Chapter 12
The Use of Videos and Classroom Artefacts in Professional Development of Teachers and Teacher Educators in Indonesia

Wanty Widjaja and Maarten Dolk

Abstract This study is grounded on adaptations of Realistic Mathematics Education, Lesson Study and design-based research in Indonesian classroom contexts. Design-based research has gained currency in educational research over the past decade due to its strength to bridge the divide between theoretical research and educational practice in naturalistic settings. Design-based approaches involve a process of designing mathematical tasks, observing the enacted design in classrooms and reflecting on the process from analysing the classroom artefacts. Video plays a central role in supporting teachers and teacher educators to study and reflect on students' mathematical thinking and in capturing the dynamic of classroom teaching and learning process. This chapter will examine and analyse practitioners' lenses in capturing the dynamic and complexity of classroom mathematical learning using video vignettes and classroom artefacts including digital photos of classroom moments and students' work. Practitioners' lenses are taken as a window to capture key teaching and learning moments from the lessons. Analysis of this selection of these video vignettes along with other classroom artefacts based on practitioners' lenses provides insights into practitioners' views on key teaching and learning moments in mathematics lessons.

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Introduction

In Indonesia, there has been a concerted effort to improve the quality of mathematics teaching and learning in classrooms. The large number of under-certified teachers and the rural nature of a large part of the country ask for professional development programmes that empower teachers through active collaborative peer involvement. In an era with increasingly more powerful mobile phones with better video- and photo-taking capabilities, collecting artefacts of teaching and learning in a classroom becomes more affordable and feasible. A critical issue is to provide a set-up that allows teachers to create self-supporting research groups.

The adaptation of Realistic Mathematics Education (RME) in Indonesia was driven by the intention to make mathematics more accessible and meaningful for students and improve students' attitude towards mathematics and quality of mathematics teaching in schools (Gijse, 2010). Indonesia is a large and culturally diverse country; an adaptation of RME – called *Pendidikan Matematika Realistik Indonesia* (PMRI) – needs to fit the diverse local contexts in Indonesia. PMRI places critical importance in learning mathematics through students' active engagement in communicating their thinking during the learning process. In contrast to the Indonesian educational tradition in which teachers are seen as the sole authority of knowledge and students are expected to learn standard algorithms and use these to solve simple mathematical applications, PMRI calls for a pedagogical approach with an emphasis on problem solving and communicating the thinking process in learning. Sembiring – one of the founding fathers of PMRI – likens the PMRI pedagogy with 'democratic teaching through mathematics' (Gijse, 2010, p. 20) as PMRI encourages students and teachers to listen to each other, defend and justify their thinking, value different opinions and discuss various mathematical approaches.

Realistic Mathematics Education started in the Netherlands at the end of the 1960s to renew Dutch mathematics education. It also formed the Dutch 'response' to the New Math movement in the United States. Freudenthal believed that mathematics should be thought of as a human activity of mathematising. To him, mathematics is a process of structuring, schematising and modelling, not a discipline of structures that could be transmitted (see, for instance, Freudenthal, 1978, 1991).

In RME, students are not seen as passive recipients of existing mathematics, but they should be guided to reinvent mathematical knowledge. The idea of guided reinvention 'is to allow students to come to regard the knowledge that they acquire as their own private knowledge, knowledge for which they themselves are responsible' (Gravemeijer & Doorman, 1999, p. 116). The learning starts with situations or problems that are 'experientially real' to the students (hence the name Realistic Mathematics Education). These are situations where the students realise what they can do and what makes sense to them in those situations. These can be 'real-world' situations but also situations from the history of mathematics, from applications of mathematics, from a fantasy world or from the world of numbers that are real to the students. These starting points for learning should not only be 'experientially real', they should also be justifiable in terms of the potential ending points of the sequence (Cobb, 2000). While working on context problems, the students develop strategies, knowledge, representations and models closely related to the situation. By offering different contexts and by generalising aspects of the contexts, the students develop more formal mathematical knowledge, and the models emerge from models of the situations into models to think with (Gravemeijer, 2004).

Gathering systematic evidence from classroom practice by research to inform teaching is perceived as a critical aspect in any reform initiative in education (Cobb, 2000; Jacobs, Lamb, & Phillip, 2010; Mason, 2002). Analysing classroom artefacts has been recognised to contribute to teachers' growth of knowledge, skills, beliefs and dispositions to improve teaching (Borko, 2004; Putnam & Borko, 2000). Mason (2002) claims that sharpening skills to notice relevant things in classrooms are vital for teachers as they constantly have to act and make pedagogical decisions in the moment. In similar vein, Jacobs, Lamb and Philipp (2010) underscore the significant role of professional noticing children's mathematical thinking as a foundation for productive learning experience for teachers. They conceived professional noticing of children's mathematical thinking as an interrelated skill comprising skills to: (a) attend to children's strategies, (b) interpret children's understanding and (c) use children's understandings as a springboard for instruction.

Extensive studies have documented the use of video as an artefact of practice to support teachers in developing their ability to notice and reflect on classroom practices (Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Sherin, 2004; van Es & Sherin, 2008). Video affords opportunities to capture the dynamics and complexity of classroom interactions and provides grounded images to hone in particular aspects of teaching (Ng, this volume). Video provides teachers and researchers the opportunity to engage in fine-grained analysis of classroom practice using multiple perspectives. Video vignettes can be examined several times with different foci and levels of insight in analysis to foster productive professional discussions for teachers' professional development (Borko, Jacobs, Eiteljorg, & Pittman, 2008). Lesh and Lehrer (2000) highlight the significance of complementing and triangulating video data in the light of the restricted lens of the camera with other evidence such as students' works and photos of the teaching and learning process.

