
Using Correspondence Analysis in Multiple Case Studies

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Résumé

Utilisation de l'analyse des correspondances dans les études de cas multiples.

Dans la recherche qualitative des études de cas multiples, Miles et Huberman ont proposé de résumer les cas distincts dans une méta-matrice qui se compose des cas croisés avec des variables. Yin a discuté de la synthèse des cas croisés pour étudier cette matrice. Nous proposons de recourir à l'analyse des correspondances (CA) pour étudier cette matrice. CA est une méthode quantitative qui donne une représentation graphique des lignes et des colonnes d'une matrice. Les lignes et les colonnes reçoivent les coordonnées qui peuvent être interprétées comme des quantifications; permettant que les cas soient comparés en utilisant ces quantifications. S'appuyant sur un exemple de la recherche qualitative en éducation concernant la philosophie, nous illustrons les deux méthodes et montrons leur complémentarité. Nous discutons des particularités de l'application de CA à l'étude de cas, tels que des moyens flexibles de codage des données, et la stabilité de la solution CA lorsque le nombre de cas est beaucoup plus petit que le nombre de variables.

Abstract

In qualitative research of multiple case studies, Miles and Huberman proposed to summarize the separate cases in a so-called meta-matrix that consists of cases by variables. Yin discusses cross-case synthesis to study this matrix. We propose correspondence analysis

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(CA) as a useful tool to study this matrix. CA is a quantitative method that yields a graphical display of the rows and of the columns of a matrix. The rows and the columns receive coordinates that can be interpreted as quantifications, hence the cases can be compared using these quantifications. Using an example from qualitative educational research into teaching philosophy, we illustrate both methods and their complementarity. We discuss special features of the application of CA to case study research, such as flexible ways of coding the data, and the stability of the CA solution when the number of cases is much smaller than the number of variables.

Mots clés

Synthèse de cas croisés, Analyse des correspondances, Recherche qualitative, Recherche en éducation, Philosophie, Etude de cas, Méthodes mixtes

Keywords

Cross-case Synthesis, Correspondence Analysis, Qualitative Research, Educational Research, Philosophy, Case Study, Mixed Methods Research

Introduction

In qualitative research an important line of research makes use of case studies. Yin (2014, chapter 1) argues that case studies are the preferred method for doing research when the main research questions are questions phrased as “how” and “why” questions, when the researcher has little or no control over behavioral events, and the focus is on a contemporary phenomenon.

Case study research can make use of a single case or of multiple cases. The use of multiple cases has the advantage that it increases the generalizability of the results found. Generalizability is attractive to make sure that events are not idiosyncratic; multiple cases also allow for more sophisticated descriptions and more powerful explanations (compare Miles and Huberman, 1994; Ragin, 2000; Byrne and Ragin, 2009; Yin, 2014). Yin (2014) brings in the notion of replication logic, being the logic for selecting more than one case. Here the term literal replication is used when one expects that the cases will produce similar results, and theoretical replication when it is expected that cases are predicted to yield contrasting findings.

In methodological texts on case study research it is usually advised to thoroughly study each of the cases separately. A comparison of the cases can be done in a next step. For the comparison of cases Miles and Huberman (1994) proposed many tools such as making matrices and displays. One of these tools is what they call a “meta-matrix”, in which the main findings for each of the cases are summarized. In such a meta-matrix cases are usually coded into separate rows, and variables in columns. In cells of the matrix many types of information can be summarized, both of a qualitative and of a quantitative nature. The word *meta* in meta-matrix stems from their proposal to first thoroughly study individual cases using matrices and displays, hence the meta-matrix is a tool for the summary and comparison of individual cases.

Miles and Huberman (1994) indicate that insight in the meta-matrix may be obtained by ordering the cases in such a way that similar cases are closer together. Over the years,

approaches for comparing multiple cases have been worked out in more detail by Yin (2014; first edition appeared in 1993), and he notices that this field is expanding rapidly. He provides an overview of five analytic strategies that can be taken in case study research. When multiple cases are available, the strategy most suited is cross-case synthesis, which makes use of the strategy pattern matching. The strategy pattern matching can already be used if there is only a single case study, and it entails comparing a theoretical pattern of dependent and/or independent variables with what is actually observed in the case for which data are collected. In cross-case synthesis this is done for multiple cases, where a stronger synthesis is obtained when rival cross-case patterns are found. This synthesis is stronger because the answers to the “how” and “why” questions about the dependent and independent variables are more convincing when formulated in terms of similarities and differences between the rival patterns.

In the meta-matrix cases are usually summarized in a qualitative way using text. However, by categorizing this text categorical variables are created and a quantitative analysis of categorical data becomes possible. Correspondence analysis (CA) (see Greenacre, 2007, for a recent overview) is a popular tool for the graphical representation of data, in particular categorical data, in a multidimensional space. Here cases are quantified into one or more scores. Differences in the scores reveal differences between the cases, and the aim of CA is to get insight into the most important differences between the cases. Yet, due to the use of quantifications, cases can be more or less similar to each other. This is different from cross-case synthesis, where cases are classified into two (or more) rival patterns. Yet the bridge between CA and cross-case synthesis is that in cross-case synthesis some cases fit more naturally in a pattern than others, and CA illustrates this graphically by using one or more quantitative scales on which the cases receive (a) score(s). By combining CA with cross-case synthesis using rival patterns, we combine quantitative and qualitative methods, and fall under the definition of mixed methods research, where the data analyses are mixed (Small, 2011).

