Digital Technology in Mathematics Education: Why It Works (Or Doesn't)

Paul Drijvers

Abstract The integration of digital technology confronts teachers, educators and researchers with many questions. What is the potential of ICT for learning and teaching, and which factors are decisive in making it work in the mathematics classroom? To investigate these questions, six cases from leading studies in the field are described, and decisive success factors are identified. This leads to the conclusion that crucial factors for the success of digital technology in mathematics education include the design of the digital tool and corresponding tasks exploiting the tool's pedagogical potential, the role of the teacher and the educational context.

Keywords Didactical function · Digital technology · Instrumentation

Introduction

For over two decades, many stakeholders have highlighted the potential of digital technologies for mathematics education. The U.S. National Council of Teachers of Mathematics, for example, in its position statement claims that "Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology" (NCTM 2008). ICMI devoted two studies to the integration of ICT in mathematics education, the second one expressing that "…digital technologies were becoming ever more ubiquitous and their influence touching most, if not all, education systems" (Hoyles and Lagrange 2010, p. 2).

However, the integration of digital technology still confronts teachers, educators and researchers with many questions. What exactly is the potential of ICT for learning and teaching, how to exploit this potential in mathematics education, does digital technology really work, why does it work, which factors are decisive in

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making it work or preventing it from working? What does a quarter of a century of educational research and development have to offer here?

Of course, these questions are not clearly articulated. What do we mean by "it works"? Does this mean that the use of digital technology improves student learning, invites deeper learning, motivated learning, more efficient or more effective learning? Does it mean that ICT empowers teachers to better teach mathematics? And, concerning the effect of educational research, do studies on digital technology "work" in the sense that they provide answers to these questions, or do they just help the researcher to better understand the phenomenon, and as such contribute only indirectly to improving mathematics education? My interpretation of "why it works" in the title of this contribution includes both learning and teaching, and also refers to learning on the part of the researcher.

In this paper I will explore the question of "why digital technology works or does not" by briefly revisiting a number of leading studies in the field, that are paradigmatic for a theme, approach, method, or type of results. For each of these studies, the focus is on what they offer on identifying decisive factors for learning, teaching and research progress. As such, this contribution reports on a concise and somewhat personal journey through—or a helicopter flight over—the landscape of research studies on technology in mathematics education.

Framework for Case Description

How to decide which studies to include in this retrospective and even somewhat historical paper? Even if somewhat subjective and personal arguments cannot be completely ignored, the case selection is based on a number of criteria. A first criterion for including a study or a set of studies is that it really contributes to the field, by providing a new perspective, a new direction or is paradigmatic for a new approach to the topic. An indication for this is that the study is frequently quoted and has led to follow-up studies. A second criterion for inclusion is that the study under consideration contributes to theoretical development in the field of integrating technology in mathematics education, and as such promotes thought in this domain. A third and final criterion for the set of cases presented in this paper as a whole, is variation. Variation does not only apply to theoretical perspectives, but also to the mathematical topic addressed in the study, the type of technological tools used, and the pedagogical functionality of the digital technology. Concerning this functionality, we use an adapted version of the model by Drijvers et al. (2010a) which distinguishes three main didactical functionalities for digital technology: (1) the tool function for doing mathematics, which refers to outsourcing work that could also be done by hand, (2) the function of learning environment for practicing skills, and (3) the function of learning environment for fostering the development of conceptual understanding (see Fig. 1). Even if these three functionalities are neither exhaustive nor mutually exclusive, they may help to position the pedagogical type of use of the technology involved. In general, the third function is the most challenging one to exploit.

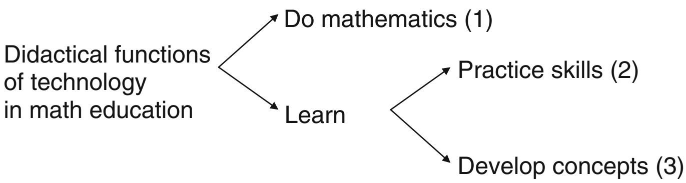

Fig. 1 Didactical functions of technology in mathematics education

How to discuss the selected studies in a short frame in a way that does justice to them and in the meanwhile serves the purpose of this paper? First, a global description of each case will be presented, including the mathematical topic, the digital tool and the type of tool use. Next, I will explain what is crucial and new in the study, and why I decided to include it. Then the theoretical perspective is addressed. Each case description is closed by a reflection on whether digital technology worked well for the student, the teacher or the researcher, and which factors may explain the success or failure.