This study was situated in the context of an intensive 5-day professional development programme for primary school teachers and mathematics teacher educators to learn design research approach for designing and enacting classroom investigations and examining evidence of mathematical learning in classroom practice. It was part of the larger project of *Pendidikan Matematika Realistik Indonesia (PMRI)* which has its genesis in 2001 (Sembiring, Hoogland, & Dolk, 2010; Zulkardi, 2013). The role of video and classroom artefacts to support teachers and teacher educators in noticing the key teaching and learning moments in mathematics classrooms will be examined. In this set-up, practitioners (primary school teachers and teacher educators) exercised their professional noticing by capturing and analysing what they considered as key teaching and learning moments in the classroom through video camera, digital still cameras and classroom artefacts such as students' works over 3 days. We contend that this set-up requires practitioners to revisit their own practice and beliefs on what constitutes critical elements of mathematics teaching and learning. Furthermore, we argue that video technology allows practitioners to ground their discussions based on collected evidence of what works in the classroom. This chapter will illuminate our learning points as teacher educators from working together with teachers and practitioners in this programme. The potentials and limitations of the set-up with respect to the role of video will be discussed.

Theoretical Framework

Lesson Study has been widely adapted in many countries as a platform for professional development of teachers (see, e.g. Doig & Groves, 2011; Lewis, Perry, & Hurd, 2009). Key elements of Lesson Study such as collaborative planning, observations of public research lessons and reflections during post-lesson discussions are recognised as valuable ways to deepen teachers' professional knowledge. Teachers who are able to orchestrate a productive whole class discussion carefully plan detailed lessons paying explicit attention to key mathematical ideas, students' anticipated solutions and students' learning trajectories (Watanabe, Takahashi, & Yoshida, 2008).

Lesson Study engages teachers as 'investigators of their own classroom practices' and 'researchers of teaching and learning in the classroom' (Takahashi & Yoshida, 2004, p. 438). Collecting specific and detailed data of students' learning including their misconceptions or difficulties, in order to have a better grasp of students' learning, is vital during the observation. Practitioners play a key role by collecting specific evidence on student learning and the teacher pedagogical move that might not be noticed by the teacher during the research lesson (Lewis & Tsuchida, 1998). Various tools including video camera, digital camera and observation record sheets are frequently utilised to document evidence of students learning in detail. With a growing concern to improve teacher professional competencies and to incorporate real classroom practice as the basis of in-service teacher training, Lesson Study has gained increased acceptance as a promising approach in Indonesia.

Aware of the common practice of evaluating teacher's performance during observations, our study draws on the Lesson Study model by predicting anticipated students' solutions as an integral part of the planning stage. In our set-up, anticipating students' solutions was made explicit to support practitioners' retrospective analysis on students' mathematical thinking and development rather than on teachers' or students' behaviours. The cyclical process of planning-doing-reflecting and the collaborative nature of capturing students' learning through observing the classroom practice in Lesson Study approach were integrated in this study.

The Use of Video and Classroom Artefacts to Facilitate Professional Noticing

It has been well documented that engaging teachers in professional development that focuses on student thinking is valuable and powerful. Skills to notice, attend and respond to students' mathematical thinking in classroom settings need to be developed. Video and classroom artefacts such as students' work are valuable tools in providing opportunities for teachers to cultivate skills to notice students' mathematical thinking.

Video has been used for a long time in education community and become increasingly popular as an artefact of practice in teacher professional development because of its unique ability to capture the richness and complexity of classrooms for later analysis (Brophy, 2004; Dolk, Faes, Goffree, Hermsen, & Oonk, 1996; Lin, 2002). Seago (2004) contends that video cases afford teachers the opportunity to develop a more complex view of teaching, new norms of professional discourse and mathematical knowledge needed for teaching by honing in particular aspect of teaching. Video also affords the luxury of time and opens up opportunities for teachers to engage in fine-grained analysis and reflection of classroom practice with varying lenses (Brophy, 2004; Sherin, 2004). Rosaen, Lundeberg, Cooper, Fritzen and Terpstra (2008) find that video affords a more tangible view of teachers' teaching that is more specific, more complex and more focused on instruction and students than respective memory-based reflections. Similarly, van Es (2012) reveals that video affords teachers to become more student centred and evidence based in their analysis and reflection of classroom interactions as they engage in collaborative inquiry of their classroom practice.

Lesh and Lehrer (2000) point out that video enables teachers and teacher educators using both theoretical and practical perspectives to work collaboratively in analysing data. Furthermore, they underscore the importance of using methods for triangulation by using other artefacts such as students' works to overcome the restricted lens of camera when analysing videotape data. Roschelle (2000) concurs with Lesh and Lehrer's point on the methodological challenges of video data and the need to devise complementary technique to attend to the potential biases of video data. He highlights the benefits of direct field observations, interviews, field notes and students' work to provide an alternative view of the capturing data and to fill some of the details missing on the video.

Hall (2000) challenges claims that videotape provides 'objective' or 'realistic' records of human action (p. 658). According to him, video records of human activity systematically miss the experience of participants but at the same time provide access to events that practitioners might miss. Brophy (2004) contends that watching classroom video does not necessarily lead to new insights for teachers. It is critical to select video vignettes with clear goals and embed these in activities that are planned carefully to help teachers in meeting those goals within a professional development programme.

In this 5-day professional development programme, video vignettes were an add-on to direct observations and field notes by the practitioners and one of the windows into the practitioners' lenses used to capture key teaching and learning moments. Therefore, the video vignette allows for analysis of the moment itself and offers insight into practitioners' observation of the classroom teaching.

