Use of Technology in Secondary Mathematics

Final Report for the International Baccalaureate

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Introduction

The International Baccalaureate (IB) commissioned us to write a report to:

> provide insights into the use and integration of technology into curriculum, classroom practice and impact on learning in secondary mathematics courses and will inform possible direction and focus for the coming curriculum review of IBDP mathematics

The study aims are:

- Aim 1. Explore types of technology used in mathematics curricula for students aged 16-19 years old in different educational systems (within a country and internationally);
- Aim 2. Examine approaches and strategies to technology integration in curriculum design, pedagogy and assessment;
- Aim 3. Indicate issues involved in the use of technology in mathematics courses;
- Aim 4. Identify factors increasing the effectiveness of technology implementation in classroom practice;
- Aim 5. Analyse the effects of using and integrating different types of technology on the development of mathematical skills and academic achievement in mathematics.

The scope of work (SoW) was to be two-fold:

> SoW_1. A literature review on a comprehensive range of theoretical and empirical studies in the use of technology in senior secondary mathematics education worldwide.
>
> SoW_2. Analysis of IB documentation including IB official curriculum documents, teacher support materials, workshop resources.

Discussions between the research team and key IB personnel led to a review of SoW_2:

> to compare the IB's intended (reflected in the official documentation) and implemented 16-19 mathematics curricula (reflected through personal communication) with the intended and implemented curricula in other countries

Research Questions

The IB stated:

> We hope to capitalise on the knowledge and interests of our research partner and thus recommend that the researchers bring their own strengths and experience to the design of the study and research questions. Research questions will address but are not limited to the following:

RQ 1. What are trends and approaches to technology in secondary mathematics education worldwide? What are differences, commonalities and variables in relation to (for example):

a) Types of technology used
b) Technology integration in curriculum
c) Learning objectives and expected outcomes
d) Pedagogy and classroom practice
e) Assessment
f) Others

RQ 2. What guidance is given on the use and integration of technology in teaching and learning mathematics in various (international) education curricula? What are emergent themes and patterns in relation to frequency, tools and applications, pedagogical strategies and so forth?

RQ 3. What are the issues, enablers and challenges of using technology in the teaching and learning of mathematics in both IB and non-IB school contexts?

RQ 4. What is the impact of using particular technologies on the development of mathematical skills and academic achievement in mathematics in both IB and non-IB school contexts?

RQ 5. To what extent do objectives, approaches to technology integration, pedagogical strategies and learning practices in DP mathematics courses reflect the contemporary trends, initiatives and strategies in the use of technology in secondary mathematics education worldwide?

We found that we could work with the IB's research questions.

In the following we first present a summary of our work. We then present our work for Scope of Work 1 and 2. We then address the research questions. Three appendices, *References for Scope of Work 1*and *Scope of Work 2, national descriptions*, complement this report.

Summary

This section summarises the Report. It does this in two parts: Findings; matters for IB consideration. This summary aims at *plain English* and does not make links to subsequent sections. The term 'six countries' refers to document analysis of 16-19 academic stream mathematics in Australia, England, France, The Netherlands, New Zealand and Singapore. The term 'technology' in this summary refers to advanced (i.e. more than a scientific calculator) digital technology.

Findings

The findings are presented in six sections which start with the intended curriculum and ends with the intended assessment. The four intermediate parts deal with aspects of the implemented curriculum: types of technology used; learning; student skills and competences; pedagogy.

Curriculum

Curricula documents for academic stream mathematics from the IB and all the six countries mention technology as an explicit element in the mathematics curricula. There has been debate on the integration of technology into mathematics in some of the documents examined. Reference to the use of technology in documents is often general, for example, "Use technology to present and communicate mathematical ideas" but is sometimes specific, for example:

- "In appropriate situations, the candidate can set up an integral, calculate its exact value and approximate it using ICT"
- "Use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities"

Types of technology used

The technology most commonly referenced in the documents from the IB and all the six countries is the graphic display calculator (GDC). In two countries calculators with symbolic manipulation facilities are used/referred to and there is some experimentation with the use of such calculators in some other countries. In mathematics lessons computer labs are not uncommon and there is a rise in class use of laptop/tablet computers in recent years. The use of video clips and online courses is also increasing. The availability and use of interactive white boards by teachers varies, from regular to occasional, over the six countries. Teachers in all countries use internet resources to find and share content; in some countries this is managed in an organised way[1].

Learning

The quality of student learning with technology is difficult to measure due, in part, to differences amongst mathematics educators: as to what 'learning' means; between those who see technology as a medium to communicate mathematics to students and those who

[1] See Scope of Work 2 for details.

see technology as a means for students to express mathematical relationships. However, reviews of research report qualified success. One review of graphic calculator use states they can aid students' understanding of concepts. A second review of all technology reported that technology is making a modest difference in learning of mathematics. Research consistently shows that the organisation of classroom resources is a crucial factor and that student learning is sensitive to small differences in the way computer-based tasks, paper-and-pen work and whole-class teaching are intertwined.

Student skills and competences

Technology introduces new skills that students must master: setting up suitable scales on graphic calculator; making mathematical rather than simply visual links between geometric objects in geometry software. Apart from skills in using technology students need to acquire skills in interpreting displays and to make connections between numeric, symbolic and graphic/geometric mathematical representations. Without these skills students may, for example, accept a graphic image uncritically, without attempting to relate it to other symbolic or numerical information.

Pedagogy

The teacher is key to the successful use of technology in the mathematics classroom but incorporating technology into teaching remains a challenge for many teachers and the degree and type of technology used in the classroom is variable. Reasons for this include teachers' proficiencies in mathematics and their perceptions of the nature of mathematical knowledge and how it should be learned; and teachers' understandings of the principles, conventions and techniques required to teach mathematics with technology. In addition to attending training courses, collaborative work to develop resources and classroom approaches are important as such work aids teachers' critical reflection on their practice and their professional development.

Assessment

The IB and all the six countries have high stakes examinations which permit the use of technology in at least some of the examination papers. All allow graphic calculators and some allow calculators with symbolic manipulation. None allow internet use or printing. There are differences with regard to: marking and grading procedures; examinations being centrally or school set; lists of approved technology (or not); the need to clear memory (or not); and whether questions are set which *expect* or simply *allow* the use of technology. Research literature on assessment with technology is sparse, particularly on formative assessment. Studies on e-assessment are in their infancy but point to the need for *principles* to avoid simply examining what current technology enables. Studies on summative assessment point to problems: focusing on a single tool with the expectation that the student will rise to the use of this tool; trivialising routine questions and thus disadvantaging lower attaining students; and the difficulty of designing 'good' technology-tasks. A study which included IB examinations concluded that there have not been major changes in the examination questions due to technology.

Matters for IB consideration

Our research into use of technology in 16-19 academic stream mathematics leads us to two sets of matters for IB consideration in their coming curriculum review of DP mathematics: tools and resources; and pedagogic practices and professional development.

Tools and resources

Whilst the IB was innovative in the past in embracing graphic calculator use, DP mathematics does, perhaps, place too much emphasis on this single tool. Coupled with this, tool use should be seen in the context of the resources available in classrooms. A challenge for the IB is to consider a wide range of tools and resources available for DP mathematics: traditional mathematical tools; techno-mathematical tools with capabilities for algebra, geometry and calculus; traditional resources such as textbooks; and digital/internet-based resources for teaching and learning[2].

There are many sub-challenges within this challenge. The use of tools with capabilities for algebra, geometry and calculus would require serious consideration of the place of techniques and concepts in DP mathematics and it may be useful to consider parallel by-hand/by-technology DP courses so that teachers may come on board when they are confident[3]. Linking graphic calculator use to the full range of classroom resources would include reviewing the tasks teachers offer to students and this may lead the IB to reconsider whether textbooks should be included in IB documentation. Consideration of internet-based resources may lead to on-line communities of teachers and students and the challenge of facilitating these communities.

Whilst IB staff who are not classroom-based have an obvious role in taking these challenges forward, it is important that teachers are a full part of finding solutions to these challenges, which leads us on to the next set of matters for IB consideration.

Pedagogic practices and professional development

The quality of IB teacher support material and workshops appears very high but the IB should consider whether this is sufficient for teacher development for the integration of technology into DP mathematics. The success of designs for such integration requires access to technology, the development of technological knowledge and assisting teachers to develop modes of organising classrooms for student learning. Our review of research points to the benefits of teachers' collaborative work in this endeavour.

A model of professional development that has had some success is to structure it around a supportive community of inquiry on everyday classroom practice where all participants are co-learners and knowledge, classroom practices and resources are developed, shared and evaluated by the group. Given the geographical distance between IB schools on-line communities can be set up to facilitate such communities of inquiry.

[2] See Scope of Work 1, section 1(b) for details.
[3] See Scope of Work 1, section 3(a) for details.

Scope of Work 1: Literature Review

Method

In this review we made a decision to concentrate on papers written from the years 2000 to 2014 (exceptions were made for key papers) from journals rated as A* and A (and some rated as B) in a recent European rating of mathematics education journals (see http://www.mathematik.uni-dortmund.de/~erme/). The journals are:

- *Educational Studies in Mathematics*
- *Journal of Mathematical Behavior*
- *Journal for Research in Mathematics Education*
- *Mathematical Thinking and Learning*
- *Mathematics Education Research Journal*
- *Recherches en Didactique des Mathématiques*
- *Research in Mathematics Education*
- *ZDM: The International Journal on Mathematics Education*

We considered every paper in these journals for 2000 – 2014 and included all papers in our review that deal with technology which impacts in some way on 16-19 mathematics. Some papers were included which were not specifically focused on but were felt to impact on 16-19 mathematics with technology. Papers focussing exclusively on elementary mathematics with technology were excluded from our review.

In addition we looked at a number of important books as well as adding relevant papers that were known to us, or written by us. These included:

Two International Handbooks of Mathematics Education:

Bishop, A.J., Clements, M.A., Keitel, C., Kilpatrick, J. & Leung, F.K.S. (Eds.) (2003). *Second international handbook of mathematics education*. Dordrecht: Kluwer Academic Publishers.

Bishop, A.J., Clements, M.A., Keitel, C., Kilpatrick, J. & Leung, F.K.S. (Eds.) (2013). *Third international handbook of mathematics education*. New York: Springer.

Two books taking a general look at ICT integration in mathematics education:

Guin, D., Ruthven, K. &Trouche, L. (Eds.) (2005), *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument*.

Gueudet, G., Pepin, B. &Trouche, L. (Eds.) (2012), *From text to 'lived' resources: Mathematics curriculum materials and teacher development.* New York: Springer.

The 17th ICMI Study:

Hoyles, C. &Lagrange, J.-B. (Eds.). (2010). *Mathematical education and digital technologies: Rethinking the terrain.* New York: Springer.

A book on teacher use of ICT:

Clark-Wilson, A. Sinclair, N. & Robutti, O. (Eds.) (2014). *The mathematics teacher in the digital era*. Dordrecht: Springer.

The references are listed in the appendix for SoW_1 as is a bibliography that includes papers not included in this review but which may be useful.

The structure of the literature review is as follows:

1. Teacher Issues and Technology Use

- *a) Some Theoretical Considerations*
- *b) Applications of the Theory to Classroom Practice*
- *c) Affordances and Constraints in Teaching with Technology*
- *d) Professional Development of Teachers*
- *e) Emerging Practice: Challenges and Implications for Curriculum Change*

2. Student Issues and Technology Use

- *a) Student Learning Outcomes*
- *b) Examples of Technology Use and Specific Curriculum Areas*
  - i. Tasks
  - ii. Geometry
  - iii. Statistics
- *c) Student and Teacher Interactions*
- *d) Classroom Organisation for Student Learning*
- *e) Emerging Practice, Challenges and Implications for Curriculum Change*

3. Technology and Assessment

- *a) Formative and Summative Assessment*
- *b) Emerging Practice, Challenges and Implications for Curriculum Change*

4. Some Implications and Predictions Related to Curriculum Change

Findings

1. Teacher Issues and Technology Use

a) Some Theoretical Considerations

There have been a number of theoretical approaches applied to issues surrounding technology use in the secondary mathematics classroom and the last decade has seen efforts aimed at using these frameworks to provide different, complementary views of the same subject. For example, Lagrange *et al.* (2003), through a review of 800 articles, distinguished seven dimensions related to ICT use in mathematics classrooms: integration, epistemological-semiotic, cognitive, institutional, instrumental, situational and the teacher-dimension. While this last dimension appears to have been quite neglected during the years 1990-2000, it has increasingly been taken into account in the last decade, including a recent

book (Clark-Wilson, Robutti & Sinclair, 2014) devoted to the mathematics teacher in the digital era. In addition, Drijvers *et al.* (2010), have suggested the need for an integrative framework in which considerations of didactical functionalities play an essential role, combining the three major dimensions of tool features, educational goals and associated potential of the tool and modalities of use in a teaching/learning process.

While it is clear that the role of the teacher is a key to the successful use of digital technology in the mathematics classroom, incorporating technology into teaching remains a challenge for many teachers and they need to be well prepared (Zbiek & Hollebrands, 2008). Hoyles *et al.* (2010) provide evidence for the critical role of teacher: as a *facilitator* who maintained and supported the interaction, as a *gatherer*, making visible on a common workspace (the screen, for example) students' production and as a mechanism for discussion; validating what did and did not make sense in terms of knowledge building.

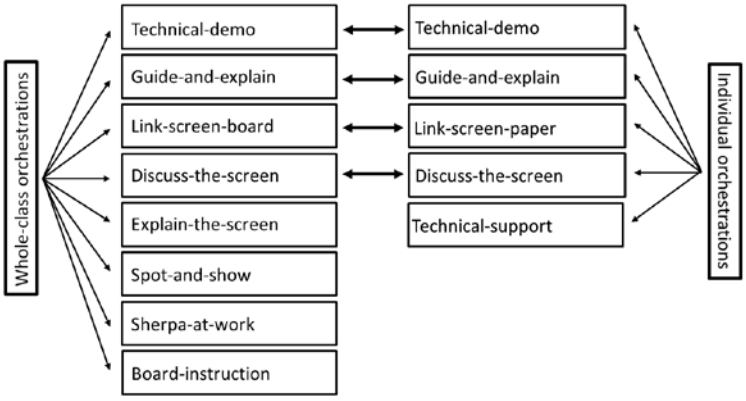
Some of the intrinsic factors contributing to the challenge facing teachers are their orientations (Schoenfeld, 2011); their instrumental genesis (Artigue, 2002; Guin & Trouche, 1999; Rabardel, 1995; Vérillon & Rabardel, 1995) and orchestrations (Trouche, 2005c); Drijvers & Trouche 2008); their perception of the nature of mathematical knowledge and how it should be learned (Zbiek & Hollebrands, 2008); their mathematical content knowledge; and their mathematical knowledge for teaching (MKT – Ball, Hill &Bass, 2005; Hill & Ball, 2004; Zbiek, Heid, Blume & Dick, 2007). The idea of MKT covers appropriate structuring of content and relevant classroom discourse and activities to form didactical situations. Thus, it is not surprising that, while many mathematics teachers claim to support the use of technology in their teaching (Forgasz, 2006a; Thomas, 2006), the degree and type of use in the classroom remains variable (Ruthven & Hennessy, 2002; Zbiek & Hollebrands, 2008).

