containing all brooms in the maximal communication tree. E.g., in the case of Figure 2, the Pareto-worst collective-action equilibrium is the one corresponding to the Sperner family of broom sets F_7 described above. At the same time, in the Pareto-best collective-action equilibrium, each player acts as soon as receiving at least a single (pen)ultimate message string, in which case in the Sperner family of broom sets describing the equilibrium, each broom in $\mathcal{B}$ is in a singleton broom set. In the case of Figure 2, the Pareto-best collective-action equilibrium is the one corresponding to the Sperner family of broom sets F_{11} described above.

4. Beneficial long communication

We now show that longer communication may make players better off in the multi-player EMG. There are two ways in which this is true. First, as shown in Proposition 1, it may be that a collective-action equilibrium only exists if $z \geq 2$, so that talking for a longer time is a necessary condition for benefits of collective action to be possible. Second, as shown in Proposition 2, even if a collective-action equilibrium exists for $z=1$, it may be that players increase the probability of collective action when *z* is increased. We start with Proposition 1, which shows that parameters exist such that for any finite $z \geq 2$, there is a collective-action equilibrium, whereas for $z=1$, such an equilibrium does not exist. Intuitively, to a player who receives just enough message strings to make collective action possible, the fact that *z* is high means that there are more ways in which other players could have received a sufficient number of message strings.[9]

Proposition 1. For the multi-player EMG, consider $z=1$, and $z=z^*$ with $z^* \geq 2$. Then for given *I*, a large (*L*/*H*) exists such that for any ε in $(0, \varepsilon^*(L/H)]$, where $\varepsilon^*(L/H)$ is a function of (*L*/*H*), a collective-action equilibrium exists for $z=z^*$, but not for $z=1$.

The result in Proposition 1 may be understood using Monderer and Samet's (1989) concept of common π-belief. An event is common π-belief if everyone believes it with probability at least π; everyone believes with probability at least π that everyone believes it with probability at least π; and so on. It is clear now that a collective-action equilibrium only exists if it is common $L/(H+L)$-belief among the players that there is an opportunity. If $(1-\varepsilon)^{I-1} < L/(H+L)$, this cannot be achieved in the unique candidate collective-action equilibrium for $z=1$. Yet, in the Pareto-best collective-action equilibrium for a $z \geq 2$, even a player who receives only a single penultimate message string *m* holds stronger beliefs that all other players received at least one ultimate message string, because this can now be achieved even if not all confirmations of *m* arrive. It follows that with $z \geq 2$, players can achieve a higher degree of common π-belief than with $z=1$. This contrasts with Rubinstein's (1989) two-player EMG, where the informed player notifies the uninformed player about the opportunity, who confirms receipt to the informed player, and so forth. In this case, if the communication process has a deadline, a unique collective-action equilibrium exists for each possible deadline, where each player acts only when receiving the (pen)ultimate message string. In each such equilibrium, players achieve the same degree of common π-belief about

[9] It should be noted that all results in this section also apply for $I = 2$. This is because, contrary to what is the case in Rubinstein's (1989) two-player EMG, in a two-player version of our game, both players may receive a message from Nature. Because of this, starting from $z \geq 2$, the communication process generates exactly two brooms, and never more. It follows that the probability of collective action necessarily decreases for $z \geq 3$, so that Proposition 2 is not valid in this case. This differs from the case with $I \geq 3$, where the number of brooms is increasing in z.

the opportunity for collective action, namely common $(1-\varepsilon)$-belief. Summarizing, while in the two-player EMG availability of additional higher-order statements does not lead to a higher degree of approximate common knowledge, it may in the multi-player EMG.

We next turn to our result that, even if collective-action equilibria exist for each *z*, being able to talk longer may still increase the probability of collective action. For simplicity, we focus on symmetric collective-action equilibria (in short: symmetric equilibria). Such equilibria can be described by a minimal number of (pen)ultimate message strings *X* which each player requires to act, where we call *X* the parameter of a symmetric equilibrium. Examples include the Pareto-best collective-action equilibrium, which has parameter $X = 1$, and the Pareto-worst equilibrium, which for $z \geq 2$ has parameter $X = I(I-1)^{z-2}$.

