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# Graviton loop corrections to vacuum polarization in de Sitter in a general covariant gauge

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## Abstract

We evaluate the one-graviton loop contribution to the vacuum polarization on de Sitter background in a 1-parameter family of exact, de Sitter invariant gauges. Our result is computed using dimensional regularization and fully renormalized with Bogoliubov, Parasiuk, Hepp and Zimmerman counterterms, which must include a noninvariant owing to the time-ordered interactions. Because the graviton propagator engenders a physical breaking of de Sitter invariance two structure functions are needed to express the result. In addition to its relevance for the gauge issue this is the first time a covariant gauge graviton propagator has been used to compute a noncoincident loop. A number of identities are derived which should facilitate further graviton loop computations.

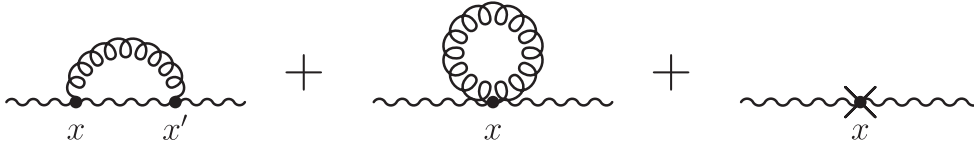
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## 1. Introduction

Observational evidence from cosmology has confronted theorists with three crucial questions:

- What drove primordial inflation?
- What is causing the current phase of acceleration?

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**Figure 1.** Feynman diagrams relevant to the one loop vacuum polarization from gravitons. Wavy lines are photons and curly lines are gravitons.

- How can we define cosmological observables which are gauge independent, infrared finite, and BPHZ renormalizable (Bogoliubov, Parasiuk, Hepp and Zimmerman [1–4]) in the sense of low energy effective field theory [5, 6]?

This paper concerns the third question, whose answer in flat space would be either the S-matrix (when only massive particles are present) or else inclusive rates and cross sections (when massless particles occur). Although a formal S-matrix can be constructed for massive scalars on de Sitter background [7], the causal structure of this geometry precludes local observers from measuring this quantity, so it cannot serve as an observable. To the tree order accuracy which has so far been resolved [8], the primordial scalar power spectrum  $\Delta_{\mathcal{R}}^2(k)$  seems to be well-represented by a 2-point quantum gravitational correlation function. This correlator can be given a gauge independent expression at tree order [9, 10], but no local extension of it can be gauge independent at higher orders [11]. Hence dependence upon the gravitational gauge has emerged as a central issue in loop corrections to cosmological observables [12–35].

The aim of this paper is to explore gauge dependence in the one graviton loop correction to the vacuum polarization  $i[\Pi^\mu]^\nu(x; x')$  on de Sitter background. This quantity, whose diagrammatic depiction is given in figure 1, explores the same issues of gauge dependence as  $\Delta_{\mathcal{R}}^2(k)$ , but is quite a bit simpler to compute and renormalize. It can be used to quantum-correct Maxwell’s equation [36],

$$\partial_\nu [\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)] + \int d^4x' [\Pi^\mu]^\nu(x; x') A_\nu(x') = J^\mu(x), \quad (1)$$

where  $A_\mu(x)$  is the vector potential,  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $J^\mu(x)$  is the current density. Quantum corrections to dynamical photons emerge from solutions with  $J^\mu(x) = 0$ , whereas quantum corrections to electrodynamic forces derive from the response to nonzero  $J^\mu(x)$ .

We define the graviton field operator  $h_{\mu\nu}(x)$  by subtracting the de Sitter background  $g_{\mu\nu}(x)$  from the full metric field operator  $\mathbf{g}_{\mu\nu}$ ,

$$\mathbf{g}_{\mu\nu}(x) \equiv g_{\mu\nu}(x) + \kappa h_{\mu\nu}(x), \quad \kappa^2 \equiv 16\pi G. \quad (2)$$

We shall compute the vacuum polarization in the 1-parameter family of exact covariant gauges that is a generalization of the de Donder condition,

$$g^{\rho\sigma} \left[ D_\rho h_{\sigma\mu} + \frac{b}{2} D_\mu h_{\rho\sigma} \right] = 0, \quad b > 2, \quad (3)$$

where  $D_\mu$  stands for the covariant derivative operator in de Sitter background, treating  $h_{\mu\nu}$  as a 2nd rank tensor. In gauges of the sort (3) the graviton propagator breaks up into a transverse-

traceless ‘spin two’ part, which does not depend upon  $b$ , and a  $b$ -dependent, ‘spin zero’ part [37, 38],<sup>5</sup>

$$i[\alpha\beta\Delta_{\gamma\delta}](x; x') = i[\alpha\beta\Delta_{\gamma\delta}^2](x; x') + i[\alpha\beta\Delta_{\gamma\delta}^0](x; x'). \quad (4)$$

The graviton contribution to the vacuum polarization on de Sitter must depend upon  $b$  because its flat space limit takes the form [39],

$$i[\mu\Pi_{\text{flat}}^\nu](x; x') = \frac{\kappa^2\Gamma\left(\frac{D}{2}\right)\Gamma\left(\frac{D}{2}-1\right)}{16(D-1)\pi^D} [C_2 + C_0(b)][\eta^{\mu\nu}\partial' \cdot \partial - \partial'^\mu\partial^\nu] \frac{1}{\Delta x^{2D-2}}. \quad (5)$$

Here  $\eta^{\mu\nu}$  is the Lorentz metric and  $\Delta x^2 \equiv \eta_{\mu\nu}(x - x')^\mu(x - x')^\nu$ . The gauge independent, spin two term contributes a coefficient which vanishes in  $D = 4$  spacetime dimensions,

$$C_2 = \frac{(D-4)(D-2)^2(D+1)(D+2)}{4(D-1)}, \quad (6)$$

and the gauge dependent, spin zero coefficient is,

$$C_0(b) = -\frac{1}{4}(D-2)^2 \left[ \left( \frac{Db-2}{b-2} \right)^2 - 4 \left( \frac{D-4}{D-2} \right) \left( \frac{Db-2}{b-2} \right) + \frac{2(D-4)^2}{(D-2)(D-1)} \right]. \quad (7)$$

Note that  $C_0(b)$  is negative semi-definite for  $D = 4$ .

It is commonplace to dismiss as unphysical any gauge dependent quantity like (5) [16]; however, that view is simplistic. The gauge-independent S-matrix of flat space arises from combining gauge-dependent Green’s functions, so the latter must possess legitimate physical information mixed in with artefacts of gauge fixing [17]. From the manner in which the flat S-matrix is constructed [40], one realizes that this physical information is distinguished by possessing momentum space poles on each external leg. In position space these poles correspond to secular growth when the Green’s function is integrated against tree order mode functions, as  $[\mu\Pi^\nu](x; x')$  necessarily is in the perturbative solution of (1) for dynamical photons. It is therefore reasonable to expect that the leading secular growth factors of de Sitter Green’s functions might be gauge independent [26]. That is what we seek to check for  $[\mu\Pi^\nu](x; x')$ . We will of course be able to compare our results for different values of  $b > 2$ . We can also compare with the result previously obtained [41] in a noncovariant gauge [42, 43].

Some comments on the physics are worthwhile before commencing this difficult computation. One might think an uncharged field like  $h_{\mu\nu}(x)$  is not capable of contributing to vacuum polarization but this ignores the role of electric and magnetic fields in transferring momentum. Virtual gravitons which interact with photons—either real or virtual ones—can alter this momentum. There is no change in how dynamical photons propagate on flat space background [39], essentially because virtual gravitons affect a single photon the same way throughout space and time. However, the interaction between charged particles on flat space background is slightly strengthened at short distances, as can be inferred from the gauge independent scattering amplitude [44]. One way to understand this effect is that the virtual photons which transfer momentum between nearby charges do not survive long enough to experience the full effect of buffeting by the longest wave length (and hence longest lived) virtual gravitons.

<sup>5</sup> Because covariant gauge propagators derive from both constrained and dynamical fields, the spin two part of (4)—whose zero components do not vanish—actually comes from both the  $\frac{1}{2}(D-3)D$  dynamical gravitons and  $D-1$  of the constrained fields. The spin zero part derives from the remaining constrained field.

On de Sitter background the vacuum polarization must of course show the same strengthening of force at short distance that is encoded in its flat space limit [39, 44]. However, it should also manifest new, secular effects arising from the inflationary production of gravitons. For example, a tree order photon redshifts as it propagates, whereas the continual replenishment of Hubble-scale gravitons should lead to a relative one loop enhancement, as momentum tends to flow from inflationary gravitons into the ever weaker photon. Similarly, the force between widely separated sources should be relatively enhanced because the highly infrared virtual photons which mediate the force are more likely to acquire momentum from, rather than lose it to, the constant pool of Hubble-scale gravitons. Both effects have been seen [45, 46] when the noncovariant gauge  $[\mu\Pi^\nu](x; x')$  [41] is used in equation (1), and it will be fascinating to learn what happens in our covariant gauge (3).

The one-loop effects of inflationary gravitons in the noncovariant gauge [42, 43] have been studied for a variety of other particles over the years:

- The graviton self-energy has been computed [47], but has not yet been used to quantum-correct the linearized Einstein equation. However, the Hartree approximation has been used to show that the Weyl curvature of dynamical gravitons experiences a secular enhancement [48].
- The self-energy has been computed for massless fermions [49] and, to first order in the mass, for massive fermions [50]. Quantum correcting the Dirac equation reveals a secular enhancement of the field strength of massless fermions [51, 52]. The result for the massive case has not yet been derived.
- The self-mass of massless, minimally coupled scalars has been computed [53]. However, quantum-correcting the Klein–Gordon equation shows no secular enhancement of the scalar field strength [54].
- A partial result has recently been obtained for the self-mass of massless, conformally coupled scalars [55], but it has so far not been used to quantum-correct the linearized field equation.

The fact that inflationary gravitons give secular enhancements to the field strengths of massless fermions, gravitons and photons, but not to massless, minimally coupled scalars, seems to be due to spin [56]. In each case energy and momentum of the physical particle under study redshifts as it propagates. If inflationary gravitons can only interact with the particle through its redshifting energy-momentum then the interaction cuts off rapidly and there can be no growing effect at late times. The presence of spin gives rise to a new interaction which does not cut off, so that particles with spin are scattered more and more as they propagate further through the sea of inflationary gravitons. Any other interaction which persists to late times should give rise to the same sort of secular enhancement, which is why it will be fascinating to see what happens to slightly massive fermions and to conformally coupled scalars.

This paper consists of five sections, of which the first is this introduction. In section 2 we give those reductions of the diagrams in figure 1 which do not depend upon the form of the graviton propagator. Section 3 derives the contribution from the spin two part of the graviton propagator, and section 4 computes the contribution from the spin zero part. Our conclusions are presented in section 5.

## 2. Preliminary reductions

The purpose of this section is to describe those parts of the computation which do not require a specific form for the graviton propagator. We begin by expressing the two primitive

diagrams of figure 1 in terms of propagators and vertices. We next point out that the form of these expressions lends itself to a simple representation for  $i[\Pi^\mu]^\nu(x; x')$  as the sum of two tensor differential operators acting on structure functions. We express the BPHZ counterterms (the third diagram of figure 1) directly in terms of their contributions to these structure functions. The section closes with the derivation of an important identity concerning the photon propagator which permits a great simplification of the more difficult, first diagram of figure 1.

### 2.1. Notation and primitive diagrams

Although we will cite original work, a unified treatment can be found in section 5.2 of [57]. We work on the spatially flat cosmological patch of de Sitter whose invariant element is,

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = a^2(-d\eta^2 + d\vec{x} \cdot d\vec{x}), \quad a(\eta) \equiv -\frac{1}{H\eta}, \quad (8)$$

where  $H$  is the Hubble constant. Note that  $g_{\mu\nu} = a^2\eta_{\mu\nu}$ , where  $\eta_{\mu\nu}$  is the Minkowski metric. We work in  $D$  spacetime dimensions to facilitate the use of dimensional regularization. Whereas the  $D - 1$  spatial coordinates  $-\infty < x^i < +\infty$  take their usual values, the conformal time  $\eta$  runs from  $\eta \rightarrow -\infty$  (the infinite past) to  $\eta \rightarrow 0^-$  (the infinite future).

The vacuum polarization  $i[\Pi^\mu]^\nu(x; x')$  is a bi-vector density which depends upon two spacetime points,  $x^\mu$  and  $x'^\mu$ . In representing functions such as propagators which depend upon these two points, we will make extensive use of the de Sitter length function,

$$y(x; x') \equiv a(\eta)a(\eta')H^2 \left[ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2 \right]. \quad (9)$$

We also need the de Sitter breaking product and ratio of the two scale factors,

$$u(x; x') \equiv \ln(aa'), \quad v(x; x') \equiv \ln\left(\frac{a}{a'}\right), \quad (10)$$

where  $a' \equiv a(\eta')$ . The de Sitter metric at  $x^\mu$  and  $x'^\mu$ , along with products of derivatives of  $y$  (without the  $i\epsilon$  term) and  $u$  furnish a convenient basis for representing bi-tensor functions of  $x^\mu$  and  $x'^\mu$ ,

$$\partial_\mu y, \quad \partial'_\nu y, \quad \partial_\mu \partial'_\nu y, \quad \partial_\mu u, \quad \partial'_\nu u. \quad (11)$$

(We do not require derivatives of  $v(x; x')$  because  $\partial_\mu v = +\partial_\mu u$  and  $\partial'_\mu v = -\partial'_\mu u$ .) It turns out that either taking covariant derivatives of any of the five derivatives (11), or contracting any two of them into one another, produces more elements of the basis [58, 59].

The Maxwell Lagrangian is  $\mathcal{L}_{\text{Max}} = -\frac{1}{4}F_{\mu\nu}F_{\rho\sigma}\mathbf{g}^{\mu\rho}\mathbf{g}^{\nu\sigma}\sqrt{-\mathbf{g}}$  and the only interactions we require descend from its second variation,

$$\frac{\delta^2 S_{\text{Max}}}{\delta A_\mu(x)\delta A_\rho(x')} = -\partial_\kappa \partial'_\lambda \left\{ \sqrt{-\mathbf{g}} \left[ \mathbf{g}^{\kappa\lambda} \mathbf{g}^{\mu\rho} - \mathbf{g}^{\kappa\rho} \mathbf{g}^{\lambda\mu} \right] \delta^D(x - x') \right\}. \quad (12)$$

The necessary vertex functions are obtained by expanding the full metric around the de Sitter background as in (2). We can take advantage of the conformal flatness of the de Sitter background ( $g_{\mu\nu} = a^2\eta_{\mu\nu}$ ) to extract the scale factors and express the result using the notation of previous work in flat space background [39] and on de Sitter, with the conformally rescaled graviton field, in the noncovariant gauge [41],

$$\sqrt{-g}(\mathbf{g}^{\kappa\lambda}\mathbf{g}^{\mu\rho} - \mathbf{g}^{\kappa\rho}\mathbf{g}^{\lambda\mu}) \equiv a^{D-4}(\eta^{\kappa\lambda}\eta^{\mu\rho} - \eta^{\kappa\rho}\eta^{\lambda\mu}) + \kappa a^{D-6}V^{\mu\rho\kappa\lambda\alpha\beta}h_{\alpha\beta} + \kappa^2 a^{D-8}U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta}h_{\alpha\beta}h_{\gamma\delta} + O(\kappa^3). \quad (13)$$

The tensor factors for the 3-point and 4-point vertices are [39, 41],

$$V^{\mu\rho\kappa\lambda\alpha\beta} = \eta^{\alpha\beta}\eta^{\kappa}[\lambda\eta^{\rho}]^{\mu} + 4\eta^{(\alpha}[\mu\eta^{\kappa}][\rho\eta^{\lambda]}^{\beta)}, \quad (14)$$

$$\begin{aligned} U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} = & \left[ \frac{1}{4}\eta^{\alpha\beta}\eta^{\gamma\delta} - \frac{1}{2}\eta^{\alpha(\gamma}\eta^{\delta)\beta} \right] \eta^{\kappa}[\lambda\eta^{\rho}]^{\mu} + \eta^{\alpha\beta}\eta^{\gamma}[\mu\eta^{\kappa}][\rho\eta^{\lambda}]^{\delta} \\ & + \eta^{\gamma\delta}\eta^{\alpha}[\mu\eta^{\kappa}][\rho\eta^{\lambda}]^{\beta} + \eta^{\kappa(\alpha}\eta^{\beta)}[\lambda\eta^{\rho}](\gamma\eta^{\delta})^{\mu} + \eta^{\kappa(\gamma}\eta^{\delta)}[\lambda\eta^{\rho}](\alpha\eta^{\beta})^{\mu} \\ & + \eta^{\kappa(\alpha}\eta^{\beta)}(\gamma\eta^{\delta})[\lambda\eta^{\rho}]^{\mu} + \eta^{\kappa}[\lambda\eta^{\rho}](\alpha\eta^{\beta})(\gamma\eta^{\delta})^{\mu} + \eta^{\kappa}[\lambda\eta^{\rho}](\gamma\eta^{\delta})(\alpha\eta^{\beta})^{\mu}. \end{aligned} \quad (15)$$

Parenthesized indices are symmetrized and indices enclosed in square brackets are anti-symmetrized, and both are normalized.

We can express the first two diagrams of figure 1 using the vertices (14) and (15), along with the graviton propagator  $i[\alpha\beta\Delta_{\gamma\delta}](x; x')$  and the photon propagator  $i[\rho\Delta_{\sigma}](x; x')$ . The leftmost diagram is formed from two 3-point vertices,

$$\begin{aligned} i[\mu\Pi_{3\text{pt}}^{\nu}](x; x') = & \partial_{\kappa}\partial'_{\theta} \left\{ i\kappa a^{D-6}V^{\mu\rho\kappa\lambda\alpha\beta} i[\alpha\beta\Delta_{\gamma\delta}](x; x') \right. \\ & \left. \times i\kappa a'^{D-6}V^{\nu\sigma\theta\phi\gamma\delta} \partial_{\lambda}\partial'_{\phi} i[\rho\Delta_{\sigma}](x; x') \right\}. \end{aligned} \quad (16)$$

The middle diagram contains a single 4-point vertex,

$$i[\mu\Pi_{4\text{pt}}^{\nu}](x; x') = \partial_{\kappa}\partial'_{\lambda} \left\{ -i\kappa^2 a^{D-8}U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta} i[\alpha\beta\Delta_{\gamma\delta}](x; x') \delta^D(x - x') \right\}. \quad (17)$$

## 2.2. Representing the tensor structure of $i[\mu\Pi^{\nu}](x; x')$

It can hardly escape notice that each of the two primitive diagrams (16) and (17) takes the form of one primed and one unprimed derivative contracted into a bi-tensor density<sup>6</sup>,

$$i[\mu\Pi^{\nu}](x; x') = \partial_{\rho}\partial'_{\sigma} \left\{ [\mu\rho T^{\nu\sigma}](x; x') \right\}, \quad (18)$$

which is anti-symmetric on each index group and symmetric under reflections,

$$[\mu\rho T^{\nu\sigma}](x; x') = -[\rho\mu T^{\nu\sigma}](x; x') = -[\mu\rho T^{\sigma\nu}](x; x') = +[\nu\sigma T^{\mu\rho}](x'; x). \quad (19)$$

(Note that because the vacuum polarization is a bi-vector *density*, there is no distinction between divergences formed with ordinary or covariant derivatives.) This form (18) and (19) is the necessary and sufficient condition for the vacuum polarization to be reflection symmetric and transverse,

$$\partial_{\mu}i[\mu\Pi^{\nu}](x; x') = 0 = \partial'_{\nu}i[\mu\Pi^{\nu}](x; x'). \quad (20)$$

The form (18) and (19) pertains to any background geometry, not just to de Sitter. When the background metric and the propagators possess isometries great restrictions on the form of the bi-tensor  $[\mu\rho T^{\nu\sigma}](x; x')$  can be imposed. For example, the flat space result (5) shows it can be reduced to the form of a single tensor times a scalar structure function,

<sup>6</sup> Note that our quantity  $[\mu\rho T^{\nu\sigma}](x; x')$  contains a factor of  $\sqrt{-g(x)} \times \sqrt{-g(x')} = (aa')^D$  that was not part of the symbol of the same name employed in [60, 61].

$$[\mu\rho T_{\text{flat}}^{\nu\sigma}](x; x') = (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) \times \mathcal{F}(x - x')^2. \quad (21)$$

This form (21) is obviously more economical than acting the derivatives in (18) and writing out all the resulting tensors. It is also more straightforward to employ in the quantum-corrected Maxwell equation (1). Not the least of this representation's advantages is that the one graviton loop contribution to the structure function  $\mathcal{F}(\Delta x^2)$  is only quadratically divergent, as opposed to the quartic divergences in the primitive diagram.