Case Descriptions

Case 1 Concept-First Resequencing by Heid (1988)

The first case description concerns a study reported by Heid (1988), which is considered as one of the first leading studies into the use of digital technology in mathematics education. The study addresses the resequencing of a calculus course for first-year university students in business, architecture and life sciences using computer algebra, table tools and graphing tools that were used for concept development (branch (3) in Fig. 1). The digital technology allowed for a 'concept-first' approach, which means that calculus concepts were extensively taught, whereas the computational skills were treated only briefly at the end of the course. The results were remarkable in that the students in the experimental group, who attended the resequenced, technology-intensive course, outperformed the control group, who attended a traditional course, on conceptual tasks in the final test, and also did nearly as well on the computational tasks that had to be carried out by hand. The subjects in the experimental group reported that the use of the computer took over the calculational work, made them feel confident about their work and helped them to concentrate on the global problem-solving process.

One of the reasons to discuss the Heid study here is that it is paradigmatic in its approach in that its results form a first 'proof of existence': indeed, it seems possible to use digital technology as a lever to reorganize a course and to successfully apply a concept-first approach, using digital technology in the pedagogical function of enhancing concept development, without a loss of student achievement on by-hand skills.

From a theoretical perspective, Heid's notion of resequencing seems closely related to Pea's distinction of ICT as amplifier and as (re-)organizer (Pea 1987). The former refers to the amplification of possibilities, for example by investigating many cases of similar situations at high speed. The latter refers to the ICT tool functioning as organizer or reorganizer, thereby affecting the organization and the character of the curriculum. In the light of that time's thinking on the role of digital tools to empower children to make their own constructions (Papert 1980), the organizing function of digital technology was often considered more interesting than the amplification.

So did technology 'work' in this case? Yes, it did at the level of learning: the final test results of the experimental group turned out to be very satisfying. And yes, it also worked at a more theoretical level, as the notions of resequencing and concept-first approach were operationalised and made concrete. Now why did it work, which factors might explain these positive results? Even if nowadays we would not consider the digital technology available in 1988 as very sophisticated, I would guess that at the time the approach was new and motivating to the students, and the representations offered by the technology did indeed invite conceptual development. Decisive, however, I believe was the fact that the researcher herself designed and delivered the resequenced course. I conjecture that she was very aware of the opportunities and constraints of the digital technology, and was skilled in carefully designing activities in which the opportunities were exploited, and in teaching the course in a way that benefitted from this. Whether the course, if delivered by another teacher, would have been equally successful, is something we will never know.

Case 2 Handheld Graphing Technology

The second case description concerns the rise of handheld graphing technology in the 1990s. For several reasons, graphing calculators became quite popular among students, teachers and educators at that time (for an overview, see Trouche and Drijvers 2010). Teaching materials were designed that made extensive use of these devices and researchers investigated the benefits of this type of technology-rich activities (Burrill et al. 2002). Very much in line with the work by Heid (see Case 1), the focus of much of this work is on the pedagogical function of concept development. The main idea seems to be that students' curiosity and motivation can be stimulated by the confrontation with dynamic phenomena that invite mathematical reasoning, in many cases concerning the relationships between multiple representations of the same mathematical object. In many cases this mathematical object is a function, but examples involving other topics, such as statistics, can also be found.

As an example, Fig. 2 shows two graphing calculator screens which students set up to explore the effect of changes in the formula of the linear functions Y1 and Y2 on the graph of the product function Y3. This naturally leads to questions about properties of the product function and the relationship with properties of the two components (Doorman et al. 1994).