Two complementary frameworks that have merit in analysing factors that may influence teacher use of technology and provide an indication of teacher readiness for implementation of technology use, are *Pedagogical Technology Knowledge* (PTK) (Hong & Thomas, 2006; Thomas & Hong, 2005b) and the *Technological Pedagogical Content Knowledge* (TPACK) (Mishra & Koehler, 2006; Koehler & Mishra, 2009). A critical review of the notion of TPACK, including an analysis of its affordances and constraints can be found in Graham (2011). Both of these frameworks build on pedagogical content knowledge (PCK) of Shulman (1986), by adding technology knowledge. In this aspect, PTK incorporates the principles, conventions and techniques required to teach mathematics through the technology. This includes the need to be a proficient user of the technology, but more importantly, to understand the principles and techniques required to build and manage didactical situations incorporating it and enable mathematical learning through the technology. According to Koehler and Mishra (2009, p. 64), the TPACK definition of technology knowledge is close to that of Fluency of Information Technology (FITness), as proposed by the Committee of Information Technology Literacy of the National Research Council (see Snyder et al., 1999). This requires that teachers understand information technology broadly enough to apply it productively at work and in their everyday lives, to recognise when information technology can assist or impede the achievement of a goal and continually to adapt to changes in information technology.

b) Applications of the Theory to Classroom Practice

In addition to technology knowledge, PTK includes teacher orientations and goals (Schoenfeld, 2011), especially beliefs about the value of technology and the nature of learning mathematical knowledge as well as crucial affective aspects, such as teacher confidence (Thomas & Palmer, 2014). Although in general teachers may agree there are potential benefits of technology for students' mathematics learning, many remain unsure or unconvinced about whether its use helps students to explore mathematical concepts or unfamiliar problems (Forgasz, 2002; Thomas, 2006; Goos & Bennison, 2008). That teacher beliefs can inhibit technology use, was demonstrated by Zuber and Anderson (2012) in a one-to-one laptop programme. They report that a prevalent belief limiting laptop use is that students only learn mathematics authentically using pen and paper. Further, cautious Adopters and Non Adopters of technology expressed beliefs that laptops exacerbate classroom management problems, especially for lower-achieving students. The study by Lagrange and Dedeoglu (2009), in the context of "ordinary" classes, reveals a high level of teacher expectation with regard to technology, but a quite low degree of integration. They analyse this phenomenon as the co-existence of two worlds, the world of teacher expectation and the world of technology potentialities. To establish links between these two worlds requires encouraging teachers' collective work and reflective practices.

One useful way to reflect on the alignment of the teacher preparedness factors with their actual classroom practice is that of instrumental orchestrations (see Drijvers & Trouche, 2008 for the theoretical framework of instrumentation and orchestration and some concrete examples), or the ways in which the teacher manages the changing classroom environment when technology is present. A primary goal of the orchestrations is to engage students in activity producing techniques with both *epistemic value*, providing knowledge of the mathematical object under study and 'productive potential' or *pragmatic value* (Artigue, 2002). The classroom orchestrations appear as a response to both the proliferation and increasing complexity of technological resources. The notion of orchestration itself evolves, from an individual and static conception (orchestrations of a mathematical situation seen through didactical configurations and exploitation modes of these configurations – Trouche 2002, 2005c, 2005d, 2007) with a social perspective (orchestrations seen as the result of teachers collaborative work – Hoyles *et al.,* 2010) to a dynamic view (including the didactical performance, teachers' adaptation on the fly – Drijvers *et al.,* 2010 and teacher adaptation over time – Drijvers, 2012). Some researchers (e.g., Drijvers, Doorman, Boon, Reed & Gravemeijer, 2010; Drijvers, Tacoma, Besamusca, Doorman & Boon, 2013) have taken the framework of instrumental orchestration as a point of departure for categorising observed teaching practices. This results in a kind of (non-exhaustive and maybe tool, topic and culture dependent) taxonomy, depicted in the Figure below. The Figure provides an overview of whole-class and individual orchestrations identified in the study; the middle arrows simply show correspondence between whole-class and individual orchestrations.



Other analyses of factors influencing teacher implementation of a technology-rich mathematics curriculum and practices include that by Assude *et al.* (2010, p. 406), who propose a typology comprising: the social, political economical and cultural level; the mathematical and epistemological level; the school and institutional level; the classroom and didactical level. Further, Ruthven (2012) identifies five key structuring features of classroom practice and shows how they relate to the constitution of digital tools and materials as classroom resources: working environment (physical surroundings where lessons take place), resource system (collection of mathematical tools and materials in classroom use), activity format (generic templates for action and interaction which frame the contributions of teacher and students to particular types of lesson segment), curriculum script (evolving teacher knowledge gained in the course of her own experience of learning and teaching a given topic) and time economy (managing the physical time available for classroom activity to be converted into a didactical time, measured in terms of advance of knowledge). Finally, a web metaphor has been used to describe the new way of teaching mathematics in the digital era (Leguay, 2011).

There appears to be a need for teachers to evolve and adapt their teaching when using technology. According to Doerr and Zangor (2000), the role, knowledge and beliefs of the teacher, along with the nature of the mathematical tasks, influenced the type of GDC use emerging, with five kinds identified: computational tool (evaluating numerical expressions, estimating and rounding), transformational tool (changing the nature of the task), data collection and analysis tool (gathering data, controlling phenomena, finding patterns), visualizing tool (finding symbolic functions, displaying data, interpreting data, solving equations) and checking tool (confirming conjectures, understanding multiple symbolic forms). Using their practitioner model, Ruthven and Hennessy (2002; 2003) provide a framework for synthetising teacher thinking about the contribution of GDC, with later analysis (Ruthven, Deaney & Hennessy, 2009) highlighting the crucial part played by teacher pre-structuring and shaping of technology-and-task-mediated student activity.

An application of the concepts of Zone of Proximal Development (ZPD), Zone of Free Movement (ZFM) and Zone of Promoted Action (ZPA) by Goos and others (Goos, 2005; Goos, Galbraith, Renshaw & Geiger, 2000, 2003) to describe teachers' thinking and modes of working with technology, uses the metaphors of technology as master, servant, partner and extension of self. In the last of these technology is "seamlessly incorporated into a teacher's pedagogical and mathematical repertoire such as through the integration of a

variety of technology resources into course planning and the everyday practices of the mathematics classroom" (Goos, 2005, p. 40). Kendal *et al.* (2005) analyse the different ways in which three pioneering Australian teachers adapted their teaching to use CAS (computer algebra system) and found it depended on their history and experiences. An example from Sinclair and Yurita (2008) focuses on changes to the instructional environment required when using a dynamic geometry system (DGS) with regard to the manner in which the teacher talks about geometric objects and uses visual artefacts and models of geometric reasoning.

c) Affordances and Constraints in Teaching with Technology

Some papers address the affordances and constraints that either promote or inhibit the ability of teachers to use technology. Heid *et al.* (2013) summarise a number of factors, including those in other research (see e.g., Forgasz, 2006a; Goos, 2005; Pierce & Ball, 2009; Thomas, 2006; Thomas *et al.*, 2008; Thomas & Chinnappan, 2008) that influence teacher adoption and implementation of technology in mathematics teaching. They mention, for example, "previous experience in using technology, time, opportunities to learn, professional development, access to technology, availability of classroom teaching materials, support from colleagues and school administration, pressures of curriculum and assessment requirements and technical supports" (Heid *et al.*, 2013, p. 630). Lack of affordability of the technology typically used for mathematics teaching has equity implications (Pierce & Ball, 2009), since the perceived advantages of this technology for supporting student learning and examination use may not be equally available for all students. With regard to classroom resources, Kieran *et al.* (2012) analysed how affordances and constraints are inherent in any given resource, noting "Resources are not neutral. They speak to different teachers in different ways." (p. 211). This influence on teacher use arises from the designers' intentions (both explicit and implicit).

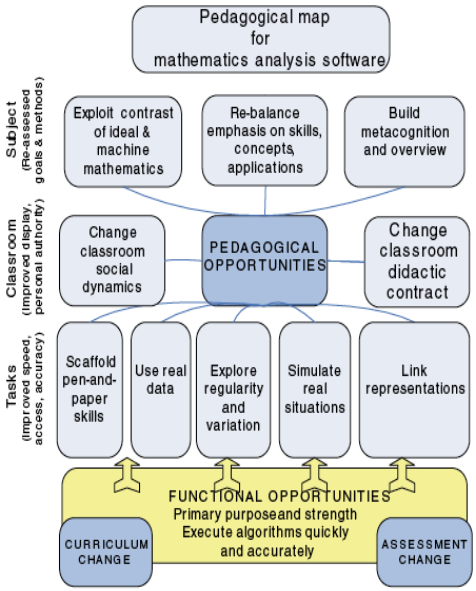
d) Professional Development of Teachers

There are clear implications arising from the literature with regard to teacher use of technology in mathematics. One major aspect is the nature of pre- and in-service professional development (PD) of teachers that will enhance their ability to incorporate technology into learning situations. It seems clear (Bennison & Goos, 2010) that participation in professional development plays a crucial role in whether and how technology is used in mathematics classrooms. In this regard there are several related aspects that PD needs to consider: issues of access to technology in schools; the promotion and development of teacher technological knowledge (based on PTK or TPACK); and assisting teachers to broaden and deepen their engagement with instrumental orchestrations. Research shows that teachers want PD that models planning and pedagogy so they can meaningfully integrate technology into their lessons in ways that help students learn mathematical concepts (Goos & Bennison, 2008).

It may be that the goals of PD are best accomplished when it is structured around a supportive community of inquiry (Jaworski, 20001, 2003) where all participants are co-learners and knowledge is developed and evaluated critically as a group. In the context of a national programme for providing digital resources to teachers, Trigueros and Lozano (2012, p. 261) acknowledge that "creating working groups inside and outside the schools constitute an essential element in professional development programs". Being part of a group that shares and reflects on their content, pedagogical and technology knowledge and explores

planning and pedagogy to integrate technology into their everyday classroom practice could be highly beneficial for teacher confidence (Goos & Bennison, 2008), especially for those inexperienced in instrumental orchestration. The potential for PD of working with teachers on mathematics curriculum materials, focusing on task analysis, was shown by Pepin (2012).

The importance of teachers' collaborative work to support the necessary evolution of resources/practices/knowledge is acknowledged by Hoyles *et al.* (2010) and the development, in France, of an online teacher association, aiming at designing and sharing digital resources, seems to be in line with this proposal (Hache, 2004). However, some facilitation is likely to be needed. Visnovska *et al.* (2012, p. 339) draw attention on the fact that, even if teachers can be considered as effective *instructional designers* through their work with and for resources, the common assumption that groups of teachers are capable of designing coherent instructional sequences from materials provided with little, if any, on-going support is a dangerous misinterpretation of both the potential of teacher collaboration and the fact that implementation is necessarily an act of design.



e) Emerging Practice, Challenges and implications for Curriculum Change

One aspect of teacher PD, raised by Mousley *et al.* (2003), is the range of technological tools now available to teacher educators in developed countries, suggesting the need to re-think teacher education programmes. They evidence the positive effects, both for in service and pre-service teacher education, of applications of videotapes, multimedia resources, internet-based communities and tutorial conferencing. A new kind of teacher education PD programme presented by Guin and Trouche (2008) is grounded on the collaborative work of teachers to co-design, experiment in real contexts and revise pedagogical resources. In this context, the goal of such an innovative programme is not to provide teachers with new knowledge, but to give them tools ("methodological assistants") to support collaboration and design. With the help of a theoretical network, Gueudet and Vandebrouck (2011) analysed the links between the integration of technologies in mathematics teaching and professional development (or evolution of practices) in a dialectic way: the integration of technologies has to be understood as the search for new equilibrium within teachers' resource systems and this equilibrium has to be established in a continuous and dynamic way. This has strong consequences for teacher education, having to combine design of resources taking into account the "déjà là", the "old resources", implementation in classrooms, redesign, etc.

This useful summary of the potential pedagogical opportunities offered by CAS technology in the classroom has been provided by Pierce, Stacey and Wander (2010).

2. Student Issues and Technology Use

a) Student Learning Outcomes

There have been several reviews of the benefits of ICT to student learning in mathematics that suggest positive effects from the use of digital technology. Looking specifically at algebra, Rakes, Valentine, McGatha and Ronau (2010) report small but significant positive effects, as do Graham and Thomas (2000). For mathematics in general, Li and Ma's (2010) literature review, which included 41 studies, led to a similar conclusion. The meta-analysis of 54 calculator studies by Ellington (2003) found a significant improvement in students' problem-solving skills and better attitudes towards mathematics when calculators were an integral part of both teaching and assessment. They also report that the calculator did not hinder the development of mathematical skills. Two reviews with higher criteria for studies to be included were those of Burrill *et al.* (2002) and Cheung and Slavin (2011). Considering graphic calculator (GDC) use, the former chose just 43 research reports (from over 180) that met the criteria of being rigorous, evidence-based and 'scientific' in approach, for inclusion in the synthesis. They concluded that GDCs can be an important factor in helping students of mathematics develop a better understanding of mathematical concepts, score higher on performance measures and raise the level of their problem solving skills. However, the type and extent of the gains are a function of how the technology is used in the teaching of mathematics. The final conclusion in the latter study speaks about a modest difference: "Educational technology is making a modest difference in learning of mathematics. It is a help, but not a breakthrough." (Cheung & Slavin, 2011, p. 20).

In a reflection on the introduction of handheld graphing calculators in mathematics education, Trouche and Drijvers (2010) agree, sketching out initially high expectations as a somewhat naïve view, whereas experiences show that the issue is more complex than expected. Assude *et al.* (2010) would concur, stating that changes, when using technology, are at first general (such as motivation for learning) and that specific changes in mathematical knowledge appear in a second phase. The relationships in the classroom appear crucial to learning, with an extensive review of the literature by Olive and Makar (2010, p. 133) concluding that "interactions among students, teachers, tasks and technologies can bring about a shift in empowerment from teacher or external authority to the students, as generators of mathematical knowledge and practices; and that feedback provided through the use of different technologies can contribute to students' learning".