Proposition 2 shows the following. Consider a multi-player EMG with given *I*, *H* and *L*, and vary the deadline *z*. Assume that the parameters are such that the unique collective-action equilibrium exists for $z = 1$, and such that for deadlines $z \geq 2$, a symmetric equilibrium with parameter *X* exists where $1 \leq X \leq (I-2)$, on which players coordinate (meaning that players require relatively few message strings). Then for sufficiently small ε, the probability of collective action is strictly larger for any finite $z \geq 2$ than for $z = 1$, and does not vanish as the finite *z* approaches infinity.

It should be stressed that Proposition 2 does not claim that there is a systematic increasing relation between the length of the deadline and the probability of collective action (for players who coordinate on a symmetric equilibrium with specific parameter *X*). E.g., for $I = 3$, $\varepsilon = 0.3$, it can be checked that the probability of collective action for players who always play the Pareto-best collective-action equilibrium is higher for $z = 2$ than it is for $z = 1$ or $z = 3$. Calculating a general expression for the probability of collective action given a symmetric equilibrium with parameter *X*, proves to be non-tractable. Instead, in the proof of Proposition 2, we calculate a lower bound on this probability, and show that, for a range of low ε, this lower bound is increasing in *z* and is larger than the true probability of collective action for $z = 1$. This shows that any longer communication can be better than the shortest possible communication.

Proposition 2. For the multi-player EMG, consider $z = z^*$ with $z^* \geq 2$, and consider $z = 1$. Let the unique collective-action equilibrium exist for $z = 1$. Let a symmetric equilibrium with parameter *X*, $1 \leq X \leq (I-2)$, exist for $z = z^*$, and let this equilibrium be played. Then a range of ε in $(0, \varepsilon^*]$ exists such that the probability of collective action is strictly larger for $z = z^*$ than for $z = 1$.

The intuition for the result in Proposition 2 is the following. Suppose that players e.g. play for each *z* the Pareto-best collective-action equilibrium. Then receipt of every (pen)ultimate message string in any single broom is sufficient for collective action to be achieved. Extending the deadline has two effects. A *first* effect is that the probability of all (pen)ultimate message strings arriving in an *individual* broom becomes smaller, as one more message string needs to arrive to achieve this. A *second* effect is that arrival of the (pen)ultimate message strings in many more brooms becomes sufficient to ensure collective action. Proposition 2 shows that the second effect may compensate the first effect. This contrasts with the two-player EMG, where the communication process cannot generate more than a single broom, so that only the first effect applies.

A fundamental requirement for the result in Proposition 2 to apply is that players coordinate on equilibria where receipt of few message strings is sufficient for acting, and that they continue to do so as the number of communication stages is increased. At the other extreme, if players instead for each *z* play the Pareto-worst collective-action equilibrium, the probability of collective action decreases as *z* gets larger, and moreover vanishes as *z* approaches infinity.

5. Conclusion

As pointed out by Geanakoplos (1992), Rubinstein's (1989) results on the two-player EMG may be interpreted as showing that two players trying to coordinate their actions need strict communication rules to avoid inefficiency. Geanokoplos' example is communication between military personnel, where a command is followed by a single acknowledgement, and nothing more.[10] Our analysis of the multi-player EMG suggests that this interpretation does not extend to more than two players. Here, the absence of strict communication rules, such that the number of confirmations, and confirmations of confirmations that is sent back and forth is only limited by a communication deadline, may increase efficiency, because it creates back-up channels which counter noise. Thus, activists on Twitter who among each other take a long

[10] Chwe (1995) similarly investigates optimal communication processes in some simple games. His focus is on whether it is optimal to use confirmations to counter noise, or to use redundancy (= repeating the same message several times). De Jaegher (2006) investigates in which circumstances positive and negative acknowledgements are efficient in some simple games, and whether they are also used in equilibrium.

time tweeting and re-tweeting calls for collective action, may be more effective at achieving collective action, as this allows for a myriad of manners in which all activists can be reached.