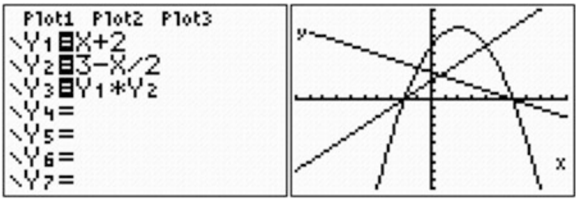

Fig. 2 Exploring the product of two linear functions

A paradigmatic study in this field is done by Doerr and Zangor (2000). The researchers report on a small-scale qualitative study, in which 15–17 year old pre-calculus students study the concept of function using a graphing calculator, with a focus on the pedagogical tool functionality of concept development (branch (3) in Fig. 1). The authors identify five modes of tool use, namely computation, transformation, data collection and analysis, visualisation, and checking. The results show that the teacher was crucial in establishing and reinforcing these modes of tool use, for example by setting up whole-class discussions 'around' the projected screen of the graphing calculator, to develop shared meaning and avoid a too individual development of tool use and mathematical insight. The researchers stress that using digital technology in mathematics teaching is not independent from the educational context and the mathematical practices in the classroom in particular.

The main reason to discuss the Doerr and Zangor study here is that it highlights the importance of the educational context in studies on the effect of digital technology, and the crucial role of the teacher in particular. The relevance of the educational context has later been elaborated in the notion of Pedagogical Map by Pierce and Stacey (2010). Concerning the teacher, she establishes a culture of discussing graphing calculator output in a format that is close to what is called a 'Discuss-the-Screen orchestration' in Drijvers et al. (2010b) and by these means contributes to the co-construction of a shared repertoire of ways to use the graphing device.

From a theoretical perspective, Doerr and Zangor use frameworks on learning as the co-construction of meaning, and that the "features of a tool are not something in and of themselves, but rather are constituted by the actions and activities of people" (p. 146). Even if this may sound somewhat trivial nowadays, during the period of initial enthusiasm these were important insights with consequences for the role of the teacher, who led the process of sharing and co-construction, particularly in the case of personal, private technology.

So did technology 'work' in this case? Doerr and Zangor did not assess learning outcomes, but it seems that the students developed a rich and meaningful repertoire of ways to use the graphing calculator for their mathematical work. Why did this work, which factors might explain these findings? My interpretation is that the use of digital tools for exploratory activities which target conceptual development is not self-evident, as it is hard for students, without the mathematical background that we as teachers have, to 'see' the mathematics behind the phenomena under consideration. It is here where the teacher comes in, and where the study becomes very informative for both teachers and researchers. In this case, I believe that the fact that the teacher herself was skilled in using the graphing calculator, was aware of its limitations, and was willing to explicitly pay attention to the co-construction of a shared and meaningful repertoire of tool techniques explains the results. As in the Heid study described in Case 1, the role of the teacher seems to be an important factor. The issue of how to deal with private, handheld technology is very relevant nowadays, as many students have smart phones with sophisticated mathematical applications, and again, teachers are faced with the danger of too individually constructed techniques and insights.

Case 3 Instrumental Genesis

By the end of the previous century, French researchers who were working on the integration of computer algebra and dynamic geometry in secondary mathematics education felt the need to go beyond the then current theoretical views. Even if they still experimented with explorative tasks, such as finding the number of zeros at the end of $n!$ (Trouche and Drijvers 2010), a theoretical perspective was needed that would do justice to the complex interaction between techniques to use the digital technology, conventional paper-and-pencil work and conceptual understanding. This led to the development of the instrumental genesis framework, or the instrumental approach to tool use (Artigue 2002; Guin and Trouche 1999; Lagrange 2000). Even if there are different streams within instrumentation theory (Monaghan 2005), it is widely recognized that the core of this approach is the idea that the co-emergence of mental schemes and tool techniques while working with digital technology is essential for learning. This co-emergence is the process of instrumental genesis. The tool techniques involved have both a pragmatic meaning (they allow the student to use the tool for the intended task) and an epistemic meaning, in that they contribute to the students' understanding. Rather than exploration, the reconciliation of digital tool use, paper-and-pencil use, and conceptual understanding is stressed (Kieran and Drijvers 2006).

A paradigmatic study in this field is the one by Drijvers (2003) on the use of handheld computer algebra for the learning of the concept of parameter. Four classes of 14–15 year old students worked on activities using a handheld computer algebra device both in its role of mathematical tool and for conceptual development (branches (1) and (3) in Fig. 1) to develop the notion of parameter as a 'super-variable' that defines classes of functions and that can, depending on the situation, play the different roles that 'ordinary' variables play as well. The results of the study include detailed analyses and descriptions of techniques that students use, and the corresponding expected mental scheme development. Figure 3 provides a schematic summary of such an analysis for the case of solving parametric equations in a computer algebra environment (Drijvers et al. 2012).