The de Sitter geometry has as many isometries as flat space so one might expect that a similar representation is possible in terms of just one structure function. However, a second structure function is required because the graviton propagator breaks de Sitter invariance [37, 38] down to just the cosmological symmetries of homogeneity and isotropy. We have chosen to represent the result in the same form which was first employed for the one loop contribution from scalar quantum electrodynamics on de Sitter background [62, 63],

$$[\mu\rho T^{\nu\sigma}](x; x') = (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}) \times F(x; x') + (\bar{\eta}^{\mu\nu}\bar{\eta}^{\rho\sigma} - \bar{\eta}^{\mu\sigma}\bar{\eta}^{\nu\rho}) \times G(x; x'). \quad (22)$$

Here and henceforth, an overlined tensor represents the suppression of temporal components, for example,  $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta_0^\mu \delta_0^\nu$ . Our representation (22) has a transparent physical interpretation [36] and is simpler to use even in cases for which a de Sitter invariant representation is possible [60]. There is also a straightforward procedure for passing from one representation to any other [61], so nothing is lost by employing the form (22).

The primitive diagrams (16) and (17) each permit one to read off a contribution to the tensor  $[\mu\rho T^{\nu\sigma}](x; x')$ , however, this contribution is not immediately in the form (22). The most general form consistent with homogeneity, isotropy and the symmetries (19) can be expressed as a linear combination of the basis tensors formed from differentiating  $y(x; x')$  and  $u(x; x')$  [60, 61],

$$\begin{aligned} \frac{[\mu\rho T^{\nu\sigma}]}{(aa')^D} &\equiv D^\mu D'^{[\nu} y D'^{\sigma]} D^\rho y \times f_1(y, u, v) + D^{[\mu} y D^{\rho]} D'^{[\nu} y D'^{\sigma]} y \times f_2(y, u, v) \\ &+ D^{[\mu} y D^{\rho]} D'^{[\nu} y D'^{\sigma]} u \times f_3(y, u, v) \\ &+ D^{[\mu} u D^{\rho]} D'^{[\nu} y D'^{\sigma]} y \times \tilde{f}_3(y, u, v) \\ &+ D^{[\mu} u D^{\rho]} D'^{[\nu} y D'^{\sigma]} u \times f_4(y, u, v) + D^{[\mu} y D^{\rho]} u D'^{[\nu} y D'^{\sigma]} u \times f_5(y, u, v), \end{aligned} \quad (23)$$

where we define the  $\tilde{f}_3(y, u, v) \equiv f_3(y, u, -v)$ . (Acting on scalars as in (23) we have  $D^\mu = g^{\mu\alpha}(x)\partial_\alpha$ ,  $D'^\nu = g'^{\nu\beta}(x')\partial'_\beta$ .) Only two combinations of the  $f_i(y, u, v)$  are independent. We call these the ‘master structure functions’  $\Phi(y, u, v)$  and  $\Psi(y, u, v)$ . Given the values of the various  $f_i(y, u, v)$ , one constructs the master structure functions according to table 1 [61].

One constructs the structure functions  $F(x; x')$  and  $G(x; x')$  of our representation according to the rules [61],<sup>7</sup>

$$F(x; x') = (aa')^{D-2} \times I[-2\Phi], \quad (24)$$

$$G(x; x') = (aa')^{D-2} \times I^2[(D-1)\Phi + y\partial_y\Phi + 2\partial_u\Phi + \Psi]. \quad (25)$$

<sup>7</sup> The formulae we give contain factors of  $(aa')^{D-2}$  which were mistakenly omitted from equations (38)–(39) of [61].



**Table 1.** The contribution to master structure functions  $\Phi(y, u, v) \equiv \sum_{i=1}^5 \Phi_i$  and  $\Psi(y, u, v) \equiv \sum_{i=1}^5 \Psi_i$  from each of the coefficient functions  $f_i(y, u, v)$  of equation (23). Note that  $\tilde{f}_3(y, u, v) \equiv f_3(y, u, -v)$ .

$i$	$4 H^{-4} \times \Phi_i(y, u, v)$	$4 H^{-4} \times \Psi_i(y, u, v)$
1	$2(D-1)f_1 - 2(2-y)\partial_y f_1$	$2(\partial_u^2 - \partial_v^2)f_1$
2	$D(2-y)f_2 + (4y-y^2)\partial_y f_2$	$(2-y)(\partial_u^2 - \partial_v^2)f_2$
3	$-(D-1)(f_3 + \tilde{f}_3) + (2-y)\partial_y(f_3 + \tilde{f}_3)$ $-2\partial_y(e^v f_3 + e^{-v} \tilde{f}_3)$	$2\partial_y \partial_u (e^v f_3 + e^{-v} \tilde{f}_3) + 2\partial_y \partial_v (-e^v f_3 + e^{-v} \tilde{f}_3)$ $-(\partial_u^2 - \partial_v^2)(f_3 + \tilde{f}_3)$
4	$\partial_y f_4$	$-(D-1)\partial_y f_4 + (2-y)\partial_y^2 f_4 - 2\partial_y \partial_u f_4$
5	$-Df_5 + 2(2-y)\partial_y f_5$ $-4 \cosh(v)\partial_y f_5$	$(D-1)f_5 - (D+1)(2-y)\partial_y f_5 - (4y-y^2)\partial_y^2 f_5 + 2\partial_u f_5$ $-[2(2-y) - 4 \cosh(v)]\partial_y \partial_u f_5 - 4\partial_y \partial_v [\sinh(v)f_5] - (\partial_u^2 - \partial_v^2)f_5$

$\infty$

Here and henceforth the symbol  $I[f]$  represents the indefinite integral with respect to  $y$ ,

$$I[f] \equiv \int^y dy' f(y', u, v). \quad (26)$$

Note from expressions (24) and (25) that the structure function  $G(x; x')$  is less divergent than  $F(x; x')$ . Whereas  $F(x; x')$  must possess quadratic ultraviolet divergences which are determined by the flat space limit,  $G(x; x')$  is only logarithmically divergent, and it vanishes in the flat space limit.

### 2.3. Renormalization

Einstein + Maxwell is not perturbatively renormalizable [64, 65], but we can still absorb the divergences using BPHZ counterterms of higher dimension in the standard sense of low energy effective field theory [5, 6, 44]. Our gauge (3) is covariant and we employ dimensional regularization, so it might be thought that only invariant counterterms are required. That turns out not to be true for three reasons:

- Our interactions are time-ordered, as is apparent from the  $i\epsilon$  prescription of the de Sitter length function (9) which enters propagators;
- Quantum gravitational interactions possess two derivatives, which allows the noninvariant ordering to contaminate expression (16); and
- The coincidence limit of the graviton propagator diverges in de Sitter background instead of vanishing like it does in flat space [39].

We are loath to change the ordering prescription because it is so embedded in the Schwinger–Keldysh formalism [66–74] that must be employed to make the nonlocal part of equation (1) both real and causal. The only alternative is to permit the same noninvariant counterterm that was necessary when using the noncovariant gauge [41],

$$\begin{aligned} \Delta\mathcal{L} = & C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} \\ & + \bar{C} H^2 F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} + \Delta C H^2 F_{ij} F_{kl} g^{ik} g^{jl} \sqrt{-g}. \end{aligned} \quad (27)$$

The term proportional to  $\bar{C}$  is the de Sitter specialization of three counterterms for which the two field strengths are contracted into  $g^{\mu\rho} g^{\nu\sigma} R$ ,  $g^{\mu\rho} R^{\nu\sigma}$  and  $R^{\mu\nu\rho\sigma}$ . The term proportional to  $\Delta C$  is the noninvariant counterterm by virtue of its indices running only over space. After some tedious tensor algebra one finds that  $\Delta\mathcal{L}$  contributes to  $F(x; x')$  and  $G(x; x')$  as [41],

$$\Delta F = 4a^{D-4} \left\{ [\bar{C} - (3D - 8)C_4] H^2 - \frac{C_4}{aa'} \partial^2 + \frac{(D - 4)C_4 H}{a} \partial_0 \right\} i\delta^D(x - x'), \quad (28)$$

$$\Delta G = 4a^{D-4} [\Delta C - (D - 6)C_4] H^2 i\delta^D(x - x'). \quad (29)$$

Although the  $\Delta C$  counterterm is strictly only required to renormalize divergences from the time-ordering we will also employ it to simplify  $G(x; x')$ . The coefficient  $C_4$  is fixed by the flat space limit [39] in terms of the quantities  $C_2$  and  $C_0(b)$  defined in expressions (6) and (7),

$$C_4 = \frac{\kappa^2 \mu^{D-4}}{64\pi^{\frac{D}{2}}} \frac{\Gamma\left(\frac{D}{2}\right)}{(D - 1)(D - 2)^2(D - 3)(D - 4)} [C_2 + C_0(b)]. \quad (30)$$

Note that the spin 2 contribution to  $C_4$  is finite by virtue of the factor of  $D - 4$  in expression (6).

Those accustomed to modern techniques of renormalization in covariant gauges sometimes find the appearance of noninvariant counterterms to be disconcerting. However, it is important to realize that they pose no problem of principle. The divergent part of the counterterm is of course fixed by the primitive divergences it is to remove, and the finite part can be determined to enforce physical symmetries. (In our case the focus on late times obviates the need for this as long as the finite part of the counterterm is assumed to be of order one.) The procedure is explained in older standard texts on quantum field theory, for example [75]. And it is important to recognize that many of the classic computations of quantum electrodynamics were in fact performed using noncovariant gauges [76].

#### 2.4. Reducing the photon propagator term

Our photon propagator is defined in exact Lorentz gauge,

$$D^\rho i[\rho \Delta_\sigma](x; x') = 0 = D'^\sigma i[\rho \Delta_\sigma](x; x'). \quad (31)$$

An early result [77] for this propagator contains a small error which was corrected 20 years later [78].<sup>8</sup> The most useful form for our purposes was obtained in a study of the graviton propagator [37],

$$i[\rho \Delta_\sigma](x; x') = -\frac{1}{2H^2} [\delta_\rho^\alpha \square - D^\alpha D_\rho] [\delta_\sigma^\beta \square' - D'^\beta D'_\sigma] [\partial_\alpha \partial'_\beta y \times i\Delta_{BBB}(x; x')]. \quad (32)$$

Here the *BBB*-type propagator is defined by inverting the 3rd power of the scalar d'Alembertian with mass  $M_S^2 = (D - 2)H^2$ ,

$$[\square - (D - 2)H^2] i\Delta_{BBB}(x; x') = i\Delta_{BB}(x; x'), \quad (33)$$

$$[\square - (D - 2)H^2] i\Delta_{BB}(x; x') = i\Delta_B(x; x'), \quad (34)$$

$$[\square - (D - 2)H^2] i\Delta_B(x; x') = \frac{i\delta^D(x - x')}{\sqrt{-g(x)}}. \quad (35)$$

Because  $M_S^2$  is strictly positive,  $i\Delta_{BBB}$ ,  $i\Delta_{BB}$  and  $i\Delta_B$  are all de Sitter invariant functions of  $y(x; x')$  [59] whose precise form can be found in [37]. We will give the expansion for  $i\Delta_B = B(y)$  in appendix A. We also give there the expansion for the propagator  $i\Delta_C(x; x') = C(y)$  of a scalar with mass  $M_S^2 = 2(D - 3)H^2$ . An important relation exists between them which we will use many times [83],

$$2C'(y) = (2 - y)B'(y) - B(y). \quad (36)$$

The differential operator used to construct the photon propagator (32) has the key property of transversality [37],

$$D^\rho [\delta_\rho^\alpha \square - D^\alpha D_\rho] = 0 = [\delta_\rho^\alpha \square - D^\alpha D_\rho] D_\alpha. \quad (37)$$

Exploiting this, the de Sitter invariance of  $i\Delta_{BBB}$  and relation (34), it is straightforward to show [37],

<sup>8</sup> There have been some recent false claims about this in the mathematical physics literature [79] so it is important to note that one really does need to employ the corrected propagator in computing standard things such as the effective potential of scalar quantum electrodynamics [80, 81]. Using the uncorrected propagator would not even recover the famous Coleman–Weinberg potential [82] in the flat space limit.

$$\begin{aligned} & \left[ \delta_\rho^\alpha \square - D^\alpha D_\rho \right] \left[ \delta_\sigma^\beta \square' - D'^\beta D'_\sigma \right] \left[ \partial_\alpha \partial'_\beta y \times i\Delta_{BBB}(x; x') \right] \\ &= \left[ \delta_\sigma^\beta \square' - D'^\beta D'_\sigma \right] \left[ \partial_\rho \partial'_\beta y \times i\Delta_{BB}(x; x') \right]. \end{aligned} \quad (38)$$

The next step is to act the remaining primed transverse projector,

$$\begin{aligned} & \left[ \delta_\rho^\alpha \square - D^\alpha D_\rho \right] \left[ \delta_\sigma^\beta \square' - D'^\beta D'_\sigma \right] \left[ \partial_\alpha \partial'_\beta y \times i\Delta_{BBB} \right] \\ &= \partial_\rho \partial'_\sigma y \times i\Delta_B - H^2 \partial_\rho \partial'_\sigma \left\{ (2 - y) i\Delta_{BB} - (D - 3) I [i\Delta_{BB}] \right\}. \end{aligned} \quad (39)$$

The final step is based on the fact that the 3-point vertex factors in expression (16) inherit an anti-symmetry from the Maxwell field strength,

$$V^{\mu\rho\kappa\lambda\alpha\beta} = -V^{\mu\lambda\kappa\rho\alpha\beta}, \quad V^{\nu\sigma\theta\phi\gamma\delta} = -V^{\nu\phi\theta\sigma\gamma\delta}. \quad (40)$$

This anti-symmetry can obviously be communicated to the differentiated photon propagator in (16),

$$D_\lambda D'_\phi i[\rho \Delta_\sigma](x; x') \longrightarrow D_{[\lambda} D'_{[\phi} i[\rho] \Delta_{\sigma]}](x; x'). \quad (41)$$

Here the single square brackets indicate anti-symmetrization under  $\lambda \leftrightarrow \rho$  while the double square brackets indicate anti-symmetrization under  $\phi \leftrightarrow \sigma$ . Because the double covariant derivative of a scalar is symmetric we can use (39) to conclude,

$$D_{[\lambda} D'_{[\phi} i[\rho] \Delta_{\sigma]}](x; x') = -\frac{1}{2H^2} \partial_{[\rho} \partial'_{[\sigma} y(x; x') \times \partial_{\lambda]} \partial'_{\phi]} B(y(x; x')). \quad (42)$$

Similar reductions of the original photon propagator structure functions down to the  $i\Delta_B$  propagator have also been noted in explicit two loop computations involving the very different interactions of scalar quantum electrodynamics [84, 85].

Identity (42) allows us to re-express the 3-point diagram (16) as,

$$i[\mu \Pi_{3pt}^\nu] = \frac{\kappa^2}{2H^2} \partial_\kappa \partial'_\theta \left\{ (aa')^{D-6} V^{\mu\rho\kappa\lambda\alpha\beta} i[\alpha_\beta \Delta_{\gamma\delta}] V^{\nu\sigma\theta\phi\gamma\delta} \partial_\rho \partial'_\sigma y \times \partial_\lambda \partial'_\phi B \right\}. \quad (43)$$

This is as far as we can get without exploiting the explicit form of the graviton propagator.

### 3. Spin 2 contributions

The purpose of this section is to work out the contributions to the renormalized structure functions from the spin two part of the graviton propagator. We begin by describing how this part of the propagator is expressed. We next work out the contributions from the 4-point diagram and the local part of the 3-point diagram which comes from the delta function part of the doubly differentiated photon propagator. A much more involved analysis is necessary to work out the nonlocal contributions to the 3-point diagram from the de Sitter breaking part of the propagator and from the de Sitter invariant part. The section closes by giving the combined results for  $F(x; x')$  and  $G(x; x')$ .

#### 3.1. Spin 2 part of the graviton propagator

The spin two part of the graviton propagator is transverse and traceless and takes a form analogous to the transverse photon propagator (32) [37, 38],

$$\begin{aligned} i[\Delta_{\mu\nu}^2]_{\rho\sigma}(x; x') &= \frac{2}{H^4} \left( \frac{D-2}{D-3} \right)^2 \times \mathbf{P}_{\mu\nu}^{\alpha\beta}(x) \times \mathbf{P}_{\rho\sigma}^{\gamma\delta}(x') \\ &\times [\partial_\alpha \partial'_\gamma y(x; x') \times \partial_\beta \partial'_\delta y(x; x') \times i\Delta_{AAAB}(x; x')]. \end{aligned} \quad (44)$$

Here  $\mathbf{P}_{\mu\nu}^{\alpha\beta}(x)$  is a 4th order differential operator which is transverse and traceless on both index groups [37, 38],

$$\begin{aligned} \mathbf{P}_{\mu\nu}^{\alpha\beta} &= \frac{1}{2} \left( \frac{D-3}{D-2} \right) \left\{ -\delta_{(\mu}^{\alpha} \delta_{\nu)}^{\beta} [\square - DH^2][\square - 2H^2] + 2D_{(\mu} [\square + H^2] \delta_{\nu)}^{(\alpha} D^{\beta)} \right. \\ &- \left( \frac{D-2}{D-1} \right) D_{(\mu} D_{\nu)} D^{(\alpha} D^{\beta)} + g_{\mu\nu} g^{\alpha\beta} \left[ \frac{\square^2}{D-1} - H^2 \square + 2H^4 \right] \\ &- \left. \frac{D_{(\mu} D_{\nu)}}{D-1} [\square + 2(D-1)H^2] g^{\alpha\beta} - \frac{g_{\mu\nu}}{D-1} [\square + 2(D-1)H^2] D^{(\alpha} D^{\beta)} \right\}. \end{aligned} \quad (45)$$

The spin two structure function  $i\Delta_{AAAB}(x; x')$  is constructed by inverting the kinetic operator for a massless scalar three times and inverting the kinetic operator for an  $M_S^2 = (D-2)H^2$  scalar twice. The order in which these inversions are performed is irrelevant but we find it convenient to alternate,

$$\square i\Delta_{AAAB}(x; x') = i\Delta_{AABB}(x; x'), \quad (46)$$

$$[\square - (D-2)H^2] i\Delta_{AABB}(x; x') = i\Delta_{AAB}(x; x'), \quad (47)$$

$$\square i\Delta_{AAB}(x; x') = i\Delta_{AB}(x; x'), \quad (48)$$

$$[\square - (D-2)H^2] i\Delta_{AB}(x; x') = i\Delta_A(x; x'), \quad (49)$$

$$\square i\Delta_A(x; x') = \frac{i\delta^D(x-x')}{\sqrt{-g}}. \quad (50)$$

Scalars with  $M_S^2 \leq 0$  inevitably break de Sitter invariance [59, 86–89] so each of the integrated propagators in relations (46) and (50) does as well<sup>9</sup>. That is, we can express each of the integrated propagators as the sum of a de Sitter invariant function of  $y(x; x')$  plus a de Sitter breaking term,

$$i\Delta_{AAAB}(x; x') = i\Delta_{AAAB}^{\text{inv}}(x; x') + i\Delta_{AAAB}^{\text{brk}}(x; x'). \quad (51)$$

Explicit forms for both terms can be found in [95]. We refer to the ‘de Sitter breaking’ and ‘de Sitter invariant’ part of the spin two graviton propagator as derived from  $i\Delta_{AAAB}^{\text{brk}}$  and  $i\Delta_{AAAB}^{\text{inv}}$ , respectively. The de Sitter breaking part is [95],

<sup>9</sup> The mathematical physics literature contains claims that there is no need for a de Sitter breaking part [90–92]. Morrison [93] has shown that constructions which purport to give a de Sitter invariant propagator differ from ours in two ways: (1) the propagator for a scalar with general mass-squared  $M_S^2$  must be considered as both de Sitter invariant and well defined for all  $M_S^2$ , except for simple poles at  $M_S^2 = -N(N+D-1)H^2$  with  $N = 0, 1, 2, \dots$ , and (2) it must be accepted that, for constructing the graviton propagator, an arbitrary constant can be added to equation (50). Both of these deviations are illegitimate, resulting in formal solutions to the propagator equation which are not true propagators in the sense of being the expectation values, in the presence of positive-normed states, of the time-ordered product of two graviton field operators [94].