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Appendix: Proofs

Proof of Theorem 1. Step 1 constructs a message string set for *i*, given a minimal sufficient set of any $j \neq i$. Parameters exist such that this constructed set is a sufficient set to *i* (Step 2), and moreover a *minimal* sufficient set (Step 3). Step 4 shows that, given these parameters, every equilibrium minimal sufficient set consists only of (pen)ultimate message strings. Step 5 shows that any collective-action equilibrium is described by a Sperner family of broom sets. Step 6 finally shows that the collection of all possible Sperner families of broom sets characterizes the set of all collective-action equilibria.

Step 1. Consider minimal sufficient set $S_j^{\min}$ of *j*. Then, denoting by a superscript above an arrow the stage at which a message is sent, for $i \neq j$, we can construct the message string set $S_i(S_j^{\min})$ containing:

(i) for any $n \stackrel{1}{\to} \ldots \stackrel{t-1}{\to} k \stackrel{t}{\to} j \in S_j^{\min}$ with $t < z$, the message string $n \stackrel{1}{\to} \ldots \stackrel{t-1}{\to} k \stackrel{t}{\to} j \stackrel{t+1}{\to} i$;

(ii) for any $n \stackrel{1}{\to} \ldots \stackrel{z-2}{\to} l \stackrel{z-1}{\to} k \stackrel{z}{\to} j \in S_j^{\min}$, in case $k \neq i$ message string $n \stackrel{1}{\to} \ldots \stackrel{z-2}{\to} l \stackrel{z-1}{\to} k \stackrel{z}{\to} i$;

in case $k = i$ message string $n \stackrel{1}{\to} \ldots \stackrel{z-2}{\to} l \stackrel{z-1}{\to} k = n \stackrel{1}{\to} \ldots \stackrel{z-2}{\to} l \stackrel{z-1}{\to} i$ itself.

Define as $\mathbb{S}_i(S_{j \neq i}^{\min})$ the set containing for each $S_j^{\min}$ of each $j \neq i$ a message string set $S_i(S_j^{\min})$.

Step 2. In any candidate collective-action equilibrium where $S_j^{\min}$ is a minimal sufficient set to *j*, it must be that *j* attaches some probability φ to obtaining L when acting after receiving every message string in $S_j^{\min}$, where $(1-\varphi)H - \varphi L \geq 0$. Furthermore, in any such equilibrium, *i* should then act as well when believing that it is sufficiently likely that *j* received every message string in $S_j^{\min}$. Let *i* receive all message strings in $S_i(S_j^{\min})$, and let $S_j^{\min}$ contain *V* message strings of the type under (ii) in Step 1. As *z* and *I* are finite, *V* is finite too. Player *i* now certainly acts if

$$(1-\varepsilon)^V\left[(1-\varphi)H-\varphi L\right]-[1-(1-\varepsilon)^V]L\geq 0. \qquad \text{(A1)}$$

Given (R1), φ is increasing in ε. It follows that for any *L*, *H* and *V*, small ε exist such that (A1) is valid, meaning that $S_i(S_j^{\min})$ is a sufficient set.

Step 3. Consider the collection of message string sets $\mathbb{S}_i(S_{j\neq i}^{\min})$ as constructed in Step 1. We show that this is a collection of *minimal* sufficient sets by, *first*, constructing a message string set $M_i \in \mathfrak{M}_i$ such that 1) there is no $S_i(S_j^{\min}) \in \mathbb{S}_i(S_{j\neq i}^{\min})$ such that $S_i(S_j^{\min}) \subseteq M_i$; 2) M_i is as conducive as possible for *i* to act; and by, *second*, showing that even when receiving all message strings in M_i, *i* prefers not to act. We construct such a M_i as follows. For any player $j \neq i$, let *each* minimal sufficient set $S_i(S_j^{\min})$ contain one message string *m*, such that $\{S_i(S_j^{\min})\setminus m\}\subseteq M_i$, but $m \notin M_i$. At the same time, let $S_j^{\min}$ contain, for a message string m' received by *k*, a message string $m'\rightarrow j$ (denoting a confirmation received by *j* of m') such that for m as defined, it is the case that $m = m'\rightarrow j \rightarrow i$, with either $k = i$, or $k \neq i$ but $m'\rightarrow i \in M_i$. Player *i* then believes that *j* did not receive message string $m'\rightarrow j$ with probability $\varepsilon\left[\varepsilon+(1-\varepsilon)\varepsilon\right]^{-1}=1/(2-\varepsilon)$; put otherwise, *i* believes that *j* received every message string in $S_j^{\min}$ with probability $1-[1/(2-\varepsilon)]$. If the sum of the cardinalities of the minimal sufficient sets held by other players is now *W*, *i* believes with probability at most $\{1-[1/(2-\varepsilon)]^W\}$ that he will obtain *H* with positive probability when acting, and with probability at least $[1/(2-\varepsilon)]^W$ that he obtains *L* with probability 1. As $\varepsilon\rightarrow 0$, the former probability approaches $\{1-0.5^W\}$, in which case *H* is obtained with probability approaching 1. It follows that as $\varepsilon\rightarrow 0$, *i* receiving message string set M_i prefers not to act if

