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Two tests for strict exogeneity in a correlated random effects panel data Tobit model

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This paper presents two tests for strict exogeneity of the covariates in a correlated random effects panel data Tobit model. The tests are applied in an analysis of hours of work of US women. Estimation procedures when a model does not pass a test for strict exogeneity are discussed.

Keywords and Phrases: strict exogeneity test, panel data, Tobit model.

1 Introduction

Wooldridge (1995) proposes a correlated random effects (CRE) estimator for a panel data Tobit model (Tobin, 1958). The advantage of this CRE estimator, opposed to a random effects estimator, is that it allows for correlations between the covariates and random effect (a time-constant unobserved covariate). The CRE estimator is easy to implement and has been used in a wide variety of applications (Sanchis-Llopis, 2001; Meyerhoefer *et al*., 2004; Sepehri *et al*., 2006; Buligescu *et al*., 2009; Hochguertel and Ohlsson, 2009; Love and Smith, 2010). A common assumption made in such applications is strict exogeneity of the covariates. This is a rather strong assumption and does not hold if, for instance, there are feedback effects; that is, the outcome variable in the current period affects covariates in a subsequent period. If the assumption of strict exogeneity does not hold, the applied estimators are inconsistent and the empirical results should not be trusted. In empirical research, it is, therefore, of major importance to test for strict exogeneity of the covariates.

The main contribution to the literature of this paper is the presentation of two tests for strict exogeneity of the covariates in a CRE Tobit model. The first test is a straightforward extension of the test for strict exogeneity of the covariates in a CRE probit model (Wooldridge, 2010, p.618). This test uses the maximum likelihood estimator of Wooldridge (1995) for a CRE Tobit model (referred to in this paper as the CRE estimator). The second test is innovative as it is based on removing the random effect from the equation of interest by taking first differences. This makes it possible to construct a test for strict exogeneity of the covariates similar to the one when using a first difference estimator for a linear panel data model (Wooldridge, 2010, p.325). This test uses the maximum likelihood estimator based on first

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differences of Kalwij (2003) for a CRE Tobit model (referred to in this paper as the FD-CRE estimator).

The outline of this paper is as follows. Section 2 presents three estimators for a panel data Tobit model proposed by Honoré (1992), Wooldridge (1995) and Kalwij (2003). Section 3 presents two tests for strict exogeneity of the covariates. Section 4 demonstrates the usefulness of the proposed exogeneity tests by re-examining an empirical analysis of hours of work of US women (Wooldridge, 2010). Section 5 concludes the paper and discusses estimation when strict exogeneity is rejected and directions for future research.

2 Three estimators for a panel data Tobit model

I consider a balanced panel of N individuals and T time periods in which the individual is indexed by i and the time period by t; hence, $i = \{1,.., N\}$ and $t = \{1,.., T\}$.[1] The dependent variable is left-censored at zero. The model of interest is

$$y_{it}^{*} = \beta_0 + \mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i + \varepsilon_{it}, \tag{1}$$

$$y_{it} = \max\left(0, y_{it}^{*}\right), \tag{2}$$

$$\varepsilon_{it}|\mathbf{x}_i, \alpha_i \sim N\left(0, \sigma_{\varepsilon}^2\right). \tag{3}$$

The outcome variable y_{it} is observed. The normally distributed error term ε_{it} is assumed to be independent across time, $\mathbf{x}_{it}$ is a vector of strictly exogenous covariates and $\boldsymbol{\beta}$ is the corresponding parameter vector, β_0 is an intercept parameter, $\mathbf{x}_i = (\mathbf{x}_{i1},.., \mathbf{x}_{iT})$ and α_i is a random effect that can be correlated with $\mathbf{x}_{it}$ and is usually assumed normally distributed. The model outlined is commonly referred to as a censored regression panel data model or, given normality of the error terms, a panel data Tobit model (Tobin, 1958). The model can be extended to allow for predetermined covariates, arbitrary correlation over time of the error terms and heteroskedasticity (Wooldridge, 2000, 2005; Kalwij, 2003). The panel data Tobit model, as specified earlier, is, however, most commonly used in empirical studies, and later I discuss three estimators for this model.