Design-Based Research Approach

Design-Based Research

Design-based research approach is grounded on a model of collaborative practitioner inquiry, which is most likely to embed pedagogical reform in practice (Design-Based Research Collective, 2003). Practitioners and researchers work closely together to implement innovative forms of learning and to study the means that are designed to support them in the contexts of practice (Cobb, 2000; Gravemeijer & van Eerde, 2009).

Design research entails a cyclical process between development stage and research stage (Gravemeijer, 1994, 2004). Theory and evidence from prior research inform and guide the development of mathematical tasks or activities, and the enactment of these tasks is evaluated based on evidence collected during research stage and fed back into a new cycle of envisioning and action. During the development stage, researchers and practitioners engage in an iterative exchange between thought and practical experiments. The iterative process of design, implementation and retrospective analysis corresponds well with the Japanese Lesson Study process of planning, observing and reflecting. Bannan-Ritland (2008) highlights the value of having first-hand experience of designing, implementing and reflecting in design-based research to teacher professional growth as adaptive experts:

> Teacher design research (TDR), whose goal is to promote the growth of teachers as adaptive experts … the instructional aspects of TDR comes not from outside experts, but, rather from the teachers' cognitive dissonance experiences as designers in design cycles. (p. 247)

Enacting Design-Based Research in a Professional Development Programme

The Study

A 5-day professional development programme took place in a private primary school in Surabaya, Indonesia. Teachers and teacher educators from PMRI centres and schools participated in the programme. The aim of this professional development was to equip participants (teachers and teacher educators) with first-hand experience of the design-based research 'mini' cycles of knowledge-designing-experimenting-retrospective analysis (Dolk, Widjaja, Zonneveld, & Fauzan, 2010) as depicted in Figs. 12.1 and 12.2.

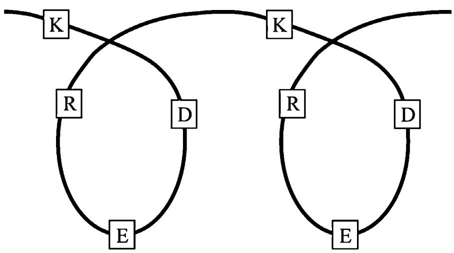


Fig. 12.1 The cyclical process of knowledge, designing, experimenting and retrospective analysis

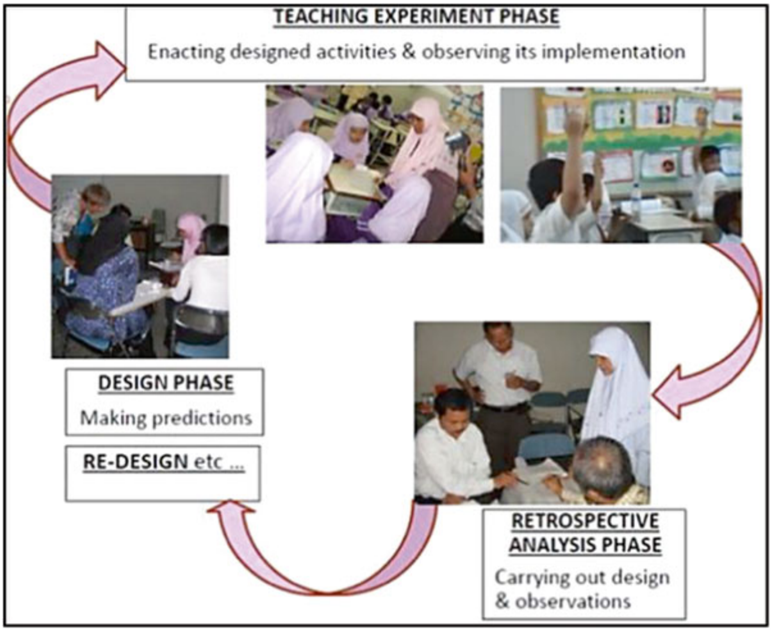


Fig. 12.2 The enactment of design-based research phases

The Participants

Eighteen teachers and teacher educators from nine PMRI centres and schools participated in this programme facilitated by the authors and two other teacher educators. Three grade 2 teachers and two grade 5 teachers from the host primary school participated in the whole programme and enacted the lessons with their respective students. In this chapter, we will refer to teachers and teacher educators who were participants in the programme as practitioners to differentiate them from the resident teachers who enacted the lessons. The resident teachers who enacted the lessons will be referred to as teachers. The authors and the other two teacher educators will be referred to as facilitators.

Structure of Groups

The practitioners worked in five small groups with two to three members in each group. Each group worked with one resident teacher and observed the same teacher and students throughout the programme. Because there were only three grade levels available, practitioners were encouraged to work with the year level that was most relevant for their own teaching post. Each member of the group was responsible for collecting specific sets of data. The member could be responsible for taking photos, videotaping lesson vignettes or writing field notes of the lessons. As practitioners collected evidence of teaching and learning in classrooms, they were requested to refrain themselves from teaching the students or talking to the teacher.

Method

Design and Planning Phase

During the design process and the planning of the lessons, practitioners, teachers and facilitators discussed the role of the context and the investigation, the role of the teacher, the role of the students, the development of mathematics by the students and the social norms in the class.

The Mathematical Investigation

The mathematical investigation was adapted from *The Young Mathematician at Work* series (Dolk & Fosnot, 2005). The facilitator chose numbers carefully to allow students to approach the problem using addition strategies involving 12 and 25 which offer potentials for students to extend their additive thinking to multiplicative thinking (see Fig. 12.3). It was also anticipated that some students might approach the problem by thinking of 25,000 as 25 thousand. Therefore, the task might be solved as 12×25 instead of $12 \times 25{,}000$. It should be pointed out that in adapting the context of the investigation, we took into account the fact that a kilogram of chicken costs about 25,000 rupiah. In the Indonesian language, this is commonly stated as '25 ribu rupiah' or 25 thousand. Formally, this is written as 'Rp. 25.000' as Indonesian used a decimal point instead of comma to mark a thousand.