The main reason to discuss this study here is that by providing elaborated examples it contributes to a concrete and operationalised view on the schemes and techniques that are at the heart of the instrumental approach. The study shows that the instrumental approach is a fruitful perspective that can provide tangible guidelines for both the design of student materials and the analysis of student behaviour.

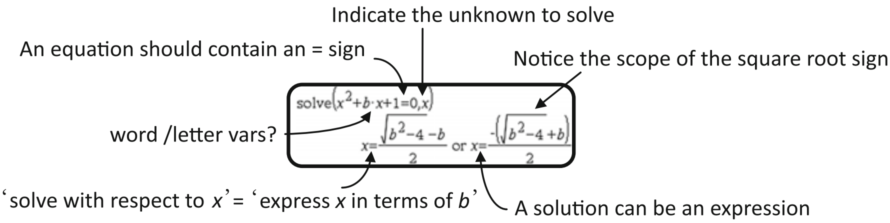

Fig. 3 Conceptual elements related to the application of the solve command

From a theoretical perspective, apart from the concretisation of the notions of schemes and techniques, the author integrated this with a more general view on mathematics education, namely the theory of realistic mathematics education (Freudenthal 1991). The two perspectives seemed to be complementary and both provided relevant guidelines for design and analysis.

So did technology 'work' in this case? No and yes. The conclusions on the learning effects of the intervention are not very clear. Even if the students learned much about the concept of parameter, their work still showed weaknesses both in the use of the tool and in the understanding of the mathematics. This suggests an incomplete instrumental genesis. Factors that may explain these findings are (1) the difficulty of the mathematical subject for students of this age, (2) the complexity of the computer algebra tool, and (3) the efforts and skills needed by the teachers to not only go through their personal process of instrumental genesis, but also to facilitate the students' instrumental genesis by their way of teaching. The latter aspect was addressed more explicitly later in the notion of instrumental orchestration (Trouche 2004; Drijvers and Trouche 2008). The study did work in the sense that it contributed to the researchers' understanding of the complexity of integrating sophisticated digital technologies in teaching relatively young students. The close intertwinement of the students' cognitive schemes and the techniques for using the digital technology is identified as a decisive factor in the learning outcomes of technology-rich mathematics education.

Case 4 Online Applications

With the growing availability and bandwidth of internet, researchers became interested in the potential of online interactive applications or applets for mathematics education. The advantages of online content include access without local software installation, ease of distribution and updating for developers, and permanent availability for users as long as the internet is accessible.

Many studies investigate this potential. For example, Boon (2009) explores the opportunities for teaching 3D geometry using online applets. Doorman et al. (2012) describe a teaching experiment in grade 8 focusing on the concept of function using

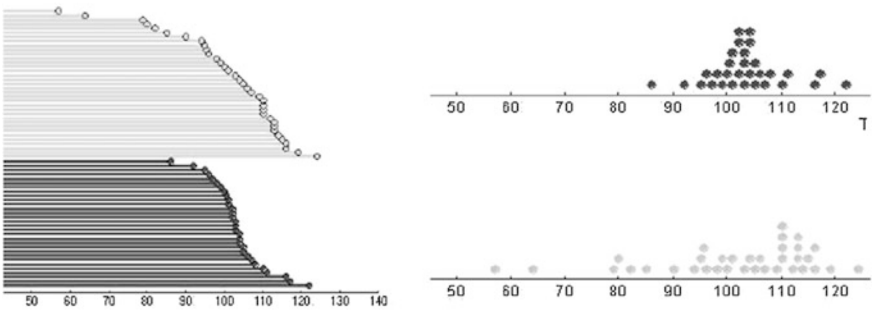

Fig. 4 Applets for investigating a small set of statistical data

an applet called Algebra Arrows[1] for building chains of operations. Apart from an instrumental perspective (see Case 3), the theoretical framework includes domain-specific theories on reification, realistic mathematics education and emergent modelling. The applet is used for concept development (branch (3) in Fig. 1). A third example is the study by Bokhove, who focuses on acquiring, practicing and assessing algebraic skills (Bokhove 2011; Bokhove and Drijvers 2012). His teaching experiments took place in grade 12 classes and made use of applets that offer means to manipulate algebraic expressions and equations.[2] The theoretical framework in this case included notions from algebra pedagogy such as symbol sense, which is expected to support skill mastery, but also elements from educational science on assessment and on feedback. In contrast to the studies described so far, the role of the digital tool in Bokhove's work includes the environment to practice skills (branch (2) in Fig. 1), which might be the easiest role, even if the design of appropriate feedback is an issue to tackle.