First, the CRE estimator of Wooldridge (1995) is a fully parametric estimator and requires the abovementioned distributional assumptions. Following Chamberlain (1984), Wooldridge (1995) parameterises the correlations between the random effect and the covariates as a time-invariant function of the covariates in all periods and a random individual specific error term. This is formalized as follows:

$$\alpha_i = h(\mathbf{x}_i; \boldsymbol{\gamma}) + \mu_i, \tag{4}$$

where $\mu_i|\mathbf{x}_i \sim N\left(0, \sigma_{\mu}^2\right)$. Equation (4) is the CRE part of the model and requires strict exogeneity to hold. Equation (4) is, in this paper, referred to as the CRE assumption. The function $h(\mathbf{x}_i; \boldsymbol{\gamma})$ needs to be chosen, and the parameter vector $\boldsymbol{\gamma}$ must also be estimated. To do so, Wooldridge (1995) assumes, for instance, that the function $h(\mathbf{x}_i; \boldsymbol{\gamma})$ is linear in $\mathbf{x}_i$ and $\boldsymbol{\gamma}$. Following Mundlak (1978), who considers a linear

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regression panel data model, a popular choice for $h(\mathbf{x}_i;\boldsymbol{\gamma})$ in empirical research is a linear function of the across-time averages of the covariates:

$$\alpha_i = \gamma_0 + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu_i, \tag{5}$$

with

$$\overline{\mathbf{x}}_i = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_{it}. \tag{6}$$

For reasons of exposition, the remainder of this paper continues using the parameterization of Equation (5), but in empirical applications, one can test for a more flexible specification (Zabel, 1992). Substituting Equation (5) into Equation (1) yields

$$y_{it}^* = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu_i + \varepsilon_{it}, \tag{7}$$

where $\alpha = \beta_0 + \gamma_0$. The maximum likelihood estimator is defined by

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \quad \sum_{i=1}^{N}\log\int_{-\infty}^{+\infty}\prod_{t=1}^{T}\left(\Phi\left(\frac{-(\alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu)}{\sigma_\varepsilon}\right)\right)^{I\{y_{it}\leq 0\}} \left(\frac{1}{\sigma_\varepsilon}\varphi\left(\frac{y_{it} - (\alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu)}{\sigma_\varepsilon}\right)\right)^{I\{y_{it}>0\}} d\Phi\left(\frac{\mu}{\sigma_\mu}\right), \tag{8}$$

where $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}, \boldsymbol{\gamma}, \sigma_\varepsilon, \sigma_\mu)$ and $I\{.\}$ is an indicator function equal to 1 if the argument is true, and 0 otherwise. The cumulative standard normal distribution function is denoted by $\Phi(.)$ and the standard normal density function by $\phi(.)$. The studies mentioned in the introduction present empirical applications of this estimator.

Second, the semi-parametric estimator of Honoré (1992) is sometimes used in empirical studies to validate the results of Wooldridge's CRE estimator (outlined earlier). Most important is that, in contrast to Wooldridge's CRE estimator, Honoré's estimator allows for arbitrary correlations between the random effects and the covariates (a fixed-effect approach) and does not require the CRE assumption of Equation (4). I therefore refer to this estimator as a fixed-effects (FE) estimator. In addition, Honoré's FE estimator does not require the assumption that the error term is normally distributed, but this comes with the cost of not being able to calculate the marginal effects of the covariates on the (observed) outcome variable.[2] Consistency of Honoré's FE estimator requires intertemporal homoskedasticity of the error terms. A formal model specification test based on these two estimators is, generally speaking, not possible because they rely on different (non-nested) sets of consistency conditions. However, the consistency conditions for the CRE and the FE estimators are satisfied in the model formulated earlier. Honoré's FE estimator is defined by minimization of