$$(1-0.5^W)H-0.5^W L<0 \qquad \text{(A2)}$$

For any finite *W* (where *W* must be finite because *I* and *z* are finite), a large (*L*/*H*) exists such that (A2) is valid. Note now that $[1/(2-\varepsilon)]^{W}$ is increasing in *ε*. It follows that if an (*L*/*H*) is imposed such that (A2) is valid, it is true for any *ε* that *i* prefers not to act when receiving message string set M_i. For such an (*L*/*H*), as long as *ε* is sufficiently low, (A1) is valid as well.

Step 4. We show by contradiction that every minimal sufficient set of any player must exclusively consists of message strings received at $(z-1)$ or *z*. Suppose that minimal sufficient set $S_j^{\min}$ contains at least one message string *m* received at stage *t*, where $t<(z-1)$. Consider now $S_i(S_j^{\min})$, and further $S_j[S_i(S_j^{\min})]$, as constructed in Step 1. Given that $S_j^{\min}$ contains a *m* received at *t* with $t<(z-1)$, $S_i(S_j^{\min})$ contains $m \rightarrow i$, and $S_j[S_i(S_j^{\min})]$ (which by Steps 2 and 3 is minimal sufficient) contains $m \rightarrow i \rightarrow j$. But this leads to a contradiction, as $S_j^{\min}$ cannot be minimal sufficient then.

Step 5. By Step 1, given a minimal sufficient set $S_j^{\min}$ of *j*, one can construct a set $S_i(S_j^{\min})$, and by Steps 2 and 3 this is a minimal sufficient set. Combining Step 4 and the construction in Step 1, it follows that $S_j[S_i(S_j^{\min})]=S_j^{\min}$, and that the mutually consistent minimal sufficient sets $S_j^{\min}$, $S_i(S_j^{\min})$ for each $i \neq j$, list for each player all the (pen)ultimate message strings that he can receive in a broom set. Repeating this exercise for each minimal sufficient set of *j*, one obtains a family of broom sets, where to each broom set corresponds a profile of mutually consistent minimal sufficient sets, with the family of broom sets describing the equilibrium. As by definition minimal sufficient sets cannot be subsets of one another, any such family of broom sets is a Sperner family of broom sets.

Step 6. As Steps 1 to 5 are valid for any profile of mutually consistent collections of minimal sufficient sets of (pen)ultimate message strings, and as the collection of all possible Sperner families of broom sets characterizes all such profiles, it follows that the collection of all possible Sperner families of broom sets characterizes all collective-action equilibria.

Proof of Proposition 1. Step 1 imposes a condition such that, for any *z*, if all $j \neq i$ play according to the Pareto-best collective-action equilibrium, *i* who does not receive (pen)ultimate message strings prefers not to act. Step 2 calculates a condition such that for $z=1$, if all $j \neq i$ act when receiving a single message string, it is a weak best response for *i* not to act. Step 3 shows that by the conditions derived in Steps 1 and 2, parameters exist such that the proposition is valid.

Step 1. We look at the case where $z \geq 3$, and where *i* receives every message string up to stage $(z-2)$, but no (pen)ultimate message strings. Player *i* is then as inclined as possible to act when not receiving (pen)ultimate message strings, and if *i* does not act even in this case, he does not act whenever he does not receive (pen)ultimate message strings.