Some individual studies by Drijvers (2000, 2002) highlight the complexity of using handheld CAS devices with grade 9 and 10 students. He concludes that even if the digital device can in principle do all the work to be done, the techniques and the idiosyncrasy of the syntax provide obstacles, which invite mathematical thinking. A study by Mitchelmore and Cavanagh (2000) considered the errors students make when using GDCs and found they were attributable to four main causes: a tendency to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; and a limited grasp of the processes used by the calculator to display graphs.

b) Examples of Technology Use and Specific Curriculum Areas

In terms of classroom learning, Hoyles and Noss (2003) distinguish two categories of digital

technologies, namely programmable microworlds and expressive tools[4], characterised by their openness and learning environments. The number of papers related to symbolic calculators around the turn of the last century, appearing as the product of the convergence of calculators and computers, seems to indicate that this new technology announced a deep evolution: towards small tools, able to perform a lot of different tasks. These handheld devices appeared to have a great future in front of them and have led to the tablets of today and whatever will follow. Trouche (2005b) introduced the instrumental approach to mathematics learning with artefacts, analysing appropriation of artefacts as dialectic processes, combining instrumentation, expressing the effects of artefacts on student activity and instrumentalisation, expressing students' creativity for transforming and adapting artefacts. The joint dynamics of instrumentation and conceptualisation are analysed in term of schemes (see Drijvers & Gravemeijer, 2005) and Trouche (2005c) underlines the fact that a more complex environment leads to a greater dispersion of the students' conceptualisation processes, which is one of the main arguments for the necessity of the instrumental orchestration described above.

Studies addressing student learning often use a particular type of technology with one or more students working on tasks in a manner indicated by a theoretical framework. They then look for some indication (such as engagement) or measure of improved learning outcomes. The most common interventions have involved the use of handheld graphing (GDC) or computer algebra system (CAS) calculators, although computer-based CAS is also used.

i. Tasks

One can distinguish several moments/phases (not successive) from the point of view of task and technology use: a first moment when an artefact (or tool, or technology), as a static entity, is taken into account, a second moment when an artefact is understood as a living entity, in the dynamics of instrumental genesis (from an artefact to an instrument), a third moment, where an artefact is understood through its links with other artefacts (the productive notion of system of artefacts, towards systems of instruments), a fourth moment where the focus is on the link artefact-task, a fifth moment where the focus is on artefact-task-orchestration and a (possibly) last moment with a holistic point of view, a *resources* point of view. The synergy between task, techniques and theory (Chevallard, 1999) in the emergence of mathematical understanding for students using CAS calculators has been investigated by Kieran and Drijvers (2006). They show that, as well as technique and theory, task design plays a crucial, fundamental role in co-emergence of by-hand and CAS techniques and theory. The epistemic value of CAS techniques may depend both on the nature of the task and the limits of students' existing understanding. Exemplifying this, a paper by Drijvers and Barzel (2012) provides a concrete example of how different technological tools and the different techniques required to use them for different tasks, affect the students' view of the underlying concept, in this case the notion of equation.

In teacher journals there are many papers illustrating the potentiality of technologies, such as for conjecturing in geometry (Caponi, 2000; Kittel & Kuntz, 2002; Sinclair & Yurita, 2008), or simulations in statistics (Fontana & Noguès, 2002) or investigating families of functions (Abu-Naja, 2008). One can also observe the increasing interest in online resources (Kuntz,

[4] Microworlds are mathematical environments (usually computer-based) designed to promote mathematical thinking. Expressive tools are those that allow the user to articulate mathematical relationships.

2003), as well as studies in this area that examine how students use them (van de Sande, 2011). Research suggests that technology can afford the opportunity to explore representations and their connections, with students finding patterns, observing links and making generalisations (Hong & Thomas, 2001; Kidron, 2001; Thomas, Monaghan & Pierce, 2004). However, Kendal and Stacey (2003) report that only the most capable students achieved the goal of developing facility with numerical, graphical and symbolic representations of functions and derivatives. Further, Burrill *et al.* (2002) saw in their survey that most students used handheld calculators as a computational tool, to move among different representational forms and as a visualising tool.

However, the primary use of handheld graphing technology was to graph functions. Heid, Thomas and Zbiek (2013) propose some major areas where CAS may be employed: the interaction of concepts and procedures; investigating new concepts, extended procedures and new structures; and the thinking and reasoning that CAS-use inspires or requires. They also provide some examples of how this might be done. Another example is provided by Arzarello and Robutti (2010), who claim that the symbolic power of a CAS-empowered spreadsheet supports the development of symbol sense and makes it easier for students to see and reason on symbolic patterns. In spite of this potential, Weigand and Weller's (2001) computer CAS study on student understanding of quadratic and trigonometric functions reported no evidence of a better understanding of function. One potential reason could be the twelve kinds of obstacles students often encounter during CAS use identified by Drijvers (2002). He highlights the view that these apparent obstacles should be perceived as learning opportunities, forming the subject of classroom discussion in which the meaning of techniques and conceptions is developed.

Artigue (2005) describes the results of two didactical engineering experiments developed by two research teams. The first project concerned exact and approximate computations and the equivalence of algebraic expressions, at grade 10 level. The second involved the teaching of the notion of derivative, for scientific grade 11 students. Using an instrumental and anthropological approach, she investigated the problems raised by the integration of symbolic calculators into secondary mathematics education and discusses the viability of such an integration. Roughly speaking, she evidences that symbolic tools can support mathematical knowledge through different categories of situations, especially in the two following ways: through the mastering of instrumented techniques, which at first appeared rather as a constraint; and through the new potential offered by instrumented work in symbolic environments.

How best to integrate by-hand and technological methods has been an issue under investigation, with Stacey, Kendal and Pierce (2002) noting that it is not clear which procedures are best executed by hand, which with technology and which in an integrated way. However, Weigand and Weller (2001) in research using CAS for understanding quadratic and trigonometric functions comment that an integrated working style is quite rare. It appears that many students use GDC technology for low level activities, such as checking by-hand working (Thomas & Hong, 2005). There is some evidence that CAS use may even undermine the learning of lower ability students (Hong, Thomas & Kiernan, 2001), since they come to rely on it, rather than learning either the concepts or the procedures. In contrast, Driver (2001) found that students who used CAS attained a higher level of achievement than would otherwise be expected of them.

ii. Geometry

Different areas of the curriculum have their own needs and the teaching of geometry is a case in point, with research on the use of dynamic geometry systems (DGS) relatively common in leading journals (e.g., Falcade, Laborde & Mariotti, 2007; Hadas, Hershkowitz & Schwarz, 2000; Jones, 2000; Leung, Baccaglini-Frank & Mariotti, 2013; Leung & Lee, 2013; Marrades & Gutierrez, 2000; Mariotti, 2000; Sinclair, 2003; Talmon & Yerushalmy, 2004) as well as mentioned in teacher journals (Caponi 2000; Kittel & Kuntz 2002; Sinclair & Yurita, 2008). The former provide evidence of the importance of DGS in the social construction of thinking about proof in geometry (Mariotti, 2000), the value of DGS in improving students' proof skills (Marrades & Gutierrez, 2000) and in supporting activities that promote contradictions to encourage a move from inductive to deductive reasoning (Hadas, Hershkowitz & Schwarz, 2000), along with means to move student explanations from imprecise expressions, through reasoning overtly mediated by the DGS environment, to mathematical explanations of the geometric situation that transcend the tool being used (Jones, 2000).

Sinclair & Robutti (2013) revisited past studies on dynamic geometry, considering particularly the epistemological nature of dragging, evidence the effect of the technology on the nature of proof, needing to rethink carefully new types of orchestration, reorganising the phases of discussions, experiments and conjectures. Similarly, Hollebrands (2007) investigated the way that DGS mediated students' understanding of geometric transformations by analysing ways in which students interpreted DGS output in terms of figures and drawings. The research identified different purposes for which students used dragging and measures and suggested that these purposes were influenced by student understandings. Talmon and Yerushalmy (2004) considered the dynamic nature of dragging and conclude dynamic behaviour is a complex phenomenon students develop different instruments for it, while Leung and Lee (2013) document student reactions to pre-designed dragging tasks and Leung, Baccaglini-Frank and Mariotti (2013) examine the relationship between invariants and dragging.

For Colmez (2009), using the full potential of dynamic geometry software is a complex task. In a number of cases, for students, there is more to learn in using 2D software, than in using 3D software. Vadcard (2002) shows that the use of an environment of dynamic geometry allows enrichment of students' conception of an angle (here: an angle as a slope), although first they mobilise angle conceptions that are closely linked to direct characteristics of the figure (edge and slides). A constructionist account of students working in a geometrical *microworld* (Psycharis & Kynigos, 2009) with dragging facilities in the area of ratio and proportion focuses on students' developing 'meanings' and discusses the learning potential of dragging for students' mathematical development. Although educators generally encourage student construction of geometric figures, Sinclair (2003) maintains that pre-constructed DGS figures by a teacher can help students notice geometric details, explore relationships and develop reasoning skills related to geometric proof. DGS have also been applied to the learning of function (Falcade, Laborde & Mariotti, 2007), where using a dynamic approach with the Trace tool to ground the meaning of function in the experience of covariation introduced both global and pointwise meanings of trajectory, leading students to grasp the notion of function better.

iii. Statistics

While there has been some research into the use of technology in learning statistics, Biehler *et al.* (2013) note that it is still necessary to integrate statistics into mathematics education more fully. Accomplishing this requires an answer not to the question What kind of mathematics will support integration of ICT? but rather What kind of technology will make statistics teaching in schools viable? In related research, Forster (2006, 2007) examined Grade 12 student learning of descriptive statistics using dynamic Java applets and trends in bivariate data using spreadsheets. She concluded that the computer was superior to the GC for graphical work due to the dynamic transformations the applets provided, along with linked representations and visual clues. Similarly the spreadsheet was superior to the GC for the work on trends, due to poor GC contrast and resolution and an absence of scales. We note that this was prior to the time when GCs had the facility for direct manipulation of graphs. In other research, Graham and Thomas (2005) considered examples to demonstrate how students can construct representational versatility in statistical thinking and some evidence for the value of a dynamic computer approach for building representational versatility in statistical thinking has also been provided (Graham, Pfannkuch & Thomas, 2009; Pfannkuch, Budgett & Thomas, 2014).

c) Student and Teacher Interactions

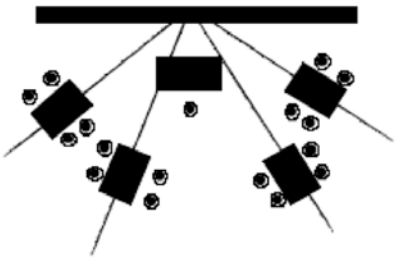
An important issue in technology use is the matter of student interactions with each other, with the teacher and the technology. Further, self-reflection on mathematical content and attitudes has been suggested (Forster & Taylor, 2000) as essential for mathematical progress. Geiger, Faragher, Redmond and Lowe (2008) propose that technology can play a role in the conceptualisation of mathematical models that can provoke a change in student–student and student–teacher interactions and has the potential to mediate collaborative approaches to mathematical enquiry (Geiger, Faragher & Goos, 2010). Doerr *et al.* (2000) and Forster & Taylor (2003) explicitly address teacher and student interactions mediated by technology and others do so implicitly (e.g., Drijvers, Doorman, Boon, Reed & Gravemeijer, 2010; Mariotti, 2000; Rivera, 2007; Trouche, 2000). According to Forster & Taylor (2003) favourable learning outcomes with GDC depend on the teachers' mode of questions and student collaboration & Rivera (2007) charts the role of student cooperation with GDCs in the development of abstract mathematical thinking. In a review of submissions to ICMI activities over the last 20 years, Betty and Geiger (2010, p. 251) suggest that "social perspectives on teaching and learning with technology have become increasingly prevalent […]

Four typologies of digital technologies and their role in collaborative practice are identified: technologies designed for both mathematics and collaboration; technologies designed for mathematics; technologies designed for collaboration; and technologies designed for neither mathematics nor collaboration". Hoyles *et al.* (2010) stress that collaboration and discussion are deeply interrelated with the construction of both individual and group knowledge; they suggest the necessity of situations, such as modelling, which will steer students toward engagement in discussion. In support of this view, Doerr and Zangor (2000) found that while the use of GDCs as a personal device could inhibit communication in a small group setting, its use as a shared device supported mathematical learning in a whole class setting. In support of this, White, Wallace and Lai (2012) report that, for students using TI-Navigator supported network of GDCs, the processes by which they came to establish mathematical meaning in algebra and to develop coordinated action approaches, were

overlapping and intertwined. In the context of the TéléCabri project, which deals with distance tutoring in geometry supported by a computer environment, Soury-Lavergne (2003) observes that scaffolding does not always lead to students' success and evidences a possible place for the teacher's intervention, needing to re-think the didactical contract. A study by Pratt and Back (2009) also focuses on discourse but on a student engaging in (and developing his identity as a mathematician) through an on-line discussion board.

d) Classroom Organisation for Student Learning

One aspect that has been receiving increasing attention relates to the implications for classroom orchestration when technology is present. For example, Kendal *et al.* (2005) comment on different ways of organising the classroom, the variety of approaches to teaching with CAS, the increased range of methods for solving problems and for teaching, the contract between using graphics and symbolic calculators, the place of paper and pencil skills, devoting time to mathematics or to technology and the curriculum and assessment changes required in schools. In their research, Doorman *et al.* (2012) report on a design–based research project on the notion of function in grade 8 using Java applets. It shows how computer-based tasks, paper-and-pen work and whole-class teaching are intertwined and need to be orchestrated. Similarly, Hivon *et al.* (2008) and Hoyles *et al*. (2010) evidence the necessity of carefully managing, or orchestrating, any network of calculators or computers present, because student work appears very sensitive to the configuration of the different available artefacts (see below, for example, two different configurations, with different orientations towards the teacher and different effects on discussions within the classroom).



Hoyles & Noss (2003) agree that learning is highly sensitive to small changes in technologies, often in unpredicted ways. One method of assisting with this organisation could be to employ a student as a sherpa, who is used by the teacher to take on the responsibility of a demonstrator (Guin & Trouche, 2002). Hoyles *et al.* (2010) provide evidence for the potential and challenge of connectivity within or between mathematics classrooms, relating some experiences from this perspective. Among them, three concerned 16 to 18 students: one (Trouche) using TI-navigator (handheld) and analysed through the lenses of instrumental orchestration, the second (Wilenski) using Netlogo (computer based) and the idea agent-based modelling and the third (Noss) evoking the WebLabs project (online), grounding on the idea of the fruitful negotiation of socio-mathematical and socio-technical norms. They illustrate the crucial role of sophisticated networks for supporting new kinds of collaboration between students, between students and teacher and between teachers themselves (within or between classrooms). A summary of possible affordances from

networking and connectedness produced by systems such as the TI Navigator for TI-Nspire are considered by Roschelle, Vahey, Tatar, Kaput and Hegedus (2003) and reasons for learning gains from these cooperative systems are also described (Hegedus & Penuel, 2008; Hegedus & Roschelle, 2013; White, Wallace & Lai, 2012).

e) *Emerging Practice, Challenges and Implications for Curriculum Change*

The influence of technology on the curriculum has taken the form of gradual change; evolution rather than revolution. Analysing the interrelation between technological and curriculum evolution, Trouche (2005a), takes into account the points of view of various actors (teachers, students, inspectors…and society) during the last decade of the previous century and underlines some critical questions, particular about assessment. Roberts *et al.* (2013, p. 525) also situate the present state of technology in the thread of a long evolution, over the 200 last years, but regard the way to be open for new curricula and new ways of teaching and learning, "where knowledge becomes both personal and communal and in which connective and explorative mathematical knowledge becomes vastly more accessible".