In this paper we have the following objectives. First, for qualitative researchers that are interested in doing a cross-case synthesis we want to explain the potential of complementing their analysis with CA. The graphical display provided by CA is similar in aim to the rival patterns derived from a cross-case synthesis and can be helpful in explaining the similarities and differences between the cases being studied. Yet, as we will discuss below, CA and cross-case synthesis using rival patterns approach the analysis differently and therefore, if their outcomes are similar, the conclusions about the data become more trustworthy. As the target reader consists of qualitative researchers, we provide a non-technical description of CA, yet providing references to the more technical literature. Also, cross-case synthesis is not discussed in much detail.

Second, we believe that sometimes CA is used in a way that is too rigid. Here we aim at the coding of the data, which is usually coding in the form of distinct and exclusive categories. Regularly such a rigorous coding does not reflect the ambiguity that may be present in the data: sometimes an aspect of a case does not fully belong to one of the categories, but is rather in between. We discuss in this paper how to deal with this situation.

Third, much quantitative research is variable-oriented and not case-oriented, as it sees the cases usually as replications. In qualitative research interest is usually case-oriented. A case-

oriented approach considers the case as a whole entity, studying effects within a case carefully before turning into a comparison of cases (compare Ragin, 2000). Typically in cross-case synthesis the number of cases is limited and the number of variables is larger than the number of cases, where the reason is that in case research rich descriptions of the cases are developed. Often one sees applications of CA where the number of cases is much larger than the number of variables (compare Phillips and Phillips, 2009). The point we want to make in this paper, however, is that it is also possible to use CA focusing on the cases. We discuss this aspect of using CA explicitly, and in particular discuss the stability of the CA solution when the number of cases is (much) smaller than the number of variables.

The use of CA has been discussed before in the context of the analysis of multiple case studies (Breiger, 2009; Phillips and Phillips, 2009), where they compare CA with qualitative comparative analysis (QCA), proposed by Ragin (2000). For the situation that the number of case studies becomes larger (Yin, 2014: 174, speaks of 15-20 cases), Ragin developed methodology for an analysis of necessity and an analysis of sufficiency. In an analysis of necessity there is a dependent variable and in the independent variables we look for the variable(s) that were necessary for manifesting the dependent variable. In an analysis of sufficiency it is investigated whether some of the independent variables always lead to the outcome of interest. Breiger (2009) proposes CA as complementary to QCA, and he reanalyzes examples of Ragin (2000) with CA to show what CA has to offer. Our aim differs from these papers, as we want to show similarity of CA with cross-case synthesis using rival patterns, and our number of cases is too small to carry out analyses as proposed by Ragin (2000).

We will illustrate our comparison of CA and cross-case synthesis in an example of qualitative research of the teaching of philosophy at secondary school in the Netherlands. Eight philosophy lessons are studied with the aim to understand how and why higher levels of students' doing philosophy in classroom teaching comes about. In what follows we will first discuss this example in section 2 and show how the eight lessons can be summarized in a meta-matrix. In section 3 cross-case synthesis using rival patterns is explained and used to analyze the meta-matrix. In section 4 CA is explained and discussed, both the step from the meta-matrix to the data that are analyzed in CA, as well as the actual analysis. In section 5 we conclude.

Data and Meta-matrix

The multiple cases in this paper are philosophy lessons on secondary schools in the Netherlands. In the Netherlands one of the aims of teaching philosophy is learning by doing philosophy. Yet little is known how this comes about. How does doing philosophy look like? How is the teacher behavior related to students' doing philosophy? And in particular, why are some lessons more effective than others in terms of reaching higher levels of doing philosophy? So these are typical "how" and "why" questions for which case study research is well suited.

Using the methodology laid out by Miles and Huberman (1994) and Yin (2014) each of the lessons separately was studied thoroughly. The lessons were observed and videotaped. After the lesson the teacher was interviewed, and both the students as well as the teacher filled in a survey. Following Yin (2014) for each lesson the analytic

technique time-series analysis was used to study the quality of doing philosophy. For the comparison of the lessons the results for all eight lessons were summarized in a meta-matrix.

Participants and Procedure

The participants in these lessons included seven teachers (one female and six males) and their students. Eight philosophy lessons were examined in their entirety (one of the instructors taught two lessons). During the 2010-11 school year, the aforementioned instructors were enrolled in a continuing education course at the first authors' university, which was intended to familiarize secondary school teachers with a newly introduced final exam topic. These eight lessons were not a part of the continuing education course, however. Note that the lessons did not start with predefined groupings, such as effective and less effective lessons.

Teachers were given a verbal overview of the study prior to beginning the course, and permission was obtained via email to videotape one of each instructor's standard lessons; teachers were also provided with information to assist them in preparing the recordings. Appointments were later made with the teachers to record said lessons, who then distributed consent forms to students for their parents to sign. The forms explained the study's aim and informed parents that their child would be videotaped. In cases wherein a parent did not provide consent, the camera was positioned in a way to ensure that their child would not be recorded. The lesson environment and content were not altered in any way for the purpose of the study, and no personal details (e.g., information pertaining to physical or mental health) were collected.