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$$\begin{aligned}\hat{\beta} &= \text{argmin}_{\beta} \quad \sum_{i=1}^{N}\sum_{t=2}^{T}(\max\{y_{it-1}, -\Delta\mathbf{x}_{it}\boldsymbol{\beta}\} - \max\{y_{it}, \Delta\mathbf{x}_{it}\boldsymbol{\beta}\} + \Delta\mathbf{x}_{it}\boldsymbol{\beta})^2 \\ &\quad +2\times I\{y_{it-1} < -\Delta\mathbf{x}_{it}\boldsymbol{\beta}\}(-\Delta\mathbf{x}_{it}\boldsymbol{\beta} - y_{it-1})y_{it} \\ &\quad +2\times I\{y_{it} < \Delta\mathbf{x}_{it}\boldsymbol{\beta}\}(\Delta\mathbf{x}_{it}\boldsymbol{\beta} - y_{it})y_{it-1}\end{aligned}, \quad (9)$$

where $\Delta\mathbf{x}_{it} = \mathbf{x}_{it} - \mathbf{x}_{it-1}$. The calculation of the asymptotic distribution and estimator performance are discussed in Honoré (1992). Charlier, Melenberg and van Soest (2000) present an empirical application of this estimator.

Third, the FD-CRE estimator of Kalwij (2003) is based on the same consistency conditions as Wooldridge's (1995) CRE estimator, including a correct parameterization of the covariates–random effects relationship as formalized in Equations (4)–(6). However, Kalwij (2003) eliminates the random effects from Equation (1) by first differencing the equation of interest and considers the following model:

$$\Delta y^*_{it} = \Delta\mathbf{x}_{it}\boldsymbol{\beta} + \Delta\varepsilon_{it}, \quad (10)$$

$$\Delta y_{it} = \begin{cases} \Delta y^*_{it} & \textit{if} \quad y^*_{it-1} > 0 \cap y^*_{it} > 0 \\ \text{not observed} & \textit{if} \quad y^*_{it-1} \le 0 \cup y^*_{it} \le 0 \end{cases}, \quad (11)$$

where $\Delta y^*_{it} = y^*_{it} - y^*_{it-1}$, $\Delta\mathbf{x}_{it} = \mathbf{x}_{it} - \mathbf{x}_{it-1}$ and $\Delta\varepsilon_{it} = \varepsilon_{it} - \varepsilon_{it-1}$. The probability of observing positive values for the dependent variable in periods $t-1$ and t is given by

$$\Pr\left(y^*_{it-1} > 0 \cap y^*_{it} > 0\right) = \Phi\left(\frac{\alpha + \mathbf{x}_{it-1}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu_i}{\sigma_\varepsilon}\right)\Phi\left(\frac{\alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mu_i}{\sigma_\varepsilon}\right). \quad (12)$$

This probability is denoted by $F(\mathbf{y}_i, \mathbf{x}_i; \boldsymbol{\theta})$ where $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}, \boldsymbol{\gamma}, \sigma_\mu, \sigma_\varepsilon)$, $\mathbf{y}_i = (y_{i1},.., y_{iT})$ and $\mathbf{x}_i = (\mathbf{x}_{i1},.., \mathbf{x}_{iT})$. The density function of the first difference $\Delta\varepsilon_{it}$ conditional on observing positive values for the dependent variable in both periods is given by