For some years now it has been proposed that CAS might provide a catalyst for a fundamental review of our traditional algebra curricula (see e.g., Stacey, Asp &McCrae, 2000). A prevailing view has been that using these calculators can facilitate implementation of a curriculum that places less emphasis on manipulation skills and more emphasis on conceptual understanding and symbol sense (Heid, 1988). However, others, such as Lagrange (2000) and Artigue (2002) consider that, while the technical dimension may be different with CAS it retains its importance in enhancing student understanding. According to Heid, Thomas and Zbiek (2013, p. 625), CAS can enrich and extend the current views of school algebra present in most curricula through: "The capability to construct and alter different symbolic expressions yields modelling possibilities. The ability to build and manipulate complex expressions and the new concepts introduced encourage generalization. Symbolic results to interpret and control provide a venue for algebra as a study of structure." Some studies have been predicated on the view that CAS can enable the design of algebra curricula that exemplify particular perspectives on the teaching and learning of algebra (Kieran & Drijvers, 2006; Kieran & Saldanha, 2008) and were situated within such a curriculum. Some researchers promote the use of a CAS-active school algebra curriculum (Flynn, Berenson & Stacey, 2002) that could privilege (Kendal & Stacey, 1999) different aspects of algebra from the traditional curriculum. According to Burrill *et al.* (2002) there were no studies on the long-term effects of using handheld graphing technology or about the potential of handheld graphing technology to change the curriculum.

One area where curriculum change might be expected is in Internet use. Borba *et al.* (2013, p. 691) underline the impact of the Internet on mathematics education, which provides "on-demand access to mathematics knowledge through the collaborative, multimodal and performative affordances of the media that it supports". However, to be effective, such changes require a deep evolution of pedagogical practices. They also provide evidence of the huge digital divide between nations and between categories of population, regarding the access to the Internet. One area where changes have been noted (Lowrie & Jorgensen, 2012) is in the changing composition of the group of students availing themselves of distance education in Australia, so that they are no longer a totally rural-based population. In their study on teachers using an on-line learning system, Cavanagh and Mitchelmore (2011) classified the teacher roles as technology bystanders, adopters, adaptors and

innovators. Their results show that all teachers made some progress toward using the system in more sophisticated ways, but the improvements were not uniform across the teachers and suggest that a critical role for professional development activities is to assist teachers develop their pedagogical technology knowledge (PTK).

The paper by Roschelle *et al.* (2008) has a focus on the issue of 'scaling-up' research results in order to evaluate robustness, while for Artigue (2010) such scaling-up of results obtained in experimental environments remains a major challenge to be faced. She claims that the term of "ICT *integration*" can be considered misleading, suggesting that there is some permanent entity to which technology has to be integrated. For her, we need to build adequate synergies between top-down and bottom-up processes and imagine dynamics that preserve all along the way an acceptable distance between the new and the old in order to be acceptable, to be viable, not to collapse or deviate (Artigue, 2010, p. 472).

Some questions raised by Heid, Thomas and Zbiek (2013) revolve around the issue of the possible curriculum implications of handheld graphing technology. They ask: What is the role of handheld graphing technology in learning mathematical content that is not part of the traditional mathematics curriculum? What is the role of handheld graphing technology in providing access to mathematics content earlier than would have traditionally been done? In what ways does the nature of the curriculum and tasks students are given influence their use of handheld graphing technology?

3. Technology and Assessment

This section focuses on the assessment of students' mathematical skills and understanding with technology and not on the assessment of students' command of technology.

a) Formative and Summative Assessment

In general, there are fewer papers addressing assessment with technology and there has been less research on formative assessment than summative. One recent exception is a paper by Bokhove and Drijvers (2012b) that reports on design principles that guided the design of an online formative assessment module on algebra. They note that the intentional creation of 'crises[5]' was found to be productive. In another example, Lumb, Monaghan and Mulligan (2000) cite an example of coursework for which CAS was suitable but for which students often chose instead to use spreadsheets, graph plotters and graphics calculators. Lumb, Monaghan and Mulligan's example raises two immediate issues. First, viewing CAS in isolation from other mathematical tools represents a somewhat limited viewpoint. Second, the suitability of CAS (or not) for assessment tasks is often an individual decision (a teacher or curriculum developer may deem a task 'suitable for CAS-use' but the student will use, or not, a CAS as they see fit). In a yearlong study, Harskamp, Suhre and Streun (2000) report that those using GDCs tended to employ graphical approaches more often, attempt to solve more problems and obtain higher test scores. They also suggest that weaker students may profit most from the use of the GDC.

Stacey and Wiliam (2013) propose a survey of the different designs of new item types, authentic assessment and automated scoring of constructed responses. Current capabilities

[5] 'crises' here are viewed as productive failures that lead to a challenge which can be met through mathematical thinking.

in terms of providing feedback to learners are discussed and supported assessment is reviewed. The chapter concludes by discussing how a more principled approach to the design of mathematics assessments can provide a framework for future developments in this field. Specifically, it is suggested that assessment in mathematics should: (a) be guided by the mathematics that is most important for students to learn (mathematics principle); enhance the learning of mathematics (learning principle); and support every student to learn important mathematics and demonstrate this learning (equity principle). The use of technology in assessment should not undermine these principles.

There is a view, recorded by Monaghan (2000), that in CAS-permitted examinations the use of CAS could automate (*trivialise*) many traditional questions and that adapting questions for CAS use may make questions more difficult for low attaining students because straightforward skill questions, which CAS can provide immediate answers to, may be reduced. However, Flynn (2003) notes the need for student algebraic insight even in apparently trivialised questions. Leigh-Lancaster and Stephens (2001) appraise the assessment situation from an examination authority point of view. They note the need to manage change responsibly with due regard to stakeholders and raise policy issues that concern equity, teacher development and the integrity of assessment procedures. They also consider various models that examination authorities can adopt:

- A *no change now model* affords minimal disturbance in the short term but may underestimate the pace of future change.
- A *dual approach* permits CAS and by-hand work through parallel CAS and non-CAS questions. An advantage of this model is that teachers may come on board when they are confident. A disadvantage is that preparing examinations that purport to offer no advantage either way is problematic.
- A *pilot curriculum and assessment approach* permits a cohort of schools/classes to follow a CAS route whilst the majority follow the traditional route. This allows time for specialist curriculum and teacher development but has the disadvantage of attempting to prepare two cohorts of students for further study.
- *CAS-permitted or CAS-required models* provide a clear endorsement for CAS use and encourage both teachers and students to consider the possibilities and constraints of CAS and non-CAS solutions. A disadvantage is possible inequities for students arising from their teachers' expertise, or not, with CAS. *(CAS-permitted* questions are likely to be written to confer no advantage to CAS users whereas *CAS-required* questions are likely to be written to make positive use of CAS.)

A comparison of CAS and non-CAS students' performance in *CAS-permitted* and CAS-*not permitted* examinations by Leigh-Lancaster (2003) showed that CAS students were not disadvantaged on the common questions. However, when CAS is used in examinations the communication required from students may change. For example, Ball and Stacey (2003) propose the rubric RIPA (Reasons–Inputs–Plan–(some)Answers) as a guide for teaching students how to record their solutions, recognising that it is no longer sufficient to instruct students to write down their working, since the CAS has done some of it for them. They found that when students in a CAS examination used words such as solve, they were communicating the plans of their solutions, rather than the detail of calculations. In support of this, Threlfall *et al.* (2007) note important differences in student responses between paper and pencil and e-assessment responses in some questions. Interestingly, Ball and Stacey (2005) interviewed five students on their types of CAS use in examinations and found that the students decided question by question whether or not to use it. Brown (2010)

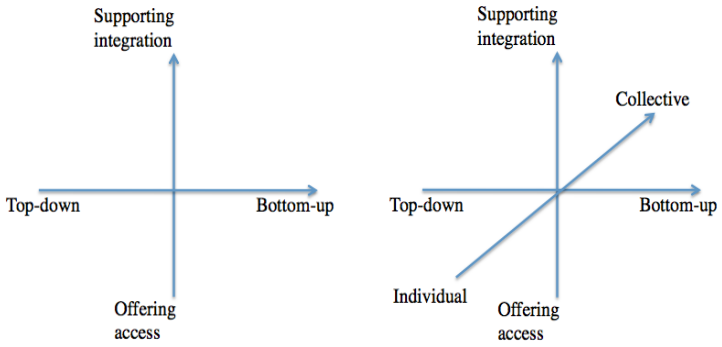
looked at GDC questions in three high-stakes examinations, including IB and concluded that there have not been major changes in the examination questions due to technology. In turn, the implications of Forster's (2002) study of GDC use in examinations were that procedures associated with graphical solutions need to be the subject of teaching, including: setting up the calculator for graphing; enhancing graphical interpretation; obtaining numerical outputs; and ensuring written answers are adequate. Further, in question setting, there should be an awareness of the demands of graphical interpretation and a balance between visual, empirical approaches and analytic methods.

b) Emerging Practice, Challenges and Implications for Curriculum Change

A survey conducted by Drijvers (2009) looked at the different policies in countries in Western Europe with respect to the use of technology in national mathematics examinations (see also the SoW2 report). His conclusion is that many countries are moving towards examinations in which technology is (at least partially) required. The design of task items, however, remains somewhat problematic. The need to assess paper and pencil skills is usually addressed through separate non-technology examinations or the use of specific vocabularies. Sangwin *et al.* (2010) describe computer-aided assessment of mathematics by focusing on the micro-level of automatically assessing students' answers. This is the moment at which a judgment takes place and so it forms the keystone of the mathematical assessment process. The paper reports some of the significant technical developments of the two last decades through examples of internet-based systems, which are important to take into account from the perspective of Massive Open Online Courses (MOOCs) A continuing problem is the affordability of technology for student use in examinations (Pierce & Ball, 2009).

4. Some Implications and Predictions Related to Curriculum Change

Many challenges remain with regard to technology use in the mathematics curriculum and finding solutions to these will involve teachers, students, examination boards, ministries of education and governments. A paper from Wong (2003), investigating the curriculum of six "western" countries (including United Kingdom, Australia, France and the Netherlands) and eight "far eastern" systems (including Singapore), provides evidence for the fact that both the mathematical field and mathematics education are pressured by the rapid development of ICT. The survey shows a general dilemma between pedagogy and administration policies: "the focus of introducing IT seems to be more on the acquisition of IT skills than on the betterment of learning of different subjects" (p. 312). Such a dilemma was evoked 10 years later, in the handbook by Trouche *et al.* (2013), arguing that "Policy measures may give priority to technological access and developments, over the intellectual growth of learners and the professional development of teachers – which should be more demanding goals of mathematics education". The same paper reveals the mathematics education landscape as an evolving one where work on resources (designing as well as offering, using or appropriating) seems to be increasingly collective. The evolution, over the ten previous years, seems to be a "paradigm shift", characterised both by a quantitative evolution on two axes (from offering access to technology to supporting integration and from top-down to bottom-up approaches) and a qualitative evolution, with the arrival of a third axis, evidencing a new balance between individual and collective, both for students and teachers work (see the two schematic diagrams below).



From the point of view of research, there have been attempts at a process of integration, considering the specific frameworks that have appeared to analyse ICT in mathematics education (such as the instrumental approach – Guin, Ruthven & Trouche, 2005), in its relationships with other approaches (Artigue, 2009; Gueudet & Vandebrouck, 2011). In France, the documentational approach of didactics (Gueudet & Trouche, 2009, 2012) has appeared as a result of such convergent theoretical processes. The notion of documentational orchestration (Sanchez, 2010) reveals this convergence between the notion of instrumental orchestration and documentational processes. Forming an understanding of how to orchestrate learning with technology is currently at the heart of the technology challenges in the mathematics education community (Joubert, 2013), along with an exploration of new and different contexts for the teaching and learning of mathematics.

One way that technology is changing is in the integration of several functionalities within one tool. Examples include GeoGebra and Cassyopée (Lagrange & Chiappini, 2007; Lagrange & Gelis, 2008). Moving away from the term CAS, some have suggested other terminologies for these tools may be appropriate, such as *collection of technologies* (COT - Holton, Thomas & Harradine, 2009) or *mathematics analysis software* (MAS - Pierce & Stacey, 2010). Newer technologies include Interactive White Boards (IWB), but some research on these, such as that of Zevenbergen and Lerman (2008), found the classrooms they observed had a restricted approach to their use of IWBs. They were used for quick introductions to lessons and whole class teaching, were teacher directed and fostered shallow learning.

Areas that would be fruitful for examination in research include the reasons underlying teacher and student adoption of technology and how they may use it to foster mathematical thinking. The use of the Internet in mathematics teaching will continue to be worthy of attention; for example, the development of the Internet and the general digitalisation of objects (a real revolution, see Borba *et al.* 2013) leads to an integrated view of ICT, towards the notion of *resources* (Guin & Trouche 2004; Gueudet & Trouche 2012).

An integrated view of modelling with mathematics and technologies is developed by Williams and Goos (2013), who propose a holistic view of the developmental needs of learners, but also teachers and the wider institutional and professional and political contexts, curriculum and assessment, pedagogy and teacher development. In addition the links between ICT integration and new links between mathematics and other sciences, in the frame of IBST (Aldon & Prieur, 2011) will need to be considered.

Governments and members of the public with an interest in education (from parents to employers), will continue to be interested in the results of large-scale research projects and so the question of scaling up local experimental results could lead to a new interest in

“ordinary” classes and practitioners in the field (Lagrange & Dedeoglu, 2009).

Some questions that may assist in thinking about what may be involved in curriculum change are: What does it mean to have a technology-integrated curriculum and do we really want one? What would it look like? How can we describe what is really meant by a technology-integrated curriculum at any level? (Heid, Thomas & Zbiek, 2013). How do epistemological obstacles vary with the kind of technology used? What is the nature of the students’ development and ability to work with technological instruments? What interactions are desirable between technology use and students’ mathematical thinking and understanding? (Thomas, Monaghan & Pierce, 2004). How do we best design and implement PD that will assist teachers to develop instrumental orchestrations?

Theoretical evolutions such as those outlined above indicate the challenges for future curricula: taking into account collective work of teachers, in and out of class work, their complex resource system including both institutional resources and resources gathered from a lot of sources, recombined, shared, etc., collaborative work of students – including cross assessment (and involving the MOOC principle).

Researchers do not advocate ‘technological blindness’ (Galbraith, 2006) in a haste to embrace technology in the classroom, but are optimistic that technology can play a key role, through epistemic and pragmatic mediation, in enhancing student mathematical thinking when teachers are well prepared, positive about its value, confident in its use and experienced in instrumental orchestrations.

Scope of Work 2: Comparative document analysis

Introduction

The main idea of SoW2 is to compare the IB's intended (reflected in the official documentation) and implemented 16-19 mathematics curricula (reflected through personal communication) with the intended and implemented curricula in other countries. The focus of this comparison lies on the role of digital technology and the aim is to inform IB on its future curriculum development in these matters.

Below, we first address the current IB curricula. Next, we summarise the curricula in the following of countries: England (EN), France (FR), the Netherlands (NL), New Zealand (NZ), Singapore (SG) and Victoria (VI). More detailed descriptions of these foreign curricula can be found in the appendices. Finally, we synthesise the findings in a conclusion.

