American Economic Journal: Microeconomics 2015, 7(4): 233–251
http://dx.doi.org/10.1257/mic.20140011

Beneficial Long Communication in the Multiplayer Electronic Mail Game†

By Kris De Jaegher*

In the two-player electronic mail game (EMG), as is well-known, the probability of collective action is lower the more confirmations and reconfirmations are made available to players. In the multiplayer EMG, however, we show players may coordinate on equilibria where they require only few of the available confirmations from each other to act. In this case, increasing the number of available confirmations may either create equilibria with positive probability of collective action when none existed before, or may increase the probability of collective action, if equilibria with positive probability of collective action already existed for fewer available confirmations. (*JEL* C70, D71, D82, D83)

Game theorists have long stressed the importance of common knowledge for solving coordination problems (e.g., Geanakoplos 1992, and Chwe 2003). Suppose that Colin and Rowena love to go to see a good movie together, but hate to go to a movie by themselves (whether good or bad), and hate to go to a bad movie (whether alone or together). When Colin now finds out that there is a good movie on, Colin and Rowena will only end up at this movie together when having a sufficient degree of *common knowledge* that they will meet to see the good movie; it is not only important that Rowena knows that Colin knows that there is a good movie on, but Colin must also know that Rowena knows that Colin knows this, and Rowena must know that Colin knows that Rowena knows that Colin knows this, and so on. One way to generate common knowledge would then seem to be that Colin and Rowena exchange confirmations, and confirmations of confirmations, where an indefinite sending back and forth of confirmations would then lead to common knowledge.

Yet, as pointed out by Rubinstein (1989) for his two-player electronic mail game—henceforth, EMG—as soon as there is the smallest chance that such confirmations get lost, the longer Colin and Rowena communicate, the less likely it is they ever meet at the cinema; if they communicate indefinitely, they will even never meet. When Rowena does not receive a first message from Colin, even though a

*Utrecht University School of Economics, Utrecht University, Kriekenpitplein 21-22, Utrecht, 3584 EC, The Netherlands (e-mail: k.dejaegher@uu.nl). I would like to thank one anonymous referee for helpful comments. Further thanks are due to Michal Bojanowski, Vincent Buskens, Rense Corten, Britta Hoyer, Jurjen Kamphorst, Stephanie Rosenkranz, and Bastian Westbrock. Special thanks are due to Fernando Vega Redondo for insightful discussion.

†Go to http://dx.doi.org/10.1257/mic.20140011 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

message could have been sent and gone lost, she considers it likely that there is no good movie on, and does not go. For this reason, Colin does not go when not receiving a confirmation of his message. This in turn induces Rowena not to act when not receiving a confirmation of her confirmation. Only when a sender's message is an ultimate message that cannot be confirmed, does the sender accept that it is likely that it was received. Yet, the longer the communication process, the less likely such an ultimate message is ever sent. Furthermore, if there is no deadline on communication, no such ultimate message exists, and coordination never takes place. It follows that Colin and Rowena are best off if communication is restricted, such that either only a single initial message can be sent, or a message and a confirmation.[1] The paradox is thus that a process that generates ever more higher-order knowledge statements, and would therefore seem to bring one closer to common knowledge, in fact reduces the probability of collective action.

The purpose of this paper is to show that this paradox need not exist in the multiplayer EMG, in that in such variants of the two-player EMG longer communication may *increase* the probability of collective action. Intuitively, let a group of activists who are active on Twitter, independently from one another find out that there is an opportunity for a successful revolt against a repressive government, conditional on all activists coordinating on revolting. First, each activist sends a tweet to each other activist. Next, each activist who receives such an initial tweet, in turn re-tweets the message to everyone else. And so on, where the number of tweets that can be generated grows exponentially as the communication process is extended. By the same reasoning as in the two-player EMG, it continues to be the case that as long as tweets are error-prone, activists only revolt when receiving them up to the end of the communication process. Looking then at one string of tweets and re-tweets in isolation, it continues to be true that the longer activists exchange tweets, the less likely they are to coordinate based on receipt of tweets in this individual string. Yet, if each activist considers receipt of a single (pen)ultimate tweet as sufficient for revolting, this is more than compensated by the fact that a longer communication process generates more strings of tweets and re-tweets. The additional strings of tweets created turn into additional back-up communication channels, which ensure that coordination is still possible even if some tweets get lost. Thus, whereas coordination in the two-player EMG benefits from strict communication rules, in the multiplayer EMG players may benefit if they are allowed to communicate in an unlimited way for a long time.

Two authors treat variants of the multiplayer EMG, but obtain results that confirm those of the two-player EMG.[2] Morris (2002a, 2002b) treats a multiplayer

[1] As pointed out by Geanakoplos (1992), this may explain why in communication between military personnel, a message is followed by a single acknowledgement, and nothing more. As an additional example, one may add the *postal acceptance rule*, saying that a contract is valid at the time when a confirmation letter is posted, and is valid even when this confirmation letter happens to get lost. The motivation is that otherwise, the parties in the contract could continue to ask confirmations from each other indefinitely (Langdell 1999, 5).

Chwe (1995) similarly investigates optimal communication processes in some simple games. His focus is on whether it is optimal to use confirmations to counter noise, or to use redundancy (i.e., repeating the same message several times). De Jaegher (2005) investigates in which circumstances positive and negative acknowledgements are efficient in some simple games, and whether they are also used in equilibrium.

[2] The following papers modify the EMG in other ways than introducing multiple players. Dulleck (2007) shows that boundedly rational players with imperfect recall can still coordinate on requiring few messages; Strzalecki

EMG where a threshold of players needs to jointly act to achieve collective action, and where at each communication stage a new sample of players smaller than the threshold is randomly picked. Either all players in this sample receive a confirmation from the sample of players taken at the previous stage, or none of them does. Corroborating Rubinstein's (1989) result, Morris shows that if the communication process continues indefinitely, the players do not achieve collective action. Coles (2009) studies a multiplayer EMG with a single informed player where uninformed players do not send confirmations to one another, and where the informed player only sends confirmations to the uninformed players when having received confirmations from all uninformed players. His results also confirm Rubinstein's. The reason for the difference between our results and those of Morris and of Coles lies in the restrictions that these authors put on their communication protocols, such that not every possible level and type of higher-order knowledge is produced. These restrictions eliminate the effect of having multiple alternative communication channels that plays a key role in our paper.[3]

The paper is structured as follows. Section I introduces our multiplayer EMG, and Section II characterizes the Pareto-best pure-strategy equilibria where collective action takes place with positive probability. Section III identifies circumstances in which extending the deadline is beneficial to players. We end with a discussion in Section IV.

I. Multiplayer Electronic Mail Game

Our multiplayer EMG is played by a set of players $\{1,2,\ldots,I\}$, with $I \geq 3$.[4] We use symbols i, j, k, l to refer to generic players. In the game, the following compound lottery takes place from stages 0 to z. At stage 0, Nature with probability p chooses state r (= there is an opportunity for collective action), and with probability $(1 - p)$ chooses state q (= there is no opportunity). In state q, no player receives messages. In state r, at stage 1, Nature (n) independently with probability $(1 - \varepsilon)$ lets each player i receive a *message string* $n \rightarrow i$ (denoting a message from Nature directed to player i), and with probability ε does not let this player receive a message string. We typically assume ε to be small. When any player i receives message string $n \rightarrow i$ at stage 1, Nature at stage 2 independently with probability $(1 - \varepsilon)$ lets each player $j \neq i$ receive message string $n \rightarrow i \rightarrow j$ (denoting that j knows that i

(2014) argues the same by applying a level-k reasoning model. Dimitri (2004) shows that when messages from different players get lost with different probabilities, coordinated action can still occur. Binmore and Samuelson (2001) and De Jaegher (2008) investigate the effect of communication being voluntary instead of automatic.

