PHYSICAL REVIEW D **92,** 044008 (2015)

# Inflation from cosmological constant and nonminimally coupled scalar

Dražen Glavan,* Anja Marunović,† and Tomislav Prokopec‡

*Institute for Theoretical Physics, Spinoza Institute and Center for Extreme Matter and Emergent Phenomena, Utrecht University, Postbus 80.195, 3508 TD Utrecht, The Netherlands*

(Received 19 May 2015; published 4 August 2015)

We consider inflation in a universe with a positive cosmological constant and a nonminimally coupled scalar field, in which the field couples both quadratically and quartically to the Ricci scalar. When considered in the Einstein frame and when the nonminimal couplings are negative, the field starts in slow roll and inflation ends with an asymptotic value of the principal slow-roll parameter, $\epsilon_E = 4/3$. Graceful exit can be achieved by suitably (tightly) coupling the scalar field to matter, such that at late time the total energy density reaches the scaling of matter, $\epsilon_E = \epsilon_m$. Quite generically the model produces a red spectrum of scalar cosmological perturbations and a small amount of gravitational radiation. With a suitable choice of the nonminimal couplings, the spectral slope can be as large as $n_s \simeq 0.955$, which is about one standard deviation away from the central value measured by the Planck satellite. The model can be ruled out by future measurements if any of the following is observed: (a) the spectral index of scalar perturbations is $n_s > 0.960$; (b) the amplitude of tensor perturbations is above about $r \sim 10^{-2}$; (c) the running of the spectral index of scalar perturbations is positive.

DOI: 10.1103/PhysRevD.92.044008 PACS numbers: 04.62.+v, 98.80.-k, 98.80.Qc

## I. INTRODUCTION

The most famous example of an inflationary model realized within a tensor-scalar theory [1–4], in which a (gravitational) scalar couples to the Ricci scalar, is Higgs inflation [5–8], in which the role of the inflaton is played by the standard model Higgs field. Tensor-scalar theories have also been extensively used to discuss the cosmological constant problem [9–12] to explain the origin of dark energy [13–16] and have been thoroughly tested on Solar System scales [17].

While many inflationary models have been considered, to our knowledge no one has investigated the model in which inflation is driven by a positive cosmological constant accompanied by a nonminimally coupled scalar field. A study of this class of models is the subject of this paper.

In Sec. II we present the model and discuss how to analyze it in the Einstein frame. In Sec. III we recall the basics of the slow-roll approximation. In Sec. IV our principal results are presented. In particular, we discuss the spectral index, its running and the amplitude of tensor perturbations. Finally, in Sec. V we shortly recapitulate our main results and discuss future directions. A particular emphasis is devoted to the graceful exit problem and to the question of falsifiability of our inflationary model.

## II. THE MODEL

In this paper we consider the following simple tensor-scalar theory of gravity, whose action in the Jordan frame reads,

$$S_J = \int d^4x\sqrt{-g_J}\left(\frac{1}{2}F(\phi_J)R_J - M_P^2\Lambda - \frac{1}{2}g_J^{\mu\nu}\partial_\mu\phi_J\partial_\nu\phi_J - V_J(\phi_J)\right), \qquad (1)$$

where $g_J = \det[g_{J\mu\nu}]$, $g_J^{\mu\nu}$ is the inverse of the (Jordan frame) metric tensor $g_{J\mu\nu}$ and $R_J$ is the Ricci scalar. In this paper we assume the following simple form for the functions $F$ and $V_J$:

$$F(\phi_J) = M_P^2 - \xi_2\phi_J^2 - \xi_4\frac{\phi_J^4}{M_P^2}, \quad \text{and} \quad V_J(\phi_J) = 0, \qquad (2)$$

where $M_P^2 = 1/(8\pi G_N)$, $G_N$ is the Newton constant and $\xi_2$ and $\xi_4$ are (dimensionless) nonminimal coupling parameters. In our conventions conformal coupling corresponds to $\xi_2 \equiv \xi_c = 1/6$, $\xi_4 = 0$, and we work with natural units in which $\hbar = 1 = c$. For the metric we take a cosmological, spatially flat, background,

$$g_{J\mu\nu} = \mathrm{diag}[-1, a_J^2(t), a_J^2(t), a_J^2(t)]. \qquad (3)$$

Even though the Jordan and Einstein frames are fully equivalent [8,18–20], cosmological perturbations are easier to analyze in the Einstein frame and when a slow-roll approximation is utilized. Therefore, we shall proceed by transforming the Jordan frame action (1) to the Einstein frame.

To get to the Einstein frame with the canonically coupled scalar, one ought to perform the following frame (conformal) transformations:

*d.glavan@uu.nl
†a.marunovic@uu.nl
‡t.prokopec@uu.nl

 

$$g_{E\mu\nu} = \frac{F(\phi_J)}{M_P^2} g_{J\mu\nu},$$
$$d\phi_E = \frac{M_P}{F(\phi_J)}\sqrt{F(\phi_J) + \frac{3}{2}\left(\frac{dF(\phi_J)}{d\phi_J}\right)^2} d\phi_J, \qquad (4)$$

where the index $E$ refers to the Einstein frame. In this frame, the scalar-tensor action (1) becomes simpler [12],

$$S_E = \int d^4x\sqrt{-g_E}\left(\frac{M_P^2}{2}R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\phi_E\partial_\nu\phi_E - \frac{M_P^6\Lambda}{F^2(\phi_J(\phi_E))}\right), \qquad (5)$$

thus coupling the cosmological constant to the scalar field. This coupling introduces a nontrivial dynamics which—as we show below—can be used to realize a viable model of primordial inflation.

In Fig. 1 we show the effective potential in the Einstein frame $V_E(\phi_E) = M_P^6\Lambda/F^2(\phi_J(\phi_E))$ as a function of the Einstein frame field $\phi_E$ for several values of $\xi_2$ and for $\xi_4$ fixed to $\xi_4 = -0.1$. When both couplings are negative, the effective potential has one local maximum (at $\phi_E = 0$) and it decays monotonically towards zero as the field $|\phi_E|$ increases (see left panel). However, when $\xi_2 > 0$ and $\xi_4 < 0$, $V_E$ develops a local minimum at $\phi_E = 0$ and two local maxima at some positive $|\phi_E|$ (right panel). In this paper we investigate the case when both couplings are negative and leave the latter case, in which tunneling from the local minimum can play an important role, for future work. While the field dependence of the potential in Eq. (5) is simple when expressed in terms of the Jordan frame field, there is no simple analytic form that describes the Einstein frame potential. This is a consequence of the fact that Eq. (4) cannot be solved analytically for $\phi_J(\phi_E)$. There are simple limits however. For small field values, $\phi_E \ll M_P$, the potential $V_E$ in Eq. (5) can be approximated by a constant plus a negative mass term (as in hilltop inflation; see e.g. Refs. [21,22]),

$$V_E(\phi_E) \simeq \Lambda[M_P^2 + 2\xi_2\phi_E^2] + \mathcal{O}(\phi_E^4), \qquad (6)$$

while for $\phi_E \gg M_P$, the potential decays exponentially with the field,

$$V_E(\phi_E) \simeq V_{E0}\exp\left(-\lambda_E\frac{\phi_E}{M_P}\right), \qquad \lambda_E = \sqrt{\frac{8}{3}}, \qquad (7)$$

where $V_{E0}$ is a constant whose value is $\sim\Lambda M_P^2$. From Eqs. (6) and (7) we see that, if the field starts from some small value near the local maximum, it will slowly roll down the hill, eventually exiting inflation when $\epsilon_E \simeq 1$.