The Indonesian mathematics curriculum introduces and develops multiplication by two digit numbers in Grade 2. The facilitators engaged practitioners and teachers to adapt a context 'The Turkey Investigation' (Dolk & Fosnot, 2005) for the Indonesian context (Fig. 12.3). Adaptation of this context of investigation includes the choice of local context and numbers that are relevant and appropriate for students. It is vital that the numbers were chosen carefully to offer multiple entry points for students and allow students to come up with alternative strategies to approach the investigation. The choice of numbers should make the investigation 'realistic' and 'enticing' for students to solve. The adaptation process also involved discussion

Turkey investigation

I need your help! My entire family is coming over for Thanksgiving dinner.

Here is the sign I saw in the supermarket: “Turkeys for sale. $1.25 per pound.”

The largest Turkey I could find was 24 pounds. With your partner, discuss how you could figure out how much will the turkey cost? How much will the turkey cost?

Adaptation of Turkey investigation

Ustadzhah wants to buy 12 kilograms of chicken. Each kilogram of chicken costs 25 thousand rupiah. How much money will Ustadzah need to bring?

Follow-up Turkey investigation

Last year when I went to my sister for Thanksgiving at her house, we waited for hours for the Turkey to be done because it was such a huge Turkey it needs to be cooked for a long long time. So last night, I was thinking I don’t want everybody to have to wait and I want to make sure that my Turkey gets cooked on time so I got out my old favourite cookbook and here marked on this page. Listen very carefully because I need your help one more time...

It says here “If you are not using a thermometer allow up to 20 minutes to the pound for a bird up to 6 pounds… For a larger bird, which my Turkey is, allow up to 15 minutes per pound”.

So it says it here on the book I have to cook it for 15 minutes per pound. What I’d like you to do now is to work out how long you think I need to cook this Turkey. With your partner, think about how you are going to figure that out.

Adaptation of follow-up the turkey investigation

Ustadzhah want to ask your help one more time.

Ustadzhah is going to cook 12 kilograms of chicken for the whole family to celebrate the Ied.

It turns out that cooking one kilogram of chicken takes about 15 minutes.

How long do you think it will take Ustadzhah to cook 12 kilograms of chicken?

Fig. 12.3 The mathematical investigations and their adaptions

about expectations of what students could do given the investigation, what approach and solution to the investigation could be and how that might support the development of students’ mathematical knowledge and insights. Because the lessons to be conducted were different compared to a traditional Indonesian classroom, it was necessary to provide the teachers some examples of the nontraditional lessons.

Short video vignettes from *Turkey investigations* (Dolk & Fosnot, 2005) were shown to the teachers and practitioners on the first day during the planning phase. In the video, the teacher presented the investigation as her own personal problem in cooking Thanksgiving dinner and invited students to help her in solving some of her problems. The video was not dubbed into *Bahasa Indonesia*, but the first author provided the translation into *Bahasa Indonesia*.

The teachers and practitioners were concerned that the Grade 2 students may find the adapted Turkey investigation too challenging because it involved solving a multiplication of 25,000 by 12. Multiplication and division involving two digit numbers (tens) are taught in the second semester of Grade 2. The three Grade 2 teachers were clearly nervous to ask their students to solve this problem. At that time, the facilitators negotiated with the teachers that the facilitators would be responsible should the students fail to engage with the investigation. The practitioners were encouraged to collect evidences that support or contradict these two questions: *what proof do we have that this context is too difficult for the children* versus *will the students be able to solve this problem using their common sense*.

Responsibilities of Facilitators The facilitators designed and supported the teachers during the teaching experiment. After each of the teaching experiments (day 2, day 3, day 4), the facilitators met with the five teachers. While the meeting was brief (10–15 min), it allowed teachers to reflect and express their thoughts and concerns. Hence, the facilitators could gather insights into teachers' personal take of how the lesson went and what support they would need for the next day. As teachers were new to the pedagogy, they might not be comfortable in sharing their concerns in public space. The facilitators also offered support for the teachers during the teaching if they were uncertain about the mathematical content or the selection of students' work to be discussed.

Responsibilities of Practitioners After each classroom teaching experiment, each group of practitioners met to discuss and analyse the evidence they collected during the teaching experiments. Based on the analysis of evidence collected during observations, each group prepared a poster containing a selection of twelve snapshots of key teaching and learning moments, a 5-min video vignette and observation notes to share their evidence and insights to other groups.

Teaching Experiment Phase

Day 1: Practitioners and teachers watched the video vignettes of Turkey investigation and solved the mathematical problem followed by discussion of adaptation of the problem into the local context. Groups of teacher-practitioners-facilitator were formed for observations on day 2, day 3 and day 4. The role of teachers, the role of practitioners, and the research questions were discussed.

Day 2: The adaption of Turkey investigation was presented to the students. Students worked on the adapted investigation in small groups and prepared their posters that contained various strategies to solve the investigation. Each group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 2.

Day 3: Students presented and shared their strategies during a whole class discussion orchestrated by the teacher. Group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 3.

Day 4: Students worked on an investigation to figure out the length of time required to cook 12 kg of chicken if it takes 15 min to cook 1 kg of chicken. Whole class discussion of students' strategies took place. Group of practitioners worked on collected evidence to prepare a group poster reporting on their classroom observation from day 4.

Day 5: Reflecting on the 4-day experience as an exercise of retrospective analysis phase. The sessions were run concurrently for teachers and teacher educators. Both authors facilitated the sessions for teacher educators with the focus on reflecting over the 4-day experiences and linking those experiences with the design research cycle. The other two facilitators ran a parallel reflection session with the teachers.