[3] Recently, Coles and Shorrer (2012) treat a variant of Coles (2009), where again uninformed players do not communicate with each other, and where the event of a message getting lost does not occur independently from other messages getting lost. As shown by the authors, because of this correlation, players are able to coordinate on equilibria where only a few messages are required.

[4] Most of our results (Lemma 1, Propositions 1 and 2) also apply for $I = 2$. This is because, contrary to what is the case in Rubinstein's (1989) two-player EMG, in a two-player version of our game, *both* players may receive a message from Nature. Because of this, starting from $z \geq 2$, the communication process generates exactly two ultimate message strings, and never more. The difference between the case $I \geq 3$ and the case $I = 2$ is that in the latter case, the lower bound on collective action that we calculate in the proof of Proposition 3, unambiguously *decreases* in z for $z \geq 3$, contrary to what is the case for the former case. Moreover, it can be directly calculated that for $I = 2$, the true probability of collective action decreases in z when $z \geq 3$. For $I \geq 3$ this need not be the case.

knows that state r occurs), and with probability ε does not let j receive this message. When any player j receives $n \rightarrow i \rightarrow j$ at stage 2, Nature at stage 3 independently with probability $(1 - \varepsilon)$ lets each player $k \neq j$ (including $k = i$) receive message string $n \rightarrow i \rightarrow j \rightarrow k$ (denoting that k knows that j knows that i knows that state r occurs), and with probability ε does not let k receive this message string. In this manner, each individual who received a message string, denoted as m, continues to be forwarded until it gets lost, or until the ultimate communication stage z is reached.

At stage $(z + 1)$, the players simultaneously choose an action from the action set $\{Q, R\}$, where R means acting ("revolting") and Q means not acting ("quitting"). The players' payoffs are summarized in Table 1. Each player obtains payoff 0 when doing Q, whatever the state, and whatever other players do. Each player incurs loss L when doing R in state q whatever other players do, and when doing R in state r when not all other players do R as well. Each player obtains benefit H when doing R in state r when *all* other players do R as well. We assume that $L > H > 0$, and typically consider L to be large.[5]

Appendix A defines the information structure, strategies, and equilibrium concept applied. In short, define by $\mathfrak{M}_i$ the collection[6] of all message string sets that player i can observe, with typical element M_i. A pure strategy α_i for player i assigns to each M_i in $\mathfrak{M}_i$ either action Q or R. We focus on pure-strategy Bayesian Nash equilibria where collective action takes place with positive probability, in short-referred to as *collective action equilibria*.[7] Within this class of equilibria, we restrict ourselves to collective action equilibria where equilibrium strategies meet the following restriction:

(R1) The strategy α_i of every player i is such that if for a $M_i \in \mathfrak{M}_i$ we have $\alpha_i(M_i) = R$, then for any $M_i' \in \mathfrak{M}_i$ with $M_i' \supset M_i$, we also have $\alpha_i(M_i') = R$; if for a $M_i \in \mathfrak{M}_i$ we have $\alpha_i(M_i) = Q$, then for any $M_i'' \in \mathfrak{M}_i$ with $M_i'' \subset M_i$, we also have $\alpha_i(M_i'') = Q$.

(R1) means excluding inefficient equilibria where players coordinate on playing Q conditional on the receipt of particular message strings. For small noise, such equilibria are obviously inefficient, as collective action is then unlikely to take place. Focusing on equilibria meeting (R1), a candidate equilibrium strategy of any player i can now be described in a more concise manner. *First*, to describe a strategy α_i it suffices to characterize the collection of all message string sets leading i to play R, where we call an individual message string set in this collection a *sufficient set*; all nonincluded message string sets then necessarily lead the player to play Q. Formally, for a given strategy α_i, define the set $\mathfrak{M}_i^R = \{M_i \in \mathfrak{M}_i: \alpha_i(M_i) = R\}$.

[5] The version of the EMG treated here is a multiplayer version of the two-player EMG of Morris and Shin (1997), which without loss of generality, differs slightly from Rubinstein's (1989) original game.

[6] "Collection" and "family" are used as synonyms for set, to avoid the expression "set of sets."

[7] The multiplayer EMG always has a pure-strategy Bayesian Nash equilibrium where players never act, and has mixed equilibria, where upon the receipt of messages players randomize over acting or not. In all of the mentioned equilibria, each player's expected payoff is zero. Our focus on collective action equilibria is based on the premise that players can coordinate on Pareto-superior equilibria.

TABLE 1—PAYOFFS OF INDIVIDUAL PLAYER AS A FUNCTION OF STATES AND ACTIONS; $L > H > 0, p < \frac{1}{2}$

	State q: Prob. $(1 - p)$	State r: Prob. p	
		One or more others play Q	All others play R
Action Q	0	0	0
Action R	$-L$	$-L$	H

Then any element of $\mathfrak{M}_i^R$, with typical element denoted as S_i, is a sufficient set for i. *Second*, by (R1), if i acts when receiving sufficient set S_i, he also acts when receiving any $S_i' \in \mathfrak{M}_i^R$ such that $S_i' \supset S_i$. For a given set S_i' leading i to act, we may therefore succinctly describe part of i's strategy by focusing only on the minimal sufficient subset of message strings, denoted $S_i^{\min}$, leading i to act (with $S_i^{\min} \subset S_i'$). Any superset of $S_i^{\min}$ then necessarily leads i to act as well. Formally, a *minimal sufficient set* is any $S_i^{\min} \in \mathfrak{M}_i^R$ such that there does not exist $S_i'' \in \mathfrak{M}_i^R$ with $S_i'' \subset S_i^{\min}$. Player i's strategy is now fully described by a collection of minimal sufficient sets $\mathcal{S}_i^{\min} = \{S_i^{\min}, \ldots, S_i'^{\min}, S_i''^{\min}, \ldots\}$, and any collective action equilibrium can be described as a profile of collections of minimal sufficient sets. The next section characterizes the Pareto-best collective action equilibrium.[8]

II. Pareto-Best Collective Action Equilibrium of the Multiplayer EMG

Assuming small noise and a large cost of acting in the wrong circumstances, Lemma 1 derives a necessary condition for a collective action equilibrium to exist. This condition says that players' minimal sufficient sets should consist exclusively of (pen)ultimate message strings. In other words, the well-known result from the two-player EMG that in order to act, players require confirmations up to the ultimate communication stage, generalizes to multiplayer EMGs. All proofs are relegated to Appendix B.

LEMMA 1: *For sufficiently large (L/H) and sufficiently small ε, in any collective action equilibrium of the multiplayer EMG, a player who does not receive (pen)ultimate message strings does not act.*

Intuitively, just as in Rubinstein's (1989) two-player EMG, if the message string set M_i received by player i does not contain message strings received later than at stage $(z - 2)$, this can never be sufficient to i for acting. If M_i would be sufficient to i anyway, given that the loss of acting in the wrong circumstances is large, when

[8] For a full characterization of all possible collective action equilibria, see Theorem 1 in the working paper version of this paper (De Jaegher 2015). Define as a *broom set* any subset of penultimate message strings, along with all ultimate confirmations of these message strings. Then to any *Sperner family* of such broom sets, corresponds a collective action equilibrium in the following way: for each broom set in any given Sperner family, the set of all (pen)ultimate message strings that any given player i can receive in a broom set, constitute a minimal sufficient set for player i. The Pareto-best collective action equilibrium, and the Pareto-worst collective action equilibrium (where each player only acts when receiving each possible (pen)ultimate message string), are two extreme cases in the set of all collective action equilibria. An attractive feature of the Pareto-best collective action equilibrium is that it exists even if players are not able to observe the string of players through which an individual message was forwarded; they just need to be able to observe whether they receive at least one (pen)ultimate message string.

available, only confirmations of the message strings in M_i would be sufficient to $j \neq i$. But, given that i is now able to receive confirmations of these confirmations received by j, i will only act when receiving these, so that M_i cannot be sufficient itself to i. It follows that players can only consider (pen)ultimate message strings as sufficient for acting.