As a paradigmatic design research study in this field, let us now describe the work done by Bakker in somewhat more detail (Bakker 2004; Bakker and Gravemeijer 2006; Bakker and Hoffmann 2005). Bakker investigated early statistical reasoning of students in grades 7 and 8. In one of the tasks, students investigate data from life spans of two brands of batteries while using applets to design and explore useful representations and symbolizations (see Fig. 4). Clearly, the digital tools' pedagogical functionality is on concept development once more (branch (3) in Fig. 1). The design of the hypothetical learning trajectory and the student materials was informed by the development of statistics in history. In his analysis of student data, Bakker uses Peirce's (1931–1935) notions of diagrammatic reasoning and hypostatic abstraction to underpin his conclusion that the teaching sequence, including the role of digital tools, invited students' reasoning about a frequency distribution as an object-like entity, as became manifest when they started to speak about the 'bump' to describe the drawings at Fig. 4's right hand side.

[1]See http://www.fisme.science.uu.nl/tooluse/en/.

[2]See http://www.algebrametinzicht.nl/.

The main reasons to discuss Bakker's work here are not only the originality of the dedicated digital tools which meet new ideas on statistical reasoning and statistics education, and which were designed in collaboration with others (Cobb et al. 2003), but also the rich relationships with the different resources and approaches, such as the historical perspective, to inform the design.

From a theoretical perspective, it is interesting to notice that even if technology plays an important role in Bakker's study, the design and analysis are driven by theoretical perspectives from outside the frame of research on the use of technology in mathematics education, but rather from the world of mathematics pedagogy and beyond. I believe that this is a meaningful and promising approach: on the one hand, as researchers we should benefit from specific results and theories from studies on the use of digital tools in mathematics education. On the other hand, we should not forget to involve theories on mathematics education and educational science in general.

So did technology 'work' in Bakker's case? Yes, it did in the sense that the author clearly reports on conceptual development by the students involved in the study. Why did this work, which decisive factors might explain these findings? I believe that an important lesson to be learnt from this study is that design research on the use of digital technology in mathematics education should not limit itself to the study of the tools alone, but should include the tasks, and their embedding in teaching as a whole, in order to understand what works and why it works. In this case, I would guess it is the combination of the digital tools, the tasks and activities, but also the whole-class discussions, the paper-and-pencil work, the established mathematical practices, in short the educational context as a whole, that explains the result. A second lesson to learn for us as researchers is that a theoretical framework which integrates different perspectives can be very powerful for generating interesting and relevant research results.

Case 5 Mobile Mathematics

Research on the use of mobile technology in mathematics education is in its early stages but its importance is rapidly growing. It is evident that mobile technology and smart phones in particular are very popular among students and more and more wide-spread. Wireless Internet access allows for the use of mobile applications (also called midlets, Mobile Information Device applications), SMS and email services offer communication and collaboration opportunities, GPS facilities allow for geographical and geometrical activities and the tool's mobile and handheld characteristics invite out-of-school activities, for example the gathering of real-life data that inform biology or chemistry lessons (Daher 2010).

As a paradigmatic example, I now address the MobileMath pilot study carried out by Wijers et al. (2010). In this study, the tool consisted of a mobile phone with GPS facilities and a 'native' application, designed for the purpose of this game, which generated the view on the game situation and arranged communication with other teams' devices. The mathematical topic involved is geometry: teams of Grades 7 and 8 students used the GPS and the application to play an outdoor game on constructing parallelograms (including rectangles and squares), and could