**Table 2.** The five functions  $J_i(y)$  in relation (56) for the de Sitter invariant part of the spin two graviton propagator. The function  $J(y)$  is given in expression (55) and  $I[f]$  stands for the indefinite integral of  $f(y)$  as in (26).

$i$	$J_i(y)$
1	$\frac{1}{2H^2}[\frac{\square}{H^2} - (D-2)]J(y)$
2	$I\left[-(2-y)\left(\frac{\square}{H^2}J\right)' + (D-1)\frac{\square}{H^2}J + (D-2)(2-y)J' - (D-1)(D-2)J\right]$
3	$I^2\left[-\frac{1}{2}\frac{\square}{H^2}J + 2\left(\frac{D-2}{D-1}\right)J'' - \frac{1}{2}(D-2)(2-y)J' + \frac{1}{2}(D-2)(D-1)J\right]$
4	$\frac{2}{D-1}\frac{\square}{H^2}J(y) + I^2\left[-\frac{1}{2}(2-y)\left(\frac{\square}{H^2}J\right)' + \frac{1}{2}(D-2)\frac{\square}{H^2}J\right]$
5	$-\frac{2}{D-1}\frac{\square}{H^4}J(y) + \frac{1}{2}(2-y)^2\frac{\square}{H^2}J(y) - \frac{1}{2}(D-3)(2-y)I\left[\frac{\square}{H^2}J\right]$

$$i[\mu\nu\Delta_{\rho\sigma}^{2\text{brk}}](x; x') = (aa')^2 \left[ 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-1}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma} \right] \times k [\ln(4aa') + A_2], \quad (52)$$

where we recall that  $\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0 \delta_\nu^0$  and the constants  $k$  and  $A_2$  are,

$$k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}, \quad A_2 \equiv 2\psi\left(\frac{D-1}{2}\right) - 4 + \frac{1}{D-1}. \quad (53)$$

(Here  $\psi(z) \equiv \frac{d}{dz} \ln[\Gamma(z)]$  is the digamma function.) The de Sitter invariant part is much more complicated. The graviton analog of relation (38) is [95],

$$i[\mu\nu\Delta_{\rho\sigma}^{2\text{inv}}](x; x') = -H^{-4} \left( \frac{D-2}{D-3} \right) \times \mathbf{P}_{\rho\sigma}{}^{\gamma\delta}(x') \left[ \partial_\mu \partial'_\gamma y \right. \\ \left. \times \partial_\nu \partial'_\delta y \times [\square - (D-2)H^2] \square i\Delta_{AA\bar{B}\bar{B}}^{\text{inv}}(x; x') \right]. \quad (54)$$

In appendix B we show that acting the derivatives gives,

$$[\square - (D-2)H^2] \square i\Delta_{AA\bar{B}\bar{B}}^{\text{inv}}(x; x') = i\Delta_{AA\bar{B}}^{\text{inv}}(x; x') + \text{constant} \equiv J(y). \quad (55)$$

The analog of relation (39) is,

$$i[\mu\nu\Delta_{\rho\sigma}^{2\text{inv}}] = \partial_\mu \partial'_{(\rho} y \partial'_{\sigma)} \partial_\nu y J_1(y) + D_{(\mu} D'_{(\rho} [\partial_{\nu)} \partial'_{\sigma)} y J_2(y)] + D_\mu D_\nu D'_\rho D'_\sigma J_3(y) \\ + H^2 [g_{\mu\nu} D'_\rho D'_\sigma + g'_{\rho\sigma} D_\mu D_\nu] J_4(y) + H^4 g_{\mu\nu} g'_{\rho\sigma} J_5(y), \quad (56)$$

where table 2 gives the functions  $J_i(y)$ .

Relation (56) is remarkably similar to relation (39). In particular, the tensor structure of the first term is provided by second derivatives of  $y(x; x')$ , and the function  $J_1(y)$  is  $i\Delta_A^{\text{inv}}/2H^4$  plus a constant. The remaining terms are all gradients and/or traces. Unfortunately for us, neither gradients nor traces drop out of the vacuum polarization the way the analogous photon gradient terms did in expression (42). This leaves no alternative but to act the various covariant derivatives in expression (56) and express the result as a linear combination of the de Sitter invariant bi-tensors given in table 3,

**Table 3.** The de Sitter invariant basis bi-tensors in relation (57). As always, indices enclosed in parentheses are symmetrized.

$i$	$[\mu\nu T^i_{\rho\sigma}](x; x')$
1	$\partial_\mu \partial'_{(\rho} y(x; x') \partial'_{\sigma)} \partial_\nu y(x; x')$
2	$\partial_{(\mu} y(x; x') \partial_{\nu)} \partial'_{(\rho} y(x; x') \partial'_{\sigma)} y(x; x')$
3	$\partial_\mu y(x; x') \partial_\nu y(x; x') \partial'_{\rho} y(x; x') \partial'_{\sigma} y(x; x')$
4	$H^2 [g_{\mu\nu}(x) \partial'_{\rho} y(x; x') \partial'_{\sigma} y(x; x') + \partial_\mu y(x; x') \partial_\nu y(x; x') g_{\rho\sigma}(x')]$
5	$H^4 g_{\mu\nu}(x) g_{\rho\sigma}(x')$

**Table 4.** The coefficient functions  $K_i(y)$  expressed first as derivatives of the functions  $J_i(y)$  from table 2, then in terms of the function  $J(y)$  defined in expression (55).

$i$	$K_i(y)$	$K_i(y)$
1	$J_1 + J'_2 + 2J''_3$	$\frac{1}{2}(\frac{\square}{H^4}J) - (2-y)(\frac{\square}{H^2}J)'$
2	$J''_2 + 4J'''_3$	$+\frac{1}{2}(D-2)(\frac{\square}{H^2}J) + 4(\frac{D-2}{D-1})J''$ $-(2-y)(\frac{\square}{H^2}J)'' + (D-2)(\frac{\square}{H^2}J)'$ $+8(\frac{D-2}{D-1})J''' - (D-2)(2-y)J'' + (D-2)DJ'$
3	$J'''_3$	$-\frac{1}{2}(\frac{\square}{H^2}J)'' + 2(\frac{D-2}{D-1})J'''$
4	$-J'_2 + [(2-y)J'_3]'' + J''_4$	$-\frac{1}{2}(D-2)(2-y)J''' + \frac{1}{2}(D-2)(D+1)J''$
5	$-(2-y)J_2 + (2-y)[(2-y)J'_3]'$ $+2(2-y)J'_4 + J_5$	$\frac{2}{D-1}(\frac{\square}{H^2}J)'' + 2(\frac{D-2}{D-1})(2-y)J''' - \frac{2(D-2)(D+1)}{D-1}J''$ $-\frac{2}{D-1}(\frac{\square}{H^4}J) + \frac{4}{D-1}(2-y)(\frac{\square}{H^2}J)'$ $-2(\frac{D-2}{D-1})(\frac{\square}{H^2}J) + 8(\frac{D-2}{D-1})J''$

$$i[\mu\nu\Delta_{\rho\sigma}^{2\text{inv}}](x; x') = \sum_{i=1}^5 [\mu\nu T^i_{\rho\sigma}](x; x') \times K_i(y). \quad (57)$$

The coefficient functions  $K_i(y)$  are given in table 4. It is worth noting that tracelessness implies two relations among the  $K_i(y)$ ,

$$4K_1 + (4y - y^2)K_4 + DK_5 = 0, \quad (58)$$

$$-K_1 + (2-y)K_2 + (4y - y^2)K_3 + DK_4 = 0. \quad (59)$$

### 3.2. The 4-point diagram

The simplest diagram is the middle one of figure 1. The spin two contribution to it comes from substituting the spin two part of the graviton propagator into expression (17). Because the spin two part of the graviton propagator is traceless at each point, we can drop terms in the 4-point vertex  $U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}$  which contain either  $\eta^{\alpha\beta}$  or  $\eta^{\gamma\delta}$ . Many of the other terms are also related when contracted into  $i[\alpha\beta\Delta_{\gamma\delta}](x; x)$  so that there are only three distinct contributions,

$$\begin{aligned} i\left[{}^{\mu}\Pi_{4\text{pt}2}^{\nu}\right](x; x') = \partial_{\rho}\partial'_{\sigma}\Bigg\{ & -\kappa^2 a^{D-4} i\delta^D(x-x')\left[-\frac{1}{2}\eta^{\mu[\nu}\eta^{\sigma]\rho}i\left[{}^{\alpha\beta}\Delta_{\alpha\beta}^2\right] \right. \\ & \left. + 2a^4 i\left[{}^{\mu[\nu}\Delta^{2\sigma]\rho}\right] + 4a^2 \eta^{[\mu}i\left[{}^{\rho]}_{\alpha}\Delta^{2\sigma]\alpha}\right]\right\}. \end{aligned} \quad (60)$$

Recall that indices enclosed in square brackets are anti-symmetrized, and that the double square brackets in the final term of (60) serves to distinguish the anti-symmetrization on  $\nu \leftrightarrow \sigma$  from that  $\mu \leftrightarrow \rho$ .

The coincidence limit of the graviton propagator takes the form [95],

$$\begin{aligned} i\left[{}^{\alpha\beta}\Delta_{\gamma\delta}^2\right](x; x) = & \left[\delta^{\alpha}_{\gamma}\delta^{\beta}_{\delta} + \delta^{\alpha}_{\delta}\delta^{\beta}_{\gamma} - \frac{2}{D}\eta^{\alpha\beta}\eta_{\gamma\delta}\right] \times i\Delta_1 \\ & + \left[\bar{\delta}^{\alpha}_{\gamma}\bar{\delta}^{\beta}_{\delta} + \bar{\delta}^{\alpha}_{\delta}\bar{\delta}^{\beta}_{\gamma} - \frac{2}{D-1}\bar{\eta}^{\alpha\beta}\bar{\eta}_{\gamma\delta}\right] \times i\Delta_2(x). \end{aligned} \quad (61)$$

Hence the three terms on the second line of (60) are,

$$\begin{aligned} -\frac{1}{2}\eta^{\mu[\nu}\eta^{\sigma]\rho}i\left[{}^{\alpha\beta}\Delta_{\alpha\beta}^2\right] = & -\frac{1}{2}[(D-1)(D-2)i\Delta_1 \\ & + (D-2)(D+1)i\Delta_2(x)] \times \eta^{\mu[\nu}\eta^{\sigma]\rho}, \end{aligned} \quad (62)$$

$$\begin{aligned} 2a^4 i\left[{}^{\mu[\nu}\Delta^{2\sigma]\rho}\right] = & -2\left(\frac{D+2}{D}\right)i\Delta_1 \times \eta^{\mu[\nu}\eta^{\sigma]\rho} \\ & - 2\left(\frac{D+1}{D-1}\right)i\Delta_2(x) \times \bar{\eta}^{\mu[\nu}\bar{\eta}^{\sigma]\rho}, \end{aligned} \quad (63)$$

$$\begin{aligned} 4a^2 \eta^{[\mu}i\left[{}^{\rho]}_{\alpha}\Delta^{2\sigma]\alpha}\right] = & \left[\frac{4(D-1)(D+2)}{D}i\Delta_1 + \frac{2(D-2)(D+1)}{D-1}i\Delta_2(x)\right] \\ & \times \eta^{\mu[\nu}\eta^{\sigma]\rho} + \frac{2(D-2)(D+1)}{D-1}i\Delta_2(x) \times \bar{\eta}^{\mu[\nu}\bar{\eta}^{\sigma]\rho}. \end{aligned} \quad (64)$$

In deriving the last of these relations we have used,

$$\eta^{[\mu}i\left[{}^{\nu\sigma]\rho}\right] = \frac{1}{2}\eta^{\mu[\nu}\eta^{\sigma]\rho} + \frac{1}{2}\bar{\eta}^{\mu[\nu}\bar{\eta}^{\sigma]\rho}. \quad (65)$$

Expressions (60) and (62) and (64) imply that the 4-point contribution already takes the form described in section 2.2,

$$i\left[{}^{\mu}\Pi_{4\text{pt}2}^{\nu}\right](x; x') = \partial_{\rho}\partial'_{\sigma}\left\{2\eta^{\mu[\nu}\eta^{\sigma]\rho} \times F_{2a}(x; x') + 2\bar{\eta}^{\mu[\nu}\bar{\eta}^{\sigma]\rho} \times G_{2a}(x; x')\right\}. \quad (66)$$

The two structure functions are,

$$\begin{aligned} F_{2a}(x; x') = \kappa^2 a^{D-4} i\delta^D(x-x') \Bigg\{ & \frac{(D+2)(D-1)(D-4)}{4D}i\Delta_1 \\ & - \frac{(D+2)(D-2)}{D}i\Delta_1 + \frac{(D-5)(D-2)(D+1)}{4(D-1)}i\Delta_2(x) \Bigg\}, \end{aligned} \quad (67)$$

$$G_{2a}(x; x') = \kappa^2 a^{D-4} i\delta^D(x-x') \left\{ -\frac{(D-3)(D+1)}{D-1}i\Delta_2(x) \right\}. \quad (68)$$

From the previous section we find the constant  $i\Delta_1$  and the time dependent function  $i\Delta_2(x)$  to be,



$$i\Delta_1 = \frac{4(D-2)D(D+1)}{(D-1)} H^4 J''(0), \quad (69)$$

$$i\Delta_2 = k[2 \ln(2a) + A_2] \equiv k[2 \ln(a) + \bar{A}_2]. \quad (70)$$

Comparison with expressions (28) and (29) suggests that we choose the ‘2a’ contributions to the  $\bar{C}$  and  $\Delta C$  counterterms to be,

$$\bar{C}_{2a} = -\frac{\kappa^2 i\Delta_1}{H^2} \frac{(D+2)(D^2-9D+12)}{16D} - \frac{\kappa^2 k\bar{A}_2}{H^2} \frac{(D-5)(D-2)(D+1)}{16(D-1)}, \quad (71)$$

$$\Delta C_{2a} = \frac{\kappa^2 k\bar{A}_2}{H^2} \frac{(D-3)(D+1)}{4(D-1)}. \quad (72)$$

The renormalized 4-point contributions to the structure functions are,

$$F_{2a}^{\text{ren}}(x; x') = -\frac{5\kappa^2 H^2}{24\pi^2} \ln(a) i\delta^4(x - x'), \quad (73)$$

$$G_{2a}^{\text{ren}}(x; x') = -\frac{5\kappa^2 H^2}{12\pi^2} \ln(a) i\delta^4(x - x'). \quad (74)$$

### 3.3. General form of the 3-3 diagram

The 3-3 diagram is the leftmost part of figure 1 and is by far the most difficult to evaluate. The first step is to substitute the 3-point vertex factor (14) into expression (43). Because the spin two part of the graviton propagator is traceless we retain only the second terms,

$$V^{\mu\rho\kappa\lambda\alpha\beta} \longrightarrow 4\eta^\alpha[\mu\eta^\kappa][\rho\eta^\lambda]^\beta, \quad V^{\nu\sigma\theta\phi\gamma\delta} \longrightarrow 4\eta^\gamma[\nu\eta^\theta][\sigma\eta^\phi]^\delta. \quad (75)$$

It is desirable to expand out the anti-symmetrizations over  $\rho \leftrightarrow \lambda$  and  $\sigma \leftrightarrow \phi$ , and also to re-label the external derivatives from  $\partial_\kappa \partial'_\theta$  to  $\partial_\rho \partial'_\sigma$ ,

$$\begin{aligned} i[\mu\Pi_{3\text{pt}2}^\nu](x; x') = \partial_\rho \partial'_\sigma \Big\{ & \frac{2\kappa^2 (aa')^D}{H^2} i[\mu_\alpha \Delta^{2[\nu}_{\beta]}(x; x') [D^{\rho 1} D'^{\sigma 1}] y \times D^\alpha D'^\beta B \\ & - D^\alpha D'^{\sigma 1}] y \times D^{\rho 1} D'^\beta B - D^{\rho 1} D'^\beta y \times D^\alpha D'^{\sigma 1}] B \\ & + D^\alpha D'^\beta y \times D^{\rho 1} D'^{\sigma 1}] B \Big\}. \end{aligned} \quad (76)$$

In expression (76) we have employed single and double square brackets to distinguish the anti-symmetrizations over  $\mu \leftrightarrow \rho$  (single) from that over  $\nu \leftrightarrow \sigma$  (double).

Recall from section 2.2 that the structure functions can be computed directly from the portion of (76) within the curly brackets. It is useful to give each of the four terms its own symbol,

$$[\mu_\rho T_2^{\nu\sigma}](x; x') \equiv +\frac{2\kappa^2 (aa')^D}{H^2} \times i[\mu_\alpha \Delta^{2[\nu}_{\beta]} \times D^{\rho 1} D'^{\sigma 1}] y \times D^\alpha D'^\beta B, \quad (77)$$

$$[\mu_\rho T_2^{\nu\sigma}](x; x') \equiv -\frac{2\kappa^2 (aa')^D}{H^2} \times i[\mu_\alpha \Delta^{2[\nu}_{\beta]} \times D^\alpha D'^{\sigma 1}] y \times D^{\rho 1} D'^\beta B, \quad (78)$$

$$[\mu_\rho T_2^{\nu\sigma}](x; x') \equiv -\frac{2\kappa^2 (aa')^D}{H^2} \times i[\mu_\alpha \Delta^{2[\nu}_{\beta]} \times D^{\rho 1} D'^\beta y \times D^\alpha D'^{\sigma 1}] B, \quad (79)$$

$$\left[ \begin{smallmatrix} \mu\rho \\ 4 \end{smallmatrix} T_2^{\nu\sigma} \right](x; x') \equiv + \frac{2\kappa^2 (aa')^D}{H^2} \times i \left[ \begin{smallmatrix} \mu \\ \alpha \end{smallmatrix} \Delta^2 \left[ \begin{smallmatrix} \nu \\ \beta \end{smallmatrix} \right] \right] \times D^\alpha D'^\beta y \times D^{\rho 1} D'^{\sigma 1} B. \quad (80)$$

Our notation is that the left hand subscript denotes which of the four permutations is intended, while the right hand subscript ‘2’ indicates that these are all contributions from the spin two part of the graviton propagator.

The next step is to act the derivatives on  $B(y)$  [96],

$$D^\alpha D'^\beta B(y) = \frac{\delta_0^\alpha \delta_0^\beta i \delta^D(x - x')}{a^{D+2}} + D^\alpha y \ D'^\beta y \times B''(y) + D^\alpha D'^\beta y \times B'(y). \quad (81)$$

It is natural to distinguish the ‘2b’ local delta function terms (section 3.4) from the nonlocal terms. We further distinguish the (nonlocal terms between the ‘2c’ ones from the de Sitter breaking part of the graviton propagator (section 3.5) and the ‘2d’ ones from the de Sitter invariant part (section 3.6).