Proposition 1 shows that parameters exist such that the multiplayer EMG has collective action equilibria, by showing the existence of a collective action equilibrium where any *single* ultimate or penultimate message string that a player may receive constitutes a minimal sufficient set to the player. This is then also the Pareto-best collective action equilibrium. Intuitively, if player i receives a penultimate message string m and believes that receipt of an ultimate confirmation of this message string is sufficient to all other players, then for sufficiently small noise m is sufficient to i, as by assumption i is not able to receive confirmations of the ultimate confirmations. At the same time, if player j receives an ultimate message string m' which is a confirmation of m, and believes that m is sufficient to i and that a confirmation of m is sufficient to each player $k \neq i, j$, then for sufficiently small noise, m' is sufficient to j. Since in the Pareto-best equilibrium collective action should be as likely as possible, in such an equilibrium each player's collection of minimal sufficient sets coincides with the collection of all (pen)ultimate message strings he can receive.

PROPOSITION 1: *For sufficiently large (L/H) and sufficiently small ε, in the Pareto-best collective action equilibrium of the multiplayer EMG, each player acts when receiving at least a single (pen)ultimate message string, and does not act when not receiving any (pen)ultimate message strings.*

Yet, it should be noted that players may also lock each other into inefficient equilibria. Indeed, for sufficiently small noise,[9] the Pareto-worst collective action equilibrium is one where each player only acts when receiving *each* possible (pen)ultimate message string. Intuitively, if player i expects that all other players only act when receiving each (pen)ultimate message string, it is a best response for player i to require each (pen)ultimate message string herself. This in turn induces the best response of the other players to require all (pen)ultimate message strings.

III. Beneficial Long Communication

We now show that longer communication may make players better off in the multiplayer EMG. There are two ways in which this is true. First, as shown in Proposition 2, it may be that a collective action equilibrium only exists if $z \geq 2$, so that talking for a longer time is a necessary condition for benefits of collective action to be possible. Second, as shown in Proposition 3, if a collective action equilibrium does exist for $z = 1$, as long as players play the Pareto-best collective action

[9] Such inefficient equilibria exist only for lower levels of noise than required for Proposition 1 (De Jaegher forthcoming). This is because the more message strings a player requires, the more ultimate confirmations of these message strings need to arrive for collective action to take place. In inefficient equilibria, a player who receives all required message strings thus faces more risk.

equilibrium, they may be better off with any higher z than they are with $z = 1$, because the probability of collective action is larger for higher z. We start with Proposition 2, which shows that parameters of the multiplayer EMG exist such that for any finite $z \geq 2$, there is a collective action equilibrium, whereas for $z = 1$, such an equilibrium does not exist. Intuitively, to a player who receives just enough message strings to make collective action possible, the fact that z is high means that there are more ways in which other players could still have received a sufficient number of message strings.

PROPOSITION 2: *For the multiplayer EMG, consider parameters such that the Pareto-best collective action equilibrium where each player acts when receiving at least one (pen)ultimate message string exists for* $z = z^*$, *with* $z^* \geq 2$. *Then within these parameters, for intermediate levels of noise, no collective action equilibrium exists for* $z = 1$.

The result in Proposition 2 may be understood using Monderer and Samet's (1989) concept of common π-belief. An event is common π-belief if everyone believes it with a probability of at least π; everyone believes with a probability of at least π that everyone believes it with a probability of at least π; and so on. It is clear now that a collective action equilibrium only exists if it is common $L/(H + L)$-belief among the players that there is an opportunity. If $(1 - \varepsilon)^{I-1} < L/(H + L)$, this cannot be achieved in the unique candidate collective action equilibrium for $z = 1$. Yet, in the Pareto-best collective action equilibrium for a $z \geq 2$, a player who receives only a single penultimate message string m holds stronger beliefs that all other players received at least one ultimate message string, because this can now be achieved even if not all confirmations of m arrive. It follows that with $z \geq 2$, players can achieve a higher degree of common π-belief than with $z = 1$. This contrasts with Rubinstein's (1989) two-player EMG, where an informed player notifies an uninformed player about the opportunity, who confirms receipt to the informed player, and so forth. In this case, if the communication process has a deadline, a unique collective action equilibrium exists for each possible deadline, where each player acts only when receiving the (pen)ultimate message string. In each such equilibrium, players achieve the same degree of common π-belief about the opportunity for collective action, namely common $(1 - \varepsilon)$-belief. Summarizing, while in Rubinstein's two-player EMG availability of additional higher-order statements does not lead to a higher degree of approximate common knowledge, it may in the multiplayer EMG.[10]

We next turn to our result that, even if collective action equilibria exist for each z, being able to talk longer may still increase the probability of collective action. Proposition 3 shows the following. Consider fixed values I, H, and L, and vary the deadline z of the multiplayer EMG. Assume that the parameters are such that the unique collective action equilibrium[11] exists for $z = 1$, and such that for a given

[10] It should be emphasized that this difference between Rubinstein's two-player EMG and our multiplayer EMG where all players are equally informed, is not due to the fact that Rubinstein has an informed player and an uninformed player, and we have multiple players who are equally well informed. All results are maintained in a multiplayer EMG with one informed player and multiple uninformed players.

[11] Note that this unique collective action equilibrium is then also the Pareto-best collective action equilibrium.

finite deadline $z = z^*$ with $z^* \geq 2$, the Pareto-best collective action equilibrium characterized in Proposition 1 exists. Then for sufficiently small ε, the probability of collective action is strictly larger for $z = z^*$ than for $z = 1$. As this result applies for *any* finite $z = z^*$, including one that approaches infinity, it follows that the probability of collective action does not vanish as z^* become very large, but on the contrary is larger than it is for $z = 1$.[12] In short, talking for a finite long time is always better than to talk for the shortest possible time.

PROPOSITION 3: *Consider parameters such that the Pareto-best collective action equilibrium of the multiplayer EMG exists both for $z = 1$ and for some $z = z^*$ (with $z^* \geq 2$ and finite). Then for sufficiently small noise, the probability of collective action is strictly larger in the Pareto-best collection action equilibrium for $z = z^*$, than it is in the unique collective action equilibrium for $z = 1$.*

The reasoning underlying the proof of Proposition 3 is the following. Call any set of message strings consisting of a single penultimate message string, and all of the confirmations of this message string, a *broom* (named after its graphical form). In the Pareto-best collective action equilibrium, receipt of all the message strings in a single such broom, is sufficient for collective action to take place. For $z = 1$, receipt of all message strings in the unique broom is also a necessary condition, so that the probability that all message strings arrive in at least one broom equals the probability of collective action in this case. For $z = z^*$, with $z^* \geq 2$, arrival of all message strings in at least one broom is a sufficient condition, but not a necessary condition for collective action, as it is possible that all players receive at least one (pen)ultimate message string without all message strings arriving in at least one broom. The probability that all messages arrive in at least one broom thus forms a lower bound on the true probability of collective action. The focus in the proof of Proposition 3 is on this tractable lower bound, because we are able to show that it is increasing in z. As for $z = 1$, the lower bound equals the true probability of collective action; this means that the probability of collective action strictly increases from $z = 1$ to $z = 2$, and is higher for any finite deadline (however large) than it is for $z = 1$.