The current IB curricula documentation

The IB 16-19 Diploma Program contains four mathematics courses (http://www.IB.org/diploma/curriculum/group5/). For each course, the main documentation consists of course guides (syllabi) and specimen examination papers. The specimen examination papers are available as pdfs, but are not online. For teacher professional development, teaching support material for graphic display calculators (GDC) is available, including guidelines such as "what should students write down?" (document 13 12 17 GDC TSM), as well as a TI-Nspire GDC workshop for teachers (document 13 12 17 TI-Nspire). The table below contains hyperlinks to the course guides.

Course	Link course guide
MS SL: mathematical studies standard level	http://ibpublishing.IB.org/math-guide/d_5_matsd_gui_1203_1/html/67.207.142.65/exist/rest/app/gui.xql@doc=d_5_matsd_gui_1203_1_e&part=1&chapter=1.html
SL: mathematics standard level	http://ibpublishing.IB.org/math-guide/d_5_matsl_gui_1203_1/html/67.207.142.65/exist/rest/app/gui.xql@doc=d_5_matsl_gui_1203_1_e&part=1&chapter=1.html
HL: mathematics higher level	http://ibpublishing.IB.org/math-guide/d_5_mathl_gui_1206_1/html/content/exist/rest/app/gui.xql@doc=d_5_mathl_gui_1206_1_e&part=2&chapter=1.html
FM HL: further mathematics higher level	http://ibpublishing.IB.org/math-guide/d_5_furma_gui_1206_1/html/content/exist/rest/app/gui.xql@doc=d_5_furma_gui_1206_1_e&part=1&chapter=1.html

The *course guides* stress the importance of using technology and the GDC in particular:

> Students are expected to have access to a graphic display calculator (GDC) at all times during the course. The minimum requirements are reviewed as technology advances and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculators allowed in examinations are provided in the Handbook of procedures for the Diploma Programme.

Digital technology is considered a powerful tool in the teaching and learning of mathematics:

> Technology can be used to enhance visualization and support student understanding of mathematical concepts. It can assist in the collection, recording, organization and analysis of data. Technology can increase the scope of the problem situations that are accessible to students. The use of technology increases the feasibility of students working with interesting problem contexts where students reflect, reason, solve problems and make decisions.

Not so much is said, however, on how teachers are expected to put these ideas into practice. Much seems to be left over to the professional skills of the individual teacher; this is an observation, not necessarily a criticism.

The *specimen papers* can be divided into two categories, the no-calculator examinations and the GDC-allowed examinations. In papers SL1 and HL1, students are not permitted access to any calculator. In the other paper, a graphic display calculator is required. Calculators with symbolic manipulation features are not allowed. It is not clear how the possible downloads of applications that do symbolic calculations (e.g., the apps Symbolic, or, more recent and more powerful, ZoomMath, that also provides stepwise solutions, see http://www.zoommath.com/products/zoom-math-500-calculus/) are dealt with; resetting the GDC is not mentioned as a requirement. Solutions found from a graphic display calculator should be supported by suitable working and candidates must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark. However, the use of calculator notation in the working is not penalised. In some cases, the specimen paper tasks explicitly refer to GDC use, for example "Use your graphic display calculator to solve $f(x) = g(x)$".

Text books are designed by publishers and authors and are not included in the official IB curriculum development. Technological resources are usually not an issue at IB schools.

Current assessment includes paper and pencil tests (paper 1 for SL and HL courses) and papers in which the use of graphing display calculators (GDC, non-CAS) is included. Practical issues concern the use of additional applications and press-to-test features. Also, different phrasings of examination tasks are considered, for example to indicate if exact answers are required or if approximations found by the GDC will do as answers.

We now present a summary of the curricula in six countries. Details for each country can be found in appendix 3.

The curricula in England, France, the Netherlands, New Zealand, Singapore and Victoria

Method

In order to compare the IB 16-19 mathematics curricula and the role of digital technology therein in particular, with the situation in some countries, we set out a questionnaire which was answered for the case of England (EN), France (FR), the Netherlands (NL), New Zealand (NZ), Singapore (SG) and Australia (with a focus on assessment policies in Victoria, VI). The research team conducted this with each member answering the questionnaire for their own country and Mike Thomas also answering the questionnaire for NZ and VI (for which he has close associations). For each country a wide range of relevant public domain documents moderated the team members' assessments.

The questionnaire has four parts. Part 1 (Place of mathematics within the national education system) asks "Please describe the place of 16-19 academic mathematics within the national system". Parts 2 – 4 address, respectively, the curriculum, assessment and implementation strategy. Parts 2 – 4 have spaces for answers with regard to: the official, intended curriculum; the real, implemented curriculum. The blank questionnaire is presented in Appendix 2 and the completed questionnaires are presented in Appendix 3. Please note that the synthesis of national responses below collapses some of the bullet points for Part 2 (Curriculum) into a single row.

Synthesis of national responses

We briefly review the main findings from this inventory with respect to curriculum, assessment and implementation strategy (including teacher professional development). In doing so, in some cases we distinguish between the intended curriculum as expressed in official documentation and the implemented curriculum reflected in real school and classroom practices.

1. Curriculum

Topic	Synthesis of the national responses
(a) ICT as an explicit part of the mathematics curricula	In the six countries under consideration, the curricula do mention ICT as an explicit element in the mathematics curricula. In all cases, the official curriculum documents mention the role of ICT in general terms ("Use … technology to present and communicate mathematical ideas", SG). Such phrasings stress the general orientation of the curriculum, but may their impact may remain limited to 'lip service'. Therefore, more explicit and concrete guidelines related to mathematical topics and/or technological tools may be appropriate in addition to these more general views. For example, "In appropriate situations, the candidate can set up an integral, calculate its exact value and approximates it using ICT" (NL), "These skills also include the abilities to use spreadsheet" (SG), "calculating probabilities, using such tools as … technology" (NZ), or "Use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities" (VI) may be more efficient guidelines that will have an impact on classroom practices than very general statements.
(b) The impact of ICT oppor-tunities on curriculum choices	"Irrational numbers can be approximated as closely as desired by rational numbers and most electronic calculators use a rational approximation when performing calculations involving an irrational number." (VI)
(c) Types of technology used, ICT infrastructure and 'ownership'	Different types of technology are used. For students, the GDC is common in many countries. CAS calculators are used in FR and in VI on a larger scale and in some other countries on a more experimental basis. Computer labs are common and laptop and tablet classes are getting more frequent but seem to be in a rather experimental phase. Video clips, MOOCs and online courses are used more and more for teaching mathematics. For teachers, IWB are widespread in EN, mentioned in NL. Teachers use Internet resources to find and share content, in some countries in a more organised way (Sesamath (FR), MEI (EN), Masterplan3 (SG)) than in other[6].
(d) The anticipation of ICT use in textbooks	In all countries, there are textbook series that anticipate the use of ICT, for example by offering additional software for the IWB or for student practice, or by including guidelines for the use of the GDC for specific procedures. Through specific icons, students are referred to additional ICT opportunities.

[6] See corresponding cells in Appendix 3 for explanations of Sesamath, MEI and Masterplan3.

2. Assessment

Topic	Synthesis of the national responses
(a) National examinations for mathematics	All six countries have national examinations, the result of which plays an important role in access to higher education. There are differences between the countries in session time and in marking schemes and grading procedures. In VI, the examination of the Mathematical Methods CAS course takes place in two sessions, a 1-hour non-calculator session and a 2-hours CAS calculator session. In NL, the national examination grade only determines half of the final grade, the other half being the result of local school examinations. The latter provides opportunities for other assessment formats, including the integration of digital technology.
(b) The use of types of ICT during the national examination	All six countries allow the use of calculators during examinations. In EN, NL, NZ and SG these are GDCs without CAS facilities. This is phrased in different ways, such as “No calculators with built-in symbolic algebra manipulations, symbolic differentiation or integration are allowed” (NZ). In FR and VI, as well as in the EN MEI approach to the curriculum and in some NZ level 3 courses, CAS calculators are allowed. In all countries, criteria are that communication (including internet access) and printing facilities are not allowed. In the EN MEI Further Pure with technology course, students are expected to have computer access during the examination.
(c) GDC in national examinations	Four countries (NL, NZ, SG, VI) provide a list of approved calculators, whereas the other two (EN, FR) don’t. In three countries (EN, NZ, SG), the calculator’s memory needs to be cleared. This is phrased in different ways, such as “Calculators must not … have retrievable information stored in them - this includes: databanks; dictionaries; mathematical formulas; text” (EN) or “No prepared programs may be taken into the examination room. Information (including text or formulae) and/or programs stored in the calculator's memory must be cleared before the examination” (NZ). In the three other countries, there is no need to clear the memory (NL, FR, VI). An argument for this may be that a calculator reset in school practice is hard to check, also because students program reset simulation programs and expert math teachers are not always around during the examination. As a result, students can bring specific applications (including CAS capabilities, e.g., the ZoomMath app for TI devices) or text files (e.g., with examination papers from earlier years). This leads to debate on the calculator’s memory size (FR) and CAS capabilities (FR, NL). Control on reset seems limited. From personal communication, the impression is that a student who brings a CAS calculator to the examination session, or a GDC with additional CAS or text resources, is unlikely to get caught in some countries.
(d) Task phrasing and	In case GDCs are allowed, some phrasing conventions are established to make clear to the student if exact by-

'magic words' to distinguish exact and approximated calculations	hand results are expected or if approximated GDC results are fine. For example, "show" may refer to exact calculations and "find an approximate value" to GDC use (EN), or "compute" versus "estimate" (FR). In NL, a list of verbs has been designed with explanations on their meanings. Here, "calculate" means that GDC facilities (including procedures such as calc intersect or zeros or nderiv) may be used. The NZ regulations state: "When graphing calculators are used to solve a problem, candidates must provide evidence of their differentiation and integration skills", so integration and differentiation should always be done by hand.
(e) Rewarding policy concerning ICT use during examinations	It is difficult to characterise the national ICT rewarding policies, but globally speaking the results show the following: EN: Calculator allowed rather than expected FR: Calculator needed for some algorithmic work, but not rewarded. Type of tasks has changed. NL: Calculator needed for procedures such as finding numerical solutions of equations. Credits assigned to this seem to decrease over the years. NZ: Calculator neutral examinations SG: Calculator supportive during examination, but their use is not rewarded VI: Standard courses; ICT is an add-on and its use is not rewarded. CAS courses: CAS-produced answers are credited.

3. Implementation strategy

Topic	Synthesis of the national responses
(a) The debate concerning the use of ICT in mathematics classes	The intensity of debate is different in the different countries. EN: Not so much debate at present. FR: A national experimentation on new ways of assessing mathematics, that not has been continued and a current debate on the memory capacities of CAS calculators during examinations. NL: Debate on equal opportunities for students concerning additional CAS and text features that GDCs can have because of the non-reset regulation. Also debate on ICT use 'versus' back-to-the-basic skills. NZ: There has been a big debate on the integration of CAS calculators. SG: No, not really. VI: Some debate on how much ICT should be in the new curriculum, but this has now been more or less settled.
(b) Support for teachers' professional development with respect to integrating ICT in mathematics teaching	In all countries there are some supported initiatives for teacher professional development concerning the use of ICT. These initiatives, however, seem to be incidental, local and small-scale, rather than structural and wide-spread. Exceptions seem to be the work done by NCETM and MEI (EN). In the frame of research projects, small scale PD initiatives exist in most countries. Online resources for teacher support are available in all countries, but the initiative to their use is mostly left over to schools and teachers.
(c) ICT use for pre- and in-service teacher courses	To a limited extent, ICT is used as a vehicle for teacher training. There are MEI initiatives using Blackboard Illuminate (EN) and projects such as Pairform@nce - M@gistère since 2013 - (FR) that use blended arrangements of face-to-face and distant exchanges.
(d) Future plans for new ICT-rich curricula	In France, interdisciplinary programs are under construction, in which ICT plays an important role. Also, the development and use of MOOCs is considered. SG has an ambitious Masterplan3 for ICT use (MP3). VI will review the new Australian curriculum.

Conclusion

In this conclusion we will briefly compare the IB's policies with the six national perspectives with respect to curriculum, assessment and implementation strategy.

Curriculum

We see the curriculum as describing what needs to be learned and how this is taught. Concerning what needs to be learned, neither the IB curriculum nor the national curricula under consideration show revolutionary developments. Traditional by-hand techniques, for example for differentiation and integration, seem to be valued such that the fact that they are essentially trivialised and encapsulated in program code available on handheld devices does not remove them from curricular goals. This being said, there seems to be a gradual movement in some countries towards more focus on modelling and applications, at the cost of complex, exact by-hand calculations. Some initiatives go beyond this gradual change. Although Scandinavia was not in our remit, some Scandinavian countries do (partial) examination sessions in which students use computers with a focus on modelling and applications. This will certainly impact on the curricula and on teaching practices. There does not seem to be a world-wide consensus on which way best to go.

For the IB, the choice for the future curriculum is between a careful, gradual approach which comes down to a shift of accents on the one hand and a more drastic scenario with a major role for technology, modelling and applications, on the other.

Assessment

The question here is how the curriculum should be assessed. Assessment practices are different in the different countries considered, but it seems common to include (at least) GDCs in national examinations. Meanwhile, in several countries this is under debate and practical issues concerning the availability for additional applications and texts are not solved in a satisfying way. Two-step examination sessions, with one part without ICT and one part with ICT access, might provide opportunities to both assess basic by-hand skills and to address higher order problem solving and modelling skills using ICT to do the calculational work. Also, school-organised local assessments offer means for other assessment formats, such as group work, extended investigation tasks, et cetera.

For IB, the choice for a future assessment policy might include an extension of the current practice, i.e. one final examination during which a GDC or a CAS calculator is allowed. In this case, the issues with resetting the device would need to be solved. Also, the GDC and CAS calculators in a sense are old technology, compared to the high-resolution screen and the sophisticated applications that smart phones, tablets and laptops offer. As a consequence, one might consider a more innovative approach. One option for such an approach would be the two-step examination format as it is used in Scandinavian countries. The second part can make use of more advanced technology and focus on higher order skills. Another option for an innovative approach is to have a central examination focusing on basic by-hand skils, in which technology is not allowed and in addition to this offer schools room for different assessment formats such as investigation tasks, group work, et cetera (cf. the situation in the Netherlands).

Implementation strategy

Concerning implementation strategies, it seems that teachers' professional development (PD) is not receiving enough attention for realising an appropriate implemented curriculum, both within the IB and in the countries addressed above. The IB PD strategy of having a train-the-trainers model seems to be not specifically training for PD in the field of mathematics and ICT. As the IB seems to have a high-level and committed corpus of mathematics teachers over the world, we would expect that dedicated, targeted and high-quality PD initiatives might be very successful. We recommend investing strong efforts into that, for example by using blended or distant learning formats, as exemplified by some initiatives in France and England.

Addressing the research questions

We address each research question (RQ) in turn. We link our comments to SoW_1 and 2 as appropriate.