Teachers were asked to utilize one philosophical exercise from among a list of 30 (see Kienstra et al., 2014a), and also told that the selected exercise should result in doing philosophy in the classroom.

Concepts Collected in the Meta-matrix

We distinguish concepts that relate to the *context* in which learning by doing philosophy is taking place, namely approaches to doing philosophy and substantive philosophical domains, *teacher behavior*, namely the choice for discussion or classroom talk, and the choice for a way of guidance, and *students doing philosophy*, in particular the level at which they do philosophy and common concept formation. For an extensive motivation, see Kienstra et al. (2015).

Approaches to doing philosophy. For philosophers truth is an important concept. Besides the question "What is truth?" a method of truth finding is also required. In earlier papers (Kienstra et al., 2014a, b) we distinguish three approaches to doing philosophy: (i) doing philosophy as connective truth finding (Ctf) or communicative action. Here truth is searched in a group through narratives and conversations; (ii) doing philosophy as test-based truth finding (Ttf); here truth is searched as it is practiced by scientists; and (iii) doing philosophy as in a juridical debate (Jd), judging truth-value and making judgment (truth-value analysis). Here truth is searched in a juridical way of finding the truth

and truth-value of the competing/different or opposite claims through analysis by a competent judge and reaching a “verdict”. Kienstra et al. (2014a) have grouped 30 philosophical exercises into these three approaches. Following curriculum theory in the educational sciences, in scoring these three approaches we distinguish the *design* of a lesson (i.e. the aim of the teacher), its *execution* (i.e. what is accomplished) and the *learning activities* (i.e. of the students).

Substantive philosophical domains. The eight lessons differed in the philosophical domain covered. Teachers selected domains that closely matched the life experiences of their students, such as philosophical anthropology, ethics, and social philosophy, in addition to more abstract domains, such as theory of knowledge, logic, and philosophy of mind.

Dialogue. For the teaching of the philosophy instructor we find two properties important: the form of *working* and the way of *guiding* the content. Teachers can work with students in an open or a closed form. A *philosophical discussion* (image: the teacher is comparable to a parent that teaches a child how to cycle: he pushes, lets the child go, runs after it and catches it before it falls) has a form that is more open than *classroom talk* (image: the teacher is comparable to a parent that helps the child to cycle, but holds the arm of the child).

Guidance. Guidance by a teacher can be strong, shared or loose. In strong guidance the teacher determines the content during the lesson most of the time (he is the only one asking questions and he is providing answers). When the teacher is aloof and the students determine the topic, we call guidance loose. When the teacher and the students have a common dialogue, jointly bringing in topics and questions, we call guidance shared.

Level of doing philosophy. Doing philosophy occurs in phases during a lesson, which we describe herein using what we have dubbed the Pearl Model (Kienstra et al., 2015). In qualifying a phase during a lesson the following levels can be distinguished: rationalizing analyzing, testing, producing criticism and reflecting (Kienstra et al., 2014a). *Rationalizing* is verbalizing some first thoughts in a logical structure. *Analyzing* is questioning, wondering; continuous questioning, interrogating; problematizing, considering. *Testing* is evaluating; making definitions and distinctions; making judgments. *Producing criticism* is reasoning being led by explanation/reason/connection; arguing (pro and con), constructing and maintaining a logical argument; debating. *Reflecting* is making meta-remarks, mirroring; making creative leaps, thinking about the thought process itself; reflecting on the pro and con arguments, on the assessment framework, and on its own application. These five levels are considered to be ordered hierarchically and conditionally from low (rationalizing) to high (reflecting). A phase during a lesson is a pearl, where it is assumed that pearls have concentric layers and these layers represent the five aforementioned activities. Depending on the layer of the pearl that is reached, doing philosophy will be more or less effective. The shine of the pearl is reached when doing philosophy has the level of reflecting.

The eight lessons generated 31 pearls. Here a pearl is operationalized as a fragment of interaction that consists of one or more utterances. An utterance is made by a single participant until (s)he is interrupted by another participant. Pearls consisted of three to

41 utterances. Each pearl is qualified by the highest level of the Pearl model that is reached. For example, when in a fragment of interaction the highest level achieved is testing, this fragment is qualified as having the third level.

For each of the lessons we denoted in the meta-matrix (i) the number of Pearls observed, (ii) the time in a lesson that Pearls are observed expressed as a percentage of the total time the lesson takes, (iii) the highest pearl level reached in that lesson.

Common concept formation. The transcripts were all evaluated in terms of common concept formation, i.e. students form concepts together. We distinguish four versions using a taxonomy of conceptual analysis: the deductive ladder (going from abstract to concrete), building of sentences (thinking out loud about how the concept can be used), defining, and searching for counterexamples and exploring boundaries. For every single lesson the usage of the methods is quantified as percentages of time that the methods were used in the Pearls, adding up to 100 percent; for each Pearl the dominant method was scored.

The Meta-matrix

The meta-matrix is displayed in Table 1, where the variables are represented in the rows of the matrix and the eight lessons in the columns. We will now analyze these lessons in two ways: in a qualitative way by doing cross-case synthesis using rival patterns (see below), and in a quantitative way by using CA.