Intuitively, extending the deadline has two effects on the lower bound. First, the probability that all message strings in a single broom arrive becomes smaller, as one more message string now needs to arrive to achieve this. Second, many more brooms can be generated, and thereby many more opportunities to achieve collective action. The proof of Proposition 3 shows that for small noise, the second effect compensates the first effect, explaining why the lower bound is everywhere increasing in the deadline. For large noise on the contrary, the first effect dominates, as the fact that each broom contains one more message string then makes it considerably less likely that all message strings in at least one broom arrive. Furthermore, note

[12] In the multiplayer EMG, there is a *discontinuity at infinity* in the following sense. With a *finite* communication deadline z, as this finite deadline approaches infinity, the probability of collective action in the corresponding Pareto-best collective action equilibrium does not vanish, and is in fact larger than for the unique collective action equilibrium in case $z = 1$. Yet, if there is no deadline (i.e., if the deadline is infinite), the probability of collective action is zero, as players always ask for one more round of confirmations.

that in Rubinstein's (1989) two-player game, only the first effect applies, as the communication protocol can then only generate one single broom.

We now further explore how the true probability of collective action changes as a function of z for $z \geq 2$, as Proposition 3 does not make any direct statement about this. To do this, we additionally consider an *upper* bound on the probability of collective action, by calculating the probability of the event that in state r, it is not the case that no penultimate message strings arrive (which is a necessary, but not a sufficient condition for collective action). For $z = 1$, there is no penultimate stage, and we may consider the probability of at least one message arriving at stage 1 as an upper bound, which equals $\left(1 - \varepsilon^N\right)$. For $z = 2$, the probability that no message string arrives at the penultimate stage 1 takes on the same value $\left(1 - \varepsilon^N\right)$. Increasing the deadline with one stage so that $z = 3$, the probability that in state r no message strings arrive at the new penultimate stage 2 is strictly higher: if no message strings arrive at the previously penultimate stage 1, no message strings will arrive at the new penultimate stage 2 either; yet if message strings arrive at stage 1, it can still occur that no message strings arrive at stage 2. It follows that for $z = 3$, the upper bound on the probability of collective action is *lower* than it is for $z = 2$. It is easy to see that this generalizes to any increase in z, so that the upper bound on the true probability of collective action always decreases in z.

The lower bound and the upper bound on the probability of collective action are sketched in Figure 1. Note that because the upper bound never increases, the lower bound must eventually flatten out as z is further increased. In the same way, because the lower bound never decreases, the upper bound must eventually flatten out. For this reason, both the upper and lower bounds in Figure 1 are sketched as concave in z. As the upper bound decreases in z and eventually flattens out, should the true probability of collective action always be increasing in z, it must still eventually flatten out itself, and can never approach one;[13] should the true probability of collective action on the contrary eventually decrease in z, the fact that the lower bound never decreases in z means that the scope for decreases of the true probability of collective action is limited. Another implication of the fact that the lower bound never decreases in z is that in order for it to be possible that the true probability of collective action decreases over a range of deadlines, it must first have increased considerably for lower deadlines—again suggesting that the scope for decreases is limited.

As an example, consider the multiplayer EMG with parameters $I = 3$, $(L/H) = 2$, $\varepsilon = 0.2$, for which it can be checked that Propositions 1 and 2 are valid. For $z = 1$, 2, and 3, the lower bound on the probability of collective action in state r equals respectively 0.512, 0.884, and 0.934. For $z = 1, 2, 3$, the upper bound equals respectively 0.992, 0.992, and 0.988 (where as noted for $z = 1$, we take as an upper bound the probability that at least one message arrives at stage 1, which is identical to the upper bound for $z = 2$). Finally, for $z = 1$, 2, and 3, the true probability of collective action, which can still feasibly be calculated for this simple case, equals respectively 0.512 (identical to the lower bound), 0.942, and 0.966. As can be seen,

[13] For any finite z, the probability of collective action does approach one if ε approaches zero. This is not specific to the multiplayer EMG, and equally applies in Rubinstein's two-player EMG, as long as the deadline is finite. Our results are thus relevant for small but positive ε.

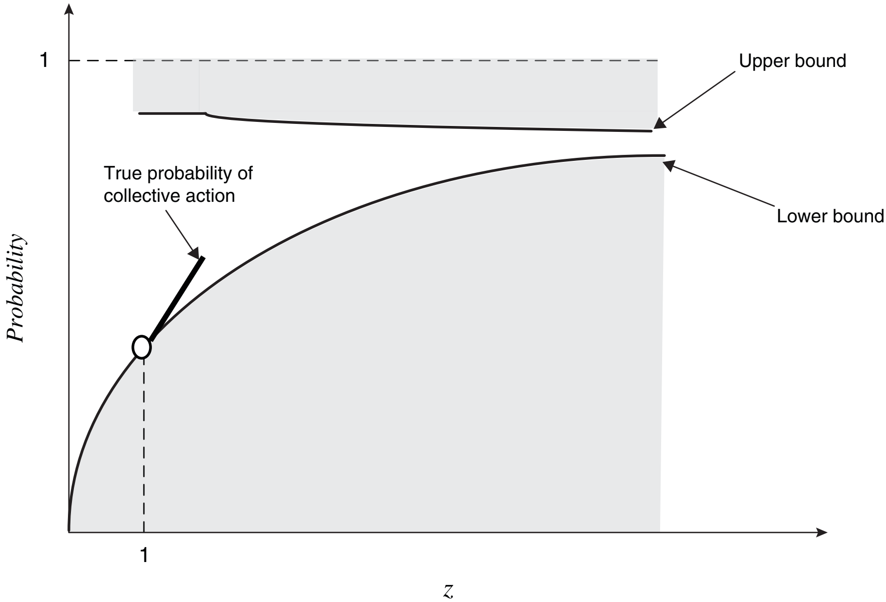

Figure 1. As Function of Deadline (z), Upper and Lower Bound on Probability of Collective Action

Notes: For $z = 1$, the true probability of collective action coincides with the lower bound and lies in the white area for larger z. The solid line departing from the lower bound for $z = 1$ indicates that the true probability must initially increase.

while the biggest increase in the true probability occurs from $z = 1$ to $z = 2$, it slightly increases from $z = 2$ to $z = 3$ as well. Furthermore, for $z = 3$, the true probability is both close to the increasing lower bound and to the decreasing lower bound. Clearly, should the true probability of collective action in this example ever decrease in z for deadlines higher than 4, the effect of such decreases will be limited. At the same time, the scope for further increases is limited as well, as the difference between the upper and the lower bound becomes narrow as z is increased.

IV. Discussion

In an inspiring treatment, Chwe (2003) analyzes common-knowledge-generating events in coordination problems. His argument is that public rituals such as ceremonies, or public buildings in the form of inward-facing circles, are designed such that everyone can observe everyone observing everyone, thereby generating common knowledge among those present. Yet, in an age of social media, where new technologies make communication cheap, the question arises whether common knowledge can also be generated in the absence of such common-knowledge-generating events. This question is of particular interest for collective action against a repressive regime, as such a regime may find it easier to disrupt public meetings than to disrupt bilateral communication. Our analysis suggests that, among multiple players, a degree of common knowledge sufficient to make collective action possible, or a higher probability of collective action if collective action was already possible before, may be generated by sending confirmations and reconfirmations back and forth in an unrestricted manner (but within a finite deadline). The intuition is that the exponential increase in the number of confirmations as one talks for a longer time, creates ever more back-up channels, which counter the effect of noise. This contrasts with two-player collective action problems, where sending confirmations and reconfirmations back and forth only decreases the probability of collective action, and where there is a need for strict communication rules.