$$\begin{aligned} f(\Delta\varepsilon_{it}|y_{it-1} > 0, y_{it} > 0; \boldsymbol{\theta}) &= \frac{1}{F(\mathbf{y}_i, \mathbf{x}_i; \boldsymbol{\theta})}\frac{1}{\sqrt{2}\sigma_\varepsilon}\varphi\left(\frac{\Delta y_{it} - \Delta\mathbf{x}_{it}\boldsymbol{\beta}}{\sqrt{2}\sigma_\varepsilon}\right) \\ \times\Phi\left(\frac{\Delta y_{it} + \mathbf{x}_{it-1}\boldsymbol{\beta} + \mathbf{x}_{it}\boldsymbol{\beta} + 2\overline{\mathbf{x}}_i\boldsymbol{\gamma} + 2\mu_i}{\sqrt{2}\sigma_\varepsilon}\right. &\left. - \frac{2\Delta y_{it}}{\sqrt{2}\sigma_\varepsilon}I\{\Delta y_{it} > 0\}\right). \end{aligned} \quad (13)$$

The maximum likelihood estimate of $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta}, \boldsymbol{\gamma}, \sigma_\mu, \sigma_\varepsilon)$ is given by

$$\begin{aligned}\hat{\boldsymbol{\theta}} &= \underset{\boldsymbol{\theta}}{\text{argmax}} \sum_{i=1}^{N}\log\int_{-\infty}^{+\infty}\prod_{t=2}^{T}\Big[(1 - F(\mathbf{y}_i, \mathbf{x}_i, \boldsymbol{\theta}))^{I\{y_{it-1} \le 0 \,\cup\, y_{it} \le 0\}} \\ &\quad +(f(\Delta y_{it} - \Delta\mathbf{x}_{it}\boldsymbol{\beta}|y_{it-1} > 0, y_{it} > 0; \boldsymbol{\theta})\times F(\mathbf{y}_i, \mathbf{x}_i, \boldsymbol{\theta}))^{I\{y_{it-1} > 0 \,\cap\, y_{it} > 0\}}\Big]d\Phi\left(\frac{\mu}{\sigma_\mu}\right)\end{aligned}. \quad (14)$$

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Kalwij (2003) provides the derivations of the likelihood contributions.[3] Kalwij and Gregory (2005) employ this estimator for analysing paid overtime work in Britain.

3 Two tests for strict exogeneity of the covariates

The first test for strict exogeneity I propose is similar to the one the literature suggests for a CRE probit model (Wooldridge 2010, p.618). The vector $\mathbf{x}_{it+1}$ is added to the model in Equation (7):

$$y_{it}^{*} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_{i}\boldsymbol{\gamma} + \mathbf{x}_{it+1}\xi + \mu_i + \varepsilon_{it}. \tag{15}$$

Equation (15) is estimated using the CRE estimator of Wooldridge (1995). A test for strict exogeneity is given by

$$\begin{aligned} &\mathrm{H}_0: \xi = 0 \ \text{(strict exogeneity)} \\ &\mathrm{H}_1: \xi \neq 0. \end{aligned} \tag{16}$$

The null hypothesis is tested using a Wald test. This test comes at the cost of losing the last time period.

The second test for strict exogeneity I propose is based on first differencing the equation of interest. This approach removes the random effect from the equation of interest and this makes it possible to construct a test for strict exogeneity similar to how one can test for strict exogeneity of the covariates in a linear panel data model. To see this, I first take the conditional expectation of Equation (10):[4]

$$E\left(\Delta y_{it}^{*}|\mathbf{x}_{it-1}, \mathbf{x}_{it}\right) = \Delta\mathbf{x}_{it}\beta + E(\varepsilon_{it}|\alpha_i, \mathbf{x}_{it-1}, \mathbf{x}_{it}) - E(\varepsilon_{it-1}|\alpha_i, \mathbf{x}_{it-1}, \mathbf{x}_{it}). \tag{17}$$