Cross-case Synthesis Using Rival Patterns

In the cross-case synthesis the meta-matrix in Table 1 is used to cluster the cases (lessons) into rival patterns in a systematic way. This is done using the following steps (compare Yin, 2014). Using a theoretical framework for doing philosophy in Step 1 a pattern of doing philosophy effectively is formulated. In Step 2 this pattern is tested against a lesson in the meta-matrix that fits the theoretical pattern well. Then, using the theoretical framework we formulate in Step 3 a rival pattern of doing philosophy less effectively. In Step 4 this rival pattern is tested against a philosophy lesson from the meta-matrix. In Step 5 we classify each of the remaining philosophy lessons into each of the two complementary patterns.

As explained in the introduction the purpose of this paper is primarily of a methodological nature and therefore we refer to Kienstra et al. (2015) for a detailed discussion of the theoretical framework and examples of (parts of) the lessons. The general idea is that in some lessons the students do philosophy more effectively than in other lessons, and this effectivity may be related to the context variables and the teacher behavior.

The aim of the first step of the cross-case synthesis is to derive a theoretical pattern of doing philosophy effectively, i.e. using more Pearls, that take longer, and have a higher level, see Pattern 1 in Table 2. At the top of Table 2 are the *context variables*, i.e. the three approaches of doing philosophy and substantive philosophical domains. In the middle we find *teacher behavior*, i.e. dialogue and guidance, and at the bottom *doing philosophy by the students*. In Pattern 1 we find, for example, that the context variables doing philosophy as juridical debate, go together with the teacher behavior philosophical

Table 1. Meta-matrix with results for Context variables, Teacher behavior and Students doing philosophy for 8 philosophy lessons

VARIABLES	Lesson 1 Oscar	Lesson 2 Marc	Lesson 3 Marc	Lesson 4 Jos	Lesson 5 Ger	Lesson 6 Marlies	Lesson 7 Frans	Lesson 8 Peter
Context variables								
<i>Approaches to doing philosophy</i>								
Connective – or Design	Jd	Jd	Ctf&Ttf	Ctf&Ttf	Ttf	Jd	Ctf	Ctf
Test-based Execution	Ttf	Ttf	Ttf	Ttf	Ttf	Ctf	Ctf	Ttf
truth finding, Learning	Ttf	Jd	Jd	Ctf	.	Ctf	Ttf	Ctf&Ttf
Judicial debate activity								
Domains								
Philosophical Anthropology and Logic	Philosophical Anthropology	Philosophy of Mind	Philosophical Anthropology	Philosophical Anthropology	Theory of knowledge	Social philosophy	Ethics	Philosophical Anthropology
Teacher behavior								
Dialogue	Philosophical discussion Shared	Philosophical discussion Shared	Philosophical discussion Shared	Classroom talk Loose/Shared	Classroom talk Shared	Classroom talk Loose	Classroom talk Strong	Classroom talk Loose
Guidance								
Students Doing philosophy								
Number of pearls	6	3	4	5	3	5	3	2
Pearls, percentage of lesson	60 %	34 %	25 %	28 %	23 %	13 %	28 %	10 %
Highest level reached (score 1-5)	5	5	5	4	4	4	4	4
Common concept formation in	Method 1	Method 1	Method 1	Method 1	Method 1	Method 1	Method 1	Method 1
Method 2	39 %	0 %	0 %	39 %	0 %	19 %	55 %	0 %
Method 3	29 %	53 %	13 %	15 %	0 %	16 %	26 %	0 %
Method 4	0 %	0 %	29 %	34 %	13 %	27 %	0 %	100 %
conceptual analysis*	32 %	47 %	58 %	12 %	87 %	38 %	19 %	0 %

Reading instruction. In the philosophy lesson by Oscar the design of the lesson falls into approach Juridical debate, but the execution and learning activity turned out to fall in Test-based truth finding. The substantive domain was Philosophical anthropology and Logic. The dialogue was a philosophical discussion with shared guidance. Six pearls were observed, that took 60 % of the time of the lesson. The pearl with the highest level had level 5 (reflecting). The total time pearls were observed consisted for 39 % by the Deductive ladder (Method 1), 29 % by Building of sentences (Method 2), 0 % by Defining (Method 3) and 32 % by Counterexamples and boundaries (Method 4).

Table 2. Theoretical patterns of doing philosophy

Variables	Pattern 1	Pattern 2
Context variables		
• Approaches to doing philosophy		
– As connective truth finding or communicative action (Ctf)	-	+
– As test-based truth finding (Ttf)	+/-	+/-
– As juridical debate (Jd)	+	-
• Substantive philosophical domains		
– Closer to the student's world	+/-	+/-
– More abstract	+/-	+/-
Teacher behavior (interaction)		
• Form of dialogue		
– Classroom talk	-	+
– Philosophical discussion	+	-
• Way of guidance		
– Strong or loose guidance	-	+
– Shared guidance	+	-
Students doing philosophy		
• Pearls of doing philosophy: quantity		
– Frequency	+	-
– Length	+	-
• Pearls of doing philosophy: quality		
– Producing criticism (and lower)	-	+
– Reflecting	+	-
• Common concept formation		
– Deductive ladder	-	+
– Defining	-	+
– Building of sentences	+	-
– Counterexamples and boundaries	+	-