Appendix

A. *States of the World*, *Information Structure*, *Strategies, and Equilibria*

The multiplayer EMG can be reformulated as a standard simultaneous moves game with incomplete information. In such a game, typically a simple lottery (Ω, π) consisting of a set of states of the word Ω and a probability distribution π, determines which state of the world occurs (where typical state of the world ω occurs with probability $\pi(\omega)$). To reduce the compound lottery in the multiplayer EMG to a simple lottery, consider the directed graph referred to as the *maximal communication tree*, an example of which is found in Figure A1 for the case $I = 3$, $z = 2$. The root node with label n has a link, represented as $n \rightarrow i$, to each of I nodes with labels 1 to I, at distance 1 from the root node. Each such node with label i has a link $i \rightarrow j$ to each of $(I - 1)$ nodes with labels $j \neq i$ at distance 2 from the root node. From each such node with label j again departs a link $j \rightarrow k$ to each of the $(I - 1)$ nodes with labels $k \neq j$, at distance 3 from the root node. And so on, where the maximal communication tree expands until terminal nodes at distance z from the root node are reached, so that for a distance t, with $1 \leq t \leq z$, it has $I(I-1)^{t-1}$ nodes at distance t from the root node. Note that to each path $n \rightarrow \ldots i \rightarrow j$ contained in the maximal communication tree starting at the root node, corresponds a possible message string.

Define as a *tree* g any connected subgraph of the maximal communication tree, that has as its root the node with label n. Denote by $\mathscr{G}$ the set of all trees contained in the maximal communication tree, where we assume this to include the empty tree. For example, in the simple case in Figure A1, the number of trees contained in the maximal communication tree equals $\sum_{x=0}^{3} \binom{3}{x} \sum_{y=0}^{(I-1)x} \binom{(I-1)x}{y} = 125.$[14] To *every* such tree, g now corresponds a state of the world where state r occurs and all message strings contained in g arrive, where in short we refer to this as state of the world g (note that in state r it is possible that no message strings arrive). In the same manner, we use symbol $\mathscr{G}$ not only for the set of all trees, but also for the set of all states of the world where state r occurs. Furthermore, as already noted, whenever state q occurs, the set of received message strings is automatically empty, where in short we

[14] This includes the empty tree; for each of the 3 cases with a single node at distance 1 from the root node, it includes 4 different trees with either 0, 1, or 2 nodes at distance 2; for each of the 3 cases with 2 nodes at distance 1, it includes 16 different trees with either 0, 1, 2, 3, or 4 nodes at distance 2; for the single case with exactly 3 nodes at distance 1, it includes 64 different trees with either 0, 1, 2, 3, 4, 5, or 6 nodes at distance 2.

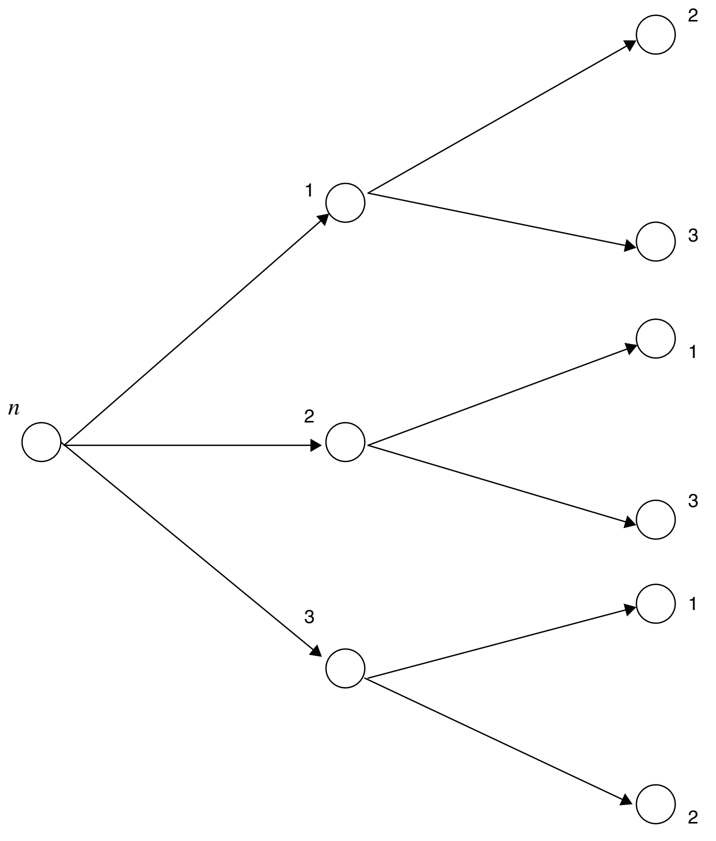

Figure A1. Maximal Communication Tree when $I = 3$, $z = 2$

refer to this as state of the world q. We thereby obtain that in the reduced-form simple lottery, Ω is the set (q, G), $\pi(q)$ equals $(1 - p)$, and for any g in $\mathscr{G}$, $\pi(g)$ equals $p(1 - \varepsilon)^{x_1}\varepsilon^{I - x_1}(1 - \varepsilon)^{x_2}\varepsilon^{(I-1)x_1 - x_2}\ldots(1 - \varepsilon)^{x_z}\varepsilon^{(I-1)x_{z-1} - x_z}$, where x_t denotes the number of message strings arriving at stage t, with $0 \leq x_1 \leq I$ and with $x_t \leq (I - 1)x_{t-1}$ for $2 \leq t \leq z$.

A signal function assigns to each state of the world a signal for each player. In particular, we denote by μ_i player i's signal function, and by $\mathfrak{M}_i$ the set of all signals he can observe. Concretely, the signal function μ_i: $(q, \mathscr{G}) \rightarrow \mathfrak{M}_i$ assigns to state of the world q the empty message string set ϕ received by i, and to each state of the world g in $\mathscr{G}$ message string set $\mu_i(g)$, consisting of each message string that i receives in g. The set of signals $\mathfrak{M}_i$, with typical element M_i, is therefore a collection of message string sets containing a $\mu_i(g)$ for each g in $\mathscr{G}$.[15] One and the same signal may be received for several states of the world. For example, in Figure A1, denote $m_1 = n \rightarrow 2 \rightarrow 1$, $m_1' = n \rightarrow 3 \rightarrow 1$, $m_2 = n \rightarrow 3 \rightarrow 2$, and $m_3 = n \rightarrow 2 \rightarrow 3$. Consider $g = \{m_1, m_1', m_2\}$. Then $\mu_1(g) = \{m_1, m_1'\}$. Yet, 1 also receives signal $\{m_1, m_1'\}$ in states of the world $\{m_1, m_1'\}$, $\{m_1, m_1', m_3\}$, and $\{m_1, m_1', m_2, m_3\}$.

[15] Note that this includes the empty message string set ϕ, which can also be received in state q.