One can show [12] that for $\xi_4 = 0$, $\xi_2 < -1/2$ and in the Einstein frame

$$\epsilon_E = \frac{-8\xi_2}{1-6\xi_2} > 1, \qquad (8)$$

with the limiting value $\epsilon_E \to 4/3$ for $\xi_2 \to -\infty$. Here we have introduced quartic nonminimal coupling $\xi_4 < 0$ in $F$ in Eq. (2) in order to be able to relax the condition on $\xi_2$ and to still be able to terminate inflation. Namely, one can show that even when the quartic coupling is arbitrarily small and negative, $\epsilon_E$ will asymptotically reach the value $4/3 > 1$, regardless of the value of negative $\xi_2$. The condition $\epsilon_E \ll 1$

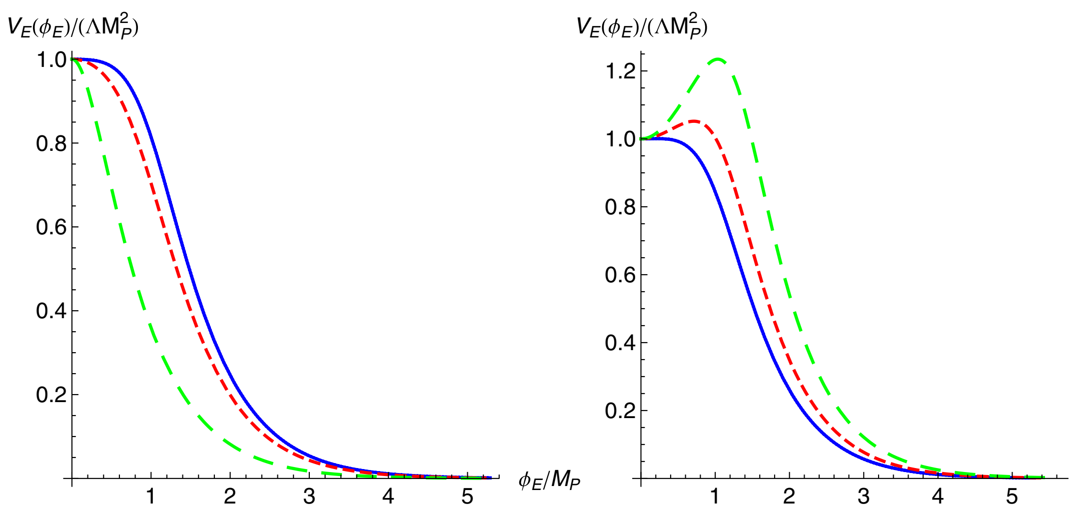


FIG. 1 (color online). The effective potential (cosmological constant) $V_E$ in the Einstein frame as a function of the Einstein frame field $\phi_E$. In this figure $\xi_4 = -0.1$. Left panel: $\xi_2 = -0.01$ (blue solid), $\xi_2 = -0.1$ (red dashes) and $\xi_2 = -1$ (long green dashes). Right panel: $\xi_2 = 0.01$ (blue solid), $\xi_2 = 0.1$ (red dashes) and $\xi_2 = 0.2$ (long green dashes). Note that when $\xi_2 < 0$, $V_E$ has a local maximum at $\phi_E = 0$ ($\phi = 0$), while for $\xi_2 > 0$, $V_E$ has a local minimum at $\phi_E = 0$ and two local maxima at some $\phi_E = \pm\phi_{E0} \neq 0$. The potential $V_E$ exhibits a $Z_2$ symmetry, i.e. it is symmetric under $\phi_E \to -\phi_E$.

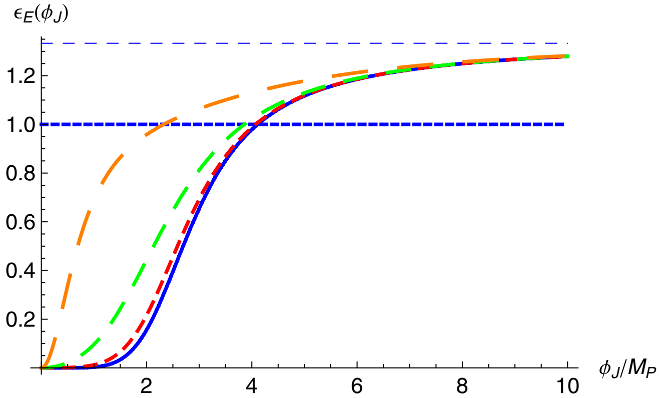


FIG. 2 (color online). Principal slow-roll parameter $\epsilon_E$ as a function of $\phi_J$ for $\xi_4 = -0.01$. Different curves show $\xi_2 = -0.01$ (blue solid), $\xi_2 = -0.02$ (short red dashes), $\xi_2 = -0.1$ (green dashes) and $\xi_2 = -0.5$ (long orange dashes). Note that, independently of the values of $\xi_2$ and $\xi_4$ (as long as they are both negative), $\epsilon_E \to 4/3$ (horizontal blue dashes) when $\phi \to \infty$. Inflation ends when $\epsilon_E \to 1$ (short horizontal blue dashes).

during inflation requires $|\xi_2| \ll 1$ which is satisfied by this setup.

One way of seeing this is to work in the adiabatic approximation and subsume the $-\xi_4\phi^4/M_P^2$ term in $F$ into a field-dependent quadratic coupling $\xi$ as follows: $\xi(\phi) \equiv \xi_2 + \xi_4\phi^2/M_P^2$. Now, when this is inserted into $\epsilon_E \simeq -8\xi/(1-6\xi)$, which is the attractor value at asymptotically large field values, one obtains, $\epsilon_E \to 4/3$ for arbitrarily small, negative values of $\xi_4$; see Fig. 2.

While inflation terminates when $\epsilon_E > 1$, $\epsilon_E = 4/3$ is not enough to explain the post-inflationary radiation and matter eras. One can show [12] that a suitable coupling to a (perfect) matter fluid can induce the decay of $\phi_E$ into matter, such that in the tightly coupled regime, the system reaches $\epsilon_E = \epsilon_m$. When matter is predominantly in the form of a relativistic fluid, for which the equation-of-state parameter $w_m = p_m/\rho_m = 1/3$, or equivalently $\epsilon_m = (3/2)(1+w_m) = 2$, one will eventually reach a post-inflationary radiation era, providing thus a graceful exit from inflation that is consistent with all observations.