### 3.4. Local contributions from the 3-3 diagram

The ‘2b’ contributions come from the replacement in expressions (77) and (80),<sup>10</sup>

$$D^\rho D'^\sigma y \times D^\alpha D'^\beta B \longrightarrow - \frac{2H^2}{a^{D+4}} \eta^{\rho\sigma} \delta_0^\alpha \delta_0^\beta i \delta^D(x - x'). \quad (82)$$

The delta function re-introduces the coincident graviton propagator expression (61). With replacement (82) the four contractions in (76) give,

$$\left[ \begin{smallmatrix} \mu\rho \\ 1 \end{smallmatrix} T_{2b}^{\nu\sigma} \right] = 4\kappa^2 a^{D-4} i \delta^D(x - x') \left\{ \eta^{\mu[\nu} \eta^{\sigma]\rho} i \Delta_1 - \delta_0^{[\mu} \delta_0^{[\nu} \eta^{\rho]\sigma]} \left( \frac{D-2}{D} \right) i \Delta_1 \right\}, \quad (83)$$

$$\left[ \begin{smallmatrix} \mu\rho \\ 2 \end{smallmatrix} T_{2b}^{\nu\sigma} \right] = 4\kappa^2 a^{D-4} i \delta^D(x - x') \left\{ \delta_0^{[\mu} \delta_0^{[\nu} \eta^{\rho]\sigma]} \left( \frac{D+2}{D} \right) i \Delta_1 \right\}, \quad (84)$$

$$\left[ \begin{smallmatrix} \mu\rho \\ 3 \end{smallmatrix} T_{2b}^{\nu\sigma} \right] = 4\kappa^2 a^{D-4} i \delta^D(x - x') \left\{ \delta_0^{[\mu} \delta_0^{[\nu} \eta^{\rho]\sigma]} \left( \frac{D+2}{D} \right) i \Delta_1 \right\}, \quad (85)$$

$$\left[ \begin{smallmatrix} \mu\rho \\ 4 \end{smallmatrix} T_{2b}^{\nu\sigma} \right] = 4\kappa^2 a^{D-4} i \delta^D(x - x') \times \left\{ \delta_0^{[\mu} \delta_0^{[\nu} \eta^{\rho]\sigma]} \left[ - \frac{(D+2)(D-1)}{D} i \Delta_1 - \frac{(D+1)(D-2)}{D-1} i \Delta_2 \right] \right\}. \quad (86)$$

We can read off the structure functions using the relation,

$$\delta_0^{[\mu} \delta_0^{[\nu} \eta^{\rho]\sigma]} = - \frac{1}{2} \eta^{\mu[\nu} \eta^{\sigma]\rho} + \frac{1}{2} \bar{\eta}^{\mu[\nu} \bar{\eta}^{\sigma]\rho}. \quad (87)$$

<sup>10</sup> Note that the analytic continuation of Barvinsky and Vilkovisky [97] would give a covariant result instead,

$$D^\rho D'^\sigma y \times D^\alpha D'^\beta B \longrightarrow + \frac{2H^2}{D} g^{\rho\sigma} g^{\alpha\beta} \frac{i \delta^D(x - x')}{\sqrt{-g}}.$$

Because physics is ultimately based on the Minkowski signature of our expression (9) we feel it is safer to work with (82), even though it entails a noncovariant counterterm. Some of the problems which can arise from analytic continuation are explained in [94].

The results are,

$$F_{2b} = \kappa^2 a^{D-4} i \delta^D(x - x') \left\{ \frac{(D+4)(D-2)}{D} i \Delta_1 + \frac{(D+1)(D-2)}{(D-1)} i \Delta_2(x) \right\}, \quad (88)$$

$$G_{2b} = \kappa^2 a^{D-4} i \delta^D(x - x') \left\{ -\frac{(D^2-8)}{D} i \Delta_1 - \frac{(D+1)(D-2)}{(D-1)} i \Delta_2(x) \right\}. \quad (89)$$

The constant  $i \Delta_1$  is divergent, which is why this computation requires a noninvariant counterterm. Comparing expressions (88) and (89) with (28) and (29) suggests that we take the ‘2b’ contributions to  $\bar{C}$  and  $\Delta C$  as,

$$\bar{C}_{2b} = -\frac{\kappa^2 i \Delta_1 (D+4)(D-2)}{H^2 4D} - \frac{\kappa^2 k \bar{A}_2 (D+1)(D-2)}{H^2 4(D-1)}, \quad (90)$$

$$\Delta C_{2b} = \frac{\kappa^2 i \Delta_1 (D^2-8)}{H^2 4D} + \frac{\kappa^2 k \bar{A}_2 (D+1)(D-2)}{H^2 4(D-1)}. \quad (91)$$

Our final results for the renormalized structure functions are,

$$F_{2b}^{\text{ren}}(x; x') = \frac{5\kappa^2 H^2}{6\pi^2} \ln(a) i \delta^4(x - x'), \quad (92)$$

$$G_{2b}^{\text{ren}}(x; x') = -\frac{5\kappa^2 H^2}{6\pi^2} \ln(a) i \delta^4(x - x'). \quad (93)$$

Note that the noncovariant divergence in expression (89) is exactly cancelled by the noncovariant counterterm (91), so there are no spurious finite terms.

### 3.5. Nonlocal de Sitter breaking 3-3 contributions

The ‘2c’ contributions derive from making the following replacements in expressions (77) and (80),

$$i \left[ {}^\mu_\alpha \Delta^{2\nu}_\beta \right](x; x') \longrightarrow \left[ \bar{\eta}^{\mu\nu} \bar{\eta}_{\alpha\beta} + \bar{\delta}_\beta^\mu \bar{\delta}_\alpha^\nu - \frac{2}{D-1} \bar{\delta}_\alpha^\mu \bar{\delta}_\beta^\nu \right] \times k [\ln(4aa') + A_2], \quad (94)$$

$$D^\alpha D'^\beta B(y) \longrightarrow D^\alpha D'^\beta y \times B'(y) + D^\alpha y \ D'^\beta y \times B''(y). \quad (95)$$

Recall that an overlined tensor indicates the suppression of its temporal components,  $\bar{\delta}_\alpha^\mu \equiv \delta_\alpha^\mu - \delta_0^\mu \delta_\alpha^0$ . These overlined tensors in expression (94) can all be represented using the standard basis described in section 2.2 [95, 98],

$$\bar{\eta}^{\mu\nu} = \frac{aa'}{2H^2} \{ -D^\mu D'^\nu y + D^\mu y \ D'^\nu u + D^\mu u \ D'^\nu y + (2-y) D^\mu u \ D'^\nu u \}. \quad (96)$$

The first step is to substitute relations (94) and (96) into the standard permutations (77) and (80) and read off the contributions to the five coefficient functions  $f_i(y, u, v)$  which were defined in expression (23). Relation (95) contains a term proportional to  $B'(y)$ , whose contributions to each  $f_i$  is given in table 5. The  $B''(y)$  contributions are listed in table 6.

The next step is to compute the master structure functions  $\Phi(y, u, v)$  and  $\Psi(y, u, v)$  according to the rules which were originally derived in [61] and which are summarized in table 1 of section 2.2. The intermediate results can be substantially simplified using the  $B$ -type propagator equation,

**Table 5.** The contribution proportional to  $B'(y)$  from each permutation type to the coefficient functions  $f_i(y, u, v)$  which were defined in expression (23). Each contribution should be multiplied by  $B'(y) \times \frac{4\kappa^2 k}{H^2} [u + 2 \ln(2) + A_2]$ .

$f_i$	$[\frac{\mu\rho}{1}T_{2c}^{\nu\sigma}]$	$[\frac{\mu\rho}{2+3}T_{2c}^{\nu\sigma}]$	$[\frac{\mu\rho}{4}T_{2c}^{\nu\sigma}]$	$[\frac{\mu\rho}{5}T_{2c}^{\nu\sigma}]$
$f_1$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$\left(\frac{D+1}{D-1}\right)$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$\frac{(D-3)(D+1)}{D-1}$
$f_2$	0	0	0	0
$f_3$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$\left(\frac{D+1}{D-1}\right)$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$-\frac{(D-3)(D+1)}{D-1}$
$\tilde{f}_3$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$\left(\frac{D+1}{D-1}\right)$	$-\frac{(D-2)(D+1)}{2(D-1)}$	$-\frac{(D-3)(D+1)}{D-1}$
$f_4$	$-\frac{(D-2)(D+1)}{2(D-1)}(2-y)$	$2(2-y) + \frac{8}{D-1} \cosh(v)$	$-\frac{(D-2)(D+1)}{2(D-1)}(2-y)$	$-\frac{(D-3)D}{D-1}(2-y) + \frac{8}{D-1} \cosh(v)$
$f_5$	0	$\left(\frac{D-3}{D-1}\right)$	0	$\left(\frac{D-3}{D-1}\right)$

$$(4y - y^2)B''(y) + D(2 - y)B'(y) - (D - 2)B(y) = 0. \quad (97)$$

The final result for the ‘2c’ contribution to  $\Phi(y, u, v)$  is,

$$\begin{aligned} \Phi_{2c}(y, u, v) = & \kappa^2 H^2 k \left( \frac{D}{2} - 1 \right) [u + 2 \ln(2) + A_2] \\ & \times \{ 4[(2 - y)B]''' \cosh(v) - 8B''' - (D - 3)[(2 - y)B]'' \}. \end{aligned} \quad (98)$$

The ‘2c’ contribution to  $\Psi(y, u, v)$  is more complicated because it involves derivatives with respect to  $u$ ,

$$\begin{aligned} \Psi_{2c}(y, u, v) = & \frac{\kappa^2 H^2 k}{D - 1} \left\{ 2(D^2 - 3D + 4)[-(2 - y)B \cosh(v) + 2B]''' \right. \\ & + (D - 3)(D - 2)(D - 1) \\ & \times [(2 - y)B]'' \} + 2\kappa^2 H^2 k \left( \frac{D - 2}{D - 1} \right) [u + 2 \ln(2) + A_2] \\ & \times \{ [8B' + (D - 1)(2 - y)B]''' \cosh(v) - 4[(2 - y)B'']'' \}. \end{aligned} \quad (99)$$

The structure function  $F(x; x')$  is constructed by integrating  $\Phi(y, u, v)$  with respect to  $y$  according to the expression (24). This is simple to do because each term in (98) is either a 2nd or 3rd derivative with respect to  $y$ ,

$$\begin{aligned} F_{2c}(x; x') = & (D - 2)\kappa^2 H^2 k (aa')^{D-2} [u + 2 \ln(2) + A_2] \\ & \times \{ -4[(2 - y)B]'' \cosh(v) + 8B'' + (D - 3)[(2 - y)B]' \}. \end{aligned} \quad (100)$$

The other structure function follows from substituting (98) and (99) in (25),

$$\begin{aligned} G_{2c}(x; x') = & \frac{2(D - 3)D}{D - 1} \kappa^2 H^2 k (aa')^{D-2} \{ [(2 - y)B]' \cosh(v) - 2B' \} \\ & + \left( \frac{D - 2}{D - 1} \right) \kappa^2 H^2 k (aa')^{D-2} \\ & \times [u + 2 \ln(2) + A_2] \{ [16B - 4(D - 1)yB]'' \cosh(v) \\ & - [16B + 4(D - 3)yB]'' - 4(D - 3)^2 B' + (D - 3)(D - 1)(yB)' \}. \end{aligned} \quad (101)$$

These results (100) and (101) are valid in  $D$  dimensions. Because the quantum-corrected Maxwell equation (1) involves integrals of  $F(x; x')$  and  $G(x; x')$  with respect to  $x'^\mu$ , we can

**Table 6.** The contribution proportional to  $B''(y)$  from each permutation type to the coefficient functions  $f_i(y, u, v)$  which were defined in expression (23). Each contribution should be multiplied by  $B''(y) \times \frac{4\kappa^2 k}{H^2} [u + 2 \ln(2) + A_2]$ .

$f_i$	$[\stackrel{\mu\rho}{1}T_{2c}^{\nu\sigma}]$	$[\stackrel{\mu\rho}{2+3}T_{2c}^{\nu\sigma}]$	$[\stackrel{\mu\rho}{4}T_{2c}^{\nu\sigma}]$	$[\stackrel{\mu\rho}{2c}T_{2c}^{\nu\sigma}]$
$f_1$	$(2-y)$	0	0	$(2-y)$
$f_2$	$-2 \cosh(v)$	$-\left(\frac{D+1}{D-1}\right)$	$\frac{(D-2)(D+1)}{2(D-1)}$	$\frac{-2 \cosh(v)}{(D^2-2D-7)}$
$f_3$	$\frac{1}{2}\left(\frac{D-3}{D-1}\right)(2-y)$	$\frac{1}{2}\left(\frac{D-3}{D-1}\right)(2-y)$	0	$2\left(\frac{D-2}{D-1}\right)(2-y)$
$\tilde{f}_3$	$-2\left(\frac{D-2}{D-1}\right)e^v - e^{-v}$	$\frac{2}{D-1}e^v - e^{-v}$	0	$-2\left(\frac{D-3}{D-1}\right)e^v - 2e^{-v}$
$f_4$	$\frac{1}{2}\left(\frac{3D-5}{D-1}\right)(2-y)$	$\frac{1}{2}\left(\frac{D-3}{D-1}\right)(2-y)$	0	$2\left(\frac{D-2}{D-1}\right)(2-y)$
	$-e^v - 2\left(\frac{D-2}{D-1}\right)e^{-v}$	$-e^v + \frac{2}{D-1}e^{-v}$		$-2e^v - 2\left(\frac{D-3}{D-1}\right)e^{-v}$
$f_5$	$2\left(\frac{D-3}{D-1}\right) + \frac{(3D-5)}{2(D-1)}(2-y)^2$	0	0	$2\left(\frac{D-3}{D-1}\right) + \frac{(3D-5)}{2(D-1)}(2-y)^2$
	$-4\left(\frac{D-2}{D-1}\right)(2-y)\cosh(v)$			$-4\left(\frac{D-2}{D-1}\right)(2-y)\cosh(v)$
	0	$-\left(\frac{3D-5}{D-1}\right)(2-y)$	$\frac{(D-2)(D+1)}{2(D-1)}(2-y)$	$\frac{(D^2-7D+8)}{2(D-1)}(2-y)$
		$-4\left(\frac{D-3}{D-1}\right)\cosh(v)$		$+4\left(\frac{D-3}{D-1}\right)\cosh(v)$

set  $D = 4$  for any part of the structure functions which diverges less strongly than  $1/(x - x')^4$  as  $x' \rightarrow x$ . Expression (204) of appendix A gives the expansion of  $B(y)$ ,

$$B(y) = \frac{H^{D-2} \Gamma\left(\frac{D}{2} - 1\right)}{4\pi^{\frac{D}{2}} y^{\frac{D}{2}-1}} + O((D-4)y^0). \quad (102)$$

Because  $y = aa'H^2\Delta x^2$ , it might seem that the terms in (100) and (101) which involve  $B''(y)$  harbor quadratic divergences, while those involving  $yB''(y)$  and  $B'(y)$  diverge logarithmically. In fact all divergences cancel. To see this first note that the factor of  $\cosh(v)$  can be rewritten as,

$$\cosh(v) = \frac{1}{2} \left[ \frac{a}{a'} + \frac{a'}{a} \right] = 1 + \frac{1}{2} aa'H^2(\eta - \eta')^2 \equiv 1 + \frac{1}{2} aa'H^2\Delta\eta^2. \quad (103)$$

This demonstrates that the quadratic divergences of  $F_{2c}$  and  $G_{2c}$  cancel,

$$-4[(2-y)B]'' \cosh(v) + 8B'' + (D-3)[(2-y)B]' = -4aa'H^2\Delta\eta^2 B'' + 4(yB)'' + 2(D-3)B' + 2aa'H^2\Delta\eta^2(yB)'' - (D-3)(yB)', \quad (104)$$

$$[16B - 4(D-1)yB]'' \cosh(v) - [16B + 4(D-3)yB]'' - 4(D-3)^2 B' + (D-3)(D-1)(yB)' = 8aa'H^2\Delta\eta^2 B'' - 8(D-2)(yB)'' - 4(D-3)^2 B' - 2(D-1)aa'H^2\Delta\eta^2(yB)'' + (D-3)(D-1)(yB)'. \quad (105)$$

Cancelling the logarithmic divergences is more subtle. Relations (102) and (103) suffice for the  $u$ -independent part of  $G_{2c}$ ,

$$[(2-y)B]' \cosh(v) - 2B' = aa'H^2\Delta\eta^2 B' - (yB)' \cosh(v) \longrightarrow -\frac{1}{4\pi^2} \frac{\Delta\eta^2}{aa'\Delta x^4}. \quad (106)$$

In the  $u$ -dependent parts we can ignore  $(yB)'$  and  $\Delta\eta^2(yB)''$  which are both integrable and vanish in  $D = 4$  dimensions. Further, we need only retain the first term of (102) in evaluating the combinations of  $\Delta\eta^2 B''$ ,  $(yB)''$  and  $B'$  which appear in expressions (104) and (105),

$$-4aa'H^2\Delta\eta^2 B'' + 4(yB)'' + 2(D-3)B' = \frac{H^{D-2}\Gamma\left(\frac{D}{2}\right)}{4\pi^{\frac{D}{2}}} \left\{ -\frac{2Daa'H^2\Delta\eta^2}{y^{\frac{D}{2}+1}} - \frac{2}{y^{\frac{D}{2}}} + \dots \right\}, \quad (107)$$

$$8aa'H^2\Delta\eta^2 B'' - 8(D-2)(yB)'' - 4(D-3)^2 B' = \frac{H^{D-2}\Gamma\left(\frac{D}{2}\right)}{4\pi^{\frac{D}{2}}} \left\{ \frac{4Daa'H^2\Delta\eta^2}{y^{\frac{D}{2}+1}} + \frac{4}{y^{\frac{D}{2}}} + \dots \right\}. \quad (108)$$

Expressions (107) and (108) are proportional to the same function which can be reduced to a form that is integrable in  $D = 4$ ,

$$\frac{Daa'H^2\Delta\eta^2}{y^{\frac{D}{2}+1}} + \frac{1}{y^{\frac{D}{2}}} = \frac{1}{(H^2aa')^{\frac{D}{2}}} \left\{ \frac{D\Delta\eta^2}{\Delta x^{D+2}} + \frac{1}{\Delta x^D} \right\}, \quad (109)$$

$$= \frac{1}{(D-2)(H^2aa')^{\frac{D}{2}}} \left\{ \partial_0^2 \left( \frac{1}{\Delta x^{D-2}} \right) + \frac{4\pi^{\frac{D}{2}} i \delta^D(x-x')}{\Gamma\left(\frac{D}{2} - 1\right)} \right\}. \quad (110)$$

**Table 7.** The functions  $(\Delta\phi_1)_i(y)$  and  $(\Delta\phi_2)_i(y)$  defined in equation (115).

$i$	$(\Delta\phi_1)_i(y)$	$(\Delta\phi_2)_i(y)$
1	$-8(D-1)B' + 2(D-2)(2-y)C'$	$4DB''$
2	$(D-2)(4y-y^2)C'$	$4(D-2)B' - 2(D-2)(2-y)C'$
3	0	$-4(D-2)(4y-y^2)C'$
4	0	$-16(D-2)C'$
5	$-8B'$	$8B''$

After some simplifications our final unregulated forms are,

$$F_{2c}^{\text{ren}}(x; x') = -\frac{\kappa^2 H^2}{16\pi^4} \left[ \ln\left(\frac{aa'}{4}\right) + \frac{1}{3} - 2\gamma \right] \nabla^2 \left( \frac{1}{\Delta x^2} \right), \quad (111)$$

$$G_{2c}^{\text{ren}}(x; x') = \frac{\kappa^2 H^2}{24\pi^4} \left\{ \frac{H^2 aa'}{4} (\partial_0^2 + \nabla^2) \ln(H^2 \Delta x^2) + \left[ \ln\left(\frac{aa'}{4}\right) + \frac{1}{3} - 2\gamma \right] \nabla^2 \frac{1}{\Delta x^2} \right\}. \quad (112)$$

### 3.6. Nonlocal de Sitter invariant 3-3 contributions

The ‘2d’ contributions derive from making the following replacements in expressions (77) and (80),

$$i \left[ {}^\mu_\alpha \Delta^{2\nu}_\beta \right](x; x') \longrightarrow \sum_{i=1}^5 \left[ {}^\mu_\alpha \mathcal{T}^{i\nu}_\beta \right](x; x') \times K_i(y), \quad (113)$$

$$D^\alpha D'^\beta B(y) \longrightarrow D^\alpha D'^\beta y \times B'(y) + D^\alpha y D'^\beta y \times B''(y). \quad (114)$$

Recall that the basis tensors  $[{}^\mu_{\nu} \mathcal{T}^i_{\rho\sigma}](x; x')$  are listed in table 3 and the functions  $K_i(y)$  are given in table 4. Because everything is de Sitter invariant, the only coefficient functions  $f_i$  from expression (23) which occur are  $f_1(y)$  and  $f_2(y)$ . It is simplest to extract a factor of  $\kappa^2 H^2$ , and to report results from each of the basis tensors  $[{}^\mu_\alpha \mathcal{T}^{i\nu}_\beta](x; x')$  in the form,

$$f_1(y) \equiv -\kappa^2 H^2 \sum_{i=1}^5 (\Delta\phi_1)_i(y) \times K_i(y), f_2(y) \equiv -\kappa^2 H^2 \sum_{i=1}^5 (\Delta\phi_2)_i(y) \times K_i(y). \quad (115)$$

Our results for the functions  $(\Delta\phi_1)_i(y)$  and  $(\Delta\phi_2)_i(y)$  are given in table 7. Relations (36) and (97) are helpful in simplifying these expressions.