If player i receives signal M_i, he learns that the true state of the world lies in the set of pre-images $\mu_i^{-1}(M_i)$ of M_i. The posterior probability that a state of the world ω in $\mu_i^{-1}(M_i)$ occurs then equals $\pi(\omega)/\pi(\mu_i^{-1}(M_i))$, where $\pi(\mu_i^{-1}(M_i))$ is the sum of the probabilities of all the states of the world in the set $\mu_i^{-1}(M_i)$. The collection of sets of states of the world obtained by assigning a set of pre-images to each M_i in $\mathfrak{M}_i$, is player i's *information partition*. A strategy for player i is a map α_i: $\mathfrak{M}_i \rightarrow \{Q, R\}$. A strategy profile $(\alpha_1, \alpha_2, \ldots, \alpha_I)$ is a pure-strategy Bayesian Nash equilibrium if for each player i, for each signal M_i, $\alpha_i(M_i)$ is such that for the action $A_i \neq \alpha_i(M_i)$,

$$\sum_{\omega\in\mu_i^{-1}(M_i)}[\pi(\omega)/\pi(\mu_i^{-1}(M_i))]\, u_i(\alpha_i(M_i), \alpha_{-i}(\mu_{-i}(\omega)), \omega)$$
$$\geq \sum_{\omega\in\mu_i^{-1}(M_i)}[\pi(\omega)/\pi(\mu_i^{-1}(M_i))]\, u_i(A_i, \alpha_{-i}(\mu_{-i}(\omega)), \omega),$$

where α_{-i} is the profile of strategies of all other players, where μ_{-i} refers to each other player's signal function, and where u_i is player i's payoff as a function of his own action, the actions of others, and the state of the world. We call a *collective action equilibrium,* any pure-strategy Bayesian Nash equilibrium where it occurs with positive probability that all players act in state r.

B. *Proofs*

PROOF OF LEMMA 1:

More formally, Lemma 1 says the following. Consider a class of multiplayer EMGs with fixed parameters I and z. In any collective action equilibrium of a game in this class, each player's minimal sufficient sets consist only of (pen)ultimate message strings if

(i) (L/H) is sufficiently large;

(ii) ε lies in $(0, \varepsilon_1(L/H, I, z)]$, where $\varepsilon_1(L/H, I, z)$ is a function of all other parameters.

PROOF:

Step 1 constructs a message string set for i, given a minimal sufficient set of any $j \neq i$. Parameters exist such that this constructed set is a sufficient set to i (Step 2), and moreover a *minimal* sufficient set (Step 3). Step 4 shows that, given these parameters, every equilibrium minimal sufficient set consists only of (pen)ultimate message strings.

Step 1: Consider minimal sufficient set $S_j^{\min}$ of j. Then, denoting by a superscript above an arrow the stage at which a message is sent, for $i \neq j$, we can construct the message string set $S_i(S_j^{\min})$ containing

(i) for any $n \xrightarrow{1} \ldots \xrightarrow{t-1} k \xrightarrow{t} j \in S_j^{\min}$ with $t < z$, the message string $n \xrightarrow{1} \ldots \xrightarrow{t-1} k \xrightarrow{t} j \xrightarrow{t+1} i$;

(ii) for any $n \xrightarrow{1} \ldots \xrightarrow{z-2} l \xrightarrow{z-1} k \xrightarrow{z} j \in S_j^{\min}$, in case $k \neq i$ message string $n \xrightarrow{1} \ldots \xrightarrow{z-2} l \xrightarrow{z-1} k \xrightarrow{z} i$; in case $k = i$ message string $n \xrightarrow{1} \ldots \xrightarrow{z-2} l \xrightarrow{z-1} k = n \xrightarrow{1} \ldots \xrightarrow{z-2} l \xrightarrow{z-1} i$ itself.

Define as $\mathcal{S}_i(S_{j \neq i}^{\min})$ the set containing for each $S_j^{\min}$ of each $j \neq i$ a message string set $S_i(S_j^{\min})$.

Step 2: In any candidate collective action equilibrium where $S_j^{\min}$ is a minimal sufficient set to j, it must be that j attaches some probability φ to obtaining L when acting after receiving every message string in $S_j^{\min}$, where $(1 - \varphi)H - \varphi L \geq 0$. Furthermore, in any such equilibrium, i should then act as well when believing that it is sufficiently likely that j received every message string in $S_j^{\min}$. Let i receive all message strings in $S_i(S_j^{\min})$, and let $S_j^{\min}$ contain V message strings of the type under (ii) in Step 1. As z and I are finite, V is finite too. Player i now certainly acts if

$$(1 - \varepsilon)^V[(1 - \varphi)H - \varphi L] - \left[1 - (1 - \varepsilon)^V\right]L \geq 0. \tag{A1}$$

Given (R1), φ is increasing in ε. It follows that for any L, H, and V, small ε exist such that (A1) is valid, meaning that $S_i(S_j^{\min})$ is a sufficient set.

Step 3: Consider the collection of message string sets $\mathcal{S}_i(S_{j \neq i}^{\min})$ as constructed in Step 1. We show that this is a collection of *minimal* sufficient sets by first constructing a message string set $M_i \in \mathfrak{M}_i$ such that (i) there is no $S_i(S_j^{\min}) \in \mathcal{S}_i(S_{j \neq i}^{\min})$ such that $S_i(S_j^{\min}) \subseteq M_i$; (ii) M_i is as conducive as possible for i to act; and by, second, showing that even when receiving all message strings in M_i, i prefers not to act. We construct such a M_i as follows. For any player $j \neq i$, let *each* minimal sufficient set $S_i(S_j^{\min})$ contain a message string m such that $\{S_i(S_j^{\min}) \backslash m\} \subseteq M_i$, but $m \notin M_i$. At the same time, let each $S_j^{\min}$ contain, for a message string m' received by k, a message string $m' \rightarrow j$ (denoting a confirmation received by j of m') such that for m as defined, it is the case that $m = m' \rightarrow j \rightarrow i$, with either $k = i$, or $k \neq i$ but $m' \rightarrow i \in M_i$. When receiving all message strings in M_i, player i then believes that j did not receive message string $m' \rightarrow j$ with probability $\varepsilon[\varepsilon + (1 - \varepsilon)\varepsilon]^{-1} = 1/(2 - \varepsilon)$; put otherwise, i believes that j received every message string in $S_j^{\min}$ with probability $1 - [1/(2 - \varepsilon)]$. If the sum of the cardinalities of the collections of the minimal sufficient sets held by other players is now W, i believes with probability at most $\left\{1 - [1/(2 - \varepsilon)]^W\right\}$ that he will obtain H with positive probability when acting, and with probability at least $[1/(2 - \varepsilon)]^W$ that he obtains L with probability 1. As $\varepsilon \rightarrow 0$, the former probability approaches $\left(1 - 0.5^W\right)$, in which case H is obtained with probability approaching 1, and the latter probability approaches 0.5^W, in which case L is obtained with certainty. It follows that as $\varepsilon \rightarrow 0$, i receiving message string set M_i prefers not to act if

$$\left(1 - 0.5^W\right) H - 0.5^W L < 0. \tag{A2}$$

For any finite W (where W must be finite because I and z are finite), a large (L/H) exists such that (A2) is valid. Note now that $[1/(2-\varepsilon)]^{W}$ is increasing in ε. It follows that if an (L/H) is imposed such that (A2) is valid (defining what is meant by "(L/H) sufficiently large" in condition (i) of the formal version of Lemma 1), it is true for any ε that i prefers not to act when receiving message string set M_i. For such an (L/H), as long as ε is sufficiently low, (A1) is valid as well (defining $\varepsilon_1(L/H, I, z)$ in condition (ii) of the formal version of Lemma 1).