Retrospective Analysis Phase

The analysis was based on ten selected key moments captured through practitioners' camera lens, a 5-min video vignette of the lesson and classroom artefacts from three Grade 2 classes.

Practitioners' Insights in Capturing the Classroom Practice

In this section, we discuss the evidence gathered by practitioners and teachers. We start by articulating the roles of the videos before presenting evidence of practitioners' insights captured through their video vignettes, a collection of 12 photos and field notes of students' works over the 3-day classroom teaching experiments. The final section discusses what the authors have learnt from working with the teachers.

The Roles of Videos in This Professional Development Programme

The videos play three important roles:

1. To cue teachers into the mathematical investigation different from conventional Indonesian classrooms

The video presented a model for teachers in this study to engage students in mathematical investigations in a real-world situation. While the choice of numbers was open for discussion, the phrasing of the investigation was left to individual teacher's personal and professional judgement. It was evident that the video vignettes of the Turkey investigation sparked discussion among teachers and practitioners.

2. To provide teachers with models of how socio-mathematical norms are being enacted and negotiated in the classroom by the teacher and students at various junctures of the lesson

   The social norms guide the class on the normative expectations of interactions such as the need not only to provide an answer but also explanations that lead to the answer. In mathematical learning, 'socio-mathematical norms' signify what counts as mathematically different or acceptable mathematical explanation (Yackel & Cobb, 1996; Yackel, Cobb, & Wood, 1998). In Indonesian classrooms, teachers are the arbiters of knowledge. They decide on whether students' answers are right or wrong. Showing video vignettes of socio-mathematical norms being enacted was vital to engage practitioners in thinking and planning ways to establish classrooms culture and socio-mathematical norms that encourage students in articulating their ideas and negotiating their interpretations. The video offers participating teachers with socio-mathematical norms that were not consistent with conventional mathematics classroom in Indonesia.

3. To gather insights about what teachers and practitioners value in mathematics lessons

   In this professional development, teachers and practitioners collected evidence from the classroom using video, a collection of 12 photos and field notes of students' works which provided insights about what teachers and practitioners valued in mathematics lessons.

Evidence Captured Through Practitioners' Lens

The collected artefacts depicted what happened in the classroom seen through lens coloured by practitioners' expectations before the lesson and by the suggested research question. All artefacts collected by the participants – video, photographs and observation reports – pointed into the same direction showing a prominent attention towards students' mathematical thinking and their work, shifts in the role of the teacher and development of socio-mathematical norms in the classroom.

Noticing Students' Mathematical Thinking and Students' Works

During the classroom experiment, practitioners could focus on what happened in class based on their expectations, and during the selection of the artefacts, they could compare what happened in class with their expectations. Given the critical

Fig. 12.4 Students' engagement captured through the practitioners' lens

debate during the design phase, we are not surprised that the practitioners pay much attention to the students' work and to students' explanations of their thinking. All groups of practitioners captured students as their focal points documenting students' working in groups and samples of students' work. The fact that practitioners noticed students' mathematical thinking as central was apparent in the proportion of snapshots that were centred on students' work and students' working on the investigation. Various strategies including some of students' misinterpretation of the task were documented. Observing these artefacts enabled group of teachers, practitioners and facilitators to make sense of students' mathematical thinking. It allowed us to notice practitioners' shift of attention to students' mathematical thinking and their expectations about students' mathematical capacity (Fig. 12.4).

The practitioners' lens focused on the students' solution of this difficult problem and balances between a focus on traditional strategies and on alternative strategies. The practitioners also balance between selecting moments in class that showed understanding by the students and moments where the students were following a standard procedure. An example of the latter was an instance when one student decomposed 63 into 0 + 60 + 3 (0 hundreds, 60 units in the tens and 3 units in the ones). Although the practitioners' artefacts still paid attention on classroom management, the focus of most artefacts shifted to students' mathematical thinking. An explanation of these shifts can be found in the earlier discussion about the suitability of the problem and the choice of the large numbers.

Students' multiple strategies in solving the problem were documented on observation notes from different teams. Contents of day 2 and day 3 posters showed that there was a strong focus on analysing students' strategies and students' works to reflect their prediction of what strategies students might use. One team recorded that students demonstrated good mathematical thinking. The team identified and classified them into three main strategies: repeated addition using doubles, direct proportion and decomposition. The team observed that students were able to record the total sum of money using appropriate notation *Rp. 300.000* and to associate this with '300 thousand' correctly. The team pointed out that they found evidence of two out of eight groups applied the anticipated strategy of using 'a table form'. Finally, the team noted that some students did not pay attention during the lesson.

Repeated Addition

1 chicken = 25 thousand.
2 chickens = 25 thousand + 25 thousand → 1 chicken + 1 chicken = 2.
4 chickens = 50 thousand + 50 thousand → 2 chicken + 2 chicken = 4.
8 chickens = 100 thousand + 100 thousand → 4 chicken + 4 chicken = 8.

Direct Proportion

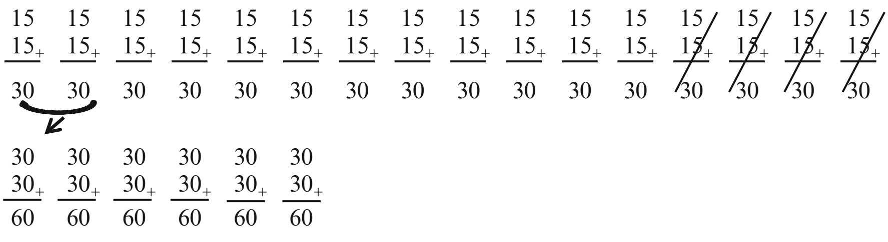
1 chicken = 25 thousand.
2 chickens = 50 thousand.
4 chickens = 100 thousand.
8 chickens = 200 thousand.
12 chickens = 100 + 200 = 300 thousand.