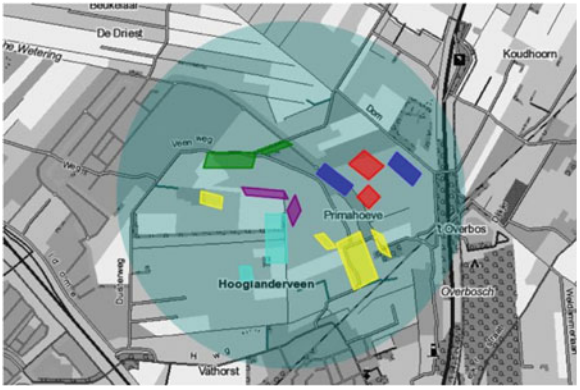

Fig. 5 Map of students' parallelogram constructions using GPS

eventually destroy other groups' geometrical shapes. This so-called MobileMath game aims at making students experience properties of geometrical figures in a lively, embodied game context. While playing the game, students look at the map to imagine where they want to make a shape, walk to the location for the first vertex to enter this location in the mobile device, which generates a dot on the map, walk to the location of the second vertex of their imagined shape which provides a line on the screen connecting the first vertex with the current (moving) location, etc. The map in Fig. 5 shows some student constructions. The deconstruction option brought extra challenge and competition into the game. From the data the researchers conclude that MobileMath adds a geometrical dimension to the world, transforming it into a game board. MobileMath also invites mathematical activity, such as the (re) discovery and use of characteristics of squares, rectangles and parallelograms, and taking notice of geometrical aspects of the world.

One reason to discuss this study here is that the digital tool—the modern smart phone with GPS facilities rather than an 'old school' computer—acts in multiple ways, and its use includes all branches of the diagram displayed in Fig. 1. The device enables the exploration of properties of quadrilaterals [branch (3)]. It also allows for practicing the construction of parallelograms, which meets branch (2). And finally, the tool also functions as an environment to outsource the mathematical work, in this case the drawing of the shapes, to, [branch (1)].

As seems to be the case in other studies on the integration of mobile technology in mathematics education, the theoretical perspective used by Wijers et al. (2010) is different from the frameworks common in most research on technology in mathematics education. It is closely associated with frameworks from studies on serious gaming, and focuses (1) on student engagement and (2) on task authenticity. Enhancing student engagement is seen as an important potential of educational games. In the MobileMath study, student engagement is stimulated by the game's hybrid reality character: on the mobile device's screen, students see the map of the reality in which they are walking, as well as the virtual geometrical shapes they are creating. Hybrid reality games are seen as beneficial for student engagement. In addition to this, the authors refer to Prensky (2001) for a model on heuristics for the design of engaging games, which include clear rules and goals, outcome and feedback, conflict, challenge and competition, and interaction. Concerning task authenticity, the authors claim that the effectiveness of learning activities can be enhanced if the tasks are authentic and realistic. In line with the framework of Realistic Mathematics Education, realistic means that problem situations presented in learning activities should be 'experientially' real to students and have meaningful, authentic problem situations as starting points, so that students experience the game's activity as making sense.

So did the digital technology 'work' in this case? As far as engagement and authenticity are concerned, the answer is 'yes'. The researchers report that the students were engaged in the game and experienced it as challenging. Apparently, the game factor, in combination with the possible attractiveness of the digital device, works out well. A second factor might be the outdoor and physical character of the game, which students may experience as a welcome change from regular classroom teaching. What is not clear yet, however, is whether these effects will persist if this type of activity were to become more common. Also, the study presented here has a small-scale pilot character and would certainly need further replication.

Case 6 Teachers' Practices and Professional Development

If we recapitulate the previous cases, in all but the last one the teachers' practices and experiences were identified as an important factor explaining why digital technology 'worked' or why it did not. Therefore, this final case focuses on the role of the teacher, teaching practices and teachers' professional development.

One of the first studies focusing on teachers' practices and professional development was the one by Ruthven and Hennessy (2002). In this study and in subsequent work (e.g., Ruthven 2007) crucial factors are identified that affect teachers integrating digital technology in their teaching. In relation to the instrumental genesis model, Trouche developed the notion of instrumental orchestration to stress the relevance of teaching practices (Trouche 2004). Case studies based on these models describe teachers' practices in relation to their opinions and beliefs (Drijvers et al. 2010b; Drijvers 2012; Pierce and Ball 2009). Another model on teachers' professional knowledge is Technological Pedagogical Content Knowledge (TPACK), which became widespread but is also criticized (Graham 2011; Koehler et al. 2007; Voogt et al. 2012).