Step 4: We show by contradiction that, under (A1) and (A2), every minimal sufficient set of any player must exclusively consist of message strings received at $(z-1)$ or z. Suppose that minimal sufficient set $S_j^{\min}$ contains at least one message string m received at stage t, where $t < (z-1)$. Consider now $S_i(S_j^{\min})$, and further $S_j[S_i(S_j^{\min})]$, as constructed in Step 1. Given that $S_j^{\min}$ contains an m received at t with $t < (z-1)$, $S_i(S_j^{\min})$ contains $m \rightarrow i$, and $S_j[S_i(S_j^{\min})]$ (which by Steps 2 and 3 is minimal sufficient) contains $m \rightarrow i \rightarrow j$. But this leads to a contradiction, as $S_j^{\min}$ cannot be minimal sufficient then. ■

PROOF OF PROPOSITION 1:

More formally, Proposition 1 is formulated as follows:

Consider a class of multiplayer EMGs with fixed parameters I and z, and fix (L/H) such that condition (i) of Lemma 1 is valid (i.e., (A2) is valid). Then a level of noise $\varepsilon_2(L/H, I, z)$ exists such that for all ε in $(0, \varepsilon_2(L/H, I, z)]$, in the Pareto-best collective action equilibrium, each player acts when receiving at least a single (pen)ultimate message string, and does not act when receiving no (pen)ultimate message strings.

PROOF:

(i) By Lemma 1, parameters exist such that in any pure-strategy collective action equilibrium, each player only acts when receiving at least one (pen)ultimate message string. It follows immediately that the candidate equilibrium where each player acts as soon as receiving *any* (pen)ultimate message string is Pareto-best among the collective action equilibria, if it exists.

(ii) Suppose that players other than i act as soon as receiving at least one (pen)ultimate message string. Let player i receive a single penultimate message string. Then player i certainly prefers to act if $(1-\varepsilon)^{I-1}H - \left[1-(1-\varepsilon)^{I-1}\right]L \geq 0$. Let player i receive a single ultimate message string. Then player i certainly prefers to act if $(1-\varepsilon)^{I-2}H - \left[1-(1-\varepsilon)^{I-2}\right]L \geq 0$. It follows that for sufficiently small ε, it is a best response for player i to act as soon as receiving at least one (pen)ultimate message string (defining $\varepsilon_2(L/H, I, z)$ in the formal representation of Proposition 1. As this is true for any given (H/L), it is also true for (H/L) such that (A2) in the proof of Lemma 1 is valid. ■

PROOF OF PROPOSITION 2:

More formally, Proposition 2 can be stated as follows:

For any finite $z^* \geq 2$, consider respectively $\varepsilon_2(L/H, I, z = 1)$ and $\varepsilon_2(L/H, I, z = z^*)$ as defined in Proposition 1. Then, $\varepsilon_2(L/H, I, z = 1) < \varepsilon_2(L/H, I, z = z^*)$, and for ε in $[\varepsilon_2(L/H, I, z = 1), \varepsilon_2(L/H, z = z^*)]$, the Pareto-best collective action equilibrium where each player acts as soon as receiving at least one (pen)ultimate message string exists for $z = z^*$, but no collective action equilibrium exists for $z = 1$.

PROOF:

Step 1 calculates a condition such that for $z = 1$, if all $j \neq i$ act when receiving a single message string, it is a weak best response for i not to act. Step 2 shows that parameters exist such that the proposition is valid.

Step 1: For $z = 1$, if all players play according to the unique collective action equilibrium, a player who receives a single message string weakly prefers not to act if and only if

$$(1 - \varepsilon)^{I-1}H - [1 - (1 - \varepsilon)^{I-1}]L \leq 0. \tag{A3}$$

Equality of (A3) defines $\varepsilon_2(L/H, I, z = 1)$ in the formal representation of Proposition 2.

Step 2: For any z, consider a player i who is supposed not to act in a collective action equilibrium. Then the most inclined this player can be to still act is in the situation described in Step 3 of Lemma 1. Note that under condition (A2) derived there, for $z = 1$ or $z = 2$, a player who does not receive (pen)ultimate message strings will also not act, as the player then does not even know whether the opportunity arises.

For $z \geq 2$, if each $j \neq i$ plays according to the Pareto-best collective action equilibrium, the least inclined a player i who is supposed to act can be to act is when observing a single penultimate message string and no other message strings. The worst event that may have occurred in this case is that none of the message strings arrived, which can be received at stage $(z - 1)$ by $j \neq i$. In this case, i's expected payoff from acting is the LHS of (A3). In all other events, where at least one penultimate message string is received by $j \neq i$, i's expected payoff is strictly larger than the LHS of (A3). Take now any (L/H) such that (A2) is valid. For the chosen (L/H), take ε such that the LHS of (A3) is zero (meaning $\varepsilon = \varepsilon_2(L/H, I, z = 1)$). Then by the above, when $z \geq 2$, if each $j \neq i$ plays according to the Pareto-best collective action equilibrium, it is a *strict* best response for i to act when receiving at least one (pen)ultimate message string. By continuity, ε exists such that the LHS of (A3) is strictly smaller than zero, meaning that no collective action equilibrium exists for $z = 1$ (i.e., $\varepsilon > \varepsilon_2(L/H, I, z = 1)$), but such that the Pareto-best collective action equilibrium does exist for $z \geq 2$ (i.e., $\varepsilon < \varepsilon_2(L/H, I, z = z^*)$). ∎

PROOF OF PROPOSITION 3:

More formally, Proposition 3 can be stated as follows:

Consider a class of multiplayer EMGs with fixed parameters I, (L/H), and ε, such that Proposition 1 is valid both for $z = 1$ and for $z = z^*$ with $z^* \geq 2$ and finite. Then there exists a $\varepsilon_3(L/H, I, z^*)$ such that for the subclass of the defined class of games with ε in $(0, \varepsilon_3(L/H, I, z^*)]$

(i) the unique collective action equilibrium exists for $z = 1$;

(ii) the Pareto-best collection action equilibrium characterized in Proposition 1 exists for $z = z^*$;

(iii) the probability of collective action is strictly *larger* in the Pareto-best collection action equilibrium for $z = z^*$ than it is in the unique collective action equilibrium for $z = 1$.

PROOF:

Step 1: For $z = z'$, consider first the probability that, conditional on a single message string having arrived at stage $(z' - 1)$, all confirmations of this message string arrive at stage z':

(A4)
$$(1 - \varepsilon)^{I-\lambda},$$

where $\lambda = 0$ when $z' = 1$ (in which case the fact that state r occurs is considered as a single message string arriving at stage $(z' - 1) = 0$), and where $\lambda = 1$ when $z' \geq 2$.

Define as a *broom* a set of message strings consisting of a single penultimate message string, and *all* ultimate confirmations of this message string. For $z = (z' + 1)$, we consider the probability of the event that all players receive all their message strings in at least *one* broom, conditional on a single message string arriving at stage $(z' - 1)$. In a Pareto-best collective action equilibrium, this event is sufficient for all players to act. As there are additional events where collective action takes place,[16] the probability of this event is a lower bound on the true probability of collective action. The proposed lower-bound probability equals:

(A5)
$$\sum_{W=1}^{I-\lambda}\left\{\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\left[\sum_{Y=1}^{W}\binom{W}{Y}\left[(1-\varepsilon)^{I-1}\right]^{Y}\left[1-(1-\varepsilon)^{I-1}\right]^{W-Y}\right]\right\},$$

where again $\lambda = 0$ when $z' = 1$, and $\lambda = 1$ when $z' \geq 2$.

[16] It may be that each player receives all his message strings in one broom, but for each player this may be a different broom. It is these events that are not considered here.