The next step is to construct the master structure functions. Only  $\Phi(y)$  is nonzero because everything is de Sitter invariant. We have found it useful to extract a factor of  $-\frac{1}{4}\kappa^2 H^6$ , and to distinguish the terms which are proportional to  $K_i(y)$  from those which are proportional to  $K'_i(y)$ ,

**Table 8.** The coefficients of  $K_i(y)$  and  $K'_i(y)$  in equation (116). An overall factor of  $(D - 2)$  has been extracted from each term.

$i$	$2(D - 1)(\Delta\phi_1)_i - 2(2 - y)(\Delta\phi_1)'_i + D(2 - y)(\Delta\phi_2)_i + (4y - y^2)(\Delta\phi_2)'_i$	$-2(2 - y)(\Delta\phi_1)_i + (4y - y^2)(\Delta\phi_2)_i$
1	$-8(D - 1)B' + 8(2 - y)C'$	$8(2 - y)B' - 8(D + 2)C' + 4(4y - y^2)C'$
2	$4(D + 2)(2 - y)B' - 32C' + 12(4y - y^2)C'$	$4(4y - y^2)B' - 4(2 - y)(4y - y^2)C'$
3	$16(4y - y^2)B' - 16(2 - y)(4y - y^2)C'$	$-4(4y - y^2)^2C'$
4	$64B' - 32(2 - y)C'$	$-16(4y - y^2)C'$
5	0	$-16C'$

$$\Phi_{2d}(y) = -\frac{1}{4}\kappa^2 H^6 \sum_{i=1}^5 \left\{ \left[ 2(D - 1)(\Delta\phi_1)_i - 2(2 - y)(\Delta\phi_1)'_i + D(2 - y)(\Delta\phi_2)_i + (4y - y^2)(\Delta\phi_2)'_i \right] \times K_i + \left[ -2(2 - y)(\Delta\phi_1)_i + (4y - y^2)(\Delta\phi_2)_i \right] \times K'_i \right\}. \quad (116)$$

Our results for the coefficients of each  $K_i(y)$  and  $K'_i(y)$  are reported in table 8. Substituting the  $K_i(y)$  from table 4 and adding everything up gives,

$$\Phi_{2d}(y) = \frac{(D - 2)(D + 1)}{D - 1} \kappa^2 H^6 \times \left\{ 8B' \left[ -\left( \frac{\square}{H^2} J \right)'' + (D - 2)(2 - y)J''' - (D - 2)(D - 1)J'' \right] + (D - 2)C' \left[ \left( \frac{\square^2}{H^4} J \right)' - 2(2 - y)\left( \frac{\square}{H^2} J \right)'' + D\left( \frac{\square}{H^2} J \right)' + 8(D - 2)J''' \right] \right\}. \quad (117)$$

The expansions of  $B'(y)$  and  $C'(y)$  are given in expressions (206) and (207) of appendix A. The function  $J(y)$  was defined in expression (55) and its expansion is given in expression (229) of appendix B. When the various factors in equation (117) are combined the result is,

$$\Phi_{2d}(y) = \frac{\kappa^2 H^{2D-2} \Gamma^2\left(\frac{D}{2}\right)}{16\pi^D} \frac{(D - 2)(D + 1)}{(D - 1)} \left\{ \frac{(D - 4)(D + 2)}{4y^D} + \frac{(3D^2 - 26D + 52)D^2}{48(D - 2)y^{D-1}} + \frac{\frac{4}{3}\ln\left(\frac{y}{4}\right)}{y^2(4 - y)^2} + \frac{\frac{1}{3}}{y^2(4 - y)} \right\}. \quad (118)$$

For this case the master structure function  $\Psi(y, u, v)$  vanishes so we have,

$$F_{2d}(x; x') = -2(aa')^{D-2} I[\Phi_{2d}(y)], \quad (119)$$

$$G_{2d}(x; x') = (aa')^{D-2} I^2[(D - 1)\Phi_{2d}(y) + y\Phi'_{2d}(y)]. \quad (120)$$

Note that the  $1/y^{D-1}$  term in (118) drops out of expression (120).



Substituting (118) into (119), performing the integration and recalling that  $y = aa'H^2\Delta x^2$  gives,

$$F_{2d}(x; x') = \frac{\kappa^2 \Gamma^2\left(\frac{D}{2}\right) (D-4)(D-2)(D+1)(D+2)}{32\pi^D (D-1)^2 aa' \Delta x^{2D-2}} \\ + \frac{\kappa^2 H^2 \Gamma^2\left(\frac{D}{2}\right) (3D^2 - 26D + 52) D^2 (D+1)}{384\pi^D (D-2)(D-1) \Delta x^{2D-4}} \\ + \frac{5\kappa^2 H^6 (aa')^2}{144\pi^4} \left\{ -\frac{1}{2} \mathcal{L}(y) - \left[ \frac{1}{y} - \frac{1}{4-y} \right] \ln\left(\frac{y}{4}\right) - \frac{2}{y} \right\}, \quad (121)$$

where we define the function  $\mathcal{L}(y)$  as,

$$\mathcal{L}(y) \equiv \text{Li}_2\left(\frac{y}{4}\right) + \ln\left(1 - \frac{y}{4}\right) \ln\left(\frac{y}{4}\right) - \frac{1}{2} \ln^2\left(\frac{y}{4}\right). \quad (122)$$

The dilogarithm function  $\text{Li}_2(z)$  is defined in (228).

Renormalization is accomplished by first localizing the ultraviolet divergence by a combination of partial integration, adding zero in the form of a delta function identity, and taking  $D = 4$  in the finite, integrable remainder [96],

$$\frac{1}{\Delta x^{2D-4}} = \frac{\partial^2}{2(D-3)(D-4)} \left[ \frac{1}{\Delta x^{2D-6}} \right], \quad (123)$$

$$= \frac{4\pi^{\frac{D}{2}} \mu^{D-4} i \delta^D(x-x')}{2(D-3)(D-4) \Gamma\left(\frac{D}{2}-1\right)} + \frac{\partial^2}{2(D-3)(D-4)} \left[ \frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right], \quad (124)$$

$$\rightarrow \frac{4\pi^{\frac{D}{2}} \mu^{D-4} i \delta^D(x-x')}{2(D-3)(D-4) \Gamma\left(\frac{D}{2}-1\right)} - \frac{\partial^2}{4} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]. \quad (125)$$

We also employ the relation,

$$\frac{1}{\Delta x^{2D-2}} \rightarrow \frac{4\pi^{\frac{D}{2}} \mu^{D-4} \partial^2 i \delta^D(x-x')}{4(D-2)^2 (D-3)(D-4) \Gamma\left(\frac{D}{2}-1\right)} - \frac{\partial^4}{32} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]. \quad (126)$$

The localized ultraviolet divergence is absorbed by a counterterm of the form (28) with the  $C_4$  and  $\bar{C}$  coefficients,

$$C_4^{2d} = \frac{\kappa^2 \mu^{D-4} \Gamma\left(\frac{D}{2}\right) (D+1)(D+2)}{64\pi^{\frac{D}{2}} (D-1)^2 (D-3)}, \quad (127)$$

$$\bar{C}^{2d} = \frac{\kappa^2 H^2 \mu^{D-4} \Gamma\left(\frac{D}{2}\right)}{96\pi^{\frac{D}{2}}} \left\{ \frac{D^2 (D+1)}{(D-1)(D-3)(D-4)} \right. \\ \left. - \frac{(3D-14)D^2 (D+1)}{4(D-1)(D-3)} + \frac{3(3D-8)(D+1)(D+2)}{2(D-1)^2 (D-3)} \right\}. \quad (128)$$

The final renormalized result is,

$$F_{2d}^{\text{ren}}(x; x') = \frac{5\kappa^2 H^2}{3^2 \pi^2} \ln(a) i\delta^4(x - x') + \frac{5\kappa^2 H^2}{2^4 3^2 \pi^4} \partial^2 \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{5\kappa^2 H^6 (aa')^2}{144 \pi^4} \left\{ -\frac{1}{2} \mathcal{L}(y) - \left[ \frac{1}{y} - \frac{1}{4-y} \right] \ln\left(\frac{y}{4}\right) - \frac{2}{y} \right\}. \quad (129)$$

The second structure function is,

$$G_{2d}(x; x') = -\frac{\kappa^2 H^2 \Gamma^2\left(\frac{D}{2}\right) (D-4)(D+1)(D+2)}{64 \pi^D (D-1)^2 \Delta x^{2D-4}} + \frac{5\kappa^2 H^6 (aa')^2}{72 \pi^4} \left\{ \frac{1}{4} (1-y) \mathcal{L}(y) + \left[ \frac{1}{4-y} - 1 \right] \ln\left(\frac{y}{4}\right) \right\}. \quad (130)$$

We go through the same procedure of localization as before, and add a counterterm of the form (29) to produce the final renormalized result,

$$G_{2d}^{\text{ren}}(x; x') = \frac{5\kappa^2 H^6 (aa')^2}{72 \pi^4} \left\{ \frac{1}{4} (1-y) \mathcal{L}(y) + \left[ \frac{1}{4-y} - 1 \right] \ln\left(\frac{y}{4}\right) \right\}. \quad (131)$$

### 3.7. The full spin 2 contribution

It remains to combine the various spin two contributions to the two structure functions. Our results for  $F_2(x; x')$  are relations (73), (92), (111) and (129),

$$F_2(x; x') = \frac{85\kappa^2 H^2}{72 \pi^2} \ln(a) i\delta^4(x - x') - \frac{\kappa^2 H^2}{16 \pi^4} \left[ \ln\left(\frac{aa'}{4}\right) + \frac{1}{3} - 2\gamma \right] \nabla^2 \left( \frac{1}{\Delta x^2} \right) + \frac{5\kappa^2 H^2}{144 \pi^4} \partial^2 \left( \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right) - \frac{5\kappa^2 H^6 (aa')^2}{144 \pi^4} \left\{ \frac{\mathcal{L}(y)}{2} + \frac{2(2-y) \ln\left(\frac{y}{4}\right)}{4y - y^2} + \frac{2}{y} \right\}. \quad (132)$$

Our results for the other structure function are given in equations (74), (93), (112) and (131),

$$G_2(x; x') = -\frac{5\kappa^2 H^2}{4 \pi^2} \ln(a) i\delta^4(x - x') + \frac{\kappa^2 H^2}{24 \pi^4} \left[ \ln\left(\frac{aa'}{4}\right) + \frac{1}{3} - 2\gamma \right] \nabla^2 \left( \frac{1}{\Delta x^2} \right) + \frac{\kappa^2 H^4 aa'}{96 \pi^4} (\partial_0^2 + \nabla^2) \ln(H^2 \Delta x^2) + \frac{5\kappa^2 H^6 (aa')^2}{72 \pi^4} \left\{ \frac{(1-y) \mathcal{L}(y)}{4} + \frac{(y-3) \ln\left(\frac{y}{4}\right)}{4-y} \right\}. \quad (133)$$

Note that there can also be arbitrary finite contributions of the form (28) and (29).

## 4. Spin 0 contributions

This section follows the analysis of section 3 applied to spin zero part of the graviton propagator. Its purpose is to work out the contributions to the renormalized vacuum

polarization structure functions coming from this part of the propagator. Firstly, the description of the spin 0 part of the graviton propagator is given. Next, we work out the contributions from the 4-point diagram ('0a') and the local part of the 3-3 diagram ('0b'). Working out the contribution from the (de Sitter invariant) nonlocal part of the 3-3 diagram ('0d') comprises the largest part of this section. In the end the full results for  $F(x; x')$  and  $G(x; x')$  are given.

Many of the tensor structure contractions performed in this section were checked with the help of the *xTensor* part of the tensor calculus package *xAct* [99] written for *Mathematica*.

#### 4.1. Spin 0 part of the graviton propagator

The spin 0 part of the graviton propagator depends on the gauge parameter  $b$  introduced in the gauge condition (3). Instead of working with  $b$ , we find it more convenient to use parameter  $\beta$ , defined to be

$$\beta = \frac{Db - 2}{b - 2}. \quad (134)$$

Note that for the range of gauges  $b > 2$  considered in this work,  $\beta$  is a monotonically decreasing function of  $b$ , with  $\beta > D$ .

The spin zero part of the graviton propagator can be written as two projectors acting on a scalar structure function [38],

$$i[\mu\nu\Delta_{\rho\sigma}^0](x; x') = \frac{-2\beta^2}{(D-1)(D-2)} \times \mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(x') \times [i\Delta_{WNN}(x; x')], \quad (135)$$

where the projector  $\mathcal{P}_{\mu\nu}(x)$  is a 2nd order differential operator,

$$\mathcal{P}_{\mu\nu}(x) = D_\mu D_\nu - \frac{g_{\mu\nu}(x)}{\beta} [\square - (\beta - D)H^2]. \quad (136)$$

The spin 0 scalar structure function  $i\Delta_{WNN}(x; x')$  is obtained by inverting twice the kinetic operator for an  $M_S^2 = (\beta - D)H^2$  scalar, and once for an  $M_S^2 = -DH^2$  scalar. The order of these inversions is irrelevant, we find the following one convenient,

$$[\square - (\beta - D)H^2]i\Delta_{WNN}(x; x') = i\Delta_{WN}(x; x'), \quad (137)$$

$$[\square - (\beta - D)H^2]i\Delta_{WN}(x; x') = i\Delta_W(x; x'), \quad (138)$$

$$[\square + DH^2]i\Delta_W(x; x') = \frac{i\delta^D(x - x')}{\sqrt{-g}(x)}. \quad (139)$$

Since one of the masses in the scalar kinetic operators is tachyonic, the scalar structure function  $i\Delta_{WNN}(x; x')$  must break de Sitter invariance. We separate it into a de Sitter invariant and a de Sitter breaking part,

$$i\Delta_{WNN}(x; x') = WNN(y) + \delta WNN(y, u, v), \quad (140)$$

the details of which are given in appendix C.

The de Sitter breaking part  $\delta WNN$  of the scalar structure function (140) is given in (248). The projectors in (135), when acted on this de Sitter breaking part give zero,

$$\mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(x') \times [\delta WNN(y, u, v)] = 0, \quad (141)$$

which means that the spin 0 part of the graviton propagator is de Sitter invariant for gauges  $b > 2$  (even though the scalar structure function is not). Therefore, we can drop the de Sitter

**Table 9.** The coefficient functions  $C_i(y)$  in the expansion (144) of the spin 0 graviton propagator in the tensor basis given in table 3.

$i$	$\left[ \frac{-2\beta^2}{(D-1)(D-2)} \right]^{-1} \times C_i(y)$
1	$2WNN''$
2	$4WNN'''$
3	$WNN''''$
4	$(2-y)WNN''' - 2WNN'' - \frac{1}{\beta} \frac{WN''}{H^2}$
5	$(2-y)^2WNN'' - (2-y)WNN' - \frac{2}{\beta}(2-y)\frac{WN'}{H^2} + \frac{1}{\beta^2} \frac{W}{H^4}$

breaking part in (135), and write

$$i[\mu\nu\Delta_{\rho\sigma}^0](x; x') = \frac{-2\beta^2}{(D-1)(D-2)} \times \mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(x') \times [WNN(y)]. \quad (142)$$

By making use of relations (256) and (257) from appendix C, the action of the projectors can be written out as

$$\begin{aligned} i[\mu\nu\Delta_{\rho\sigma}^0](x; x') = & -\frac{2\beta^2}{(D-1)(D-2)} D_\mu D_\nu D'_\rho D'_\sigma WNN(y) \\ & + \frac{2\beta}{(D-1)(D-2)} [g_{\mu\nu}(x) D'_\rho D'_\sigma WN(y) + g_{\rho\sigma}(x') D_\mu D_\nu WN(y)] \\ & - \frac{2}{(D-1)(D-2)} g_{\mu\nu}(x) g_{\rho\sigma}(x') W(y). \end{aligned} \quad (143)$$

It is convenient to expand this propagator in a de Sitter invariant tensor basis defined in table 3,

$$i[\mu\nu\Delta_{\rho\sigma}^0](x; x') = \sum_{i=1}^5 [\mu\nu\mathcal{T}_{\rho\sigma}^i](x; x') \times C_i(y), \quad (144)$$

where the coefficient functions in this expansion are given in table 9.

#### 4.2. The 4-point diagram

The contribution to the vacuum polarization from the spin 0 part of the 4-point diagram (middle one in figure 1) is

$$i[\mu\Pi_{4pt0}^\nu](x; x') = \partial_\kappa \partial'_\lambda \left\{ -i\kappa^2 a^{D-8} U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta} i[\alpha\beta\Delta_{\gamma\delta}^0](x; x) \delta^D(x - x') \right\}. \quad (145)$$

Here the spin 0 part of the graviton propagator was substituted in the full contribution (17), and the 4-point vertex function is given in (15). The coincidence limit of the spin 0 graviton propagator is

$$i[\alpha\beta\Delta_{\gamma\delta}^0](x; x) = 4(aH)^4 \eta_{\alpha(\gamma} \eta_{\delta)\beta} C_1(0) + (aH)^4 \eta_{\alpha\beta} \eta_{\gamma\delta} C_5(0), \quad (146)$$

where the coincidence limits of the relevant coefficient functions from table 9 are

$$\begin{aligned}
 C_1(0) = & \frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}} \times \frac{\Gamma(D+2)}{4(D-1)(D-2)\Gamma\left(\frac{D}{2}+2\right)} \\
 & \times \left\{ 1 + \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(1-\frac{D}{2}\right)\Gamma\left(\frac{D+3}{2}+b_N\right)\Gamma\left(\frac{D+3}{2}-b_N\right)}{\Gamma(D+2)\Gamma\left(\frac{1}{2}+b_N\right)\Gamma\left(\frac{1}{2}-b_N\right)} \right. \\
 & \times \left[ 1 + \frac{\beta}{2b_N} \left[ \psi\left(\frac{D+3}{2}+b_N\right) - \psi\left(\frac{D+3}{2}-b_N\right) \right. \right. \\
 & \left. \left. - \psi\left(\frac{1}{2}+b_N\right) + \psi\left(\frac{1}{2}-b_N\right) \right] \right] \right\}, \tag{147}
 \end{aligned}$$

$$\begin{aligned}
 C_5(0) = & 2C_1(0) + \frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}} \times \frac{\Gamma(D+1)}{(D-1)(D-2)\Gamma\left(\frac{D+1}{2}\right)} \\
 & \times \left\{ -\frac{D+1}{D} + \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(1-\frac{D}{2}\right)\Gamma\left(\frac{D+1}{2}+b_N\right)\Gamma\left(\frac{D+1}{2}-b_N\right)}{\Gamma(D+1)\Gamma\left(\frac{1}{2}+b_N\right)\Gamma\left(\frac{1}{2}-b_N\right)} \right. \\
 & \times \left[ 1 - \frac{\beta}{2b_N} \left[ \psi\left(\frac{D+1}{2}+b_N\right) - \psi\left(\frac{D+1}{2}-b_N\right) \right. \right. \\
 & \left. \left. - \psi\left(\frac{1}{2}+b_N\right) + \psi\left(\frac{1}{2}-b_N\right) \right] \right] \right\}. \tag{148}
 \end{aligned}$$

They are easily calculated from explicit forms of scalar propagators (235) and (240), and expressions (251) and (253) in appendix C, where, by rules of dimensional regularization, all  $D$ -dependent powers of  $y$  vanish at coincidence.