## III. SLOW-ROLL INFLATION

We do not know what was the state of the Universe before inflation. It seems reasonable to assume that the Universe was expanding and that it was in a chaotic state, whose energy-momentum tensor was dominated by field fluctuations of various (energy and distance) scales. Even if not in equilibrium, such a state could be approximated by a nearly perfect fluid, whose equation of state is well approximated by the radiation equation of state, $w \simeq 1/3$. In such a state nonminimal couplings do not play a significant role (since $\langle R\rangle \sim 0$), and thence it is natural to take the expectation value of the (quantum) field $\hat{\phi}$ to be close to zero, $\langle\hat{\phi}(x)\rangle = \phi_0 \simeq 0$.

As the Universe expands, the amplitude of fluctuations decreases, and the corresponding energy density and pressure decrease accordingly, reaching eventually the point when the contribution from the cosmological constant (whose origin may be both geometric and vacuum fluctuations of quantum fields) becomes significant. At that moment the Universe enters an inflationary phase, whereby the field feels a hilltop-like potential (6) and starts rolling down the hill. As it rolls, the contribution from fluctuations will rapidly redshift, becoming less and less important for the Universe's dynamics. Thus we see that in our inflationary model the Universe enters inflation from a broad range of initial conditions without any need for (fine-) tuning. Of course, it is still true that the cosmological constant and the nonminimal couplings have to have the right values (set by the COBE normalization of the amplitude of the scalar spectrum of cosmological perturbations and by the Planck value of the corresponding spectral slope). As we show below, these values can be obtained in our model by quite a natural choice of the parameters.

The Einstein frame action (5) implies the following equations of motion:

$$\ddot{\phi}_E + 3H_E\dot{\phi}_E + V_E' = 0, \tag{9}$$

$$H_E^2 = \frac{1}{3M_P^2}\left(\frac{\dot{\phi}_E^2}{2} + V_E(\phi_E)\right), \tag{10}$$

$$\dot{H}_E = -\frac{\dot{\phi}_E^2}{2M_P^2}, \tag{11}$$

where the metric tensor is now, $g_{E\mu\nu} = \mathrm{diag}[-1, a_E^2(t), a_E^2(t), a_E^2(t)]$. While these equations can be solved numerically [12] without resorting to the slow-roll approximation (in which the Hubble parameter and possibly some of its time derivatives can be treated as adiabatic functions of time), it is instructive to use the slow-roll approximation because one can use analytical techniques that allow us to get a better grasp of the parameter dependences of the observables. One can check the predictions of the slow-roll approximation by studying (approximate or exact) solutions of the attractor equation,

$$H_E^2(\phi_E) = \frac{2}{3}M_P^2\left(\frac{dH_E}{d\phi_E}\right)^2 + \frac{V_E(\phi_E)}{3M_P^2}, \tag{12}$$

which is more general than the slow-roll approximation. This equation can be derived as follows. In general $H_E = H_E(\phi_E, \dot{\phi}_E)$. However, it is often the case that the dependence on $\dot{\phi}_E$ can be neglected because the initial conditions for $\dot{\phi}_E$ are forgotten or $\dot{\phi}_E$ is a function of $\phi_E$ (as it is, for example, in slow roll). More generally this will be the case when there is a phase-space attractor towards which trajectories $(\phi_E(t), \dot{\phi}_E(t))$ rapidly converge.[1] In this

[1]An attractor behavior is opposite from a chaotic behavior, in which phase-space trajectories repulse each other in the sense that they (exponentially) diverge from each other.

case $H_E = H_E(\phi_E)$ and Eq. (12) can be easily obtained by rewriting Eq. (11) as $\dot{\phi}_E = -2M_P^2 dH_E/d\phi_E$ and inserting it into Eq. (10). With these caveats in mind, solving Eq. (12) is equivalent to solving the full system of equations (10)–(11) [Eq. (9) does not provide any new information as it can be obtained from the other two equations].

In the slow-roll approximation one neglects the first term in Eq. (9) and the kinetic term in Eq. (10) (the last equation is irrelevant because it is not independent). The memory of the initial conditions is neglected (because $\dot{\phi}_E$ and $\dot{H}_E$ are not independent variables and slow roll is an attractor). Moreover, the dependence on the initial field value $\phi_{J0} = \phi_J(t_0)$ is irrelevant, because one measures the number of $e$-folds from the end of inflation $\phi_J(t_e) = \phi_{Je}$ (at which the principal slow-roll parameter $\epsilon_E = 1$), and during inflation we are in the attractor. With these in mind, we can define the number of $e$-folds as,

$$N(\phi_J) = \int_t^{t_e} H_E(\tilde{t})d\tilde{t} \simeq \frac{1}{2}\int_{\tilde{\phi}_J}^{\phi_{Je}} d\phi_J \left[\frac{3}{2}\frac{F'}{F} + \frac{1}{F'}\right] = \frac{3}{4}\ln\left(\frac{F(\tilde{\phi}_J)}{M_P^2}\right) + \frac{1}{8\xi_2}\ln\left(\frac{M_P^2 F'(\tilde{\phi}_J)}{\tilde{\phi}_J^3}\right)\Bigg|_{\phi_J}^{\phi_{Je}}, \tag{13}$$

where we made use of,

$$\frac{\dot{\phi}_E}{H_E} = \frac{2M_P F'(\phi_J)}{\sqrt{F(\phi_J) + \frac{3}{2}F'(\phi_J)^2}}. \tag{14}$$

Next, the principal slow-roll parameter, $\epsilon_E \equiv \epsilon_1 = -\dot{H}_E/H_E^2$ reads in the slow-roll approximation,

$$\epsilon_E(\phi_J) \simeq \frac{2F'^2}{F + \frac{3}{2}F'^2}. \tag{15}$$

The other two slow-roll parameters can be defined in terms of the rate of change of $\epsilon_E$ as, $\eta_E \equiv \epsilon_2 = \dot{\epsilon}_E/(\epsilon_E H_E)$, $\xi_E \equiv \epsilon_3 = \dot{\eta}_E/(\eta_E H_E)$. In the slow-roll approximation they read,

$$\eta_E(\phi_J) \simeq \frac{2F(2FF'' - F'^2)}{(F + \frac{3}{2}F'^2)^2},$$
$$\xi_E(\phi_J) \simeq \frac{2F'}{F + \frac{3}{2}F'^2}\left(\frac{2F^2F''}{2FF'' - F'^2} - \frac{F'(F - \frac{3}{2}F'^2 + 6F'')}{F + \frac{3}{2}F'^2}\right). \tag{16}$$