Theoretical patterns of doing philosophy effectively (Pattern 1, in column 1) and doing philosophy less effectively (Pattern 2, in column 2). '+' means: that the row label appears relatively more often in a pattern, '-' that a row label appears relatively less often in a pattern, and '+/-' that a row label does not appear more or less often in a pattern.

discussion and shared guidance and higher levels of students doing philosophy (in Table 2 labeled with a '+'; a plus indicates that a row label appears relatively more often in a pattern, while a minus, '-', indicates that a row label appears relatively less often. Both a plus and minus is indicative of a row label whose frequency of appearance is neutral). In the second step we identify one lesson in the meta-matrix (Table 1) that fits well in Pattern 1. This turns out to be lesson 2 by teacher Marc (compare the meta-matrix in Table 1).

In the third step of the cross-case synthesis a rival pattern is derived from the theoretical framework (in this section), see Pattern 2 in Table 2. In this pattern doing philosophy has a (relatively) lower level. In the fourth step one lesson is chosen from the meta-matrix (Table 1) that fitted well in Pattern 2. This is lesson 7 by teacher Frans.

In the last step of the cross-case synthesis each of the remaining six lessons is classified into one of the two rival patterns: in the end lessons 1, 2 and 3 are classified in the effective pattern 1 and lessons 4 to 8 in pattern 2, the less effective pattern. Here we make

the remark that lessons 4 and 5 also have characteristics of the effective pattern: in lesson 4 there is a large number of Pearls, yet the highest level is not reached; in lesson 5 the number of Pearls is low but in the three Pearls there is common concept formation as in searching for counterexamples and exploring boundaries (Method 4); in both lessons there is a dialogue with shared guidance. We note that cross-case synthesis with rival patterns is popular in educational research, see for example Meirink et al. (2010).

CA of the Meta-matrix

The meta-matrix (Table 1) is analyzed with CA (CA; for introductions, see Benzécri, 1992; Gifi, 1990; Greenacre, 2007; Nishisato, 2007; Le Roux and Rouanet, 2010). CA is a statistical tool for descriptive analysis of data. It provides a graphical display of the rows of a matrix, where the rows are represented as points in a multidimensional space, and simultaneously it provides a graphical display of the columns of a matrix, where the columns are represented as points in a multidimensional space. The coordinates for the points in multidimensional space can also be interpreted as quantifications. This geometric, visual interpretation of CA is also found in Breiger (2009) and Phillips and Phillips (2009).

Apart from this geometric interpretation of CA, other interpretations are also possible. CA can be interpreted as principal component analysis for categorical data (Gifi, 1990). It is closely related to loglinear modelling (van der Heijden et al., 1989). CA is also closely related to latent class analysis in the sense that CA gives insight into an optimal clustering of the cases (here: the lessons) (compare van der Heijden et al., 1999). In this way the aim of CA resembles the aim of cross-case synthesis using rival patterns: finding the rival patterns.

Creating the Matrix to be Analyzed

A first step in analyzing the meta-matrix with CA is to code the meta-matrix into a so-called super-indicator matrix that will serve as an input matrix for the analysis. We will discuss this in some detail as it is an important step in the analysis, where different choices can be made, possibly leading to different outcomes of the analysis. These choices are simple indicator coding, fuzzy coding and equality constraints. The super-indicator matrix is in Table 3.

Simple indicator coding. Typically there are variables where a lesson falls into only one category. This happens, for example, in the variable *Dialogue*, where a lesson follows either a philosophical discussion or classroom talk (dis and crt, see Table 1). This leads to an indicator matrix of 8 lessons by 2 categories. Lessons 1, 2 and 3 score (1, 0) as they fall into the category philosophical discussion and lessons 4 to 8 score (0, 1) as they fall into classroom talk.

Simple indicator coding is also found for *the number of pearls* (2 and 3 versus 4, 5 and 6), *the percentage of time* pearls take in the full lesson (low, middle or high) and *the highest pearl level* found (4 or 5). For the number of pearls the choice for 2-3 versus 4-6 was a bit arbitrary: by making this split the lessons are categorized in two groups of 4 lessons.

Table 3. Super-indicator matrix of the meta-matrix in Table 1

Approaches		Philosophical domains					Dialogue			Guidance			# Pearls			Pearls % of time			Highest P			Method of c. concept		
jd	ttf	ctf	ant	min	tok	soc	eth	log	dis	crt	loo	sha	str	p23	p46	%lo	%mi	%hi	Le4	Le5	me1	me2	me3	me4
1.	1	2	0	.5	0	0	0	.5	1	0	0	1	0	0	1	0	0	1	0	1	.39	.29	0	.32
2.	1	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0	1	0	.53	0	.47
3.	1	1.5	.5	1	0	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	0	.13	.29	.58
4.	0	1.5	1.5	1	0	0	0	0	1	.5	.5	0	0	0	1	0	1	0	1	0	.39	.15	.34	.12
5.	0	2	0	0	1	0	0	0	1	0	1	0	1	1	0	0	1	0	1	0	0	0	.13	.87
6.	1	0	2	0	0	1	0	0	1	1	1	0	0	0	1	1	0	1	0	1	.19	.16	.27	.38
7.	0	1	2	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	1	0	.55	.26	0	.19
8.	0	1.5	1.5	1	0	0	0	0	1	1	1	0	0	1	0	1	0	0	1	0	0	0	1	0