The contractions of the tensor structures of the spin 0 graviton propagator coincident limit (146) with the 4-vertex (15) are

$$U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}\eta_{\alpha(\gamma}\eta_{\delta)\beta} = \frac{1}{4}[12 - (D-4)^2]\eta^{\mu[\nu}\eta^{\lambda]\kappa}, \tag{149}$$

$$U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}\eta_{\alpha\beta}\eta_{\gamma\delta} = \frac{1}{4}(D-4)(D-6)\eta^{\mu[\nu}\eta^{\lambda]\kappa}, \tag{150}$$

so that,

$$\begin{aligned}
 U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}i[\alpha\beta\Delta_{\gamma\delta}^0](x; x) = & \frac{(aH)^4}{2}(\eta^{\mu\nu}\eta^{\kappa\lambda} - \eta^{\mu\lambda}\eta^{\nu\kappa}) \\
 & \times \left\{ [12 - (D-4)^2]C_1(0) + \frac{(D-6)(D-4)}{4}C_5(0) \right\}. \tag{151}
 \end{aligned}$$

From here it is straightforward to see that the vacuum polarization is in the form given by (18) and (22) in section 2.2,

$$i[\mu\Pi_{4pt0}^\nu](x; x') = \partial_\kappa\partial'_\lambda\left\{ 2\eta^{\mu[\nu}\eta^{\lambda]\kappa} \times F_{0a}(x; x') + 2\bar{\eta}^{\mu[\nu}\bar{\eta}^{\lambda]\kappa} \times G_{0a}(x; x') \right\}, \tag{152}$$

where the structure functions are

$$F_{0a}(x; x') = \kappa^2 a^{D-4} i \delta^D(x - x') \times \left\{ -6 H^4 C_1(0) - \frac{(D-4)(D-6)}{8} H^4 C_5(0) + \frac{(D-4)^2}{2} H^4 C_1(0) \right\}, \quad (153)$$

$$G_{0a}(x; x') = 0. \quad (154)$$

In the  $D = 4$  limit the first term in the brackets in (153) diverges as  $1/(D-4)$ , the second term is finite, and the third term vanishes in this limit. This is a consequence of the coefficient functions (147) and (148) diverging as  $1/(D-4)$ . Comparing with (28) and (29) we see that the entire contribution of (153) and (154) can be absorbed into the counterterms by choosing the coefficient

$$\bar{C}_{0a} = -\frac{\kappa^2}{4} \left\{ -6 H^2 C_1(0) + \frac{(D-6)(D-4)}{4} H^2 C_5(0) + \frac{(D-4)^2}{2} H^2 C_1(0) \right\}. \quad (155)$$

Therefore, the renormalized 4-point contributions to the structure functions are,

$$F_{0a}^{\text{ren}} = 0, \quad G_{0a}^{\text{ren}} = 0. \quad (156)$$

#### 4.3. Local contributions from the 3-3 diagram

The ‘0b’ local part of the 3-3 diagram (left one in figure 1) descends from isolating the delta function coming from two derivatives acting on the  $B$ -type scalar propagator as in (82), which we reproduce here (with all indices lowered),

$$D_\rho D'_\sigma y \times D_\lambda D'_\phi B \longrightarrow -\frac{2H^2}{a^{D-4}} \eta_{\rho\sigma} \delta_\lambda^0 \delta_\phi^0 i \delta^D(x - x') \quad (157)$$

and by making the substitution in (43),

$$i \left[ {}^\mu \Pi_{3\text{pt}0b}^\nu \right](x; x') = \partial_\kappa \partial_\theta \left\{ -i \kappa^2 a^{D-8} \tilde{U}^{\mu\nu\kappa\theta\alpha\beta\gamma\delta} i \left[ {}^\alpha \Delta_{\gamma\delta}^0 \right](x; x) \delta^D(x - x') \right\}. \quad (158)$$

Note that this contribution has the same structure as the 4-point diagram contribution (145). The effective 4-vertex  $\tilde{U}^{\mu\nu\kappa\theta\alpha\beta\gamma\delta}$  is constructed by contracting the tensor structure in (157) with the two 3-vertices in (43),

$$\begin{aligned} \tilde{U}^{\mu\nu\kappa\theta\alpha\beta\gamma\delta} &= V^{\mu\rho\kappa\lambda\alpha\beta} \eta_{\rho\sigma} \delta_\lambda^0 \delta_\phi^0 V^{\nu\sigma\theta\phi\gamma\delta} \\ &= -\delta_0^{[\mu} \eta^{\kappa][\nu} \delta_0^{\theta]} \eta^{\alpha\beta} \eta^{\gamma\delta} + 2 \left[ \delta_0^{[\mu} \eta^{\kappa](\gamma} \eta^{\delta)][\nu} \delta_0^{\theta]} \eta^{\alpha\beta} + \delta_0^{[\mu} \eta^{\kappa](\alpha} \eta^{\beta)][\nu} \delta_0^{\theta]} \eta^{\gamma\delta} \right] \\ &\quad + 2 \left[ \delta_0^{[\mu} \eta^{\kappa][\nu} \eta^{\theta](\gamma} \delta_0^{\delta)} \eta^{\alpha\beta} + \delta_0^{(\alpha} \eta^{\beta)][\mu} \eta^{\kappa][\nu} \delta_0^{\theta]} \eta^{\gamma\delta} \right] - 4 \delta_0^{[\mu} \eta^{\kappa](\alpha} \eta^{\beta)(\gamma} \eta^{\delta)][\nu} \delta_0^{\theta]} \\ &\quad - 4 \delta_0^{(\alpha} \eta^{\beta)][\mu} \eta^{\kappa][\nu} \eta^{\theta](\gamma} \delta_0^{\delta)} - 4 \left[ \delta_0^{[\mu} \eta^{\kappa](\alpha} \eta^{\beta)][\nu} \eta^{\theta](\gamma} \delta_0^{\delta)} + \delta_0^{(\alpha} \eta^{\beta)][\mu} \eta^{\kappa](\gamma} \eta^{\delta)][\nu} \delta_0^{\theta]} \right]. \end{aligned} \quad (159)$$

Contractions of this effective 4-vertex with the tensor structures in the coincident graviton propagator (146) are

$$\tilde{U}^{\mu\nu\kappa\theta\alpha\beta\gamma\delta} \eta_{\alpha(\gamma} \eta_{\delta)\beta} = -2 \eta^{\mu[\nu} \eta^{\theta]\kappa} - (3D-8) \delta_0^{[\mu} \eta^{\kappa][\nu} \delta_0^{\theta]}, \quad (160)$$

$$\tilde{U}^{\mu\nu\kappa\theta\alpha\beta\gamma\delta} \eta_{\alpha\beta} \eta_{\gamma\delta} = -(D-4)^2 \delta_0^{[\mu} \eta^{\kappa][\nu} \delta_0^{\theta]}. \quad (161)$$

Using the identity

$$\delta_0^{[\mu} \eta^{\kappa][\nu} \delta_0^{\theta]} = \frac{1}{2} \eta^{\mu[\nu} \eta^{\theta]\kappa} - \frac{1}{2} \bar{\eta}^{\mu[\nu} \bar{\eta}^{\theta]\kappa}, \quad (162)$$

we can write the contribution to vacuum polarization as described in section 2.2,

$$i \left[ {}^\mu \Pi_{3\text{pt}0b}^\nu \right](x; x') = \partial_\kappa \partial'_\theta \left\{ 2\eta^{\mu[\nu} \eta^{\theta]\kappa} \times F_{0b}(x; x') + 2\bar{\eta}^{\mu[\nu} \bar{\eta}^{\theta]\kappa} \times G_{0b}(x; x') \right\}, \quad (163)$$

where the structure functions are

$$F_{0b}(x; x') = \kappa^2 a^{D-4} i \delta^D(x - x') \times \left\{ (3D - 4) H^4 C_1(0) + \frac{(D - 4)^2}{4} H^4 C_5(0) \right\}, \quad (164)$$

$$G_{0b}(x; x') = \kappa^2 a^{D-4} i \delta^D(x - x') \times \left\{ -(3D - 8) H^4 C_1(0) - \frac{(D - 4)^2}{4} H^4 C_5(0) \right\}, \quad (165)$$

and  $C_1(0)$  and  $C_5(0)$  are given in (147) and (148) (recall that they diverge as  $1/(D - 4)$  in  $D \rightarrow 4$  limit). Again, we can completely absorb these contributions into the counterterms (28) and (29) by choosing the coefficients

$$\bar{C}_{0b} = -\frac{\kappa^2}{4} \left\{ (3D - 4) H^2 C_1(0) + \frac{(D - 4)^2}{4} H^2 C_5(0) \right\}, \quad (166)$$

$$\Delta C_{0b} = \frac{\kappa^2}{4} \left\{ (3D - 8) H^2 C_1(0) + \frac{(D - 4)^2}{4} H^2 C_5(0) \right\}, \quad (167)$$

making the renormalized contributions to vacuum polarization scalar structure functions vanish,

$$F_{0b}^{\text{ren}} = 0, \quad G_{0b}^{\text{ren}} = 0. \quad (168)$$

Note that had we not introduced a noninvariant counterterm in (27) we would not have been able to remove the divergence in (165).

#### 4.4. Nonlocal contributions from the 3-3 diagram

The nonlocal ‘0d’ contribution from the 3-3 diagram derives from making the replacement for the two derivatives acting on the  $B$ -propagator,

$$D_\lambda D'_\phi B \longrightarrow D_\lambda D'_\phi y \times B' + D_\lambda y \ D'_\phi y \times B'', \quad (169)$$

in expression (43). This makes the bi-tensor density defined in (18),

$$\begin{aligned} \left[ {}^{\mu\kappa} T_{0d}^{\nu\theta} \right](x; x') &= \frac{\kappa^2}{2H^2} (aa')^{D-6} V^{\mu\rho\kappa\lambda\alpha\beta} i \left[ {}_{\alpha\beta} \Delta_{\gamma\delta}^0 \right](x; x') V^{\nu\sigma\theta\phi\gamma\delta} D_\rho D'_\sigma y \\ &\quad \times \left[ D_\lambda D'_\phi y \times B' + D_\lambda y \ D'_\phi y \times B'' \right]. \end{aligned} \quad (170)$$

We use the recipe developed in [61] and outlined in section 2.2 to find the ‘0d’ contribution to  $F(x; x')$  and  $G(x; x')$  structure functions. The first thing is to calculate coefficient functions  $f_i$  from (23). We do this by contracting the tensor structure of the graviton propagator with the vertices and the tensor structure of the photon part. The contraction rules can be found in [95].

**Table 10.** The functions  $(\Delta\varphi_1)_i(y)$  and  $(\Delta\varphi_2)_i(y)$  defined in equation (171). Function  $B(y)$  is defined in (204), and  $C'(y)$  in (36).

$i$	$(\Delta\varphi_1)_i(y)$	$(\Delta\varphi_2)_i(y)$
1	$-2(5D - 13)B' - \frac{1}{2}(2 - y)^2B'$	$2(3D - 8)B'' + (2 - y)B' - 3(D - 2)C'$
2	$+2(D - 2)(2 - y)C'$ $-\frac{1}{2}(2 - y)(4y - y^2)B'$ $+(D - 2)(4y - y^2)C'$	$4(D - 3)B' + (4y - y^2)B'$ $+(D - 2)(2 - y)C'$
3	$-\frac{1}{2}(4y - y^2)^2B'$	$\frac{1}{2}(4y - y^2)^2B''$
4	$-(D - 4)(4y - y^2)B'$	$-2(D - 4)[(2 - y)B' - (D - 2)C']$
5	$-\frac{1}{2}(D - 4)^2B'$	$\frac{1}{2}(D - 4)^2B''$

**Table 11.** The coefficient functions of  $C_i(y)$  and  $C'_i(y)$  from expression (172). An overall factor of  $(D - 2)$  has been extracted from each term.

$i$	$2(D - 1)(\Delta\varphi_1)_i - 2(2 - y)(\Delta\varphi_1)'_i$ $+D(2 - y)(\Delta\varphi_2)_i + (4y - y^2)(\Delta\varphi_2)'_i$	$-2(2 - y)(\Delta\varphi_1)_i + (4y - y^2)(\Delta\varphi_2)_i$
1	$2(2 - y)C' - 8(D - 3)B'$	$8(2 - y)B' + (4y - y^2)C' - 4(3D - 4)C'$
2	$4(D - 2)(2 - y)B'$ $+3(4y - y^2)C' - 8C'$	$(4y - y^2)[4B' - (2 - y)C']$
3	$-4(2 - y)(4y - y^2)C'$	$-(4y - y^2)^2C'$
4	$4(D - 4)[(2 - y)C' - 4B']$	$2(D - 4)(4y - y^2)C'$
5	0	$-(D - 4)^2C'$

This contribution is de Sitter invariant so only  $f_1$  and  $f_2$  can appear (the de Sitter breaking part ‘0c’ is zero in this case), and we find it convenient to express them as a sum over spin 0 graviton coefficient functions defined in table 9,

$$f_1(y) \equiv -\kappa^2 H^2 \sum_{i=1}^5 (\Delta\varphi_1)_i(y) \times C_i(y), f_2(y) \equiv -\kappa^2 H^2 \sum_{i=1}^5 (\Delta\varphi_2)_i(y) \times C_i(y), \quad (171)$$

where factors  $-\kappa^2 H^2$  are extracted for convenience. The coefficient functions  $(\Delta\varphi_1)_i$  and  $(\Delta\varphi_2)_i$  are given in table 10, where identities (36) and (97) have been used to simplify the expressions.

The next step is to construct the master structure functions defined in table 1. Because of de Sitter invariance  $\Psi(y, u, v)$  has to be zero, and  $\Phi(y)$  is, according to table 1 and expression (171),

$$\begin{aligned} \Phi_{0d}(y) = & -\frac{1}{4}\kappa^2 H^6 \sum_{i=1}^5 \left\{ \left[ 2(D - 1)(\Delta\varphi_1)_i - 2(2 - y)(\Delta\varphi_1)'_i + D(2 - y)(\Delta\varphi_2)_i \right. \right. \\ & \left. \left. + (4y - y^2)(\Delta\varphi_2)'_i \right] \times C_i + \left[ -2(2 - y)(\Delta\varphi_1)_i + (4y - y^2)(\Delta\varphi_2)_i \right] \times C'_i \right\}. \end{aligned} \quad (172)$$

The results for coefficients of  $C_i$  and  $C'_i$  in this expansion are presented in table 11. Plugging in  $C_i(y)$  from table 9 and coefficient functions from table 11 into (172) we get for the master



function,

$$\begin{aligned}
\Phi_{0d}(y) = & \frac{\kappa^2 H^6 \beta^2 B'}{2(D-1)} \times \left\{ 16(4y - y^2) WNN'''' + 48(2 - y) WNN''' \right. \\
& + 16(D - 5) WNN'' + \frac{16}{\beta} (D - 4) \frac{WN''}{H^2} \Big\} \\
& + \frac{\kappa^2 H^6 \beta^2 C'}{2(D-1)} \times \left\{ -(4y - y^2)^2 WNN'''' \right. \\
& + 2(D - 8)(2 - y)(4y - y^2) WNN''' \\
& + \left[ (D^2 - 18D + 70)(4y - y^2) - 4(D^2 - 6D + 32) \right] WNN'' \\
& + (D - 6)(3D - 14)(2 - y) WNN' - (D - 4)^2 WNN' \\
& - \frac{2}{\beta} (D - 4)(4y - y^2) \frac{WN''}{H^2} + \frac{2}{\beta} (D - 4)(D - 6)(2 - y) \frac{WN''}{H^2} \\
& \left. - \frac{2}{\beta} (D - 4)^2 \frac{WN'}{H^2} - \frac{(D - 4)^2}{\beta^2} \frac{W'}{H^4} \right\}. \tag{173}
\end{aligned}$$

Using the identity for the d'Alembertian operator acting on a scalar function that depends only on  $y$ ,

$$\frac{\square}{H^2} S(y) = (4y - y^2) S''(y) + D(2 - y) S'(y), \tag{174}$$

and derivatives of it with respect to  $y$ , we can rewrite the above expression for master function as

$$\begin{aligned}
\Phi_{0d}(y) = & \frac{\kappa^2 H^6 \beta^2 B'}{2(D-1)} \times \left\{ 16\partial_y^2 \left[ \frac{\square}{H^2} WNN \right] - 16(D + 1)(2 - y) WNN''' \right. \\
& + 48(D - 1) WNN'' + 16 \frac{(D - 4)}{\beta} \frac{WN''}{H^2} \Big\} + \frac{\kappa^2 H^6 \beta^2 C'}{2(D-1)} \\
& \times \left\{ -\partial_y \left[ \left( \frac{\square}{H^2} \right)^2 WNN \right] + 4(D - 2)(2 - y) \partial_y^2 \left[ \frac{\square}{H^2} WNN \right] \right. \\
& + 4(D - 2)(D - 3) \partial_y \left[ \frac{\square}{H^2} WN \right] - \frac{2}{\beta} (D - 4) \partial_y \left[ \frac{\square}{H^2} \frac{WN}{H^2} \right] \\
& - 16(D - 2)(D + 1) WNN''' - 4(D - 2)^2 (D - 1)(2 - y) WNN'' \\
& + 4(D - 2)^2 (D - 1) WNN' + \frac{4}{\beta} (D - 4)(D - 2)(2 - y) \frac{WN''}{H^2} \\
& \left. - \frac{4}{\beta} (D - 4)(D - 2) \frac{WN'}{H^2} - \frac{(D - 4)^2}{\beta^2} \frac{W'}{H^4} \right\}. \tag{175}
\end{aligned}$$

Next, using identities (256) and (257) from appendix C, we can further simplify this expression,

$$\begin{aligned}
\Phi_{0d}(y) = & \frac{\kappa^2 H^6 \beta^2 B'}{2(D-1)} \times \left\{ -16(D+1)(2-y)WNN''' \right. \\
& + 16[\beta + (2D-3)]WNN'' + 16\left[1 + \frac{(D-4)}{\beta}\right]\frac{WN''}{H^2} \Big\} \\
& + \frac{\kappa^2 H^6 \beta^2 C'}{2(D-1)} \times \left\{ -16(D-2)(D+1)WNN''' \right. \\
& + 4(D-2)\left[\beta - (D^2 - 2D + 2)\right](2-y)WNN'' \\
& - \left[\beta^2 - 2(2D^2 - 9D + 12)\beta + (D-4)^2\right]WNN' \\
& + 4(D-2)\left[1 + \frac{D-4}{\beta}\right](2-y)\frac{WN''}{H^2} \\
& - 2\left[\beta - 2(D^2 - 5D + 8) + \frac{(D-4)^2}{\beta}\right]\frac{WN'}{H^2} \\
& \left. - \left[1 + \frac{D-4}{\beta}\right]^2\frac{W'}{H^4} - \frac{4}{\beta^2}(D-2)^2\frac{W'}{H^4} \right\}. \tag{176}
\end{aligned}$$

As argued in section 3.5, we can set  $D = 4$  in any part of the structure functions  $F(x; x')$  and  $G(x; x')$  that diverges less strongly than  $1/\Delta x^4$ , as  $\Delta x \rightarrow 0$ . Since, according to (24) and (25),  $F$  is an integral of  $\Phi$  with respect to  $y$ , and  $G$  is a double integral of  $\Phi$  and  $\Psi$  with respect to  $y$ , in the master functions we can set  $D = 4$  in any of the parts which diverge less strongly than  $y^{-3}$  as  $y \rightarrow 0$  (recall that  $y = H^2 aa' \Delta x^2$ ). Also, we can throw away terms containing  $(D-4)^2$ . Taking into account the expansion of  $B'$  and  $C'$  given in (206) and (207) in appendix A, and functions  $W$ ,  $N$  and  $NN$  defined in appendix C, we get for the master function

$$\Phi_{0d}(y) = \frac{\kappa^2 H^{2D-2} \Gamma^2\left(\frac{D}{2}\right)}{8\pi^D} \left\{ \frac{\ell_1(D)}{y^D} + \frac{\ell_2(D)}{y^{D-1}} + \frac{\mathcal{N}(y)}{y^2} \right\}, \tag{177}$$

where the two  $D$ -dependent coefficients are

$$\ell_1(D) = -\frac{1}{8} \left[ (D-2)\beta^2 - 4(D-4)\beta + \frac{2(D-4)^2}{(D-1)} \right], \tag{178}$$

$$\ell_2(D) = \frac{1}{72} \left[ \beta^2(5-\beta) - \frac{\beta}{6}(4\beta^2 - 83\beta + 180)(D-4) \right] + \mathcal{O}((D-4)^2). \tag{179}$$