RQ 1. What are trends and approaches to technology in secondary maths?

What are differences, commonalities and variables in relation to:

a) Types of technology used
b) Technology integration in curriculum
c) Learning objectives and expected outcomes
d) Pedagogy and classroom practice
e) Assessment

RQ1a (types of technology used) is addressed in SoW_2(2c):

> Different types of technology are used. For students, the GDC is common in many countries. CAS calculators are used in FR and in VI on a larger scale and in some other countries on a more experimental basis. Computer labs are common and laptop and tablet classes are getting more frequent but seem to be in a rather experimental phase. Video clips, MOOCs and online courses are used increasingly for teaching mathematics. For teachers, IWBs are widespread in EN, are common in France and are mentioned in NL. Teachers use Internet resources to find and share content, in some countries in a more organised way (Sesamath (FR), MEI (EN), Masterplan3 (SG)) than in other.

RQ1b (technology integration in curriculum) is addressed in SoW_2(2a). The following is a summary:

> In the six countries under consideration, the curricula do mention ICT as an explicit element in the mathematics curricula and in FR there was a debate in 2007 about integrating ICT into baccalaureat mathematics (see appendices, SoW_2, FR). Other than the debate in FR the official curriculum documents mention the role of ICT in general terms ("Use … technology to present and communicate mathematical ideas", SG). Such phrasings stress the general orientation of the curriculum, but their impact may remain limited to 'lip service'. Therefore, more explicit and concrete guidelines related to mathematical topics and/or technological tools may be appropriate in addition to these more general views. For example, "In appropriate situations, the candidate can set up an integral, calculate its exact value and approximates it using ICT" (NL), "These skills also include the abilities to use spreadsheet" (SG), "calculating probabilities, using such tools as … technology" (NZ), or "Use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities" (VI) may be more efficient guidelines that will have an impact on classroom practices than very general statements.

RQ1c (learning objectives and expected outcomes):

This is quite a difficult RQ to address due to a longstanding division, rarely made explicit, in the ranks of mathematics educators concerned with mathematical learning with technology. This division can be viewed as a polarity between those who see technology as a medium to communicate mathematics to students and those who see technology as a means for students to express mathematical relationships. This division is discussed, with regard to types of software used in mathematics instruction, in an influential book that predates our SoW_1 literature review, Noss and Hoyles (1996, p.54):

> Software which fails to provide the learner with a means of expressing mathematical ideas also fails to open any window on the processes of mathematical learning. A student working with even the very best simulation, is intent on grasping what the simulation is demonstrating rather than attempting to articulate the relationships involved. It is the *articulation* which offers some purchase on what the learner is thinking and it is in the process of articulation that a learner can create mathematics.

With regard to curriculum documentation, RQ1c overlaps with RQ1b (technology integration in curriculum) and we refer the reader to our discussion above on RQ1b. With regard to research, the words of caution expressed in SoW_1(2a) are important to keep in mind:

> Studies addressing student learning often use a particular type of technology with one or more students working on tasks in a manner indicated by a theoretical framework. They then look for some indication (such as engagement) or measure of improved learning outcomes. The most common interventions have involved the use of handheld graphing (GDC) or computer algebra system (CAS) calculators, although computer-based CAS is also used.

Nevertheless, studies which have reviewed research on learning mathematics with technology report partial success. One review of GDC use states that they can aid students' understanding of concepts and improve problem solving skills but gains are a function of how the technology is used in the teaching. A second review which included all ages and all forms of digital technology concluded with, "Educational technology is making a modest difference in learning of mathematics. It is a help, but not a breakthrough."

SoW_1(2a) goes on to consider learning with technology with respect to tasks, geometry and statistics; we consider these matters in our response to RQ4 below. A matter that has received increasing attention is the organisation of classrooms for student learning. In the literature this is often referred to as 'instrumental orchestrations', the ways in which the teacher manages the classroom learning environment when technology is present, see SoW_1(2b). The following is a summary of SoW_1(2d):

> Several studies evidence the necessity of carefully orchestrating student tools and classroom resources because student work appears very sensitive to the configuration of the different available artefacts. This is illustrated in a project on the notion of function in grade 8 using

Java applets which shows how computer-based tasks, paper-and-pen work and whole-class teaching are intertwined and need to be orchestrated. The need to carefully orchestrate people, tools and resources is, almost certainly, increased when we consider connectivity within or between mathematics classrooms.

RQ1d (pedagogy and classroom practice) is addressed in SoW_1(1a & 1b). The following is a summary:

The teacher is key to the successful use of digital technology in the mathematics classroom but incorporating technology into teaching remains a challenge for many teachers and the degree and type of technology used in the classroom is variable. Reasons for this include: teachers' proficiencies in mathematics and their perceptions of the nature of mathematical knowledge and how it should be learned; teachers' understandings of the principles, conventions and techniques required to teach mathematics through the technology; the need for teachers to collaboratively reflect on their actual classroom practices, i.e. their instrumental orchestrations. A grounded analysis of pedagogy and classroom practice revealed five key features of classroom practice: working environment; resource system; activity format; curriculum script; and time economy.

We further consider these matters with regard to teacher professional development in RQ3 below.

RQ1e (assessment) is addressed in SoW_2(all sections) and SoW_1(3a & 3b). The following are summaries:

SoW_2

All six countries have high stakes national examinations. There are differences between the countries in session time, marking schemes and grading procedures. In VI, the examination of the Mathematical Methods CAS course takes place in two sessions, a 1-hour non-calculator session and a 2-hours CAS calculator session. In NL, the national examination grade only determines half of the final grade, the other half being the result of local school examinations. The latter provides opportunities for other assessment formats, including the integration of digital technology.

All six countries allow the use of calculators during (some/most) examinations; in EN, NL, NZ and SG these are GDCs without CAS facilities In FR and VI, as well as in some EN MEI and some NZ level 3 courses, CAS calculators are allowed. In all countries, criteria are that communication (including internet access) and printing facilities are not allowed. In the EN MEI Further Pure with technology course, students are expected to have computer access during the examination. Four countries (NL, NZ, SG, VI) provide a list of approved calculators, whereas the other two (EN, FR) do not. In three countries (EN, NZ, SG), the calculator's memory needs to be cleared. This is phrased in different ways.

In the three other countries, there is no need to clear the memory (NL, FR, VI). An argument for this may be that a calculator reset in school practice is hard to check, also because students program reset simulation programs and expert math teachers are not always around during the examination. As a result, students can bring specific applications

(including CAS capabilities, e.g., the ZoomMath app for TI devices) or text files (e.g., with examination papers from earlier years). This leads to debate on the calculator's memory size (FR) and CAS capabilities (NL). Where GDCs are allowed, some phrasing conventions are established to make clear to the student if exact by-hand results of GDC approximations are expected.

SoW_1

Overall there are fewer research papers addressing assessment than other issues and there is less research on formative assessment than summative. With regard to e-assessment one study notes the danger of assessing what current technology can assess and argues that assessment should be guided by principles: be true to the mathematics; enhance the learning of mathematics; and support every student's learning. Three other studies note: there are important differences in student responses between paper and pencil and e-assessment responses in some questions; provoking 'crises' in student work can be productive; focusing on the micro-level, the moment at which a judgment takes place, is important.

With regard to summative assessment, one paper notes two important considerations with regard to students using a single tool: using a single tool in isolation from other mathematical tools is limited; the suitability of a single tool for assessment tasks is not something that can be determined solely by teachers or curriculum developers. Another study notes that using advanced mathematical technology in examinations could automate (*trivialise*) many traditional questions, thus making these questions more difficult for low attaining students. Several papers note the difficulty of designing 'good' technology-tasks for examinations and one study looked at GDC questions in three high-stakes examinations, including IB and concluded that there have not been major changes in the examination questions due to technology. Another study (of GDC use in examinations) noted that procedures associated with graphical solutions need to be the subject of teaching, including: setting up the calculator for graphing; enhancing graphical interpretation; obtaining numerical outputs; and ensuring written answers are adequate.

Long standing work in Australia (Victoria) on the use of CAS-calculators in examinations is reported in the two papers in SoW_1. They note the need to manage change responsibly with due regard to stakeholders and raise policy issues that concern equity, teacher development and the integrity of assessment procedures. They also consider various models that examination authorities can adopt: a no change now model; a dual approach; a pilot curriculum and assessment approach; a CAS-permitted or CAS-required model (see the inset text in SoW_1, section 3(a) for details). Affordances and problems with each of these models are noted. They note the need to teach students how to record their technology-based solutions and, when this is done, students using technology were not disadvantaged on common questions

RQ 2. What guidance is given on the use and integration of technology in teaching and learning mathematics in various (international) education curricula? What are emergent themes and patterns in relation to frequency, tools and applications, pedagogical strategies and so forth?

In FR there is an emphasis on providing a range of resources, including Ministry websites, for teachers (see appendices, SoW_2, FR). Other than this the message that comes through from SoW_2 on the guidance that is given on the use and integration of technology in

teaching and learning mathematics is minimal: guidance is restricted in curricula documentation to generalities along the lines of 'technology can be useful in teaching and learning' and to specifications of 'what is allowed and not allowed in examinations'.

In the research literature (SoW_1) 'integration' is often mentioned but almost always with regard to the difficulty of integrating technology into 'ordinary' lessons. Further to this the term 'integration' is questioned:

> "the integration of technologies has to be understood as the search for new equilibrium within teachers resource systems" (SoW_1(1e))
>
> "[Artigue] claims that the term of "ICT *integration*" can be considered misleading, suggesting that there is some permanent entity to which technology has to be integrated. For her, we need to build adequate synergies between top-down and bottom-up processes and imagine dynamics that preserve all along the way an acceptable distance between the new and the old in order to be acceptable" (SoW_1(2e)).

However, recent literature (see (SoW_1(4)) suggests that incorporating several functionalities within one tool (as GeoGebra does) and teachers working collaboratively on general resource-bases to support learning and teaching may be productive towards this *holy grail* of integration.

RQ 3. What are the issues, enablers and challenges of using technology in the teaching and learning of mathematics in both IB and non-IB school contexts?

Our response arises from our interpretation of research outlined in SoW_1(1d, 1e, 2e, 3a) and extends our comments under RQ2 above.

Whilst the IB was innovative in the 1990s in embracing GDCs it has, perhaps, put too much emphasis on this one tool in isolation. A challenge for the IB now is for teachers, researchers and curriculum-assessment personnel to consider the full range of technological tools now available for 16-19 academic stream mathematics learning and teaching: traditional mathematical tools and resources such as rulers, compasses, textbooks, schemes of work, etc.; techno-mathematical tools such as GDCs, CAS-calculators, dynamic geometry systems, etc.; digital resources such as IWBs, video files, multimedia resources, internet-based communities and tutorial conferencing. This consideration should lead to collaborative work to co-design, experiment in real contexts and revise pedagogical resources. In this context, the goal of such an innovative programme is not so much to provide teachers with new knowledge, but to give them tools to support collaboration and design. This consideration of the full range of tools and resources can be viewed as a search for new equilibrium within teachers' resource systems. The enablers for this challenge are classroom mathematics teachers in IB schools supported by IB personnel. The pedagogical map provided displayed in SoW_1(1e) could form a starting point for IB supported teacher collaborations.

A second challenge is to allow CAS and DGS to be a part of the above range of technological tools available, for examination as well as classroom use. This challenge would have to be approached with serious consideration of the place of techniques and concepts in the

mathematics curriculum. As SoW_1(2e) shows, there are conflicting points of view in the literature: a curriculum that embraces CAS can place less emphasis on techniques and more on concepts vs the view that, while the technical dimension may be different with CAS it retains its importance in enhancing student understanding. An enabler for this challenge would be for the IB to adopt the 'dual approach' described in SoW_1(3a).

A third challenge is teacher professional development (PD). This is addressed in SoW_1(1d). The following is a summary:

> Successful PD requires the consideration of several interrelated issues: access to technology; the development of technological knowledge; and assisting teachers to broaden their conceptions and engagement with new modes of organising classrooms for student learning. A model of PD that has had some success is to structure it around a supportive community of inquiry on everyday classroom practice where all participants are co-learners and knowledge is developed and evaluated by the group.
>
> The importance of teachers' collaborative work to support the necessary evolution of resources, practices, knowledge and tasks is acknowledged by an international study and by an online teacher association in France which aimed to design and share digital resources. Research also shows, however, that support for such teacher collaborative work is essential.

There are no 'quick fix' enablers for setting up supportive networks of teachers. A problem for the IB is the geographical distance between its schools but the work in France was conducted between geographically distant school and in England the MEI blends distance and live PD. The English and French authors of the team writing this Report would be happy to put IB personnel in touch with people who could advise them on implementing distance PD strategies.

RQ 4. What is the impact of using particular technologies on the development of mathematical skills and academic achievement in mathematics in both IB and non-IB school contexts?

We note:

- the discussion, above, of RQ1c is relevant background information to considerations of RQ4;
- very little (nothing?) is offered towards addressing RQ4 through an examination of curricula documentation (SoW_2);
- the IB places particular emphasis on GDCs.

The following is a summary of the literature with regard to GDCs from SoW_1(2a).

> GDCs can be an important factor in helping students develop a better understanding of mathematical concepts and raise the level of their problem solving skills. This, however, depends on how the technology is used in the teaching of mathematics and initial high expectations may be somewhat naïve. Classroom interactions (students, teachers, tasks and technologies) are important and changes in learning occur over time. A study of errors students make when using GDCs attribute them to four main causes: a tendency to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; a limited grasp of the processes used by the calculator to display graphs.

SoW_1(2b) offers insight into RQ4 through its subsections on tasks, geometry and statistics, which we now consider.

Tasks

The impact of using particular technologies, e.g. a GDC, on the development of mathematical skills and academic achievement in mathematics cannot be gleaned from a consideration of tools in isolation. Tools needs to be considered in relation to: the user (the student or the teacher); the environment the agent-tool are in (the institute and the classroom with other people and resources); and the task (or activity) at hand. The task is a crucial and often neglected feature of student learning with technology. Consider, for example, two tasks concerned with quadratic functions: Task 1, sketch the graph of $y = x^2 - 3x + 1,\ 0 \le x \le 3$; Task 2, where $y = x^2 - 3x + 1,\ 0 \le x \le 3$ is presented graphically and the student is asked to reflect the graph in the *x*-axis. Task 1 is more challenging using a pencil and graph paper than it is using a GDC but Task 2 is more challenging using a GDC than it is using a pencil and graph paper. Further to this the *epistemic* (concerning understanding) and *pragmatic* (concerning breadth of possible applications) values in the techniques required to perform these two tasks with pencil and paper/GDC differ markedly. These two tasks also illustrate that the role of tools in students making connections between different representations (Task 1 presents an algebraic representation and asks for a graphic representation but to achieve this with pencil and paper students must go via a numeric representation such as a table of values; Task 2 presents a graphic presentation and asks for another graphic representation but to achieve this with a GDC the initial graphic representation must be transformed into an algebraic representation).

Ways to co-ordinate by-hand and by-technology solutions to tasks are a matter of current debate.