Decomposition

$2 = 1 + 1 \rightarrow 25 + 25 = 50.$
$4 = 2 + 2 \rightarrow 50 + 50 = 100.$
$8 = 4 + 4 \rightarrow 100 + 100 = 200.$
$12 = 8 + 4 \rightarrow 200 + 100 = 300.$

Observation record from day 3 continued to have a strong focus on students' strategies but also highlighted the fact that students were getting used to working in groups and to sharing ideas among group members. Multiple strategies including an incorrect strategy that indicated lack of place value knowledge in working out the multiplication algorithm were documented below:

Repeated addition and doubling



One to one correspondence and counting on by 15 s

1 chicken: 15 min
2 chickens: 15 + 15 = 30 min
3 chickens: 15 + 15 + 15 = 45 min
⋮
24 chickens: $\underbrace{15+15+15+\ldots+15}_{\text{24 times}} = 360$ minutes

Multiplication and direct proportion

$24 \times 15 = 300$.
1 chicken requires 15 min.
2 chickens require 30 min.
⋮
24 chickens requires 360 min.

Confusion about place value in multiplication algorithm

$$\begin{array}{r} 25 \\ \underline{12}\ \times \\ 39 \\ \underline{15}\ + \\ 54 \end{array}$$

The practitioners' focal lens was not directed on the teacher but mainly on the students and their work. The video vignettes captured the practitioners' lens of key teaching and learning moments on day 2 of the teaching experiment whereby a significant portion of the lesson was devoted to group presentations. More than 5 min of the video was focused on the process of students presenting their solution strategies documented in the posters in front of the classrooms. Students' multiple strategies were evident, and the teacher invited some groups to explain their strategies to the teacher, the practitioners and their peers.

The video vignette captured the teacher asking a student to work on decomposition strategy in adding two whole numbers at the end of the lesson. The student showed some understanding of decomposing a number 63 into 60 and 3 and addition strategy. At this point, the practitioners chose to focus on students' written work on the board (Fig. 12.5) which documented the use of addition sign instead of an equal sign. Capturing this moment on the video provided evidence of students' knowledge of place value as shown in the decomposition of $63 = 0 + 60 + 3$ and $6 = 0 + 0 + 6$. The teacher ended the lesson by posing some problems (Fig. 12.5) to help students notice the relationships between $63 + 20 = 83$ and $63 + 19 = 82$. This action was intended to consolidate students' learning on place value and addition and to extend their addition strategy by looking at the relationships between the addends (i.e. 20 and 19) and the sums (i.e. 83 and 82).

Shifts in the Role of the Teacher and the Students

Both videos and photos captured the teacher at work mainly at the beginning of the lesson when she introduced the problem, set up groups and managed the classroom. Hence, the teacher was not out of the picture, but only one or two snapshots were centred on the teachers, and these were mainly recorded on day 1 and day 3. Other moments that featured the teacher at the centre were when the teacher practised

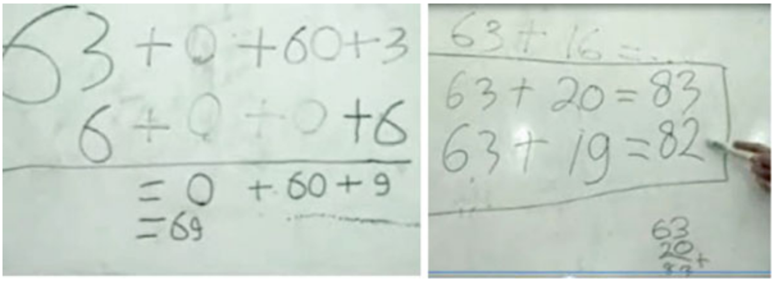


Fig. 12.5 Revisiting addition strategy

classroom norms and socio-mathematical norms such as dealing with disengaged students, getting students to contribute their explanations and posing questions to set a new norm. There was a clear shift in the teacher's role with more attention to encourage students to articulate their thinking and ideas to the teacher and the rest of the class. Rather than telling students the main learning point from yesterday's lesson, the teacher invited students to identify those learning points. Students listed addition, multiplication, story problem, division (that the teacher crossed out from the list) and addition with carrying over as their learning points.

Students had a more active role by working together in groups to explore multiple solutions and preparing a poster to document their solution strategies and presenting them to their peers. As students were engaged in the investigation, the video captured the teacher spending time to observe students' strategies and listening to their discussion. The teacher dealt with classroom management issues such as calling students' attention back to the lesson. Although the practitioners' artefacts paid attention to classroom management, the focus of most artefacts shifted to students' mathematical thinking.

The teacher was – given the design process – able to create a classroom that was different from the traditional Indonesian classroom. She encouraged students to explain and justify their strategies and spent time on different strategies including 'incorrect' solutions and negotiated the 'new' social norms with the students in the classroom. Schoenfeld (2002) underscores the significant role of the teacher in shaping students' understanding of mathematics through a carefully crafted design and plan. He argued that productive exchanges among students were not a spontaneous act but rather a reflection of a consistent practice of a classroom discourse community. While the evidence indicated that the classroom discourse might not yet been part of a consistent practice in these classrooms, the artefacts reflected attempts that mathematical discursive habits were being negotiated by the teacher and the students (Sfard, 2000; Yackel & Cobb, 1996). Based on the collected artefacts, we noticed a growing appreciation and attention to other aspects of teaching such as active role of the students and how students develop mathematical knowledge and reasoning, and the use of alternative strategies.