If strict exogeneity holds, the last terms in Equation (17) are equal to 0 and should not depend on $\mathbf{x}_{it}$. In this case, one expects that when adding $\mathbf{x}_{it}$ to Equation (10), its corresponding parameter estimates will be jointly insignificant. To perform a test for strict exogeneity, the vector $\mathbf{x}_{it}$ is added to Equation (10):

$$\Delta y_{it}^{*} = \Delta\mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{x}_{it}\boldsymbol{\xi} + \Delta\varepsilon_{it} \tag{18}$$

$$\Delta y_{it} = \begin{cases} \Delta y_{it}^{*} & \textit{if} \quad y_{it-1}^{*} > 0 \cap y_{it}^{*} > 0 \\ \text{not observed} & \textit{if} \quad y_{it-1}^{*} \leq 0 \cup y_{it}^{*} \leq 0 \end{cases}. \tag{19}$$

This model is estimated using the FD-CRE estimator of Kalwij (2003), and the test for strict exogeneity is given by

$$\begin{aligned} &\mathrm{H}_0: \boldsymbol{\xi} = 0 \ \text{(strict exogeneity)} \\ &\mathrm{H}_1: \xi \neq 0. \end{aligned} \tag{20}$$

The null hypothesis is tested using a Wald test. This test is equivalent to a test for time-invariant coefficients when only two time periods are available, and one may

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therefore consider only the case when more than two time periods are available or when one can assume time-invariant coefficients.

Although, as mentioned in section 2, the consistency conditions for the CRE and FD-CRE estimators are identical, in practice these estimators may produce significantly different empirical results. Monte Carlo evidence in Kalwij (2003) provides a possible explanation for this as it suggests that, compared with the CRE estimator, the FD-CRE estimator is less sensitive to an incorrect CRE assumption and one may, therefore, favour the second test. Nevertheless, I focus in this paper on the test for strict exogeneity and assume the CRE assumption to hold. If one would like to test the CRE assumption of Equation (5) using the FD-CRE estimator, the added variables should not enter Equation (10), as they are 'differenced out', but would enter the indices in Equation (12) and the second term in Equation (13).

4 Empirical example

The usefulness of the proposed tests for exogeneity of the covariates is demonstrated by re-examining an empirical analysis of hours of work of US women. This example is taken from a textbook of Wooldridge (2010, p.711). The data are taken from 1980–1992 waves of the Panel Study of Income Dynamics and contain information on women's annual hours of work, non-labour income (*nwifeinc*), number of children aged 0–2 (*ch0_2*), number of children aged 3–5 (*ch3_5*), number of children aged 6–17 (*ch6_17*) and marital status (*marr*). The variable *nwifeinc* is defined as total family income minus labour income of the women and is in measured in thousands of US dollars. The variable *marr* is equal to 1 if legally married, and 0 otherwise. Table 1 presents sample averages by year.

I start with reproducing the table of results as presented in Table 17.3 of Wooldridge (2010). These results are reported in Table 2. Compared with the results in Wooldridge (2010), the results in column (1) are identical and the results in columns (2) and (3) are almost identical.[5] The three sets of results in Table 2 show rather similar effects in terms of directions but some differences in the sizes of these effects.

Table 1. Sample averages by year

Year	*N*	*hours*=0	*hours*	*nwifeinc*	*ch0_2*	*ch3_5*	*ch6_17*	*marr*
1980	898	0.26	1050	23.24	0.23	0.23	0.80	0.861
1981	898	0.26	1045	25.28	0.22	0.24	0.78	0.863
1982	898	0.29	1027	26.44	0.25	0.22	0.79	0.864
1983	898	0.28	1062	28.25	0.19	0.23	0.80	0.865
1984	898	0.25	1154	30.95	0.18	0.23	0.80	0.864
1985	898	0.26	1156	32.97	0.14	0.24	0.80	0.864
1986	898	0.26	1162	34.24	0.14	0.20	0.84	0.862
1987	898	0.27	1152	37.01	0.12	0.17	0.86	0.864
1988	898	0.27	1166	39.13	0.09	0.14	0.88	0.864
1989	898	0.26	1184	40.71	0.07	0.13	0.87	0.864
1990	898	0.25	1193	41.20	0.05	0.12	0.86	0.864
1991	898	0.25	1198	41.88	0.04	0.09	0.82	0.864
1992	898	0.27	1155	43.58	0.04	0.07	0.79	0.864