Geometry

Different areas of the curriculum have their own needs and the teaching/learning of geometry is a case in point. There is a great deal of research into student learning with dynamic geometry systems (DGS), which are available on some advanced calculators. A

number of research papers focus on the epistemic value of 'dragging' in a DGS and its contribution to students' understanding of geometric proofs. Dragging is considered important because invariant geometric relationships (e.g. perpendicular lines) remain unchanged when a geometric figure is 'dragged'. An ongoing debate is the importance of the move from inductive to deductive reasoning in such dragging: there are those who claim this move is often realised and there are those that say dragging is a useful activity to 'see' geometric invariants prior to working out a proof without the DGS (but, interestingly, there is no one who says that dragging, in relevant tasks, is unimportant at the level of students' understanding of geometric relationships).

Student-DGS actions are usually studied via students' own constructions of 2D geometric figures but DGSs have the potential for 3D geometric work and studies have shown: that pre-constructed DGS figures can help students notice geometric details, explore relationships and develop reasoning skills related to geometric proof; that DGSs can advance students' understanding of ratio and proportion and of function.

Statistics

The study of advanced statistics and the practices of working statisticians have undergone tremendous changes in the last 50 years due to the use of computers. There is also an international debate on the relationship between mathematics and statistics. A debate in statistics education is *What kind of technology will make statistics teaching in schools viable?* One study examined Grade 12 student learning of descriptive statistics using dynamic Java applets and trends in bivariate data using spreadsheets. This study concluded that the computer was superior to the GDC for graphical work due to the dynamic transformations the applets provided, along with linked representations and visual clues. Other studies have provided evidence for the value of a dynamic computer approach for building 'representational versatility' in statistical thinking.

RQ 5. To what extent do objectives, approaches to technology integration, pedagogical strategies and learning practices in DP mathematics courses reflect the contemporary trends, initiatives and strategies in the use of technology in secondary mathematics education worldwide?

There is no straightforward answer to this compound question. SoW_1 shows that there is considerable research into technology integration, pedagogical strategies and learning practices in academic stream 16-19 mathematics but research is constrained by the opportunities for research and these are invariably local initiatives in a specific country. This constraint places limits on the extent to which SoW_1 can be used to evaluate DP practices. With regard to SoW_2, the countries considered have, like the IB, respected curricula and assessment regimes. The report for SoW_2 shows variation in approaches to technology integration but, considering the report as a whole, it is clear that the IB is certainly not 'lagging behind' the initiatives in these countries. Further to noting constraints and making this general comment we do offer some comparisons with regard to approaches to

technology integration, pedagogical strategies and learning practices between the six countries considered in SoW_2 and DP mathematics courses.

With regard to the integration of technology in DP mathematics courses, the IB has a very clear role for the GDC in its curriculum and assessment and this role goes beyond the 'add on' approach of some countries but the DP mathematics courses do, perhaps, place too much emphasis on this single tool. Further to this, GDC use in DP mathematics courses appears to be limited to graphic and numeric use. This leads us to two recommendations: (i) that the IB considers extending the use of hand-held technology to include dynamic geometry and symbolic manipulation software; (ii) that the IB considers widening the network of resources that the handheld technology resides in (perhaps along the line taken in France, see appendices, SoW_2, FR).

With regard to pedagogical strategies, the IB documentation that we have access to does not say a great deal about how teachers are expected to teach with technology though it does provide us with teacher support material and details of workshops (both for GDCs). The documentation from other countries (SoW_2) also says very little about how teachers are expected to teach with technology but our comments on the third challenge in response to RQ 3 above may be relevant to future IB pedagogical strategies. We point in particular to teachers' collaborative work (in addition to support material and workshops) to support the development of resources, online teacher associations in France and blended learning in England (MEI).

With regard to learning practices, we are obviously constrained by a lack of classroom observation but all the documentation (from the IB and the six countries) points to 'conservative' changes in practice; this is a comment, not an evaluation, and there are dangers in short term radical attempts to change practice[7]. Nevertheless SoW_2 suggests that NL, NZ and VI are using the presence of technology to enhance modelling and application learning practices and the IB appears to do this too. Further to this, SoW_1 draws attention to the importance to reconceptualise the nature of mathematical tasks in the presence of technology with particular regard to *epistemic* and *pragmatic* dimensions of tasks (see RQ 4 above); the fundamental ideas behind the words 'epistemic/pragmatic' is not evident in any curricula documents we have examined.

[7] Two dangers are: (i) variation in teacher expertise in using technology to enhance the learning of mathematics leading to 'unfair' variation in student learning; (ii) short term problems in examination grade distributions as examination question writers learn how to set examination questions using technology which allow the vast majority of students to succeed and also differentiate fairly with regard to students' expected attainment levels.

Appendix1: References[8] for Scope of Work 1 (Literature review)

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[8] Note: Papers marked with * are short, practice-based accessible articles, for example those found in journals for teachers, those marked with ** are relatively short, close-to-practice research articles, such as conference papers, and those marked with *** are long research papers, found in scientific journals.

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Appendix 2: Scope of Work 2 (comparative document analysis), questionnaire

Respondent name: _____ **Country described in the response: _____**

Part 1: Overall description

Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?		
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones, …)?		
Is ICT used in mathematics classes on a regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?		
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?		
Do textbooks anticipate the availability of ICT?		
Are internet resources used in mathematics courses?		

Are there any plans to extend the use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?		
Please add other comments and information that you consider relevant but that is not addressed in the questions		

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)		
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?		
If GDC are allowed, do they need to be reset before the start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?		
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?		
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers		

that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?		
Please add other comments and information that you consider relevant but that is not addressed in the questions		

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?		
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?		
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?		
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?		
Please add other comments and information that you consider relevant but that is not addressed in the questions		

References
Examples

Appendix 3: Scope of Work 2, national descriptions

Questionnaire 1

Respondent name: John Monaghan **Country described in the response: England**

Part 1: Overall description of the place of 16-19 academic mathematics within the national system

Answer
Please describe the place of 16-19 academic mathematics within the national system We describe academic mathematics in England; the descriptions for Northern Ireland and for Wales (but not for Scotland) would be similar. This section reports on the situation circa February 2014 and ends with notes on planned changes. Compulsory schooling, which includes mathematics, ends at 16 years of age. Most students take a GCSE Mathematics examination (grades A*-G) where the grade that filters higher mathematics study is C in the intended curriculum but B, in most cases, in the implemented curriculum. After GCSE most academic stream students study for two further years taking, typically, four subjects in their first year and continuing with three of these subjects in their second year. Although a range of curriculum/assessment options are available (including IBO Diplomas) the vast majority of academic stream students take GCE courses/examinations (graded A*-E; grade A was divided into A and A* in 2010): Advanced Supplementary (AS), one year; Advanced level (A-level), two year. GCEs are available for Mathematics and for Further Mathematics (FM). Three private organisations (AQA, Edexcel and OCR) called Examination Boards (EB) publish GCE curricula and associated examinations. These EBs are monitored by a government 'watch dog' Ofqual (http://ofqual.gov.uk/). Students studying A-level (respectively AS) Mathematics take 6 (resp.3) modules of which 4 (resp. 2) are 'pure mathematics core' and 2 (resp. 1) are chosen from 'applied' module options: decision (discrete mathematics), mechanics and statistics. FM allows advanced content in 'pure' and 'applied' module options. A-level Mathematics curricula are similar in content to that of the IBO Mathematics Higher Level Diploma Programme. England, along with Northern Ireland and Wales, is an outlier in OECD countries with regard to participation rates in 16-19 mathematics and most students of this age study no mathematics at all. The following 2009 age cohort percentages summarise data from Hodgen et al. (2010): ≈80% are in education and training; 43% take A-levels; 11% take A-level Mathematics; 1% take FM. Mathews & Pepper (2007) provides a wealth of 1999-2006 statistics on GCE Mathematics and five comparator subjects which shows a roughly even A-E grade percentage (the grade A* did not exist at the time) pass over the six subjects but a marked difference in A grades, with Mathematics students achieving two to three times the percentage of A Grades of comparator subjects; an interpretation of this difference is that there is an 'facility threshold' beyond which A-level Mathematics is 'easy'. A significant 'player' in GCE Mathematics is the charitable foundation *Mathematics in Education and Industry* (MEI) which publishes textbooks, runs CPD and has its own AS/A-level (through OCR) for Mathematics. MEI also operates the national *Further Mathematics Support Programme* project which aims to give every student who could benefit from the study of FM the opportunity to do so through face-to-face and/or online tuition. There is a sense in which GCE Mathematics, other than MEI, is conservative with regard to embracing changes in Mathematic curricula/assessment arising from digital technology. In October 2013 the Department for Education (DfE) and Ofqual announced plans for new A-level regulatory requirements. Mathematics is in phase 2 (first teaching 2016) of these plans and there is no mention of the use of technology. In December 2013 the DfE published a policy statement on 16-18 core maths qualifications in response to the 'outlier' label of Hogden et al. (2010); the only mention of technology is "the content of Core Maths qualifications can be taught and learnt through the use of appropriate technology" (DfE, 2013, p.9).

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	Yes, an aim of all AS/A-level specification includes "acquire the skills to use technology such as calculators and computers effectively, to recognise when such use may be inappropriate and to be aware of limitations" and the assessment objectives includes "use contemporary technology and other permitted resources … understand when not to use such technology, and its limitations"	There is great variation. All students have, at least, a scientific calculator and many have a GDC. Schools, and teachers within schools, vary in the extent in which they embrace the use of technology. The writer has anecdotal evidence to suggest that where technology is used extensively it is often 'local', that is a particular teacher shows students how to use a GDC to solve a specific type of question.
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones, …)?	Nothing of significance in the 'pure mathematics core' modules since logarithmic and trigonometric tables stopped being used in the 1980s though some topics, such as fixed point iteration, have greater prominence than they did 20+ years ago. The situation is similar in the option/applied modules. For example, modules in statistics do not use a computer for, say, general linear modelling, but do expect students to use the statistical function in their GDCs.	Although there is variation over students and teacher use of technology I have no evidence to suggest that even high use of technology impacts on curriculum choices; it is more the case of a few individuals choosing to investigate isolated topics within the intended curriculum.
Is ICT used in mathematics classes on a regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?	Beyond the aim, stated above, for students to "acquire the skills to use technology …" and the expectation to use scientific calculators in the course of studying certain topics (e.g. the sine rule), there is no exhortation in the intended curriculum to use technology. There is reluctance in England to tell teachers how to teach their subject.	IWBs are very common in English mathematics classrooms and their use is regular. It is often, but not always, the teacher who uses the IWB. There is variation over teachers in the use of PowerPoint demonstrations of topics and the use of mathematical software on IWBs. All schools have computer labs but the use of these by the Mathematics Department, again, varies over schools and teachers within schools. The number of departments with a class set of laptops (and, recently, tablet PCs) continues to increase. Very few mathematics classes have inbuilt computer clusters.
Is there any funding, e.g. by governmental institutions, for ICT integration? Or other	In the past, yes, but in recent years, no. The British Education and Technology Agency (funded	A few schools make extra money available for Mathematics Departments to purchase computers. Virtually all state

kinds of resources?	by the DfE) was the lead UK agency for the promotion and integration of ICT into education but it went in to liquidation in 2011.	schools provide other curriculum resources (e.g. textbooks) at no cost to students.
Do textbooks anticipate the availability of ICT?	There are a great many textbooks available for AS/A-level Mathematics. The DfE does not comment on the suitability of textbooks. An increasing (since≈2000) trend is the publications of textbooks for AQA, Edexcel and OCR (MEI and non-MEI) for specific modules or pairs of modules. The only series of textbooks known to the writer that embraces technology is the MEI series. For example, Hanrahan et al. (2010) has 13 instances in the first 124 pages where readers are encouraged to use a range of mathematical soft/hardware: investigate mathematics; explore specific buttons (e.g. x!) and sketch/fit graphs.	The writer does not have experience of textbooks, other than the MEI series, being used by teachers as a source for using technology in AS/A-level mathematics. Internet resources appear to be the main source of ideas.
Are internet resources used in mathematics courses?	Neither government agencies or Examination Boards (other than OCR-MEI, see adjacent cell) provide internet resources for AS/A-level mathematics.	There are a great many sites (many designed/updated by enthusiasts) which provide resources suitable for AS/A-level mathematics. I do not attempt to list them except: MyMaths (http://www.mymaths.co.uk/) appears to be popular; MEI (http://www.mei.org.uk/) does provide extensive resources, including ICT resources, for teachers and students (see Button et al., 2008, for details circa 2008).
Are there any plans to extend the use of digital technology in mathematics classes in the near future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?	There are no plans to extend the use of digital technology in AS/A-level mathematics classes in the near future.	There does appear (in the writer's experience) a recent increase in the number of teachers (and, to a lesser extent, Mathematics Departments) incorporating technology (notably the software *GeoGebra* and *Autograph*) into AS/A-level mathematics.
Please add other comments and information that you consider relevant but that is not addressed in the questions	At the government level education, including 16-19 academic stream mathematics education, is in a period of de-centralisation and a return to values of the last century. There are plans for innovation in 16-19 mathematics education (as described in Part 1) but technology is merely paid lip service in these plans.	

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	As stated in Part 1, students studying A-level (respectively AS) Mathematics take 6 (resp.3) modules of which 4 (resp. 2) are 'pure mathematics core' and 2 (resp. 1) are chosen from 'applied' module options. FM also has 'pure' and 'applied' modules. All of the modules have timed examinations set by the Examination Boards (AQA, Edexcel and OCR) with pass grades A* - E (grade A was divided into A and A* in 2010) at a specific date/time. These are high stakes as university offer places to students on the basis of their grades in these examinations (though some other examinations, notably those of the IBO, can be offered). In 1990s AS/A-level mathematics became modular with examinations with options to take individual module examinations at several points in the year and for students to re-sit examinations. This practice is in the process of being curbed with the intention that students will take end of two year examinations from 2015.	English education and mathematics in particular is often accused of "teaching for the test" (see Ofsted, 2012, paragraphs 210-212). In the writer's experience there is truth in this statement but it varies in a mix of teaching practices which include what the IBO would see as 'good practice'.
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	In all except Core 1 Pure Mathematics examinations students may use a scientific calculator or a GDC but not a CAS calculator. No computer of any kind is allowed in an AS/A-level mathematics examination. There is no list of allowed/disallowed brands of calculators. MEI has a unit Further Pure with Technology (approved 2013) where "Students are expected to have access to software for the teaching, learning and assessment that features a graph-plotter, spreadsheet, CAS and programming language … assessed by a timed written paper that assumes that students have access to the technology. For the examination each student will need access to a computer with the software installed and no communication ability. See http://www.mei.org.uk/?section=teachers&page=fpt	Core 1 examinations appear to be a vehicle for the examination of paper and pencil techniques, e.g. simple co-ordinate geometry, simplification of surd forms and simple calculus techniques.
If GDC are allowed, do they need to be reset before the start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in	The Joint Council for Qualifications (2013) states: "Calculators must not … have retrievable information stored in them - this includes: databanks; dictionaries; mathematical formulas; text."	The situation is not known.

schools?		
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?	Questions often state that answers should be left in surd form or in terms of π. Further to this an example of 'magic words' is Q2a from the AQA Core 2 (Jan 2012) paper, "… to find an approximate value to <definite integral> giving your answer to three significant figures." In Q9bii from the same paper we see the word "show" (not "find") which indicates that GDC is not appropriate: "Show that the equation of the tangent at $A(8,0)$ is $y+8x=64$".	The writer has been in Examination Board meetings dedicated to finalising the wording of examination questions and mark schemes where specific questions are revised so as to not disadvantage students without GDCs.
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?	Although AS/A-level specifications mention 'inappropriate' use of technology, there is no government or Examination Board documentation regarding what appropriate use of technology is.	In the language of the IBO, the impression of the writer is that examination questions are set in the style of "calculator allowed" and not "calculator expected". MEI examinations may be an exception to this but the writer has no experience of MEI examiner meetings. At the school level teachers certainly do prepare their students for expected solution methods including calculator methods. However, due to wide variation in teachers' familiarity with GDCs, it is expected that there is wide variation in teachers' scaffolding of students' solution-with-GDC methods.
Please add other comments and informations that you consider relevant but that is not addressed in the questions	The OCR has a Maths Council whose members are Professors of Mathematics, teachers and independent bodies associated with mathematics. This is an ongoing review of matters relating to future 16-19 mathematics curriculum and assessment matters and its brief includes advising OCR on "how best to harness new technologies in the delivery of mathematics".	