Introducing the investigation | *Listening to students' group discussion* | *Introducing the investigation*

Fig. 12.6 The teacher at work captured through the practitioner's lens

Min 2:25 video | Min 3:06 video | Min 3:20

Fig. 12.7 Dynamic interaction of students at work (screen grabs extreme *left* and *centre*, and the teacher checking herself, extreme *right*)

Development of Social Norms and Socio-mathematical Norms

The practitioners chose the moments that indicated the progress of development of social norms in the classroom. Evidence of the development of socio-mathematical norms in the classroom being practised and negotiated by the teacher and the students was consistent in both the videos and the collection of 12 photos by the practitioners. The video vignettes captured in a Grade 2 class revealed that the teacher initiated the norms to explain and justify students' strategies by saying 'Please explain to *Ustadzah*, to *Uztadzah's* friends and to all your friends how do you solve the problem using those strategies' (Fig. 12.6).

Establishing new classroom norms and socio-mathematical norms is neither easy nor straightforward. In fact, the process might create tensions for both the teacher and the students when these norms are not yet accepted as part of the classroom culture (Yackel et al., 1998). The grounded images from the video captured the tension experienced by the teacher and the students as the new norms were being negotiated and enacted in the classroom. After inviting a group whose work indicated the use of repeated addition and multiplication to explain their work, the teacher asked one student to demonstrate the use of multiplication algorithm to multiply 12 and 25. When the student finished his work, the teacher posed a question to the student but quickly checked herself by asking '*Oh why Ustadzah was the one who ask questions*?' (min 3:20, Fig. 12.7). Here, the moment when the teacher noticed the need to practise a new norm to invite other students in posing questions to other students was captured in the practitioner's lens.

The development of socio-mathematical norms was reflected as the main focus of the whole class discussion. It was clear that the goal of the whole class discussion was not to find out whether the answer was correct or incorrect but to understand students' strategies and the thinking behind those strategies. The video vignettes documented the teacher's attempt to probe into student thinking albeit students' responses did not reveal much.

Teacher: Oh according to '*mbak*' Ani (pseudonym) this is incorrect.
Ani: That is incorrect.
Teacher: How about this one? Is this correct or incorrect? Why did you hide behind? Why does a beautiful girl hide behind? Okay so why is this incorrect?
Ani: Because the answer is only 37.
Teacher: Because the money is only 37. Why is 37 not acceptable?
Ani: Because the money is not enough.
Teacher: Oh according to Ani, 37 is not enough money. Now, please help explain why there are so many twenty fives; Ustadzah is still confused. Please explain your thinking behind one and two.

The grounded images brought to light the importance of looking out for the socio-emotional needs of children who were overtly shy and hid behind the poster during her group presentation. In contrast, there were students who dominated the presentation or made a scene in front of the class during the group presentations. These artefacts were indicative of challenges students faced when new social norms and socio-mathematical norms were enacted and being negotiated. We anticipated that having all members of the group share their work would alleviate the pressure for shy students. The evidence suggests the new classroom social norms take time to develop and teachers need to be prepared to deal with such group dynamics.

The development of socio-mathematical norms was evident when the teacher invited another student to explain another group's strategy whose work was shown in Fig. 12.8. Furthermore, the teacher underlined the importance of understanding other students' ideas by saying 'It was *a must* to pose questions if you don't understand'. The student explained the second strategy from group 2 by articulating '25 plus 25 is 50, 25 plus 25 is 50 so the sum of these two is 100'. She continued with repeated addition of 25s until she reached 300.

The collected artefacts captured some students struggling with having to explain and justify their thinking during presentations (see, e.g. Fig. 12.8). Students were not used to speaking up during the presentation, so a microphone was used to ensure that their peers could follow other students' explanations and engaged in the discussion after the presentation. It was not surprising that the practitioners focused on the development of social norms in the classroom. What was remarkable was that the practitioners' lens was on the students when they were explaining their strategy and did not shift towards the teacher even when she asked a question. The focal point on the student was indicative that students were the dominant source of attention for the practitioners.

Min 3:57 video Min 4:36 video Min 4:40

Fig. 12.8 Dynamic collaboration among students (Group 2) as they worked on the board

Learning Points from Working with Teachers and Practitioners

In this chapter, we describe a programme where a group of teachers and practitioners became design researchers. This programme was based on a hybrid of Lesson Study, design research and Realistic Mathematics Education. We had the teachers and the practitioners codesign an investigation and anticipate what they thought would happen in class. Furthermore, the teachers were asked to formulate one question they liked to be able to answer after the class observation and to find evidence that would help to understand that question better and to answer the question in this particular case. Specific roles and tasks were assigned to every team member in collecting grounded images from the classroom using video, cameras or observation notes of students' works. The expectations to use these grounded images to explain students' written work, their discussions and their thinking to the other teams were made very clear throughout the programme. As the participants were active codesigners of the investigation, they had insights what to expect in these lessons as discussed in Section 4.2. These classroom artefacts informed our next design, teaching experiment and retrospective analysis.