In addressing the questions of how to prepare teachers for technology-rich teaching and how to enhance their professional development in this field, in line with the work done by Wenger (1998) on communities of practice, it is suggested that the participation in a community of teachers who co-design and use resources for teaching, can contribute to this (e.g., see Fuglestad 2007; Jaworski 2006). Digital technology in such an enterprise acts on two levels: first, the professional development concerns its use in mathematics education, and second, digital technology may support the community's work by offering online and virtual facilities for exchange. Digital technology is both the subject at stake and the vehicle to address it. Efforts have been done to exploit digital technology's potential for teachers' professional development by designing online courses.[3]

As a paradigmatic design research study in this field, let us now describe the work done by Sabra (2011) in somewhat more detail. In his PhD dissertation, Sabra describes two case studies of teachers' collaborative process of professional development in detail. In the first case ten teachers in the same school collaborate on the design of a final assessment training session and a mathematics investigation task while integrating the use of TI Nspire in their teaching. The second case study concerns a project in which eleven teachers, all members of the Sesamath community from all over France, collaboratively design resources on the concept of function that are part of the course manual. The analysis shows that the two communities develop in quite different ways, but that in both developments some critical incidents—called documentary incidents in the thesis—are decisive. The digital tools in this case include web facilities for collaborative work, file exchange and communication; the role they play for the participating teachers is best characterized by branch (1) in Fig. 1, the role of a tool for doing mathematics, or rather a tool for collaborating on the design of mathematical resources.

The main reason to discuss this study here is that its rich data including interviews, blogs and observations and its fine-grained data analysis provide a detailed insight in how communities of teachers may work (or may not) and how technology may support this.

From a theoretical perspective, Sabra uses the notion of documentational genesis as a main concept. Figure 6, taken from Gueudet and Trouche (2009), shows how this is analogue to the notion of instrumental genesis, but now addressing the level of teachers using and designing digital resources. The interesting point here, in my opinion, is that a similar framework is applied to and elaborated for different situations and different levels of technology integration.

So did technology 'work' in the Sabra study? Maybe the answer is different for the two cases that are described. In the case of the team of teachers within the same school, it seems that the digital technology does not have so much to offer, and that the professional interest of the community members does not invite a real engagement in an effective collaboration. As a result, one can wonder whether the targeted professional development really took place, and whether the community really contributed to it. In the second case of the teachers all over France, the analysis shows a very lively process of collaboration, which is clearly afforded by the digital technology and would not have been possible without it. Similar to the other cases described in this paper, it seems that decisive factors that explain the phenomena go beyond the straightforward point of the available technology. My impression is that for a school team of teachers, collaboration is far from self-evident, whereas teachers who volunteer for a role in the Sesamath project share a

[3]E.g., see http://www.edumatics.eu/.

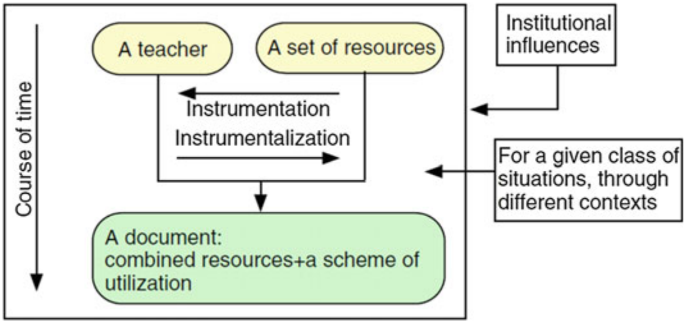

Fig. 6 Schematic representation of a documentational genesis (Gueudet and Trouche 2009)

professional interest to engage in a virtual community and in a shared process of distant collaboration. This, I would conjecture, might be the main explanation for the different results in the two cases Sabra describes.

Conclusion and Discussion

Conclusion

The—slightly provocative—question raised in the title of the paper is why digital technology in mathematics education 'works' or does not. The underlying aim was to identify factors that promote or hinder the successful integration of digital technology in mathematics education. The analysis of the six cases described in this paper show that the integration of technology in mathematics education is a subtle question, and that success and failure occur at levels of learning, teaching and research. In spite of this complexity, three factors emerge as decisive and crucial: the design, the role of the teacher, and the educational context.