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Table 2. Estimation results: a reproduction of Table 17.3 in Wooldridge (2010, p.711)

	(1)		(2)		(3)	
Model estimation method	Linear model fixed effects		RE Tobit MLE		CRE Tobit MLE	
nwifeinc	−0.775	(0.343)	−2.349	(0.320)	−1.525	(0.376)
ch0_2	−342.38	(26.65)	−465.07	(22.30)	−471.58	(22.91)
ch3_5	−254.13	(25.88)	−321.13	(18.71)	−329.22	(19.40)
ch6_17	−42.96	(14.89)	−35.99	(10.07)	−46.24	(10.86)
marr	−634.80	(286.17)	−522.20	(52.83)	−781.54	(154.09)
Constant	1786.02	(247.30)	1634.99	(50.98)	1657.52	(54.65)
$\hat{\sigma}_\mu$	701.66		938.13		922.71	
$\hat{\sigma}_\varepsilon$	503.92		619.97		617.42	
Scale factor			0.80		0.82	
Log likelihood			−11228.5		−11194.3	
Number of women	898		898		898	
Number of time periods	13		13		13	

Note: Standard errors are in parentheses. All specifications include a full set of time dummies. The standard errors in column (1) are robust to arbitrary serial correlation and heteroskedasticity. In the RE Tobit model, $\boldsymbol{\gamma} = 0$. The scale factor is given by $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\Phi\left(\frac{\hat{\alpha} + \mathbf{x}_{it}\hat{\boldsymbol{\beta}} + \overline{\mathbf{x}}_i\hat{\boldsymbol{\gamma}}}{\sqrt{\hat{\sigma}_\mu^2 + \hat{\sigma}_\varepsilon^2}}\right)$. RE, random effects; CRE, correlated random effects; MLE, maximum likelihood estimates.

To get an idea of the differences between these models, the coefficient in the FE linear model is equal to −0.78, which is interpreted as a 0.78 reduction in annual hours of work for every thousand dollars of additional non-labour income (*ceteris paribus*). This can be compared with the average partial effects (APEs; Wooldridge 2010) obtained for the other models, which is for continuous variables equal to the scale factor multiplied with the slope coefficient. The APE of *nwifeinc* on hours of work is about −1.25 in the random effects Tobit model and −1.89 in the CRE Tobit model. Arguably, the results with the CRE estimator are to be preferred, as possible corner solutions (zero hours of work) are taken into account and pre-specified correlations between the random effect and explanatory variables are allowed for. Nevertheless, all three estimators of Table 2 require strict exogeneity of the covariates, and one can, for instance, argue that this is unlikely to hold, as past shocks in women's hours of work might influence men's income and household savings, hence future *nwifeinc* (feedback effects).

In columns (1) to (3) of Table 3 are the results for the three estimators discussed in section 2. The APEs presented in the middle of the table cannot be calculated when using Honeré's FE estimator. Under the assumptions outlined in section 2, all three estimators are consistent, and one would not expect large systematic differences between the parameter estimates across the first three columns. However, there are some rather large differences, and the results with the FD-CRE estimator are in between those with the FE estimator and the CRE estimator. One reason for these differences could be, as discussed earlier, that the common assumption of strict exogeneity does not hold.