When *i* receives all message strings up to stage $(z-2)$ and no (pen)ultimate message strings, for each $m = ... l \overset{z-2}{\rightarrow} k \overset{z-1}{\rightarrow} j$ that $j \neq i$ can receive at stage $(z-1)$, *i* can calculate the probability that *m* was received. When $k = i$, the probability that *m* got lost equals $\varepsilon / [\varepsilon + (1-\varepsilon)\varepsilon] = \pi_1$. When $k \neq i$, then either $l = i$, or $l \neq i$ and by assumption *i* receives message string $...l \overset{z-2}{\rightarrow} i$ at stage $(z-2)$. In case $k \neq i$, the probability that *m* was not received therefore equals $[\varepsilon + (1-\varepsilon)\varepsilon^2] / \{\varepsilon + (1-\varepsilon)\varepsilon[\varepsilon + (1-\varepsilon)\varepsilon]\} = \pi_2$, where for any ε, $\pi_2 > \pi_1$. Given this fact, to derive a condition such that *i* who receives all message strings up to stage $(z-2)$ and no (pen)ultimate message string does not act, it suffices to look at the fictitious case where for each of the $(I-1)^{z-1}$ message strings of the type *m* (i.e. $(I-1)^{z-2}$ message strings received by each of the $(I-1)$ players other than *i*) it is true that $m = ... i \overset{z-1}{\rightarrow} j$.

As $\varepsilon \rightarrow 0$, π_1 approaches 0.5. Furthermore, when *j* receives a message string *m* at stage $(z-1)$, this makes it possible that up to $(I-2)$ players receive $m \rightarrow k$, with $k \neq i, j$. As $\varepsilon \rightarrow 0$, the probability that $(I-2)$ players receive a confirmation of *m* and act approaches 1. It follows that as $\varepsilon \rightarrow 0$, *i* who receives all message strings up to $(z-2)$ but no (pen)ultimate message strings, certainly prefers not to act if:

$$(1-0.5^W)H - 0.5^W L < 0 \Leftrightarrow (H/L) < 0.5^W / (1-0.5^W). \qquad \text{(A3)}$$

where $W = (I-1)^{z-1}$. We now show that when (A3) is valid, *i* who does not receive (pen)ultimate message strings but receives all other available message strings, prefers not to act for any positive ε. The expected payoff of acting in these circumstances consists of probabilities $\binom{W}{Y}(1-\pi_1)^Y \pi_1{}^{W-Y}$, with $W = (I-1)^{z-1}$, attached to the event of each number

Y of penultimate message strings arriving, and for each such an event an expected payoff *E*[*Y*] depending on *Y*, where *E*[*Y*] is weakly larger the larger *Y*.[11] Note that $\partial[(1-\pi_1)^Y \pi_1^{W-Y}]/\partial\varepsilon$ = $\pi_1^{Y-1}(1-\pi_1)^{W-Y-1}(\partial\pi_1/\partial\varepsilon)[W(1-\pi_1)-Y]$, meaning that this derivative is larger than zero for $Y<W(1-\pi_1)$, and is smaller than zero for $Y>W(1-\pi_1)$. It follows that increasing ε gives more weight to the lower *E*[*Y*], corresponding to lower *Y*. Second, it is clear that each individual *E*[*Y*] is weakly decreasing in *ε*, as it is less likely that other players receive ultimate message strings. It follows that the expected payoff of acting for *i* is weakly decreasing in *ε*, meaning that condition (A3) on (*L*/*H*) suffices as a condition to ensure that for any *ε*, a player who does not receive (pen)ultimate message strings does not act.

Step 2. For $z=1$, if all players play according to the unique collective-action equilibrium, a player who receives a single message string weakly prefers not to act iff

$$(1-\varepsilon)^{I-1}H-[1-(1-\varepsilon)^{I-1}]L\leq 0. \qquad \text{(A4)}$$

Step 3. For any *z*, the most inclined *i* who is supposed not to act in a collective-action equilibrium can be to act, is when he is in the situation described in Step 1. Note that under the condition derived there, for $z=1$ or $z=2$, a player who does not receive (pen)ultimate message strings will also not act, as the player then does not even know whether the opportunity arises.