The first factor concerns *design*. Cases 1, 3, and 4 reveal the crucial role of design. This concerns not only the design of the digital technology involved, but also the design of corresponding tasks and activities, and the design of lessons and teaching in general, three design levels that are of course interrelated. In terms of the instrumental genesis model, the criterion for appropriate design is that it enhances the co-emergence of technical mastery to use the digital technology for solving mathematical tasks, and the genesis of mental schemes that include the conceptual understanding of the mathematics at stake. As a prerequisite, the pedagogical or didactical functionality in which the digital tool is incorporated (see Fig. 1) should match with the tool's characteristics and affordances. Finally, even if the digital technology´s affordances and constraints are important design factors, the main guidelines and design heuristics should come from pedagogical and didactical considerations rather than being guided by the technology's limitations or properties.

The second factor concerns the role of the *teacher*, which is crucial in cases 1, 2, and 6. The integration of technology in mathematics education is not a panacea that reduces the importance of the teacher. Rather, the teacher has to orchestrate learning, for example by synthesizing the results of technology-rich activities, highlighting fruitful tool techniques, and relating the experiences within the technological environment to paper-and-pencil skills or to other mathematical activities. To be able to do so, a process of professional development is required, which includes the teacher's own instrumental genesis, or, in terms of the TPACK model, the development of his technological and pedagogical content knowledge. Case 6 suggests that technology can help the teacher to advance on this, together with colleagues in technology-supported collaboration. What seems to be an open question is how the role of the teacher changes if we consider the use of technology in out-of-school learning, gaming, and other forms of informal education (see case 5).

The third and final factor concerns the *educational context*, which includes mathematical practices and the elements of the Pedagogical Map designed by Pierce and Stacey (2010). Case 2 reveals how important it is that the use of digital technology is embedded in an educational context that is coherent and in which the work with technology is integrated in a natural way. Case 5, the MobileMath example, shows that taking into account the educational context includes attention for important aspects such as student motivation and engagement. Another factor that is not so much elaborated in the case descriptions but is important to mention here, is assessment, which should be in line with the students' activities with technology; not doing so would suggest that in the end the use of digital technology is not important. Finally, the use of digital technology may lead to an extension of the educational context towards out-of-school settings, as exemplified in case 5.

The three factors identified above may seem very trivial, and to a certain extend they are quite straightforward indeed; however, their importance, I believe, can hardly be overestimated and to really take them into account in educational practice is far from trivial.

Discussion

Let me first acknowledge that the study presented here clearly has its limitations. The discussion of the studies addressed cannot be but somewhat superficial in the frame of this paper. Also, the number of studies is small, and the choice of the studies included is not neutral. This being said, I do believe the article provides a—very rough—map of the landscape of research studies on technology in mathematics education and reveals some trends in the domain over the previous decades.

So what trends can be seen in retrospective? Globally speaking, a first trend to identify is that from optimism on student learning in the early studies towards a more realistic and nuanced view, the latter acknowledging the subtlety of the relationships between the use of digital technology, the student's thinking, and his paper-and-pencil work. A second trend is the focus not only on learning but also on teaching. The importance of the teacher is widely recognized and models such as TPACK, instrumental orchestration and the pedagogical map help to understand what is different in teaching with technology and to investigate how teachers can engage in a process of professional development. The third and final trend I would like to mention here concerns theoretical development. Whereas many early studies mainly use theoretical views that are specific for and dedicated to the use of digital technology (e.g., Pea's notions of amplifier and reorganizer in the Heid study), recent studies often include more general theories on mathematics education or learning in general, and also combine different theoretical perspectives (e.g., see the work by Bakker, using Pierce, RME, and other theoretical views).

To close off this discussion, I would like to express my strong belief that these theoretical developments are crucial for the advancements in the field. The studies addressed in this paper show strong relationships between the theoretical frameworks, the digital tools and the mathematical topics (Kieran and Drijvers 2012). We now have a myriad of theoretical approaches available in our work, including very specific theories on the use of technology in mathematics education, domain-specific instruction theories, and very general views on teaching and learning. One of the challenges in our work, therefore, is to combine and contrast the lenses each of these approaches offer (Drijvers et al. 2012). The notion of networking theories (Bikner-Ahsbahs and Prediger 2010) provides a good starting point that may help to better understand the role of digital technology in mathematics education and, as a consequence, to contribute to the learning and teaching of the topic.

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