Approaches to doing philosophy; juridical debate (jd), test-based truth finding (ttf), connective truth finding (ctf); Philosophical domains: philosophical anthropology (ant), philosophy of mind (min), theory of knowledge (tok), social philosophy (soc), ethics (eth), logic (log); Dialogue: philosophical discussion (dis), classroom talk (crt); Guidance: loose (loo), shared (sha) and strong (str); # Pearls: 2 or 3 (p23), 4, 5 or 6 (p46); % lesson: low (%lo), middle (%mi), high (%hi); Highest Pearl level: level 4 (le4), level 5 (le5); Method of common concept formation: method 1 (me1), method 2 (me2), method 3 (me3), method 4 (me4).

For Percentage of time we made a split between 10 and 13 percent; 23, 25 and 28 percent; and 34 and 60 percent. We decided for this grouping because a well-known property in CA is that if there are many categories that are empty, the CA solution is prone to produce outliers and by taking 34 together with 60 instead of with 23-28, the first lesson (with 60 percent) is less likely to become an outlier. Although it is interesting to investigate why certain cases become outliers, outliers have the drawback that they obscure relations between the other cases and thus hinders us in studying the big picture of the cases.

Fuzzy coding. (Gifi, 1990; van Rijckevorsel, 1987). In fuzzy coding a lesson does not fall into only one category but the lesson is distributed over more categories. A first example is Lesson 1 in the variable. In qualitative comparative analysis the same concern is evident (see Ragin, 2000; Breiger, 2009). *Philosophical domain*, where for the first case in the table 0.5 is attributed to the category Philosophical anthropological and 0.5 to Logic. A second example is *Guidance* where in Lesson 4 there were elements of loose as well as shared guidance. More extreme is *Method of common concept formation*, where the proportion of time is filled in in the four levels Methods 1 to 4, and the proportions add up to 1 for each lesson.

Equality constraints. (van Buuren and de Leeuw, 1992). A last possibility is to set equality constraints on categories. As can be seen in the meta-matrix, the Approaches to doing philosophy where scored from three perspectives, namely as design, execution and as learning activities. Each of the three perspectives can be coded separately in three columns (Ctf, Ttf and Jd). By adding up the codings of the three perspectives, one score for each column is obtained and be interpreted as the score for, for example, Ctf where the Ctf score is constrained to be identical for design, execution and learning activities.

Non-technical Introduction to CA

Here we introduce CA in a non-technical way, i.e. without any formulas. There are different ways to describe CA. One way, that we will use here, is a geometric approach. Both the rows as well as the columns of the super-indicator matrix can be plotted jointly as points in a multidimensional space. As there are eight cases, each of the columns, i.e. the categories of the variables, can be plotted in an 8-dimensional space using the scores of the cases in this column as coordinates.¹ In this 8-dimensional space columns will be closer together when their eight coordinates are similar, and they will be further apart when their coordinates are dissimilar. Thus, for example, the categories jd, dis and Le5 will be close together because they occur in the same lessons. We simple human beings can imagine at most three dimensions simultaneously and therefore a method is discussed to approximate the distances in this eight-dimensional space by a lower-dimensional space. This method is CA.

To study the cloud of column points in the 8-dimensional space CA finds new axes using a mathematical tool called generalized singular values decomposition (SVD). These new axes simplify the interpretation of the 8-dimensional space by focusing on the directions where most of the information is. Thus the aim is to approximate the

distances in the 8-dimensional space as good as possible by distances in a lower-dimensional space. First notice that, as two points span a one-dimensional sub-space in a two-dimensional space (i.e. they are on a line), the eight points (the lessons) span a 7-dimensional space. The SVD provides as the first dimension of the CA solution the direction in the 7-dimensional subspace in which the (weighted) variation of the points is maximal. Thus the first dimension shows the direction in the 7-dimensional subspace where the distances between the categories are largest. Then the second dimension is found as the dimension with maximal variation of the points in the remaining 6-dimensional subspace, and so on for further dimensions.

The matrix has 25 columns. Therefore, in a similar way as for the columns, for the eight rows (the lessons) of the meta-matrix, there is a 25-dimensional space in which the 8 points lie. It can be proven that, as the dimensionality of full-dimensional subspace for the columns is 7, this is also the dimensionality of the full-dimensional subspace for the rows. The SVD finds the first dimension in which the 8 lessons differ most, and this is the first CA dimension. The second dimension for the rows is found in a similar way as in the subspace for the columns.

In CA the dimensions of the rows are closely related to the dimensions of the columns: the row points (the lessons) can be placed in the weighted average of the column points (the categories that they have). For example, row 5 in the meta-matrix is in the average of (twice) ttf, tok, crt, sha, p23, %mi, Le4, .13 times me3 and .87 times me4. Similarly, (up to a multiplicative constant) the column points (the categories) are in the weighted averages of the rows (the lessons) by which they are used: for example, the category dis is in the weighted average of lessons 1, 2 and 3.