The strict exogeneity test at the bottom of column (2) is as formalized in Equations (15)–(16), and the strict exogeneity test at the bottom of column (3) is as formalized

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Table 3. Estimation results of a CRE panel data Tobit model, APEs and results of tests for strict exogeneity

	(1)		(2)		(3)		(4)	
Model estimation method	FE estimator by Honoré (1992)		CRE Tobit MLE by Wooldridge (1995)		FD-CRE Tobit MLE by Kalwij (2003)		Pooled Tobit MLE	
nwifeinc	−2.017	(0.441)	−1.525	(0.376)	−1.744	(0.286)	−4.875	(1.700)
ch0_2	−149.50	(39.36)	−471.58	(22.91)	−304.49	(19.21)	−440.48	(57.79)
ch3_5	−121.35	(40.79)	−329.22	(19.40)	−225.63	(16.94)	−301.16	(47.64)
ch6_17	10.32	(32.37)	−46.24	(10.86)	−10.19	(10.06)	−12.48	(28.26)
Marr	−240.60	(164.11)	−781.54	(154.09)	−559.95	(138.74)	−139.06	(371.82)
Constant			1657.52	(54.65)	1074.20	(64.34)	1565.18	(75.40)
$\hat{\sigma}_\mu$			922.71		637.66			
$\hat{\sigma}_\varepsilon$			617.42		388.98			
$\hat{\sigma}$							1081	
Scale factor			0.82		0.79		0.80	
Log (pseudo) likelihood			−11,194		−9111		−15,913	
Average partial effect								
nwifeinc			−1.245		−1.383		−3.900	
ch0_2			−366.24		−228.54		−352.38	
ch3_5			−261.09		−172.82		−240.93	
ch6_17			−37.91		−8.09		−9.98	
Marr			−682.86		−480.24		−111.25	
Strict exogeneity test								
Wald test statistic			139.94		26.84		25.10	
Degrees of freedom			5		5		5	
p-value			0.000		0.0001		0.0001	

Note: The number of women and time periods are equal to, respectively, 989 and 13 in all estimations. Standard errors are in parentheses and are robust to arbitrary serial correlation and heteroskedasticity in column (1) and are clustered at the individual level in column (4). The log (pseudo) likelihood values are not comparable. All specifications include a full set of time dummies. The scale factor is given by $\frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\Phi\left(\frac{\hat{\alpha}+\mathbf{x}_{it}\hat{\boldsymbol{\beta}}+\bar{\mathbf{x}}_i\hat{\boldsymbol{\gamma}}}{\sqrt{\hat{\sigma}_\mu^2+\hat{\sigma}_\varepsilon^2}}\right)$. In column (4), I use $\alpha_i = \gamma_0 + \mathbf{x}_{1i}\boldsymbol{\gamma} + \mu_i$ for Equation (4), a strict exogeneity test similar to the one in column (2), and the standard deviation of the error term σ equals $\sqrt{\sigma_\mu^2+\sigma_\varepsilon^2}$. FE, fixed effects; CRE, correlated random effects; MLE, maximum likelihood estimates; FD-CRE, first-difference estimator of Kalwij (2003) for a CRE Tobit model.

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in Equations (18)–(20). These test statistics show that the null hypothesis of strict exogeneity of the covariates is rejected. The test statistic when using the CRE estimator is considerably higher than when using the FD-CRE estimator. One possible explanation for this, as discussed in section 3, is that the test statistic using the FD-CRE estimator is less sensitive to an incorrect CRE assumption (Equation (4)) than the CRE estimator. The estimates of ξ (not reported here) show that in particular the children variables cause the rejection of strict exogeneity. Because strict exogeneity does not hold, none of the three estimators of columns (1)–(3) of Table 3 (and the estimators of Table 2 for that matter) are consistent, and the empirical results should not be trusted or at least interpreted with extreme caution.

5 Summary and discussion

This paper has presented two tests for strict exogeneity of the covariates in a CRE panel data Tobit model: the first test uses the CRE estimator of Wooldridge (1995) and the second test uses the FD-CRE estimator of Kalwij (2003). Their usefulness has been demonstrated with an empirical analysis of hours of work of US women taken from Wooldridge (2010, p.711).