In (A5), $\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}$ reflects the probability that W equal to one or more message strings arrive at stage z'. $\binom{W}{Y}\left[(1-\varepsilon)^{I-1}\right]^{Y}\left[1-(1-\varepsilon)^{I-1}\right]^{W-Y}$ reflects the probability that after W message strings have arrived at stage z', all message strings arrive at $(z'+1)$ in Y brooms, with Y equal to one or higher (with W as a maximum). In some consecutive steps, we now show that the expression in (A4) is smaller than the expression in (A5) for small ε. We show this for (A4) equal to $(1-\varepsilon)^{I-1}$, which suffices as $(1-\varepsilon)^{I-1} > (1-\varepsilon)^{I}$.

$$\text{(A6)} \qquad (1-\varepsilon)^{I-1} < \sum_{W=1}^{I-\lambda}\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\left[\sum_{Y=1}^{W}\binom{W}{Y}\left[(1-\varepsilon)^{I-1}\right]^{Y}\left[1-(1-\varepsilon)^{I-1}\right]^{W-Y}\right]$$

$$\Leftrightarrow$$

$$\left[\varepsilon^{I-\lambda}+\sum_{W=1}^{I-\lambda}\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\right](1-\varepsilon)^{I-1}<\sum_{W=1}^{I-\lambda}\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\left[1-\left[1-(1-\varepsilon)^{I-1}\right]^{W}\right]$$

$$\Leftrightarrow$$

$$\varepsilon^{I-\lambda}(1-\varepsilon)^{I-1} < \sum_{W=1}^{I-\lambda}\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\left[1-(1-\varepsilon)^{I-1}\right]\left[1-\left[1-(1-\varepsilon)^{I-1}\right]^{W}\right]$$

$$\Leftrightarrow$$

$$\varepsilon^{I-\lambda}(1-\varepsilon)^{I-1}$$

$$< \sum_{W=1}^{I-\lambda}\binom{I-\lambda}{W}(1-\varepsilon)^{W}\varepsilon^{I-\lambda-W}\left[\sum_{T=0}^{I-2}\binom{I-1}{T}(1-\varepsilon)^{T}\varepsilon^{I-2-T}\right]\left[1-\left[1-(1-\varepsilon)^{I-1}\right]^{W}\right]$$

As ε approaches zero, both for $\lambda = 0, 1$, the left-hand side of (A6) approaches zero, while the right-hand side approaches one. By continuity, it follows that for a range of small ε, (A6) is valid.

Step 2: Consider $z = t + y$, with $t \geq 2$, and let players play the Pareto-best collective action equilibrium as characterized in Proposition 1. Let players also play such an equilibrium when $z = t + y + 1$. Suppose that we have been able to show for a specific given t that the lower bound is larger for $z = t + y + 1$ than for $z = t + y$, when $y = 0$. Then we show that this is also true for $y = 1$.

Note that in case $z = t + 2$ or $z = t + 1$, once exactly one message string has arrived at stage 1, it is as if we have $z = t + 1$, respectively $z = t$. Given our assumption that the lower-bound probability is larger with $z = t + 1$ than it is with $z = t$, this also applies for $z = t + 2$ versus $z = t + 1$, in the event that exactly one message string has arrived at stage 1. If this applies in the event that a single message string arrives at stage 1, it applies for any number of message strings that arrive at stage 1.

Step 3: Step 1 implies that the lower-bound probability is larger for $z = 2$ when players play the Pareto-best collective action equilibrium than it is in the unique collective action equilibrium with $z = 1$ (where for $z = 1$, the lower-bound probability and the true probability of collective action are identical). Also, Step 1 implies that, conditional on a single message string having arrived at stage 1, the lower-bound probability is larger when $z = 3$ than it is when $z = 2$, assuming that players each time play the Pareto-best collective action equilibrium. If this applies in the event that a single message string arrives at stage 1, it applies for any number of message strings that arrive at stage 1, so that the lower-bound probability in the Pareto-best collective action equilibrium is larger in case $z = 3$, than in case $z = 2$ (base case). Step 2 (inductive step) showed that if, for a specific t with $t \geq 2$, it is true for $y = 0$ that the lower-bound probability is larger in any case where $z = t + y + 1$, than in case $z = t + y$, then this is also true for $y = 1$ (assuming again that each time players play the Pareto-best collective action equilibrium). It follows that the lower-bound probability always increases in the deadline. As the true probability of collective action is always at least as large as the lower-bound probability, the proposition follows. ∎

REFERENCES

Binmore, Ken, and Larry Samuelson. 2001. "Coordinated Action in the Electronic Mail Game." *Games and Economic Behavior* 35 (1–2): 6–30.

Chwe, Michael Suk-Young. 1995. "Strategic Reliability of Communication Networks." http://www.chwe.net/michael/p.pdf.

Chwe, Michael Suk-Young. 2003. *Rational Ritual: Culture, Coordination and Common Knowledge.* Princeton: Princeton University Press.

Coles, Peter. 2009. "Coordination and Signal Design: The Electronic Mail Game in Asymmetric and Multiplayer Settings." http://www.people.hbs.edu/pcoles/papers/coord.pdf.

Coles, Peter A., and Ran Shorrer. 2012. "Correlation in the Multiplayer Electronic Mail Game." *B. E. Journal of Theoretical Economics* 12 (1): Article 1576.

De Jaegher, Kris. 2005. "Game-Theoretic Grounding." In *Game Theory and Pragmatics*, edited by Anton Benz, Gerhard Jäger, and Robert van Rooij, 220–47. Basingstoke-UK: Palgrave Macmillan.

De Jaegher, Kris. 2008. "Efficient Communication in the Electronic Mail Game." *Games and Economic Behavior* 63 (2): 468–97.

De Jaegher, Kris. 2015. "Beneficial Long Communication in the Multi-Player Electronic Mail Game." http://www.uu.nl/en/files/rebousedp201515-09pdf.

Dimitri, Nicola. 2004. "Efficiency and equilibrium in the electronic mail game: The general case." *Theoretical Computer Science* 314 (3): 335–49.

Dulleck, Uwe. 2007. "The E-Mail Game Revisited: Modelling Rough Inductive Reasoning." *International Game Theory Review* 9 (2): 323–39.

Geanakoplos, John. 1992. "Common Knowledge." *Journal of Economic Perspectives* 6 (4): 53–82.

Langdell, C. C. 1999. *A Selection of Cases of the Law of Contracts: With References and Citations*. Boston: Lawbook Exchange.

Monderer, Dov, and Dov Samet. 1989. "Approximating common knowledge with common beliefs." *Games and Economic Behavior* 1 (2): 170–90.

Morris, Stephen. 2002a. "Coordination, Communication, And Common Knowledge: A Retrospective On The Electronic Mail Game." *Oxford Review of Economic Policy* 18 (4): 433–45.

Morris, Stephen. 2002b. "Faulty Communication: Some Variations on the Electronic Mail Game." *B. E. Journal of Theoretical Economics* 1 (1): 1–26.

Morris, Stephen, and Hyun Song Shin. 1997. "Approximate Common Knowledge and Co-ordination: Recent Lessons from Game Theory." *Journal of Logic, Language and Information* 6 (2): 171–90.

Rubinstein, Ariel. 1989. "The Electronic Mail Game: Strategic Behavior under 'Almost Common Knowledge.'" *American Economic Review* 79 (3): 385–91.

Strzalecki, Tomasz. 2014. "Depth of Reasoning and Higher Order Beliefs." http://scholar.harvard.edu/tomasz/files/strzalecki-depth.pdf.