Thus there are few principles that guide the interpretation: (1) preferably only the first few dimensions are studied of the full dimensional space. This is allowed when higher dimensions start showing only peculiarities in the data; (2) rows (lessons) are close together when they use the same columns (categories) and they are further apart when they use different columns (categories) (3) similarly, columns are closer together when they are used by the same lessons, and they are further apart when they are used by different lessons, and (4) rows (lessons) are closer to columns (categories) that they use, and columns are closer to the rows by which they are used. Breiger (2009), in his explanation of CA where he compares CA with qualitative comparative analysis, emphasizes points (3) and (4) by the phrase “variables are constituted by the cases that comprise them and vice versa”.

We also note that, as CA focuses on directions in which the rows and the columns differ most, CA reveals *departures* from the average. The average is located in the origin of the space (the average has coordinate 0). So rows (lessons) with a positive coordinate differ in an opposite way from the average as rows with a negative coordinate. Also, when all cases fall into the same category, this will not be revealed by CA as CA focuses on *differences* between cases.

Another point of interest is that the origin (i.e. the point with coordinates zero), represents the average row point (average over the case studies) and the average column point. For a variable coded using simple indicator coding, the averages of the categories are proportions, for example for Dialogue the average is $(3/8 =) .375$ for philosophical discussion (dis) and $(5/8 =) .625$ for class room talk (crt; see Table 3). In the full

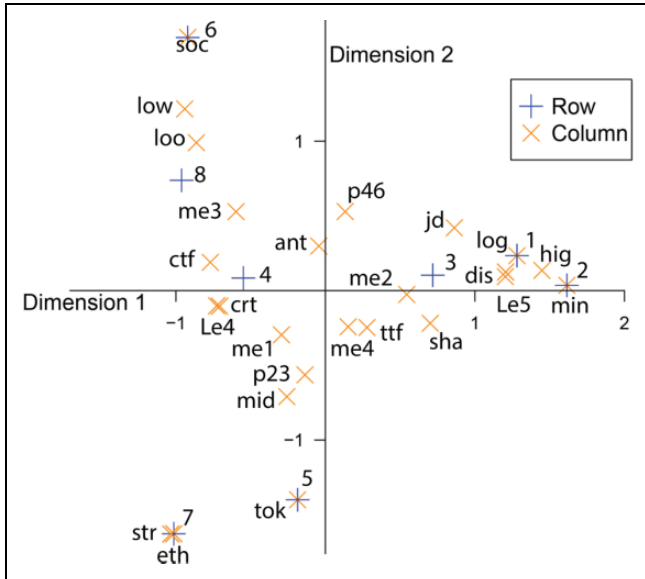


Figure 1. First two dimensions of CA solution.

dimensional space cases 1-3 depart in one way from this average and cases 4-8 in another way, and in the full-dimensional graphical representation cases 1-3 will be relatively closer to the column point dis and cases 4-8 will be relatively closer to the column point crt. For Method of common concept formation the average of the four categories are the average proportions of the eight cases (for me1-me4 these average proportions are .19, .19, .25, .37), and distances from the origin in full dimensional space show how much rows (cases) depart from this average. For example, case 4, with proportions .39, .15, .34 and .12, will in full dimensional space lie relative closer to me1 and me3 (as .39 is larger than .19, and so on) and relatively further away from me4. We note that the averages will change when new cases are added or taken out of the data. A last remark is that, in full-dimensional space, if a row departs more from the origin than another row, this means that the former row departs more from the average than the latter row. This holds approximately also for the first dimension, to the extent that the first dimension represents the distances in full-dimensional space better. Thus CA provides an ordering of the cases on the first dimension that reveals the magnitude of departure from the average, where on the left of the origin the cases depart in one way and on the right in the opposite way.

CA of the Super-indicator Matrix of Lessons by Categories

We carried out the analysis using IBM SPSS version 22, using the routine ANACOR². The first two dimensions of the CA solution are displayed in Figure 1. We have chosen for a representation where the columns (categories) are exactly in the weighted average of the rows (lessons) by which they are used. The rows are indicated by labels 1 to 8,

and the columns by the labels provided in the second line of Table 3. We note that the row coordinates per se have no meaning, but it is their relative distances that matter. A normalization is chosen that, for each dimension, the sum of the squares of the coordinates is approximately equal to 1.³

The lessons are labeled by their number and the columns by an abbreviation found in Table 3. The first dimension shows lessons 1, 2 and 3 on the right of the origin and the other lessons on the left. If we focus on categories that are used by at least two lessons, we see on the right a high percentages of time of doing philosophy (hig), the highest level of the pearl use is 5 (Le5), a philosophical discussion (dis), juridical debate (jd), shared guidance (sha), and the second method of common concept formation (me2). On the left of the first dimension we find lessons 4 to 8, where the second dimension seems to be taken by peculiarities in the data: on the bottom right we find lesson 7 with the domain ethics (eth) and strong guidance (str), which are only used in lesson 7; similar peculiarities are found for lesson 5 and 6. As the focus of attention is not data peculiarities concerning individual lessons, but overall structure of all lessons, we do not discuss dimension 2 any further but focus on the position of the categories on the first dimension. Then we find on the left of dimension 1 loose and strong guidance (loo and str), a low percentage of time that pearls appear (low), the approach connective truth finding (ctf) and class room talk (crt), the highest level found is 4 (Le4). Overall we may conclude that the first dimension of the CA comes to the same division as cross-case synthesis of relatively more effective lessons 1, 2 and 3 versus less effective lessons 6, 7 and 8, with lessons 4 and 5 a bit in the middle.