For $z\geq 2$, if each $j\neq i$ plays according to the Pareto-best collective-action equilibrium, the least inclined a player *i* who is supposed to act, can be to act is when observing a single penultimate message string, and no other message strings. The worst event which may have occurred in this case is that none of the message strings arrived which can be received at stage $(z-1)$ by $j\neq i$. In this case, *i*'s expected payoff from acting is the LHS of (A4). In all other events, where at least one penultimate message string is received by $j\neq i$, *i*'s expected payoff is strictly larger than the LHS of (A4). Take now any (*L*/*H*) such that (A3) is valid. For the chosen (*L*/*H*), take *ε* such that the LHS of (A4) is zero. Then by the above, when $z\geq 2$, if

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[11] *E*[*Y*] involves a distribution reflecting, conditional on *Y* message strings having arrived at $(z-1)$, how many different players receive message strings at $(z-1)$, and an expected payoff depending on each such event. Yet, this distribution does not depend on *ε*. Moreover, taking a given number of different players who receive message strings at $(z-1)$, as you increase *Y*, collective action is more likely to be achieved. At the same time, increasing *Y* means increasing the weight put on different players receiving message strings at $(z-1)$, where for large *Y* it becomes unnecessary for players still to receive message strings at *z* to make them act.

each $j \neq i$ plays according to the Pareto-best collective-action equilibrium, it is a strict best response for *i* to act when receiving at least one (pen)ultimate message string. By continuity, ε exists such that the LHS of (A4) is strictly smaller than zero, meaning that no collective-action equilibrium exists for $z=1$, but such that the Pareto-best collective-action equilibrium does exist for $z \geq 2$.

Proof of Proposition 2.

Step 1. For $z = z^*$, consider first the probability that, when a single message string arrives at at stage (z^*-1), all confirmations of this message string arrive at stage z^*:

$$(1-\varepsilon)^{I-\lambda} \tag{A5}$$

where $\lambda = 0$ when $z^* = 1$ (in which case the fact that state *r* occurs is considered as a single message string arriving at stage $(z^*-1)=0$), and $\lambda = 1$ when $z^* \geq 2$.

For $z = (z^*+1)$, we consider the probability of the event that all players receive all their message strings in at least *V* brooms, conditional on a single message string arriving at stage (z^*-1). In a symmetric equilibrium with parameter *X* = *V*, this event is sufficient for all players to act (yet, it should be noted that there are additional events where collective action takes place, not considered here[12]). The proposed probability equals:

$$\sum_{W=V}^{I-\lambda} \left\{ \binom{I-\lambda}{W} (1-\varepsilon)^W \varepsilon^{I-\lambda-W} \left[\sum_{Y=V}^{W} \binom{W}{Y} \left[(1-\varepsilon)^{I-1}\right]^Y \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y} \right] \right\} \tag{A6}$$

where again $\lambda = 0$ when $z^* = 1$, and $\lambda = 1$ when $z^* \geq 2$.

In (A6), $\binom{I-\lambda}{W}(1-\varepsilon)^W \varepsilon^{I-\lambda-W}$ reflects the probability that *W* equal to *V* or more message strings arrive at stage z^*. $\binom{W}{Y}\left[(1-\varepsilon)^{I-1}\right]^Y \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y}$ reflects the probability that, after *W* message strings have arrived at stage z^*, all message strings arrive at (z^*+1) in *Y* brooms, with *Y* equal to *V* or higher (with *W* as a maximum). In some consecutive steps, we

[12] It may be that each player receives all his message strings in one broom set, but for each player this may be a different broom set. It is these events that are not considered here.

now show that the expression in (A5) is smaller than the expression in (A6) for small ε. We show this for (A5) equal to $(1-\varepsilon)^{I-1}$, which suffices as $(1-\varepsilon)^{I-1} > (1-\varepsilon)^{I}$.