The scalar and tensor perturbations are of the form,

$$\Delta_s^2(k) = \Delta_s^2(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}, \qquad \Delta_s^2(k_*) = \frac{H_E^2}{8\pi^2\epsilon_E M_P^2},$$
$$\Delta_t^2(k) = \Delta_t^2(k_*)\left(\frac{k}{k_*}\right)^{n_t}, \qquad \Delta_t^2(k_*) = \frac{2H_E^2}{\pi^2 M_P^2}, \tag{17}$$

where (to the leading order in the slow-roll approximation) the spectral indices $n_s$ and $n_t$ can be determined from the variation of $\Delta_s^2(k)$ and $\Delta_t^2(k)$ with respect to $k$ at the first horizon crossing during inflation (where $k = k_* = Ha$) as follows:

$$n_s = 1 + \left(\frac{d\ln[\Delta_s^2(k)]}{d\ln(k)}\right)_{k=k_*} = \frac{dt}{d\ln(Ha)}\frac{d\ln[\Delta_s^2(k_*)]}{dt} \simeq -2\epsilon_E - \eta_E, \tag{18}$$

$$n_t = \left(\frac{d\ln[\Delta_t^2(k)]}{d\ln(k)}\right)_{k=k_*} = \frac{dt}{d\ln(Ha)}\frac{d\ln[\Delta_t^2(k_*)]}{dt} \simeq -2\epsilon_E. \tag{19}$$

Next, Eq. (17) implies that the ratio of the tensor and scalar spectra is,

$$r(k_*) \equiv \frac{\Delta_t^2(k_*)}{\Delta_s^2(k_*)} = 16\epsilon_E. \tag{20}$$

Finally, the running of the spectral index $n_s$ is,

$$\alpha(k_*) = \left[\frac{d(n_s)}{d\ln(k)}\right]_{k=k_*} = -(2\epsilon_E + \xi_E)\eta_E. \tag{21}$$

This completes the calculation of the quantities required for the slow-roll analysis, which is used in the remainder of the paper. In our plots we shall sometimes express our quantities in terms of the Jordan frame field $\phi = \phi_J$, and sometimes in terms of the number of $e$-folds in the Einstein frame, $N_E$. For the latter it is useful to know how to calculate the field value $\phi_{Je}$ at the end of inflation, which is by convention defined as the field value at which $\epsilon_E = 1$. A cursory look at Eq. (15) reveals that $\epsilon_E = 1$ when $F = F'^2/2$, which is equivalent to the zeros of the following cubic equation for $\phi_J^2$:

$$8\xi_4^2\phi_J^6 + (8\xi_2 + 1)\xi_4 M_P^2\phi_J^4 + (2\xi_2 + 1)\xi_2 M_P^4\phi_J^2 - M_P^6 = 0. \tag{22}$$

Two of the zeros of this equation are complex and hence unphysical and one zero is real and positive, representing hence the unique physical solution defining the end of inflation. In the following analyses we use that solution to signify the end of inflation.

An important question that needs to be addressed is the validity of the slow-roll approximation. When inflation lasts much longer than $N \simeq 60$, it is to be expected that the field will be extremely close to the attractor regime described by Eq. (12). Under that assumption Eq. (12) can be used to test the slow-roll approximation. To get some insight into that question, we shall now study the early-time evolution of the field, which is described by

Eq. (12). Inserting the ansatz, $H_E^2 = H_0^2(1+\zeta\phi_E^2/M_P^2)$ into Eq. (12) yields a quadratic equation for $\zeta$,

$$\zeta^2 - \frac{3}{2}\zeta + 3\xi_2 = 0, \tag{23}$$

where we made use of $V_E(\phi_E) \simeq 3H_0^2[M_P^2 + 2\xi_2\phi_E^2]$; see Eq. (6). The two solutions are,

$$\zeta_\pm = \frac{3}{4}\left(1 \pm \sqrt{1 - \frac{16}{3}\xi_2}\right). \tag{24}$$

The physically relevant solution is the negative one, $\zeta = \zeta_-$, as $H_E^2(\phi_E)$ must decrease as $\phi_E^2$ increases. When $|\xi_2| \ll 1$, $\zeta = 2\xi_2 + \mathcal{O}(\xi_2^2)$, so the leading-order result in $\xi_2$ reproduces the slow-roll result, and the higher-order powers in $\xi_2$ are corrections to slow roll. Thus, as long as $|\xi_2| \ll 1$, the slow-roll results should be trustable. This is, of course true, provided the attractor behavior (discussed above) is realized and $H_E = H_E(\phi_E)$ does not depend on $\dot{\phi}_E$. At late times, when $\phi_E^2 \gg M_P^2$, the Einstein frame effective potential reduces to Eq. (7). It is well known that solutions to the Friedmann equations in such an exponential potential exhibit an attractor behavior [12,23,24] in which, while $\xi_2$ dominates the dynamics, $\epsilon_E = -8\xi_2/(1-6\xi_2)$, and asymptotically (when $\xi_4$ dominates), $\epsilon_E = 4/3$. The slow-roll approximation again reproduces the leading-order results: at intermediate times, $\epsilon_E = -8\xi_2$, and at late times, $\epsilon_E = 4/3$, leading to an identical conclusion: as long as $|\xi_2| \ll 1$, the slow-roll approximation yields approximately correct results. With this in mind, we are ready to proceed to analyze our inflationary model in the slow-roll approximation. We leave the analysis that goes beyond slow roll for future work.

## IV. RESULTS

In this section we present the principal results for the most important inflationary observables, which include the amplitude of the scalar spectrum $\Delta_s^2$, the scalar spectral index $n_s$ and its (logarithmic) running $\alpha$ and the ratio of tensor and scalar perturbations, $r = \Delta_t^2/\Delta_s^2$. We do not discuss separately the tensor spectral index $n_t$ and its running, but observe in passing that (within our approximations) the latter satisfies a consistency relation, $n_t = -2\epsilon_E = -r/8$ and thus, up to a constant rescaling, $n_t$ is captured by the analysis of $r$.

Figure 3 shows the dependence of the spectral index $n_s$ on the number of $e$-folds $N$, taking $\xi_2$ and $\xi_4$ as parameters. The figure shows that $n_s$ peaks for $\xi_2 \simeq -0.002$ (short red dashes), and it is very weakly dependent on $\xi_4$ (in the left panel $\xi_4 = -0.1$ and in the right panel $\xi_4 = -0.01$). Decreasing $|\xi_4|$ further would lead to smaller values of $n_s$. Since the peak value of $n_s$ in our model is smaller [by about one standard deviation (dashed blue horizontal line)] than the central value of $n_s$ obtained by the Planck Collaboration [25], we conclude that our model gives the best results when preheating is instant and when $\xi_2 \sim -0.002$ and $\xi_4 \sim -0.1$.