It may be helpful for the interpretation of the first dimension in CA to see how much variables contribute to this dimension. Here the contribution is a function of distances of categories to the origin, weighted by the number of lessons that used these categories (compare Greenacre, 2007). There are eight variables, but the variable Approaches in the super-indicator matrix has values adding up to 3, so it makes the potential contribution of this variable thrice as large. This makes the average contribution of a variable 10 percent (i.e. 100 percent divided by 7 variables plus 3 for Approaches), and for Approaches it is 30 percent. In order of importance, variables Dialogue (17 percent) and Highest Pearl level (17 percent), Pearls percent of time (15 percent), Philosophical domains (13 percent) and Guidance (13 percent) contribute more than the average contribution. We notice that the contribution of Philosophical domains is not that interesting as in this study most domains are used in only one lesson, and therefore Philosophical domain will not help in placing similar lessons together. A tentative interpretation in one sentence is: lessons 1, 2 and 3 make use of a philosophical discussion and shared guidance, thus producing a larger percentage of time of doing philosophy as well as doing philosophy on a higher level.

Stability

There are eight variables in the super-indicator matrix. As the focus in this paper is on the synthesis of cases, we investigated the stability of the scores of the lessons (cases). This was done by eliminating single variables, thus doing eight analyses with seven variables. For dimension 1 the scores of the eight analyses had correlations with the scores of the

original analysis ranging from .986 to .994 for dimension 1 and .906 to .997 for dimension 2. For both dimensions the correlations were lowest (i.e. .986 and .906) when the variable guidance was omitted. This had impact in particular on the position of Lesson 7, which was the only lesson where strong guidance was used. We conclude that, even though the CA is carried out on only eight lessons, the solution of the lessons is very stable, probably due to the large number of variables used.

We did not carry out similar analyses for the stability of the column scores. A reasonable approach to investigate this would be to leaving out lessons one by one. However, as 6 of the 28 categories were used by only one lesson, these 6 category points will disappear when the lesson involved is removed. Also, because of the property that the category points are in the average of the lessons, these averages can change substantially when lessons involved in the category point are removed, in particular when the lesson involved lies far from the average of the lessons. Also, it is likely with this small number of cases that leaving out one or two cases may lead to another dimensional representation than the current triangle.

Concluding Remarks

We conclude that CA can be used in the context of cross-case synthesis in a useful way. It can complement cross-case synthesis using rival patterns. Both methods study which cases are similar and which are dissimilar. Both methods allow to go back easily to the original data to see why individual cases behave as they do in the analysis. Yet cross-case synthesis using rival patterns is doing this by making clusters of cases on both theoretical as well as empirical grounds, whereas CA is data driven and only uses the empirical grounds (even though one could argue that the categorizations chosen in CA will be based on theoretical considerations). A second distinction is that cross-case synthesis using rival patterns provides clusters of cases whereas CA provides one or more quantifications of the cases (see Figure 1, where the quantifications are used as coordinates to make a graphical display). Because of these distinctions it is not our aim to propagate one of the two methods but rather to let them complement each other. In particular the theoretical input needed in cross-case synthesis using rival patterns can be a valuable part of the analysis, and is an advantage over the data-driven approach of CA. Yet the graphical display of CA can be very useful to illustrate the findings in cross-case synthesis, and in many instances quantifying cases is more natural than clustering them.

We make final comments about the use of CA in this context. One commonly sees applications of CA where the number of cases is much larger than the number of cases that we used here. However, we have illustrated that this is not problematic as the quantifications of the cases can be very stable even though the number of cases is small. The reason is that in statistical analyses, solutions for the rows will be stable when they are based on many columns (i.e. the matrix is broad), whereas solutions for the columns will be stable when they are based on many rows (i.e. the matrix is long, which is seen more often in statistical analyses). Therefore we think that CA can be useful in situations that the number of cases is even smaller than eight. Actually, although the example that we

discuss consists of eight case studies, we think that CA can be used fruitfully when the number for cases goes down to even three or four.

Last, as discussed in section 4, the CA solution can be used in the scoring of new lessons; that is, if a new lesson is observed, and (part of the) column variables in the super-indicator matrix are scored for this new lesson, then this new lesson can be projected into Figure 1 as a so-called passive row point (passive as the lesson does not actively influence the configuration of the other points, compare Gifi, 1990; IBM SPSS). This allows to compare new lessons with the eight lessons analyzed in this paper, and such a comparison can be helpful in supporting teachers.

Notes

1. A point with two coordinates can be represented in a two-dimensional space. For three coordinates it is a point in three-dimensional space. For eight coordinates there is an eight-dimensional space.
2. This routine only allows to use whole numbers. For fuzzy coded variables that leads to a problem. To circumvent this problem we multiplied all scores in the super-indicator matrix with 100. This multiplication leaves the CA solution invariant.
3. We use the word “approximately” because there was one missing value, namely for case 5 on the variable Approaches, and CA therefore reduces the contribution of point 5 to the solution somewhat. If all cases would have no missing values, the word “approximate” should be taken out of the sentence.

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