$$(1-\varepsilon)^{I-1} < \sum_{S=V}^{I-\lambda} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} \left[\sum_{Y=V}^{W} \binom{W}{Y} \left[(1-\varepsilon)^{I-1}\right]^{Y} \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y} \right]$$

$$\Leftrightarrow$$

$$\left[\sum_{W=0}^{V-1} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} + \sum_{W=V}^{I-\lambda} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} \right] (1-\varepsilon)^{I-1} <$$

$$\sum_{W=V}^{I-\lambda} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} \left[1 - \sum_{Y=0}^{V-1} \binom{W}{Y} \left[(1-\varepsilon)^{I-1}\right]^{Y} \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y} \right]$$

$$\Leftrightarrow$$

$$\sum_{W=0}^{V-1} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} (1-\varepsilon)^{I-1} <$$

$$\sum_{W=V}^{I} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} \left[1-(1-\varepsilon)^{I-1}\right] \left[1 - \sum_{Y=0}^{V-1} \binom{W}{Y} \left[(1-\varepsilon)^{I-1}\right]^{Y} \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y-1} \right]$$

$$\Leftrightarrow$$

$$\sum_{W=0}^{V-1} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-1-W} (1-\varepsilon)^{I-1} <$$

$$\sum_{W=V}^{I-\lambda} \binom{I-\lambda}{W} (1-\varepsilon)^{W} \varepsilon^{I-\lambda-W} \left[\sum_{T=0}^{I-2} \binom{I-1}{T} (1-\varepsilon)^{T} \varepsilon^{I-2-T} \right]$$

$$\left[1 - \sum_{Y=0}^{V-1} \binom{W}{Y} \left[(1-\varepsilon)^{I-1}\right]^{Y} \left[1-(1-\varepsilon)^{I-1}\right]^{W-Y-1} \right] \tag{A7}$$

As ε approaches zero, for $V \leq (I-2)$, both for $\lambda = 0,1$ the left-hand side of (A7) approaches zero, while the right-hand side approaches one.

Step 2. Consider $z = t + y$, with $t \geq 2$, and let players be able to achieve collective action if the message strings in at least one broom are all received. Consider $z = t + y + 1$ and let players be able to achieve collective action if the message strings in at least X brooms are received, with $X \leq (I-2)$. Suppose that we have been able to show for a specific given t that the probability of collective action is larger for $z = t + y + 1$ than for $z = t + y$, when $y = 0$. Then we show that this is also true for $y = 1$.

Note that in case $z = t + 2$ or $z = t + 1$, once exactly one message string has arrived at stage 1, it is as if we have $z = t + 1$, respectively $z = t$. Given our assumption that the probability of collective is larger with $z = t + 1$ if $X \leq (I - 2)$ brooms suffice, than it is with $z = t$ if one broom suffices, this also applies for $z = t + 2$ versus $z = t + 1$, in the event that exactly one message string has arrived at stage 1. If this applies in the event that a single message string arrives at stage 1, it applies for any number of message strings that arrive at stage 1.

Step 3. Step 1 (base case) implies that the probability of collective action is larger in any case where $z = 2$ and arrival of all (pen)ultimate message strings in $X \leq (I - 2)$ brooms suffices, than in the symmetric equilibrium with $z = 1$ and parameter $X = 1$. Also, Step 1 implies that, conditional on a single message string having arrived at stage $(z - 2)$, the probability of collective action is larger in any case where $z = 3$ and arrival of all (pen)ultimate message strings in $X \leq (I - 2)$ brooms suffices, than in case $z = 2$ and arrival of all (pen)ultimate message strings in one broom suffices. If this applies in the event that a single message string arrives at stage 1, it applies for any number of message strings that arrive at stage 1, so that the probability of collective action is larger in case $z = 3$ and $X \leq (I - 2)$ brooms suffice, than in case $z = 2$ and one broom suffices. Step 2 (inductive step) showed that if, for a specific *t* with $t \geq 2$, it is true for $y = 0$ that the probability of collective action is larger in any case where $z = t + y + 1$ and $X \leq (I - 2)$ brooms suffice, than in case $z = t + y$ and one broom suffices, then this is also true for $y = 1$. Given that for any given *z*, the probability of collective action is weakly larger in case $X \leq (I - 2)$ brooms suffice, than in case one broom suffices, the proposition follows.