In Fig. 4 (left panel) we show the dependence of $n_s$ and $r$ on $\xi_2$ with the number of $e$-folds $N$ as a parameter (from the bottom up, the curves corresponding to $N = 50$, 60 and 65 are shown). Figure 4 shows that the optimal value of $\xi_2$ is about $-2\times10^{-3}$, which is the value at which $n_s$ peaks. The peak value of $n_s$ is very weakly dependent on $\xi_4$ and decreases slowly as $|\xi_4|$ decreases. The right panel of Fig. 4 shows the ratio of the spectral amplitudes of tensor and scalar cosmological perturbations $r$ as a function of $\xi_2$: $r$ is typically small and peaks for $\xi_2 \simeq -0.005$. Contrary to $n_s$, $r$

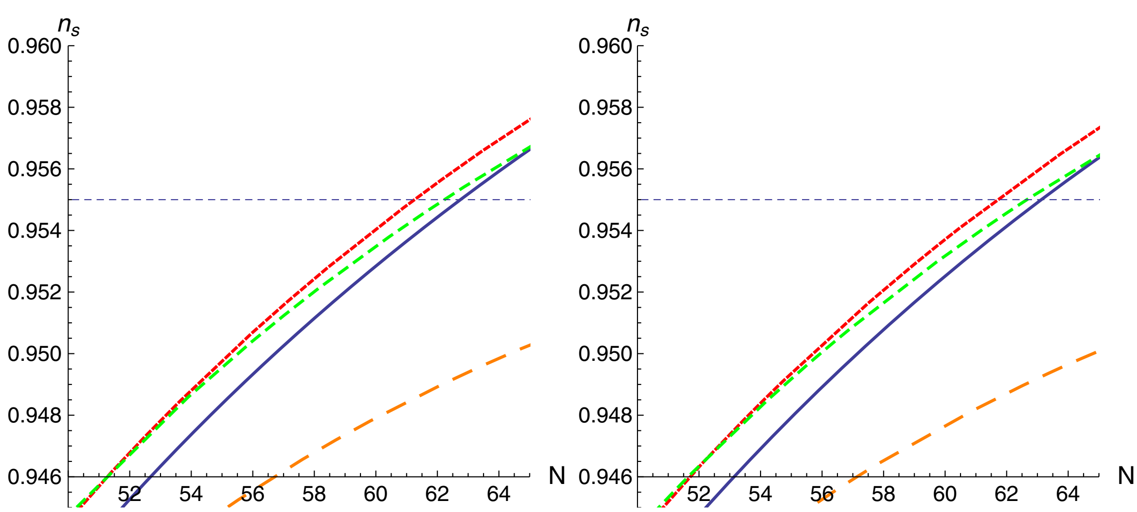


FIG. 3 (color online). Spectral index $n_s$ as a function of the number of $e$-folds $N$ (from bottom to top) for $\xi_2 = -0.005$ (long orange dashes), $\xi_2 = -0.001$ (blue solid), $\xi_2 = -0.003$ (green dashes) and $\xi_2 = -0.002$ (short red dashes). Left panel: $\xi_4 = -0.1$. Right panel: $\xi_4 = -0.01$. Numerical investigations show that the maximum value of $n_s$ is very weakly dependent on $\xi_4$, and peaks for $\xi_4 \in (-10^{-2}, -10^{-1})$. The dependence on $\xi_2$ is much more pronounced, and $n_s$ peaks around $\xi_2 \simeq -0.002$ (short red dashes in both left and right panels).

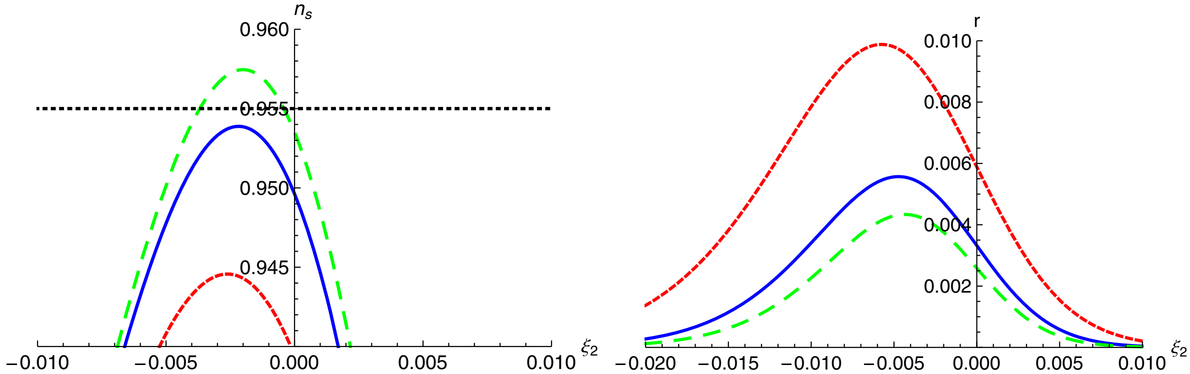

FIG. 4 (color online). Left panel: The spectral index $n_s$ as a function of $\xi_2$ for $N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashed). The maximum value of $n_s$ is very weakly dependent on $\xi_4$, and peaks for $\xi_4 \in (-10^{-2}, -10^{-1})$. The dependence on $\xi_2$ is much more pronounced, and $n_s$ peaks around $\xi_2 \simeq -0.002$: $n_s = 0.955$ (the value which is about $1\sigma$ lower than the Planck satellite best-fit value) when $N \simeq 62$ at the pivotal scale $k_* = 0.05$ Mpc. In this graph $\xi_4 = -0.02$. Right panel: The ratio of the spectra of tensor and scalar cosmological perturbations $r$ as a function of $\xi_2$ for $N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashes). The maximum value of $r$ is attained for $\xi_2 \sim -0.005$, and it is approximately inversely proportional to $|\xi_4|$. This means that, to get values that are observable by the near-future experiments, $|\xi_4|$ needs to be sufficiently small. Roughly, we have (when $N = 60$) $r \sim 10^{-6}/|\xi_4|$, such that in order to get an observable $r$ one needs $|\xi_4| \leq 10^{-3}$. In this graph $\xi_4 = -0.0002$.

shows a strong (approximately inversely proportional) dependence on $\xi_4$, such that one can get $r$ as large as $10^{-2}$ when $|\xi_4| \sim 10^{-4}$.

Also from Fig. 4 one sees that the dependence of $n_s$ and $r$ on the number of $e$-folds $N$ (for a sufficiently large $N$) is approximately,

$$n_s - 1 \sim -\frac{p_{n_s}(\xi_2, \xi_4)}{N} \quad \text{and} \quad r \sim \frac{p_r(\xi_2)/|\xi_4|}{N^3} \tag{25}$$

where $p_{n_s}$ is weakly dependent on $\xi_2$ and $\xi_4$, and for the typical choice of the nonminimal couplings taken in this paper, $p_{n_s} \simeq 2.5$. Likewise, $p_r$ is weakly dependent on $\xi_2$ and $p_r \sim 4$.

In Fig. 5 we show the ratio of the spectra of tensor and scalar cosmological perturbations $r$ as a function of the spectral index $n_s$ for the number of $e$-folds, $N = 50$ (short red dashes), 60 (blue solid) and 65 (long green dashes). We see that the maximum value of $n_s$ increases as the number of $e$-folds increases, and it touches the lower $1\sigma$ observed bound on $n_s$ (taken from Fig. 4 of Ref. [25], from where we took the $1\sigma$ contours obtained at the optimal value of the running spectral index) when $N \simeq 62$. The question whether this high value of $N$ can be obtained within the standard cosmology (inflation followed by radiation and matter era) is discussed in the paragraph below. The model favors small values of $r$. An $r$ that is large enough ($r \sim 10^{-3} - 10^{-2}$) to be observable by the near-future planned missions (such as CORE+ and PRISM [26]) can be obtained at the price of slightly decreasing $n_s$, thus moving it further away (to about $1.5\sigma$) from the sweet spot, $n_s \simeq 0.965$.