The de Sitter invariant function  $\mathcal{N}(y)$  is defined to be

$$\begin{aligned}
\mathcal{N}(y) \equiv & \frac{\beta^2}{48} \overline{N}'_1 + \frac{5}{3} y \overline{N}''_1 - \frac{5}{3} \overline{N}''_2 + \frac{5}{3} y \overline{N}'''_2 - \frac{20}{3} \overline{N}'''_3 + \frac{\beta^2(\beta-16)}{48} \overline{NN}'_1 \\
& + \frac{\beta(\beta-10)}{6} y \overline{NN}''_1 - \frac{\beta(2\beta-5)}{3} \overline{NN}''_2 - \frac{5\beta}{3} y \overline{NN}'''_2 + \frac{20\beta}{3} \overline{NN}'''_3 \\
& = \frac{\beta^2}{6} \frac{\partial}{\partial \beta} \left\{ \left[ \frac{\beta}{8} - 2 \right] \overline{N}'_1 + \left[ -\frac{10}{\beta} + 1 \right] y \overline{N}''_1 \right. \\
& \left. + \left[ \frac{10}{\beta} - 4 \right] \overline{N}''_2 - \frac{10}{\beta} y \overline{N}'''_2 + \frac{40}{\beta} \overline{N}'''_3 \right\}, \tag{180}
\end{aligned}$$

where the definitions in  $D$  dimensions of functions  $\overline{N}_i(y)$  and  $\overline{N\overline{N}}_i(y)$  are given in (259) and (260) in appendix C, and the limit  $D \rightarrow 4$  in (261), which is taken here. The power series representation of this function is

$$\mathcal{N}(y) = \frac{\beta^2}{6} \frac{\partial}{\partial \beta} \sum_{n=0}^{\infty} q_n y^n \left[ A_n \ln\left(\frac{y}{4}\right) + B_n \right], \quad (181)$$

where the coefficients are

$$q_n = \frac{\Gamma\left(\frac{5}{2} + b_N + n\right) \Gamma\left(\frac{5}{2} - b_N + n\right)}{4^{n+1} (n+1)! (n+2)! \Gamma\left(\frac{1}{2} + b_N\right) \Gamma\left(\frac{1}{2} - b_N\right)}, \quad (182)$$

$$A_n = \frac{(n+1)}{8(n+3)(n+4)\beta} \left[ n(n-1)\beta^2 - 4(n-1)(3n+2)\beta + 40n(n+1) \right], \quad (183)$$

$$\begin{aligned} B_n = & \frac{(n+1)}{8\beta(n+3)(n+4)} \left[ n(n-1)\beta^2 - 4(n-1)(3n+2)\beta + 40n(n+1) \right] \\ & \times \left[ \psi\left(\frac{5}{2} + b_N + n\right) + \psi\left(\frac{5}{2} - b_N + n\right) - \psi(n+2) - \psi(n+3) \right] \\ & + \frac{1}{8(n+3)^2(n+4)^2\beta} \left[ \beta^2(n^4 + 14n^3 + 37n^2 - 12) \right. \\ & - 4\beta(3n^4 + 42n^3 + 125n^2 + 52n - 22) \\ & \left. + 40(n+1)(n^3 + 13n^2 + 36n + 12) \right]. \end{aligned} \quad (184)$$

Since the master function  $\Psi(y, u, v)$  vanishes, according to (24) and (25), we have for the vacuum polarization structure functions,

$$F_{0d}(x; x') = -2(aa')^{D-2} \times I[\Phi_{0d}(y)], \quad (185)$$

$$G_{0d}(x; x') = (aa')^{D-2} \times I^2[(D-1)\Phi_{0d}(y) + y\Phi'_{0d}(y)]. \quad (186)$$

Performing the integrals above gives the following,

$$\begin{aligned} F_{0d}(x; x') = & \frac{\kappa^2 \Gamma^2\left(\frac{D}{2}\right)}{8\pi^D} \left\{ \frac{2\ell_1(D)}{(D-1)aa'\Delta x^{2D-2}} + \frac{2\ell_2(D)H^2}{(D-2)\Delta x^{2D-4}} \right\} \\ & - \frac{\kappa^2 H^6}{4\pi^4} (aa')^2 I[y^{-2}\mathcal{N}(y)], \end{aligned} \quad (187)$$

$$\begin{aligned} G_{0d}(x; x') = & \frac{\kappa^2 \Gamma^2\left(\frac{D}{2}\right)}{8\pi^D} \left\{ -\frac{\ell_1(D)H^2}{(D-1)(D-2)\Delta x^{2D-4}} \right\} \\ & + \frac{\kappa^2 H^6}{8\pi^4} (aa')^2 \left\{ 2I^2[y^{-2}\mathcal{N}(y)] + I[y^{-1}\mathcal{N}(y)] \right\}, \end{aligned} \quad (188)$$

where we have plugged in the definition  $y = aa'H^2\Delta x^2$ .

Next we need to localize the divergences using relations (125) and (126), which we reproduce here,

$$\frac{1}{\Delta x^{2D-4}} \longrightarrow \frac{4\pi^{\frac{D}{2}}\mu^{D-4}\delta^D(x-x')}{2(D-3)(D-4)\Gamma\left(\frac{D}{2}-1\right)} - \frac{\partial^2}{4} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right], \quad (189)$$

$$\frac{1}{\Delta x^{2D-2}} \longrightarrow \frac{4\pi^{\frac{D}{2}}\mu^{D-4}\partial^2\delta^D(x-x')}{4(D-2)^2(D-3)(D-4)\Gamma\left(\frac{D}{2}-1\right)} - \frac{\partial^4}{32} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]. \quad (190)$$

The divergences in (187) and (188) are absorbed into the counterterms (27) by choosing the following coefficients

$$C_4^{0d} = \frac{\kappa^2 \mu^{D-4}}{32\pi^{\frac{D}{2}}} \frac{\Gamma\left(\frac{D}{2}\right)\ell_1(D)}{(D-1)(D-2)(D-3)(D-4)}, \quad (191)$$

$$\bar{C}^{0d} = (3D-8)C_4^{0d} - \frac{\kappa^2 \mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma\left(\frac{D}{2}\right)\ell_2(D)}{(D-3)(D-4)}. \quad (192)$$

Note that, since  $\ell_1(D) = C_0(b)/2(D-2)$ , where  $C_0(b)$  was defined in (7), the  $C_4^{0d}$  coefficient indeed coincides with the spin 0 part of the coefficient (30) as inferred from the flat space limit [39]. What remains is to perform explicitly the integrals in second lines of (187) and (188),

$$\begin{aligned} \frac{\beta^2}{6} \mathcal{N}_F(y) &\equiv I[y^{-2}\mathcal{N}(y)] \\ &= \frac{\beta^2}{6} \frac{\partial}{\partial \beta} \left\{ -\frac{q_0 A_0}{y} \ln\left(\frac{y}{4}\right) - \frac{q_0(A_0 + B_0)}{y} + \frac{q_1 A_1}{2} \ln^2\left(\frac{y}{4}\right) + q_1 B_1 \ln\left(\frac{y}{4}\right) \right. \\ &\quad \left. + \sum_{n=0}^{\infty} \frac{q_{n+2}}{(n+1)} y^{n+1} \left[ A_{n+2} \ln\left(\frac{y}{4}\right) + B_{n+2} - \frac{A_{n+2}}{(n+1)} \right] \right\}. \end{aligned} \quad (193)$$

$$\begin{aligned} \frac{\beta^2}{6} \mathcal{N}_G(y) &\equiv 2I[y^{-2}\mathcal{N}(y)] + I[y^{-1}\mathcal{N}(y)] \\ &= \frac{\beta^2}{6} \frac{\partial}{\partial \beta} \left\{ -\frac{q_0 A_0}{2} \ln^2\left(\frac{y}{4}\right) - q_0(2A_0 + B_0) \ln\left(\frac{y}{4}\right) \right. \\ &\quad + q_1 A_1 y \ln^2\left(\frac{y}{4}\right) \\ &\quad + q_1(2B_1 - A_1)y \ln\left(\frac{y}{4}\right) + q_1(A_1 - B_1)y \\ &\quad + \sum_{n=0}^{\infty} \frac{q_{n+2} y^{n+2}}{(n+1)(n+2)} \left[ (n+3)A_{n+2} \ln\left(\frac{y}{4}\right) + (n+3)B_{n+2} \right. \\ &\quad \left. \left. - \frac{n^2 + 6n + 7}{(n+1)(n+2)} A_{n+2} \right] \right\}. \end{aligned} \quad (194)$$

The following integrals have been used to calculate the two functions above,

$$I\left[y^n \ln\left(\frac{y}{4}\right)\right] = \frac{y^{n+1}}{(n+1)} \left[ \ln\left(\frac{y}{4}\right) - \frac{1}{(n+1)} \right] \quad (n \neq -1), \quad (195)$$

$$I\left[y^{-1} \ln\left(\frac{y}{4}\right)\right] = \frac{1}{2} \ln^2\left(\frac{y}{4}\right), \quad (196)$$

$$I\left[\ln^2\left(\frac{y}{4}\right)\right] = 2y\left[1 - \ln\left(\frac{y}{4}\right) + \frac{1}{2} \ln^2\left(\frac{y}{4}\right)\right], \quad (197)$$

$$I[y^n] = \frac{y^{n+1}}{(n+1)} \quad (n \neq -1), \quad I[y^{-1}] = \ln\left(\frac{y}{4}\right), \quad (198)$$

where the choice of integration constants is immaterial since any dependence on them drops out when the derivatives in (18) are acted with. Here we list explicitly just the first few coefficients in (193) and (194) of the most singular terms in  $y \rightarrow 0$  limit, which will be relevant for quantum-correcting Maxwell's equation (1),

$$\frac{\partial}{\partial\beta}(q_0 A_0) = \frac{(\beta-5)}{48}, \quad \frac{\partial}{\partial\beta}(q_1 A_1) = \frac{(\beta-5)}{96}, \quad (199)$$

$$\begin{aligned} \frac{\partial}{\partial\beta}(q_0 B_0) = & -\frac{5}{4\beta^2} - \frac{(\beta-2)(9\beta-86)}{2304} \\ & + \frac{(\beta-5)}{48} \left[ \psi\left(\frac{5}{2} + b_N\right) + \psi\left(\frac{5}{2} - b_N\right) + 2\gamma_E - \frac{5}{2} \right] \\ & + \frac{(\beta-6)(\beta-4)}{96} \left( \frac{-1}{2b_N} \right) \left[ \psi'\left(\frac{5}{2} + b_N\right) - \psi'\left(\frac{5}{2} - b_N\right) \right], \end{aligned} \quad (200)$$

$$\begin{aligned} \frac{\partial}{\partial\beta}(q_1 B_1) = & -\frac{43}{384} + \frac{29\beta}{640} - \frac{3\beta^2}{512} + \frac{\beta^3}{3840} \\ & + \frac{(\beta-5)}{96} \left[ \psi\left(\frac{7}{2} + b_N\right) + \psi\left(\frac{7}{2} - b_N\right) + 2\gamma_E - \frac{10}{3} \right] \\ & + \frac{(\beta-6)(\beta-4)}{192} \left( \frac{-1}{2b_N} \right) \left[ \psi'\left(\frac{7}{2} + b_N\right) - \psi'\left(\frac{7}{2} - b_N\right) \right], \end{aligned} \quad (201)$$

where  $b_N = [25/4 - \beta]^{1/2}$ . Even though the rest of the coefficients can be calculated from (182)–(184), and the remaining series in (193) and (194) summed into generalized hypergeometric functions, these will give irrelevant contributions when used in (1), so we do not do it here.

The renormalized contribution to the structure functions is

$$\begin{aligned} F_{0d}^{\text{ren}}(y) = & \left( \frac{2b-1}{b-2} \right)^2 \left\{ \frac{\kappa^2}{48\pi^2} \frac{\ln(a)}{aa'} \partial^2 i\delta^4(x-x') - \left( \frac{b-8}{b-2} \right) \frac{\kappa^2 H^2}{72\pi^2} i\delta^4(x-x') \right. \\ & - \frac{\kappa^2 H}{48\pi^2 a} \partial_0 i\delta^4(x-x') + \frac{\kappa^2}{384\pi^4} \frac{\partial^4}{aa'} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \\ & \left. + \left( \frac{b-8}{b-2} \right) \frac{\kappa^2 H^2}{576\pi^4} \partial^2 \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] - \frac{\kappa^2 H^6}{6\pi^4} (aa')^2 \mathcal{N}_F(y) \right\}, \end{aligned} \quad (202)$$

$$G_{0d}^{\text{ren}}(y) = \left( \frac{2b-1}{b-2} \right)^2 \left\{ \frac{\kappa^2 H^2}{24\pi^2} [1 - \ln(a)] i\delta^4(x-x') - \frac{\kappa^2 H^2}{192\pi^4} \partial^2 \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2 H^6}{12\pi^4} (aa')^2 \mathcal{N}_G(y) \right\}. \quad (203)$$

Since the renormalized ‘0a’ and ‘0b’ contributions (156) and (168) vanish, the ‘0d’ contribution above constitutes the full contribution to the vacuum polarization structure functions from the spin 0 part. Note that there also might appear finite contributions of the form (28) and (29).

## 5. Discussion

We have evaluated the one graviton loop contribution to the photon vacuum polarization  $i[\mu\Pi^\nu](x; x')$  on de Sitter background in the 1-parameter family of exact, de Sitter invariant gauges (3). The result is represented in terms of two structure functions  $F(x; x')$  and  $G(x; x')$ , whose relation to  $i[\mu\Pi^\nu](x; x')$  was defined in equations (18) and (22). Our graviton propagator has a gauge independent, de Sitter breaking spin two part and a gauge dependent but de Sitter invariant (for  $b > 2$ ) spin zero part. Each part makes distinct contributions to the structure functions. The spin two structure functions are (132) and (133); the corresponding spin zero results are (202) and (203).

The point of this exercise was to check the conjecture [26] that the leading secular dependence of solutions to effective field equations such as (1) might be gauge independent. The full solutions certainly contain unphysical gauge and field variable dependent information because the flat space limit does [39]. However, they also contain physical information because one can use them to construct the flat space S-matrix [44]. What we need is a filter to distinguish physical effects from unphysical ones. The S-matrix provides this in flat space, but there is as yet no analog for cosmology. There is no question that such a filter exists because astronomers are measuring *something*. Identifying what theoretical quantity represents these measurements is one of the central problems of cosmological quantum field theory [57].

Note that the vacuum polarization will play an essential role *whatever* is the outcome. If the conjecture proves to be correct then one can extract physical information from the leading secular dependence of solutions to (1). If the conjecture proves false then one must resort to some form of gauge invariant Green’s function, which would inevitably consist of the expectation value of the field (which is what solving the effective field equations gives) plus some extra terms to filter out the gauge dependence [11].

Unfortunately, we are not yet in a position to answer this fascinating question. Had this computation produced the same kinds of terms, with different numerical coefficients, as occur with the noncovariant gauge [41], it would not have been necessary to do much work to solve equation (1). In that case we would simply have read off the result for each term from the previous solutions [45, 46]. Those analyses show that the largest effects derive from the  $\kappa^2 H^2 \ln(a) i\delta^4(x-x')$  part of  $F(x; x)$ . It is interesting to note that the coefficient of this term is independent of the parameter  $b$ , although it does not agree with the noncovariant result. We find a coefficient of  $+\frac{85}{72\pi^2}$  in equation (132) whereas the coefficient was  $+\frac{1}{8\pi^2}$  in equation (136) of the noncovariant gauge analysis [41]. However, our computation also produced some quantitatively different terms at the end of expressions (132), (133), (202) and (203). Each of those terms contains multiplicative factors of  $(aa')^2$  which should compensate for the measure

of the  $d^4x'$  integration. If these terms acquire an extra logarithm they can contribute just as strongly as the local logarithms. So there seems no alternative to working out another set of complicated integrations for these new terms, multiplied by the classical field strengths for dynamical photons and for the response to charges and currents. Key questions are whether or not the leading secular effects depend on the gauge parameter  $b$ , and whether or not they agree with the noncovariant gauge. It would also be interesting to work out the spin 0 structure functions for  $b < 2$ .

One novel feature of our computation is the need for a noninvariant counterterm, despite our use of an invariant regularization and a de Sitter invariant gauge. As was explained in section 2.3, the problem arises from the time ordering of the  $h\partial A\partial A$  interaction and from the fact that the coincident graviton propagator contains a logarithmic divergence (the famous ‘tail term’) which gives rise to a factor of  $1/(D - 4)$  in dimensional regularization. There is nothing we can do about the derivative interactions of quantum gravity or the logarithmic divergence of the coincident graviton propagator on de Sitter. We *might* impose a covariant ordering prescription on the interactions but this is problematic in view of the need to keep the effective field equation (1) real and causal by using the Schwinger–Keldysh formalism [66–74].<sup>11</sup> The only alternative is to resort to the same noninvariant counterterm—the term proportional to  $\Delta C$  in expression (27)—that was needed with the noncovariant gauge [41].

One major spin-off from our work is the great simplification that was made in section (3.1) for representing the spin two part of the graviton propagator. This should facilitate checking the gauge dependence of other one loop computations involving gravitons such as the fermion self-energy [49–52] and the graviton self-energy [47, 48].

The photon propagator identities (38) and (39) and their graviton analogs (54) and (56) are strikingly similar. Equation (39) expresses the photon propagator as a longitudinal term plus the tensor  $-\frac{1}{2H^2}\partial_\rho\partial'_\sigma y(x; x')$  times the  $B$ -type propagator, whereas equation (56) gives the (de Sitter invariant part of the) graviton propagator as a collection of traces and gradients plus the tensor  $\frac{1}{2H^4}\partial_\mu\partial'_\rho y \times \partial'_\sigma\partial_\nu y$  times the (invariant part of the)  $A$ -type propagator. So the tensor structure of the graviton propagator is the square of the photon tensor structure, up to terms which drop out when contracted into the appropriate polarizations. This seems like the remarkable insight that on-shell gravitational scattering amplitudes are essentially the squares of gauge scattering amplitudes [100–103]. This has usually been thought to arise from simplifications that occur from going on shell but perhaps it is partly due to working in exact gauges.

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<sup>11</sup> Note that our wish to use the in-in formalism is only the *motivation* for employing time-ordering. Once that is done, the ultraviolet divergences of the in-in formalism are the same as those of the in-out formalism.