If strict exogeneity is rejected – like in section 4 – one has several options when estimating a panel data Tobit model. One way to proceed is to use a pooled estimator and choose not to control for possible correlations between the random effect and the covariates. Like for a linear panel data model, a pooled estimator requires the assumption of weak (or contemporaneous) exogeneity and not the assumption of strict exogeneity. One of the drawbacks of using a pooled estimator is that one may need to refrain from making causal interpretations, as the covariates may be correlated with the random effect. Another way to proceed is to use a pooled estimator and control for possible correlations between the random effect and the covariates by assuming strict exogeneity is rejected because of feedback effects. Such an approach requires the conventional assumption concerning the parameterization of the correlations between the random effect and the covariates as formalized in Equation (5) to be replaced by a parameterization that depends only on covariates of the first period; that is, Equation (5) is replaced by $\alpha_i = \gamma_0 + \mathbf{x}_{1i}\boldsymbol{\gamma} + \mu_i$. The estimation results when using this estimation procedure are presented in column (4) of Table 3. They show that in particular the estimated effect of *nwifeinc* on hours of work becomes much larger, which may suggest important feedback effects from the wife's hours of work to husband's earnings.[6] If there are important feedback effects and the model estimates are used for policy relevant predictions, one would like to explicitly model these feedback effects and identify the variance of the random effect. The latter is, for instance, needed if one would like to condition predictions or marginal effects on a specific value of the random effect. In short, although estimation procedures using a pooled estimator do not require the assumption of strict exogeneity, they may not yield all results needed to answer specific policy questions. Wooldridge (2000) has suggested a possible solution within the framework of CRE models to

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allow for feedback effects. His approach requires distributional assumptions to model the relations between the covariates and the lagged outcome variable. In addition, and similar to the discussion earlier, it requires a different assumption concerning the parameterization of the correlations between the random effect and the covariates. As this approach can be computationally intensive, the advice is to first test for strict exogeneity. If the model does not pass this test, and it is reasonable to assume this rejection is due to feedback effects, one could follow the approach of Wooldridge (2000). Nevertheless, given the fact that there are, to my knowledge, no empirical studies that test for strict exogeneity in a panel data Tobit model and, when rejected, apply various alternative models and estimation procedures, more research on these issues is needed to obtain insights into the empirical importance of the assumption of strict exogeneity.

Another avenue for future research is the development of maximum likelihood estimators based on taking first differences of equations of interest for other nonlinear CRE models and to obtain more insights into their sensitivity with respect to the CRE assumption. For instance, a model for which two identical tests for strict exogeneity can be carried out is the CRE panel data selection model for which a (conventional) CRE maximum likelihood estimator (Wooldridge 1995) and a FD-CRE maximum likelihood estimator (Rochina-Barrachina 1999) are available.[7]

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Notes

1. If panel attrition is exogenous, the estimators presented in this section can be applied to an unbalanced panel.
2. This cost is not an issue when controlling for data censoring due to, for instance, top-coding.
3. The model in this paper is a restricted version of the model of Kalwij (2003) as the error terms are uncorrelated over time.
4. A similar reasoning is followed when setting up the test for strict exogeneity in a linear panel data model (Wooldridge 2010, p.325).
5. Some random differences are expected because of the choice of the integral approximations. All results in Table 2 are exactly equal to the ones obtained when using STATA software (including the values of the likelihood function).
6. When using a CRE Tobit maximum likelihood estimator, the parameterization $\alpha_i = \gamma_0 + \mathbf{x}_{1i}\boldsymbol{\gamma} + \mu_i$ yields almost the same results as in column (2) of Table 3. For instance, the estimated coefficient corresponding to *nwifeinc* is -1.754 (SE 0.328).
7. Kyriazidou (1997) has developed an estimator for a panel data selection model using a conditional exchangeability assumption. A similar assumption is used by Lee (2001) who presents a semi-parametric first-difference estimator for panel censored regression models.

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