In conclusion, our analysis shows that, even though our model is slightly (at $1\sigma$) disfavored by the current data, it is a viable model of inflation.

A simple calculation shows that the number of $e$-folds during inflation that corresponds to some pivotal scale $k_*$ is (see e.g. Appendix B in Ref. [13]),

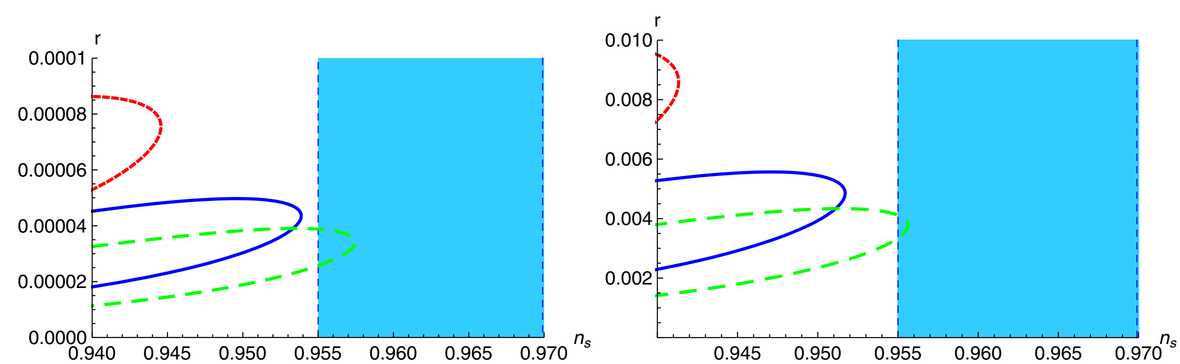

FIG. 5 (color online). The ratio of the spectra of tensor and scalar cosmological perturbations $r$ versus the spectral index $n_s$ for the number of $e$-folds $N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashes). Left panel: $\xi_4 = -0.02$. Right panel: $\xi_4 = -0.0002$. From the figure we see that the maximum value of $n_s$ grows as $N$ increases, favoring thus models with a large number of $e$-folds, such as models with instant preheating and models which have a post-inflationary period of kination. We also see that $r$ decreases as $N$ increases and as $\xi_4$ increases. However, making $|\xi_4|$ smaller has as a consequence a slight reduction of $n_s$.

$$N_I = \frac{1}{2-\bar{\epsilon}_I}\left\{\ln\left[\frac{H_*}{H_0}\left(\frac{k_0}{k_*}\right)^{\frac{1}{1-\bar{\epsilon}_I}}\right] - \frac{1}{2}N_m\right\}, \tag{26}$$

where instant preheating is assumed. More accurately: an instant transition from inflation (during which $\epsilon_E = \epsilon_I$) to radiation (during which $\epsilon_E = 2$) is assumed. In the above formula $H_0 \simeq 68$ km/Mpc/s is the Hubble rate today, $H_* \simeq 3.4\times 10^{13}$ GeV$/\hbar \simeq 1.6\times 10^{60}$ m/Mpc/s is the Hubble rate at the time when the pivotal comoving momentum $k_* = 0.05$ Mpc$^{-1}$ exits the Hubble radius during inflation, $k_0 = 0.00026$ Mpc$^{-1}$ is the comoving momentum corresponding to the Hubble scale today, and $N_m \simeq 8.1$ is the number of $e$-folds during the matter era. Once $N_I$ is known, the number of $e$-folds during the radiation era is easily calculated from $N_r = (1-\bar{\epsilon}_I)N_I - \frac{1}{2}N_m$. Taking $\epsilon_I = \bar{\epsilon}_I \simeq 0.02$ during inflation gives $N_I \simeq 59.4$; this result is correct provided $\epsilon_E = \epsilon_I$ stays constant during inflation and then relatively suddenly (within one or at most a few $e$-folds) changes at the end of inflation to $\epsilon_E = 2$. More realistically, $\epsilon_E$ changes gradually during inflation. Indeed, typical inflationary models predict $\epsilon_E \sim q/N$, where $q$ is a constant of the order of unity. In these models $\epsilon_I$ needs to be replaced by its average value, $\bar{\epsilon}_I \simeq 0.1$. In this case Eq. (26) gives, $N_I \simeq 62.2$. This is the maximum number of $e$-folds one can attain during inflation that corresponds to the pivotal momentum $k_* = 0.05$ Mpc$^{-1}$ in standard cosmology.

However, there are nonstandard cosmologies [23,27,28] which include a period of kination (during which the kinetic energy of a scalar field dominates the energy density such that $\rho \propto 1/a^6$ and $\epsilon_E \simeq \epsilon_k = 3$). During kination comoving modes approach the Hubble scale faster than during the radiation or matter era, increasing thus the number of required inflationary $e$-folds. For example, when the number of $e$-folds of kination is 20% of that in radiation, the number of $e$-folds (corresponding to $k_* = 0.05$ Mpc$^{-1}$) increases from $N_I \simeq 62.2$ to $N_I \simeq 66.4$.

In conclusion, a careful calculation shows that the number of inflationary $e$-folds corresponding to the pivotal scale $k_* = 0.05$ Mpc$^{-1}$ used by the Planck Collaboration is at most $N_I \simeq 62$ (for standard cosmology), while in nonstandard cosmologies (with e.g. a period of kination) it can be larger. For these reasons in our figures we show results not just for $N = 50$ and $N = 60$, but also for $N = 65$.

Let us now try to figure out what the current data can tell us about the magnitude of the cosmological constant $\Lambda$ in our inflationary model. Recall that we know [25] that the amplitude of the scalar power spectrum (the COBE normalization) at the pivotal scale $k_* = 0.05$ Mpc$^{-1}$ is $\Delta_s^2(k_*) = (2.20 \pm 0.09)\times 10^{-9}$. On the other hand, combining Eqs. (10), (17) and (20) gives,

$$\frac{\Lambda}{M_P^2} = \frac{F^2}{M_P^4}\frac{3\pi^2}{2}\Delta_s^2(k_*)r \simeq \frac{3\pi^2}{2}\Delta_s^2(k_*)r$$
$$= (3.25\pm 0.13)\times 10^{-8}r, \qquad r \sim \frac{10^{-6}}{|\xi_4|}, \tag{27}$$

where in the second equality we used the approximation, $F \simeq M_P^2$. To investigate whether the value of this cosmological constant is at the grand unified scale (GUT), let us define the GUT energy density as, $\rho_{\rm GUT} \equiv E_{\rm GUT}^4 = \Lambda M_P^2$, from which one gets,

$$\frac{\Lambda}{M_P^4} \simeq 2.84\times 10^{-10}\left(\frac{E_{\rm GUT}}{10^{16}\ {\rm GeV}}\right)^4. \tag{28}$$

Comparing this with Eq. (27) gives the following estimate of the grand unified scale producing $\Lambda$:

$$\frac{E_{\rm GUT}}{10^{16}\ {\rm GeV}} \simeq 3.27\times r^{1/4} \tag{29}$$

which yields $E_{\rm GUT} \simeq 10^{16}$ GeV when $r \sim 10^{-2}$. Thus it is fair to say that for a rather broad range of $r$'s the cosmological constant in our model is at the grand unified scale.