## Appendix A. Photon propagator functions

The propagator for minimally coupled scalar with mass  $M_S^2 = (D-2)H^2$  obeys equation (35) and consists of a de Sitter invariant function of  $y(x; x')$  called  $i\Delta_B(x; x') \equiv B(y)$ . It is closely related—by equation (36)—to the propagator for a minimally coupled scalar of  $M_S^2 = 2(D-3)H^2$ , which is called  $i\Delta_C(x; x') \equiv C(y)$ . Their expansions are,

$$B(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\frac{D}{2}-1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \sum_{n=0}^{\infty} \left[ \frac{\Gamma(n+D-2)}{\Gamma\left(n+\frac{D}{2}\right)} \left(\frac{y}{4}\right)^n - \frac{\Gamma\left(n+\frac{D}{2}\right)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}. \quad (204)$$

$$C(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\frac{D}{2}-1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \sum_{n=0}^{\infty} \left[ \frac{(n+1)\Gamma(n+D-3)}{\Gamma\left(n+\frac{D}{2}\right)} \left(\frac{y}{4}\right)^n - \frac{\left(n-\frac{D}{2}+3\right)\Gamma\left(n+\frac{D}{2}-1\right)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}. \quad (205)$$

They obviously agree in  $D = 4$  spacetime dimensions. Our work requires only the first two terms in the expansions of their derivatives,

$$B'(y) = -\frac{H^{D-2}\Gamma\left(\frac{D}{2}\right)}{4\pi^{\frac{D}{2}}} \left\{ \frac{1}{y^{\frac{D}{2}}} + \frac{(D-4)}{8y^{\frac{D}{2}-1}} + O((D-4)y^0) \right\}, \quad (206)$$

$$C'(y) = -\frac{H^{D-2}\Gamma\left(\frac{D}{2}\right)}{4\pi^{\frac{D}{2}}} \left\{ \frac{1}{y^{\frac{D}{2}}} + \frac{(D-6)(D-4)}{8(D-2)y^{\frac{D}{2}-1}} + O((D-4)y^0) \right\}. \quad (207)$$

## Appendix B. Spin two propagator functions

The propagator for a massless, minimally coupled scalar obeys equation (50) and consists of a de Sitter invariant function of  $y(x; x')$  plus a de Sitter breaking logarithm [96],

$$i\Delta_A(x; x') = A(y) + k \ln(aa'), \quad k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma\left(\frac{D}{2}\right)}. \quad (208)$$

The expansion for  $A(y)$  is,

$$A(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\frac{D}{2}-1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma\left(\frac{D}{2}+1\right)}{\frac{D}{2}-2} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} + A_1 + \sum_{n=1}^{\infty} \left[ \frac{\Gamma(n+D-1)}{n \Gamma\left(n+\frac{D}{2}\right)} \left(\frac{y}{4}\right)^n - \frac{\Gamma\left(n+\frac{D}{2}+1\right)}{\left(n-\frac{D}{2}+2\right)\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}, \quad (209)$$



where the constant  $A_1$  is,

$$A_1 = \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\psi\left(1 - \frac{D}{2}\right) + \psi\left(\frac{D-1}{2}\right) + \psi(D-1) + \psi(1) \right\}. \quad (210)$$

The full,  $D$ -dimensional series expansion for  $i\Delta_{AAAB}^{\text{inv}}(x; x')$  has been derived [95] but we here require only the de Sitter breaking part,

$$\begin{aligned} i\Delta_{AAAB}^{\text{brk}} = & \frac{kH^{-8}}{(D-2)^2} \left\{ \frac{\ln^3(4aa')}{6(D-1)^2} + \left[ \frac{\psi\left(\frac{D-1}{2}\right)}{(D-1)^2} - \frac{\left(D - \frac{3}{2}\right)D}{(D-2)(D-1)^3} \right] \ln^2(4aa') \right. \\ & + \left[ \frac{\psi'\left(\frac{D-1}{2}\right) + 2\psi^2\left(\frac{D-1}{2}\right)}{(D-1)^2} - \frac{2(2D-3)D\psi\left(\frac{D-1}{2}\right)}{(D-2)(D-1)^3} \right. \\ & \left. \left. + \frac{3}{(D-2)^2} \right] \ln(4aa') \right\}. \end{aligned} \quad (211)$$

It is simple to act the scalar d'Alembertian on functions of  $\ln(aa')$ ,

$$\square f(\ln(aa')) = -H^2 \{ (D-1)f'(\ln(aa')) + f''(\ln(aa')) \}. \quad (212)$$

Because  $i\Delta_{AAAB}^{\text{inv}} = i\Delta_{AAAB} - i\Delta_{AAAB}^{\text{brk}}$  we can use relations (46) and (47), (211) and (212) to conclude,

$$\begin{aligned} [\square - (D-2)H^2] i\Delta_{AAAB}^{\text{inv}}(x; x') = & i\Delta_{AAB}(x; x') - \frac{kH^{-4}}{(D-2)(D-1)} \\ & \times \left\{ \frac{1}{2} \ln^2(4aa') + \left[ 2\psi\left(\frac{D-1}{2}\right) - \left(\frac{D-1}{D-2}\right) \right] \ln(4aa') + \text{constant} \right\}, \end{aligned} \quad (213)$$

where the constant is,

$$\psi'\left(\frac{D-1}{2}\right) + 2\psi^2\left(\frac{D-1}{2}\right) - \left(\frac{D-1}{D-2}\right)\psi\left(\frac{D-1}{2}\right) + \left(\frac{D-1}{D-2}\right) + \frac{2D-3}{(D-2)^2(D-1)^2}. \quad (214)$$

This completes the demonstration of equation (55).

Because our structure functions are at most quadratically divergent we require only the leading two terms of  $J(y)$  to be kept in  $D$  dimensions. Rather than acting the derivatives of (213) on the complicated series expansion of  $i\Delta_{AAAB}(x; x')$  that has been derived [95] it is simplest to construct  $J(y)$  by integrating the differential equation  $i\Delta_{AAB}(x; x')$  obeys,

$$\square i\Delta_{AAB}(x; x') = i\Delta_{AB}(x; x') = \frac{[i\Delta_B(x; x') - i\Delta_A(x; x')]}{(D-2)H^2}. \quad (215)$$

From the expansions (204) and (209) one finds,

$$i\Delta_{AB}(x; x') = H^2 \left\{ \frac{\alpha_1}{y^{\frac{D}{2}-2}} + \alpha_2 + \frac{\beta_1}{y^{\frac{D}{2}-3}} + \beta_2 y + O((D-4)y^2) \right\}, \quad (216)$$

where the coefficients are,

$$\alpha_1 = -\frac{H^{D-6}}{4\pi^{\frac{D}{2}}} \frac{\Gamma\left(\frac{D}{2}\right)}{(D-2)(D-4)}, \quad (217)$$

$$\alpha_2 = \frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-2)}{\Gamma\left(\frac{D}{2}\right)} \left\{ \frac{2}{D-4} + \psi\left(3 - \frac{D}{2}\right) - \psi\left(\frac{D-1}{2}\right) - \psi(D-2) - \psi(1) \right\}, \quad (218)$$

$$\beta_1 = -\frac{H^{D-6}}{16\pi^{\frac{D}{2}}} \frac{\Gamma\left(\frac{D}{2} + 1\right)}{(D-2)(D-6)}, \quad (219)$$

$$\beta_2 = -\frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-2)}{2\Gamma\left(\frac{D}{2}\right)}. \quad (220)$$

Expression (212) defines how to integrate the de Sitter breaking  $\ln(aa')$  term on the right hand side of (215). The de Sitter invariant analog is,

$$\square g(y) = H^2 \left[ (4y - y^2)g''(y) + D(2 - y)g'(y) \right]. \quad (221)$$

Equation (221) suggests that we can solve  $\square g(y) = h(y)$  as a double integral with respect to  $y$ ,

$$\square g(y) = h(y) \quad \Rightarrow \quad g(y) = \frac{1}{H^2} I \left[ \frac{I \left[ (4y - y^2)^{\frac{D}{2}-1} h(y) \right]}{(4y - y^2)^{\frac{D}{2}}} \right]. \quad (222)$$

However, relation (222) is neither tractable in  $D$  dimensions, nor even correct. The problem with tractability is obvious from the  $D$ -dependent powers of  $(4y - y^2)$ . The problem with validity derives from the impossibility of avoiding poles at either  $y = 0$  or  $y = 4$  that would introduce delta functions into relation (221) which are not present in the desired source function  $h(y)$ .

We solve the first problem by extracting the two leading powers of  $y$  from the solution and then taking  $D = 4$  on the remainder,

$$\begin{aligned} i\Delta_{AAB}(x; x') &= -\frac{\alpha_1}{4(D-6)} \frac{1}{y^{\frac{D}{2}-3}} + \frac{\alpha_2}{2D} y - \frac{[(D+4)\alpha_1 + 16\beta_1]}{96(D-8)} \frac{1}{y^{\frac{D}{2}-4}} \\ &+ \frac{[\alpha_2 + 2\beta_2]}{8(D+2)} y^2 + \frac{kH^{-4}}{(D-1)(D-2)} \left[ \frac{1}{2} \ln^2(aa') - \frac{\ln(aa')}{D-1} \right] + \Delta g(x; x'). \end{aligned} \quad (223)$$

The remainder  $\Delta g(x; x')$  obeys,

$$\frac{\square}{H^2} \Delta g = \frac{(D+6)[(D+4)\alpha_1 + 16\beta_1]}{384y^{\frac{D}{2}-4}} + \frac{(D+1)(\alpha_2 + 2\beta_2)y^2}{4(D+2)} + O(D-4), \quad (224)$$

$$\longrightarrow \frac{5H^{-2}}{384\pi^2} \left[ \ln\left(\frac{y}{4}\right) - \frac{143}{60} + 2\ln(2) + 2\gamma \right] y^2. \quad (225)$$

We solve the second problem by first setting the lower limit to  $y = 4$  on the inner integral of (222), which means there are no poles at  $y = 4$ . Then the poles at  $y = 0$  are cancelled by adding a constant times the  $D = 4$  limit of the  $i\Delta_A(x; x')$ . Because expression (225) consists of terms proportional to  $y^2$  and to  $y^2 \ln\left(\frac{y}{4}\right)$  we only need solutions for these two sources,

$$\frac{\square}{H^2} g_1(x; x') = y^2 \quad \Rightarrow \quad g_1(x; x') = -\frac{3}{5}y - \frac{1}{10}y^2 - \frac{8}{5} \ln(aa'), \quad (226)$$

$$\begin{aligned} \frac{\square}{H^2} g_2(x; x') = y^2 \ln\left(\frac{y}{4}\right) &\Rightarrow g_2(x; x') = -\frac{8}{5} \left[ \text{Li}_2\left(\frac{y}{4}\right) + \ln\left(1 - \frac{y}{4}\right) \ln\left(\frac{y}{4}\right) \right] \\ &+ \left[ \frac{\frac{4}{5}y}{4-y} - \frac{3}{5}y - \frac{1}{10}y^2 \right] \ln\left(\frac{y}{4}\right) + \frac{67}{100}y + \frac{7}{100}y^2 + \frac{18}{25} \ln(aa'). \end{aligned} \quad (227)$$

Here  $\text{Li}_2(z)$  is the dilogarithm function,

$$\text{Li}_2(z) \equiv - \int_0^z dt \frac{\ln(1-t)}{t}. \quad (228)$$

The de Sitter breaking factors of  $\ln(aa')$  in expressions (223), (226) and (227) obviously do not belong in  $J(y)$ ,

$$\begin{aligned} J(y) = & -\frac{\alpha_1}{4(D-6)} \frac{1}{y^{\frac{D}{2}-3}} + \frac{\alpha_2}{2D} y - \frac{[(D+4)\alpha_1 + 16\beta_1]}{96(D-8)} \frac{1}{y^{\frac{D}{2}-4}} + \frac{[\alpha_2 + 2\beta_2]}{8(D+2)} y^2 \\ & + \text{constant} + \frac{5H^{-2}}{384\pi^2} \left[ -\frac{143}{60} + 2\ln(2) + 2\gamma \right] \times g_1(y) + \frac{5H^{-2}}{384\pi^2} \times g_2(y). \end{aligned} \quad (229)$$

Recall that the four  $D$ -dependent constants  $\alpha_i$  and  $\beta_i$  are given in expressions (217) and (220). The functions  $g_1(y)$  and  $g_2(y)$  are just the de Sitter invariant parts of (226) and (227),

$$g_1(y) \equiv -\frac{3}{5}y - \frac{1}{10}y^2, \quad (230)$$

$$\begin{aligned} g_2(y) \equiv & -\frac{8}{5} \left[ \text{Li}_2\left(\frac{y}{4}\right) + \ln\left(1 - \frac{y}{4}\right) \ln\left(\frac{y}{4}\right) \right] \\ & + \left[ \frac{\frac{4}{5}y}{4-y} - \frac{3}{5}y - \frac{1}{10}y^2 \right] \ln\left(\frac{y}{4}\right) + \frac{67}{100}y + \frac{7}{100}y^2. \end{aligned} \quad (231)$$

### Appendix C. Spin zero propagator functions

The spin-0 scalar structure function  $i\Delta_{WNN}(x; x')$ , introduced in (135) is constructed out of two scalar propagators,  $i\Delta_W(x; x')$  and  $i\Delta_N(x; x')$  [95]. They satisfy propagator equations for a massive minimally coupled scalar,

$$[\square + DH^2] i\Delta_W(x; x') = \frac{i\delta^D(x - x')}{\sqrt{-g(x)}}, \quad (232)$$

$$[\square - (\beta - D)H^2]i\Delta_N(x; x') = \frac{i\delta^D(x - x')}{\sqrt{-g(x)}}. \quad (233)$$

The  $W$ -type scalar propagator has a tachyonic mass  $M_S^2 = -DH^2$ , and necessarily breaks de Sitter invariance. We split it into a de Sitter invariant and a de Sitter breaking part,

$$i\Delta_W(x; x') = W(y) + \delta W(y, u, v). \quad (234)$$

The de Sitter invariant part is

$$\begin{aligned} W(y) = & \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\frac{D}{2} - 1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma\left(\frac{D}{2} + 2\right)}{\left(\frac{D}{2} - 2\right)\left(\frac{D}{2} - 1\right)} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} \right. \\ & + \frac{\Gamma\left(\frac{D}{2} + 3\right)}{2\left(\frac{D}{2} - 3\right)\left(\frac{D}{2} - 2\right)} \left(\frac{4}{y}\right)^{\frac{D}{2}-3} + W_1 + W_2\left(\frac{y-2}{4}\right) \\ & \left. + \sum_{n=2}^{\infty} \left[ \frac{\Gamma\left(n + \frac{D}{2} + 2\right)\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}}{\left(n - \frac{D}{2} + 2\right)\left(n - \frac{D}{2} + 1\right)(n+1)!} - \frac{\Gamma(n+D)\left(\frac{y}{4}\right)^n}{n(n-1)\Gamma\left(n + \frac{D}{2}\right)} \right] \right\}, \quad (235) \end{aligned}$$

where

$$W_1 = \frac{\Gamma(D+1)}{\Gamma\left(\frac{D}{2} + 1\right)} \left\{ \frac{D+1}{2D} \right\}, \quad (236)$$

$$W_2 = \frac{\Gamma(D+1)}{\Gamma\left(\frac{D}{2} + 1\right)} \left\{ \psi\left(-\frac{D}{2}\right) - \psi\left(\frac{D+1}{2}\right) - \psi(D+1) - \psi(1) \right\}. \quad (237)$$

The de Sitter breaking part of the  $W$ -type propagator is

$$\delta W(y, u, v) = k \left\{ (D-1)^2 e^u + \left(\frac{D-1}{2}\right)(2-y)u - 2 \cosh(v) \right\}, \quad (238)$$

$$k = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma\left(\frac{D}{2}\right)}. \quad (239)$$

In general, the  $N$ -type propagator contains de Sitter breaking parts as well, but for the choice of gauge in this work  $b > 2$  ( $\beta > D$ ), its mass is  $M_S^2 = (\beta - D)H^2 > 0$ . Therefore, it is completely de Sitter invariant,  $i\Delta_N(x; x') = N(y)$ ,

$$\begin{aligned}
N(y) = & \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\frac{D}{2} - 1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right. \\
& - \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(1 - \frac{D}{2}\right)}{\Gamma\left(\frac{1}{2} + b_N\right)\Gamma\left(\frac{1}{2} - b_N\right)} \sum_{n=0}^{\infty} \left[ \frac{\Gamma\left(\frac{3}{2} + b_N + n\right)\Gamma\left(\frac{3}{2} - b_N + n\right)}{\Gamma\left(3 - \frac{D}{2} + n\right)(n+1)!} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right. \\
& \left. \left. - \frac{\Gamma\left(\frac{D-1}{2} + b_N + n\right)\Gamma\left(\frac{D-1}{2} - b_N + n\right)}{\Gamma\left(\frac{D}{2} + n\right)n!} \left(\frac{y}{4}\right)^n \right] \right\}, \tag{240}
\end{aligned}$$

where

$$b_N = \sqrt{\frac{(D-1)}{4} \left[ (D-1) + \frac{8}{2-b} \right]} = \sqrt{\left(\frac{D+1}{2}\right)^2 - \beta}. \tag{241}$$

Note that even though the terms in the series in (240) cancel in  $D = 4$ , the whole series is multiplied by a factor diverging as  $1/(D-4)$  giving a finite contribution in this limit.

The spin 0 scalar structure function in (135) is solved for by inverting (137) and (139), the solution of which is

$$i\Delta_{WNN}(x; x') = \frac{i\Delta_{NN}(x; x') - i\Delta_{WN}(x; x')}{\beta H^2}, \tag{242}$$

where

$$i\Delta_{WN}(x; x') = \frac{i\Delta_N(x; x') - i\Delta_W(x; x')}{\beta H^2}, \tag{243}$$

$$i\Delta_{NN}(x; x') = -\frac{1}{2b_N H^2} \frac{\partial}{\partial b_N} i\Delta_N(x; x') = \frac{1}{H^2} \frac{\partial}{\partial \beta} i\Delta_N(x; x'). \tag{244}$$

The functions in (242) and (244) we can split into de Sitter invariant and de Sitter breaking parts,

$$i\Delta_{WNN}(x; x') = WNN(y) + \delta WNN(y, u, v), \tag{245}$$

$$i\Delta_{WN}(x; x') = WN(y) + \delta WN(y, u, v), \tag{246}$$

$$i\Delta_{NN}(x; x') = NN(y) + \delta NN(y, u, v). \tag{247}$$

The de Sitter breaking parts receive contribution only from the de Sitter breaking part of the  $W$ -type propagator (238), since the  $N$ -type propagator is de Sitter invariant ( $\delta N(y, u, v) = 0$ ),

$$\delta WNN(y, u, v) = \frac{\delta W(y, u, v)}{\beta^2 H^4}, \tag{248}$$

$$\delta WN(y, u, v) = -\frac{\delta W(y, u, v)}{\beta H^2}, \tag{249}$$

$$\delta NN(y, u, v) = 0. \tag{250}$$

The de Sitter invariant parts are

$$WNN(y) = \frac{NN(y) - WN(y)}{\beta H^2}, \tag{251}$$

$$WN(y) = \frac{N(y) - W(y)}{\beta H^2}, \quad (252)$$

$$NN(y) = \frac{\partial}{\partial \beta} \frac{N(y)}{H^2}. \quad (253)$$

The de Sitter breaking part in (234) of the W-type scalar propagator is not its homogeneous part, but rather it satisfies

$$\left[ \frac{\square}{H^2} + D \right] \delta W(y, u, v) = -\frac{k}{2} (D^2 - 1)(2 - y) \equiv w(y), \quad (254)$$

which can be calculated by acting with a d'Alembertian on (238). Furthermore,

$$\left[ \frac{\square}{H^2} + D \right] w(y) = 0. \quad (255)$$

Using these relations (254) and (255) together with (242) and (244) we can derive useful identities for d'Alembertians acting on de Sitter invariant functions (251) and (253),

$$\frac{\square}{H^2} WNN(y) = (\beta - D) WNN(y) + \frac{WN(y)}{H^2} - \frac{w(y)}{\beta^2 H^4}, \quad (256)$$

$$\frac{\square}{H^2} WN(y) = (\beta - D) WN(y) + \frac{W(y)}{H^2} + \frac{w(y)}{\beta H^2}, \quad (257)$$

$$\frac{\square}{H^2} NN(y) = (\beta - D) NN(y) + \frac{N(y)}{H^2}. \quad (258)$$

It is convenient to define a dimensionless function which is a part of the infinite sum in the  $N$ -type propagator (240),

$$\begin{aligned} \bar{N}_i(y) = & - \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(1 - \frac{D}{2}\right)}{\Gamma\left(\frac{1}{2} + b_N\right)\Gamma\left(\frac{1}{2} - b_N\right)} \sum_{n=i}^{\infty} \left[ \frac{\Gamma\left(\frac{3}{2} + b_N + n\right)\Gamma\left(\frac{3}{2} - b_N + n\right)}{\Gamma\left(3 - \frac{D}{2} + n\right)(n+1)!} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right. \\ & \left. - \frac{\Gamma\left(\frac{D-1}{2} + b_N + n\right)\Gamma\left(\frac{D-1}{2} - b_N + n\right)}{\Gamma\left(\frac{D}{2} + n\right)n!} \left(\frac{y}{4}\right)^n \right], \end{aligned} \quad (259)$$

where the sum starts at  $n = i$ , and similarly for the  $NN$ -type propagator,

$$\overline{NN}_i(y) = \frac{\partial}{\partial \beta} \bar{N}_i(y). \quad (260)$$

The  $D \rightarrow 4$  limit of function (259) is

$$\begin{aligned} \bar{N}_i(y) = & \sum_{n=i}^{\infty} \frac{\Gamma\left(\frac{3}{2} + b_N + n\right)\Gamma\left(\frac{3}{2} - b_N + n\right)}{\Gamma\left(\frac{1}{2} + b_N\right)\Gamma\left(\frac{1}{2} - b_N\right)(n+1)!n!} \left(\frac{y}{4}\right)^n \\ & \times \left\{ \ln\left(\frac{y}{4}\right) + \frac{1}{n+1} - 2\psi(n+2) + \psi\left(\frac{3}{2} + b_N + n\right) \right. \\ & \left. + \psi\left(\frac{3}{2} - b_N + n\right) \right\}, \end{aligned} \quad (261)$$

where

$$\beta = \frac{4b - 2}{b - 2}, \quad b_N = \sqrt{\frac{3(14 - 3b)}{4(2 - b)}} = \sqrt{\frac{25}{4} - \beta}. \quad (262)$$

No terms arise in this limit from expanding the  $D$ -dependence of  $b_N$  or  $\beta$  in (259) (they all cancel). Therefore, the definition of  $\overline{NN}_i$  (260) is still valid where now (261) is differentiated.

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