The lessons carried out in this programme were not an average mathematics lesson in the Indonesian context. The majority of mathematics lessons in Indonesia are still teacher centred. Traditionally, in Indonesia, the teacher explains the mathematics and sets the students work practising and applying the new knowledge. During this time, the teacher walks around to support students, to check upon their work and to indicate if the answers are correct (Dolk et al., 2010; Widjaja, Dolk, & Fauzan, 2010). Artefacts from such a lesson would focus on the teacher, on the correct explanation of the mathematics at hand, on the support the teacher gives to the students and on the correct answers by the students. The artefacts collected during this lesson are different. The artefacts not only focused on the teacher but also showed *a balance* between the teacher and her work and the students and their work. They signal practitioners' redefining the role of teacher and students' work in classrooms and what teachers and practitioner value in their classrooms. We contend that three simultaneous processes involved in the design phase are working towards this:

- Design of the lesson emphasising students' mathematical reasoning, the use of alternative strategies and not correct or incorrect answers
- The emphasis during the design phase to 'other' aspects to teaching (e.g. active role of the students, how students develop mathematical knowledge, social norms in a classroom)
- Aspects of the educational theory and knowledge that the facilitators emphasised during the design process and the way the facilitators supported the teacher in her preparation of the lesson

The teachers and the practitioners were triggered by the discussion, by the facilitators' confidence that bigger numbers will serve as a challenge and not as an obstacle and by the arguments by the facilitators that the chosen numbers allowed the students to use their common sense to solve this problem in multiple ways. Furthermore, practitioners were triggered by an explicit call to focus on evidence in the classroom. Their attempt to understand if the problem was within reach of the students implied that they had to focus on students thinking, actions and work. The students and their work supported our claim that the large numbers were not an obstacle and that the careful choice of numbers enabled students to use common sense in developing alternative strategies rather than rely only on the formal algorithm. The fact that the practitioners focused on the students' thinking and work had two effects. Firstly, they focused less on behavioural aspects of the teacher's role, aspects like classroom management. Secondly, they focused more on the development of the mathematics by the students. This was the change in the practitioners' lens that the facilitators anticipated with this programme. In particular, there was a clear shift on the use of student works as a springboard for analysing teaching and revisiting the plan for the subsequent task.

The set-up of this professional development proved to be rich. During the course of this professional development programme, the teacher was supported by a teacher educator who facilitated the programme on a day-to-day basis. The collaborative effort between the teacher and the teacher educator in carrying out this classroom practice was critical in creating an environment that supports the learning of mathematics. It is important that teachers feel that they are being supported and not 'judged' by the presence of other teachers and teacher educators in the classrooms. To create this, the practitioners were not allowed to talk with the teacher or to teach the students. If they felt the teacher had to do something different or if they had a suggestion for the teacher, they could inform the co-teacher (one of the teacher educators). It was up to the co-teacher's discretion how best to proceed. Furthermore, shifting the attention from the teacher to the students' mathematical thinking is really critical, particularly in Indonesian context where there is a strong tendency to focus on teachers' actions during classroom observations (Saito, Hawe, Hadiprawiroc, & Empedhe, 2008).

We argue that this set-up is powerful for three reasons. Firstly, the division of several observation tools and detailed observation 'rules' created a wide range of artefacts of the lesson. The different tools supported and complemented each other. For instance, the limitations of video (video footage does not show the whole situation and is not as ‘wide’ as real-time observation) were compensated by the other observation tools. Secondly, the discussion about their expectations and the focus on evidence towards a research question the practitioners formulated before the lesson created a backdrop against what the practitioners could reflect on the lesson. Thirdly, the nature of the set-up provides opportunities for teachers and teacher educators to engage in collaborative learning to examine and reflect on their classroom practice as members of a community of inquiry (Jaworski, 2008). Such opportunities are vital for sustained professional learning of teachers and teacher educators.

Future Directions

Analysis of practitioners’ video vignettes along with photos of key teaching and learning moments and students’ works suggests that teachers and teacher educators in this study paid an increased attention to students’ works and their mathematical thinking and less on the action of the teachers. This concurs with findings reported by van Es and Sherin (2008) that teachers increase their focus on interpreting students’ mathematical thinking in detailed ways after sharing and discussing vignettes of their video-recorded classroom practices. While realising the affordances of video in capturing key teaching and learning moments and supporting teachers’ and teacher educators’ reflections, it is critical to complement the use of videos with photos and students’ work. This is in line with points raised by Lesh and Lehrer (2000) and Roschelle (2000) on the importance of complementing classroom video data with students’ works and field notes from the classroom observation. The nature of this study did not allow us to trace the impact of this programme when the participants returned to their regular classroom practice. However, our work with some teachers in Indonesia following a similar set-up documented teachers’ pedagogical growth in noticing and engaging students in developing their mathematical thinking (Widjaja, 2012; Widjaja, Dolk, & Fauzan, 2010).

This set-up situated in a classroom setting empowered practitioners through active involvement in designing, observing and analysing data using video, camera and field notes. Participation in this professional development programme provided opportunities for teachers and teacher educators to further their collaborations in order to improve classroom practices in their PMRI centres and schools. Through collaborative work, teachers and teacher educators construct educational knowledge on how to design a learning trajectory for a mathematics topic, to establish classroom norms that support learning and to create a situation in which students can construct mathematical knowledge. This study shows that the input of the teacher educator was still crucial. Further research and development is needed to design a system that allows teachers in rural settings to establish self-supporting design groups that utilise classroom video and photo artefacts to empower their knowledge about children’s learning and to improve their noticing of mathematical moments in class.

The use of grounded images employing a combination of video vignettes of lessons, a collection of 12 photos and observation notes allowed the teachers and practitioners to work together in researching students' learning, teachers' learning and classroom practices. The videos enabled all participants to analyse students' thinking and teachers' actions in depth. Sharing the short video vignettes and other classroom artefacts with other teachers and teacher educators who observed different classrooms offered opportunities and challenges for the practitioners to 'tell a story' grounded on evidence rather than relying on their personal views or beliefs.

The process of designing, observing, selecting classroom video vignettes and photos and analysing offers participants a high level of commitment to and ownership of the professionalisation. The in-service discourse was to a high extent about their thinking and their professional noticing (Mason, 2002, 2011). It has to be acknowledged that technical issues of selecting and creating a 5-min video segment on a daily basis for 3 days were quite challenging for some participants. In upcoming courses of intensive professional development with limited time, we would provide more support in both the process of selecting and in the technical aspect of the selection.

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