Figure 6 shows how the running of the spectral index $\alpha = dn_s/d\ln(k)$ depends on the spectral index $n_s$. While the dependence of $n_s$ on $\xi_2$ and $\xi_4$ is by now familiar, we

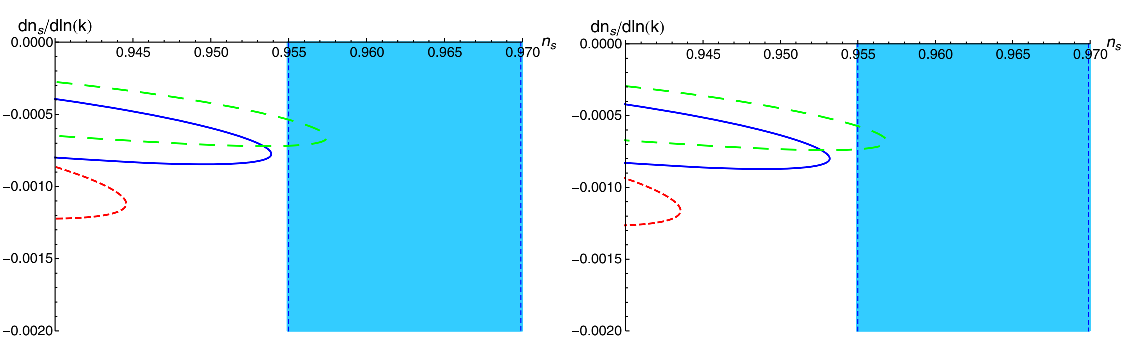


FIG. 6 (color online). Left panel: The running of the spectral index $\alpha = dn_s/d\ln(k)$ as a function of $n_s$ for $N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashed). Both the maximum value of $n_s$ and $\alpha$ are very weakly dependent on $\xi_4$. The typical value of the running is about $\alpha \sim -10^{-3}$, which is consistent with the current Planck data and it is about a factor of a few smaller in value than the central value favored by the Planck (and other) data, $\alpha \sim -0.003 \pm 0.007$. In this graph $\xi_4 = -0.02$. Right panel: The same as in the left panel but with $\xi_4 = -0.2$. The value of the running $|\alpha|$ grows slowly as $|\xi_4|$ increases.

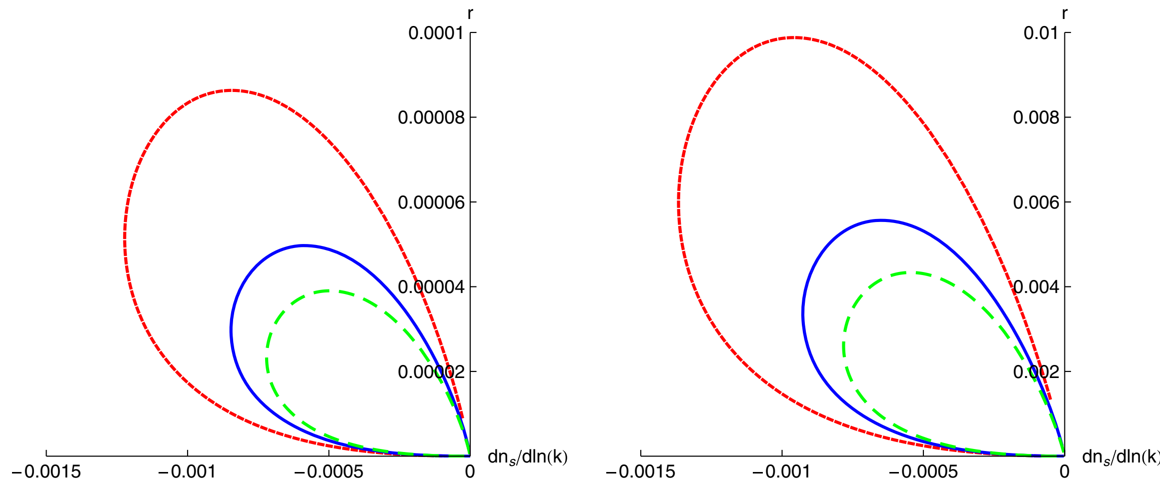


FIG. 7 (color online). Left panel: The tensor-to-scalar ratio $r$ versus the running of the spectral index $\alpha = dn_s/d\ln(k)$ for $N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashed). For these curves $\xi_4 = -0.02$. Right panel: The same as in the left panel but with $\xi_4 = -0.0002$. Note that as $N$ increases the values of $r$ and $\alpha$ decrease. A comparison of the left and right panels reveals that $r \propto 1/|\xi_4|$, which was already pointed out above. For wide ranges of values of $\xi_2$ and $\xi_4$ the values of $\alpha$ and $r$ are small enough to be consistent with the current observations.

see from the figure that $\alpha$ depends very weakly on $\xi_4$ (increasing slowly as $|\xi_4|$ increases), and for $N \simeq 60$ peaks at a value, $\alpha \sim -0.0008$, which is to be compared with the value observed by the Planck Collaboration, $\alpha = -0.003 \pm 0.007$ [25]. Therefore, the spectral index running in our model is consistent with the current data and it is potentially observable provided the error bars decrease by about a factor of 10. It is unlikely that such an accuracy in $\alpha$ can be attained by the near-future cosmic microwave background (CMB) missions. Therefore, observing a running different from zero in the near future would be tantamount to ruling out our model.

In Fig. 7 we show the dependence of $r$ on $\alpha = dn_s/d\ln(k)$ with $N$ as a parameter [$N = 50$ (short red dashes), $N = 60$ (blue solid) and $N = 65$ (long green dashed)]. In the left panel $\xi_4 = -0.02$, while in the right panel $\xi_4 = -0.0002$. The figure shows that, while the running $\alpha$ very weakly depends on $\xi_4$, $r$ is approximately inversely proportional to $\xi_4$. If future observations show that $r \sim 10^{-2}$ that would mean that $\xi_4$ would have to be small (e.g. $\xi_4 \sim -10^{-4}$) and that $n_s$ would have to be below about 0.950.

## V. DISCUSSION

In this paper we analyzed a novel inflationary model, where inflation is driven by a (Jordan frame) cosmological constant and a nonminimally coupled scalar field plays the role of the inflaton. The model is inspired by the recent work [12], where it was argued that, when viewed in the context of a nonminimally coupled scalar, a Jordan frame cosmological constant can be dynamically relaxed to zero (from the point of view of the Einstein frame observer). The model was analyzed in the slow-roll approximation, whose accuracy was (to a certain extent) tested. Our analysis shows that we can get the spectral index consistent with current observations, albeit the maximum value of the spectral index is about one standard deviation below the observed value; see Fig. 3. The value of the tensor-to-scalar ratio $r$ is typically small; see Fig. 4. Since $r$ is inversely proportional to the quartic nonminimal coupling $|\xi_4|$, it can be enlarged by decreasing the value of $|\xi_4|$ to obtain an $r$ that is observable by the planned CMB experiments, but the price to pay is a smaller $n_s$. The running of the spectral index $\alpha$ is negative (see Figs. 6 and 7), but the typical amplitude of the running is by about 1 order of magnitude below the sensitivity of the current CMB data.

It is worth noting that the value of the cosmological constant is to a large extent determined by the COBE normalization and it is of the order of the GUT scale, i.e. $\Lambda/(8\pi G_N) \sim E_{\rm GUT}^4 \sim 10^{16}$ GeV$^4$, and hence it can be nicely attributed to the value attained at a GUT transition (both from the Higgs potential as well as from the contributions generated by the particle masses). A second nice feature of the model at hand is that the model works for a large class of initial conditions. Namely, inflation naturally begins from a chaotic state, in which the total (averaged) energy in fluctuations scales as $\langle\rho\rangle \propto 1/a^4$ and during which the average field value is naturally small, $\langle\phi\rangle \ll M_P$ (this is so because the nonminimal coupling plays no role as the Ricci scalar is small, $\langle R\rangle \sim 0$). Sometime after the GUT transition the cosmological constant starts dominating the energy density, and one enters (slow-roll) inflation. Inflation is terminated as $\epsilon_E \sim 1$; at asymptotically late times $\epsilon_E \simeq 4/3$. Graceful exit and preheating is solved by suitably coupling the scalar field to matter; for details see Ref. [12]. Another advantage of our model is in that there is no need to fine-tune the potential to zero at the end of inflation, thus getting rid of one of the major fine-tuning problems of scalar inflationary models.

From our analysis the accuracy of the slow-roll approximation utilized in this paper is not completely clear. For that reason we are working on studying predictions of the inflationary model presented here by using exact solutions of the Friedmann equations (9)–(11). One hope is that, performing an exact analysis will allow us to obtain values for $n_s$ and $r$ that are closer to the (central) values favored by current observations, and thus get an even better agreement with the data.

## ACKNOWLEDGMENTS

This work is part of the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW). A. M. is funded by NEWFELPRO, an International Fellowship Mobility Programme for Experienced Researchers in Croatia and by the D-ITP.

[1] P. Jordan, *Schwerkraft und Weltall* (Friedrich Vieweg and Sohn, Braunschweig, 1955).
[2] C. Brans and R. H. Dicke, Mach's principle and a relativistic theory of gravitation, Phys. Rev. **124**, 925 (1961).
[3] P. G. Bergmann, Comments on the scalar tensor theory, Int. J. Theor. Phys. **1**, 25 (1968).
[4] R. Fakir and W. G. Unruh, Improvement on cosmological chaotic inflation through nonminimal coupling, Phys. Rev. D **41**, 1783 (1990).
[5] D. S. Salopek, J. R. Bond, and J. M. Bardeen, Designing density fluctuation spectra in inflation, Phys. Rev. D **40**, 1753 (1989).
[6] F. L. Bezrukov and M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, Phys. Lett. B **659**, 703 (2008).
[7] F. Bezrukov, The Higgs field as an inflaton, Classical Quantum Gravity **30**, 214001 (2013).
[8] T. Prokopec and J. Weenink, Naturalness in Higgs inflation in a frame independent formalism, arXiv:1403.3219.
[9] A. D. Dolgov, An attempt to get rid of the cosmological constant, in *The Very Early Universe: Proceedings of the Nuffield Workshop, Cambridge, 21 June–9 July, 1982*, edited by G. W. Gibbons, S. W. Hawking, and S. T. C. Siklos (Cambridge University Press, Cambridge, England, 1985), p. 449.
[10] L. H. Ford, Cosmological constant damping by unstable scalar fields, Phys. Rev. D **35**, 2339 (1987).
[11] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).
[12] D. Glavan and T. Prokopec, Nonminimal coupling and the cosmological constant problem, arXiv:1504.00842.
[13] D. Glavan, T. Prokopec, and D. C. van der Woude, Late-time quantum backreaction from inflationary fluctuations of a nonminimally coupled massless scalar, Phys. Rev. D **91**, 024014 (2015).
[14] D. Glavan, T. Prokopec, and A. Starobinsky (to be published); D. Glavan, T. Prokopec, and T. Takahashi (to be published).
[15] E. J. Copeland, M. Sami, and S. Tsujikawa, Dynamics of dark energy, Int. J. Mod. Phys. D **15**, 1753 (2006).
[16] P. J. E. Peebles and B. Ratra, The Cosmological constant and dark energy, Rev. Mod. Phys. **75**, 559 (2003).
[17] C. M. Will, The confrontation between general relativity and experiment, Living Rev. Relativity **9**, 3 (2006).
[18] J. Weenink and T. Prokopec, Gauge invariant cosmological perturbations for the nonminimally coupled inflaton field, Phys. Rev. D **82**, 123510 (2010).
[19] T. Prokopec and J. Weenink, Frame independent cosmological perturbations, J. Cosmol. Astropart. Phys. 09 (2013) 027.
[20] T. Prokopec and J. Weenink, Uniqueness of the gauge invariant action for cosmological perturbations, J. Cosmol. Astropart. Phys. 12 (2012) 031.
[21] D. H. Lyth and A. Riotto, Particle physics models of inflation and the cosmological density perturbation, Phys. Rep. **314**, 1 (1999).
[22] L. Boubekeur and D. H. Lyth, Hilltop inflation, J. Cosmol. Astropart. Phys. 07 (2005) 010.
[23] M. Joyce and T. Prokopec, Turning around the sphaleron bound: Electroweak baryogenesis in an alternative post-inflationary cosmology, Phys. Rev. D **57**, 6022 (1998).
[24] B. Ratra and P. J. E. Peebles, Cosmological consequences of a rolling homogeneous scalar field, Phys. Rev. D **37**, 3406 (1988).
[25] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XX. Constraints on inflation, arXiv:1502.02114.
[26] P. Andr *et al.* (PRISM Collaboration), PRISM (Polarized Radiation Imaging and Spectroscopy Mission): An extended white paper, J. Cosmol. Astropart. Phys. 02 (2014) 006.
[27] B. Spokoiny, Deflationary universe scenario, Phys. Lett. B **315**, 40 (1993).
[28] M. Joyce and T. Prokopec, Baryogenesis from 'electrogenesis' in a scalar field dominated epoch, J. High Energy Phys. 10 (2000) 030.