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	Yes and no. There was much debate on issues concerned with mathematics education for students aged 14-19 following the publication of Smith (2004) and this resulted in the DfE commissioning several studies related to the use of ICT in mathematics which are reported in Monaghan (2006) but this debate ceased around the onset of 'austerity'. The Joint Mathematical Council (JMC) of the UK published a report (Clark-Wilson et al., 2011) but the writer is not aware that this has generated debate outside of the JMC.	Cornerstone Mathematics (www.cornerstonemaths.co.uk) concerns 11-14 rather than 16-19 mathematics but it is worthy of comment as it aims to integrate digital technology into mathematics lessons to present mathematical ideas using dynamic representations and simulations. A pilot study, involving 19 teachers and 490 pupils, is being 'scaled up' to 100 schools across England (see Hoyles et al., 2013).
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?	There are many professional development (PD) providers in England. In 2005 the National Centre for Excellence in the Teaching of Mathematics (NCETM) was established and one of its main briefs was to co-ordinate and validate the diverse provision of PD. Some of this PD relates to the use of technology in mathematics but the writer is not aware of any relating to technology in 16-19 mathematics other than offers for bespoke PD and the MEI. MEI's provision includes a year long part-time course, Teaching Advanced Mathematics, for 11-16 teachers who are starting to teach 16-19 academic stream mathematics; technology is integrated into this course.	Two independent organisations have a strong interest in technology in 16-19 academic mathematics education are: *Technology for Secondary/College Mathematics (see* http://www.tsm-resources.com/). This is centred around the software *Autograph* and provides training, resources and an annual residential workshop. *Wolfram Research*, the founders of *Mathematica*. Conrad Wolfram is vocal in his support of the use of technology in mathematics education and organises events for teachers (see http://www.wolfram.com/events/cambridge-feb-2014/)
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	Lee (2014) reports on MEI's blended learning for students and teachers involved in the Further Mathematics Support Programme. This involves face-to-face tuition, live online tuition and a mixture of these forms. The live online tuition	The writer is not aware of developments in this area.

	uses Blackboard Collaborate's Elluminate software.	
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	Apart from further development of MEI units, there are no future plans that the writer is aware of to implement new curricula with a different role for ICT than is the case at present.	It is certainly the case that individual teachers and Mathematics Departments have future plans for the use of ICT to enhance learning and teaching 16-19 academic stream mathematics.
Please add other comments and informations that you consider relevant but that is not addressed in the questions	None.	

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Questionnaire 2

Respondent name: Luc Trouche **Country described in the response: France**

My response will be based on the official curriculum (MEN 2009, 2010 & 2011), inspectors prescription for use (IGEN 2004), official resources provided to teachers for ICT integration (MEN 2012, 2013), resources provided by a teacher association, as an advanced point for new ICT practices in math education (Sésamath 2014) and on two official reports: the first one, by the inspectors (IGEN 2010), concerns the implementation of the grade 10 program, the second one is a survey on mathematics teaching, also in grade 10 (MEN 2014). About this last survey, to be noticed: the sample of highschool has been made by the Ministry of education to be representative, but only one or two math teachers has answered. Then it cannot be considered as fully representative, just a point of view giving some recent indications of the real situation. I have added to personal (with G. Gueudet) papers (Gueudet & Trouche, 2011a & b).

Part 1: Overall description

Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	The 16-19 academic mathematics take place, in France, in highschools (grades 10 to 12), with a final assessment (baccalaureat), opening the doors of the universities. Some numbers: Number of grade 9 students (last year of French college – middle school): **776,481** Number of grade 10 students (first year of French highschools): **517,983** - Two categories of highschool: vocational, and "general and technological": **406, 429** students in the grade 10 in general and technological highschool (the same program for all students at this level) In grade 11, students have to chose a major topic: **136,000** choose a scientific section, 42,801 a literary section, 80,000 a economics section, 100,000 a technological section In grade 12, students from scientific sections have to choose an option: **37,000** chose mathematics, 52,000 chose physics-chimistry, 61,000 a biology-earth science. I have insisted (in bold) on mathematics in classes privileging this matter, but, of course, there is some mathematics in every section. All the numbers, for the year 2012, at http://cache.media.education.gouv.fr/file/2013/49/1/DEPP-RERS-2013-eleves-second-degre_266491.pdf Even if mathematics remain, certainly, in France, a prestigious discipline, attracting many good students, it appears both as "a way opening the best opportunities for future professions", and "a very hard discipline, mostly dedicated to good students". The number of students privileging mathematics has decreased. There is nowadays a national debate on "mathematics attractiveness", see: http://www.cfem.asso.fr/debats/attractivite-mathematiques, as well as on the respective places of boys and girls regarding to mathematics learning. http://www.maths-a-venir.org/2009/en-france-les-mathématiques-attendent-plus-de-femmes

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	YES The use of ICT in education is a priority for the French Ministry of Education in the frame of his digital strategy http://www.education.gouv.fr/pid29064/ecole-numerique.html In mathematics, ICT is explicitly part of the curriculum at each level: - grade 10: one of the main objectives is "using digital tools (calculators as well as computers) supporting problem solving" (MEN 2009) - grade 11: « using software, tools for visualisation or simulation, of computing (both CAS and scientific) and of programming, deeply changes the nature of teaching in favouring inquiry based learning » (MEN 2010) ; - grade 12 : same sentence (MEN 2011). Besides, the inspectors are very supportive for ICT integration in teaching: « a reasonable use of different kinds of software is particularly fitted to mathematics teaching: it is the case for calculators, spreadsheets, CAS and DGS » (IGEN 2004). More information in (Gueudet & Trouche 2011a) To be noticed : there is, since September 2013, a new teaching (optional) for grade 12 students, dedicated to « Informatique et sciences du numérique » [computer sciences and digital sciences], mainly taught by mathematics teachers.	« At a large scale, teachers considerer that the new curriculum supports ICT integration. More and more teachers see ICT as real pedagogical tools. However ICT usages changes according to the highschool. DGS are more and more used in classes, but there is not really analysis of their effects. Teachers are waiting for an assessment of such tools during the final official examination (baccalaureat) » (IGEN 2010)
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones,...)?	YES Examples: "Computing derivatives in simple cases is compulsory ; but, for more complex situations, one will use CAS […]. Introducing sequences with the support of different registers, et with a large mobilization of various software » (MEN 2010). In grade 12, integration by parts is no longer needed. For a lot of activities, the official program calls for the design of an algorithm (equation solving, computation of an integral…) leading to approximated results (MEN 2011)	Difficult to evaluate precisely what teachers do. What appears quite clearly is that teachers need time to change their lessons…
Is ICT used in mathematics classes on a regular basis? If yes, what	It is clearly prescribed « The utilisation of ICT intervene under three modalities - by the teacher, during the lesson, with a devise of collective visualization ; - by the students, during practicals ;	Software are used only occasionally by 60% of the teachers (see schema 47). 25% of the teachers use software for at least one lesson among 4. The use of overhead projector, as well a computer lab, seems

type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?	- by students, for their homework » (MEN 2011). To be noticed: for the vocational highschool, the official program states that: using software for displaying graphs, simulating random experiences, solving systems of equations, conjecturing properties of geometrical figures, is a **compulsory training**	to be quite easy. Only 28,5% of the teachers have an easy access to an IWB (TNI in French), see schema 45 (both schema from MEN 2014) To be noticed: there is a clear difference between vocational highschool and general highschool (due to the content of the programs): in the vocational highschool, the usage of software, in mathematics lesson, are fully integrated in day to day ordinary practices.
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?	There are a lot funding (Ministry of education, local authorities: regions of highschool), in the frame of a global strategy of the ministry for entering the digital era	The main problem is the lack of human resources for maintaining the material… the lack of teacher training for using it, and the lack of relevant resources for a relevant usage (my personal opinion!).
Do textbooks anticipate the availability of ICT?	YES, there is a clear evolution of textbook, following the strategy of the Ministry. Textbooks take into account ICT in conceiving their presentation (including CD), coherent with the curricula, and develop now digital textbooks, available (not freely) online (see a commercial of the Nathan company below). To be noticed: the work of a French teacher association (Sésamath 2014), funded in the beginning of this century, designing on a collaborative way e-textbooks freely available online including a lot of interactive exercises. Such textbooks have already be designed (and regularly renewed) for grades 6 to 9. Grade 10 is in progress.	Teachers, when they look for exercises using ICT mainly use their textbooks, that seems well adapted to the curriculum (see schemas 83, 84 and 60 below, from MEN 2014), but they adapt them: the part of personal construction appear very important (69,6%)
Are internet resources used in mathematics courses?	There is a clear institutional incitation to use internet resources (mainly those of the ministry, but also those of the IREM) The French Ministry develops specific websites (names eduscol or edubases) dedicated to the use of ICT in mathematics (see MEN 2012 and MEN 2013), integrating "resources for supporting curriculum implementation" see an example at http://maths.ac-reunion.fr/IMG/pdf/Annexe_derivee.pdf See examples proposed by the national inspector in charge of ICT in mathematics: Probability and statistics in grade 11: http://cache.media.eduscol.education.fr/file/Mathematiques/59/6/Ressource_Statistiques_Probabilites_1eres_208596.pdf Numerical sequences in grade 11: http://cache.media.eduscol.education.fr/file/Mathematiques/81/7/Ress_Math_1ere_STMG_fiche3_255817.pdf	Schema 60 evidences that, among mathematics teachers, 25% use their "academique"website (meaning the institutional website in the region) , and 25% other institutional website. Other interesting results: - what kind of extra resources do you need from the ministry(schema 88): the most demanded are resources for integrating TICE; - what are the ministry online resources the most useful (schema 90 to 97): those related to new elements of the program (algorithm, probability, statistics and logic), and those related to chapter related to modelling (functions).

	Functions for grade 10: http://cache.media.eduscol.education.fr/file/Programmes/18/1/Doc_ressource_fonctions_109181.pdf « you could remark that the use of spreadsheet, programming calculators, etc… are mentioned as current practices, during the phase of searching a problem, or as a tool for visualisation, experimentation, simulation » (inspector speaking)	
Are there any plans to extend the use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?	YES (see above the digital strategy of the Ministry, including: - equipment (the regions are in charge of such an equipment for highschools- and sometimes students themselves, see for my own region http://www.laregion.fr/18-education-et-lycees.htm ; the equipment by tablets is just at the beginning, but is constitutes a very field of experimentations	There is clearly a divide between the objectives and the reality
Please add other comments and information that you consider relevant but that is not addressed in the questions	To be noticed: we are clearly, one these questions of ICT and Internet resources, prescribed and used, in a period of transition, and we have not so many information on the current evolution. The French ministry for research and higher education has launched an national program of research (2014-2018), named ReVEA (Ressources vivantes pour l'enseignement et l'apprentissage), aiming to analyse these processes, on a quantitative and qualitative way, for 4 disciplines: mathematics, physics, English, and technology. This program is leaded by Eric Bruillard, and Ghislaine Gueudet and me are among the 5 members of the coordination team. Then… In 2018, we will have (probably) more relevant information!	

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	There is a national examination at the end of grade 12 (baccalauréat), opening doors of the university. Precise definition of the content and modality of the mathematics test for the scientific section online (new organization since June 2013, see other comments below) Duration: 4 hours Coefficient: 7 (or 9 for students having chosen math speciality) among a total of 32 for the whole exam Assessing the if the main objectives have been reached by the student: - acquiring and organizing knowledge - designing a research process on a autonomous way - reasoning - having a critical attitude towards results obtained - communicating by writing Content: 3 to 5 independent exercises (marks from 3 to 10, over a total of 20).	See the text and corrections of the 2013 mathematics test on the APMEP website, see an emblematic extract of this baccalauréat ("exercice 1, commun à tous les candidats", below). There is a real evolution, leading to discussion on a given curve (instead of a lot of computations for having the curve). It constitutes, to me, a good illustration of "having a critical attitude towards results obtained", or 'communicating by writing", or "reasoning"…
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	The same institutional reference states that all kinds of calculators are allowed: "The mastering of the usage of calculators is an important goal for the education of students. Then using such a material is allowed, in the conditions fixed by the official rules: The designers of the test have to notify if this usage is allowed, or not, at the beginning of the text of the test". Practically, such a restriction never happened for mathematics test in baccalaureat.	The question is: are the calculators really useful during the test? Seeing the "exercice 1, commun à tous les candidats", below, we could imagine that the text has been conceived for "making unnecessary" the calculators, giving curves, etc. See also discussion lists among teachers about such a text http://www.les-mathematiques.net/phorum/read.php?6,850036

	The precise rules had been defined by an official text, in 1999, still relevant today : « All kinds of calculators are allowed, on two conditions : their use has to be autonomous (no Internet connexion, no connexion with other calculator), and no printer.	
If GDC are allowed, do they need to be reset before the start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?	They do not need to be reset. So it is possible to have downloaded in her personal calculators specific applications or text files.	
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?	YES, tasks as phrased in such a way. Magic words: "give an **approximate value**", "**estimate** a result". Implicitly, when the question is "compute", without any precision, it is a matter of exact computation.	See examples on the APMEP website,
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?	Complex question: - without a calculator, some tasks are impossible to ne done (example for applying an algorithm, see below, extract of the baccalaureat 2013, see the APMEP website); - the calculators are mainly needed for performing an algorithm, or computing an approximate value; - it seems to be a kind of mechanical use of a calculator; - roughly speaking, the spirit of the tasks have changed, due to the calculators (see "exercice 1 commun à tous les candidats" below, but performing the task needing a calculator does not need a special creativity… (see "other comments" below).	
Please add other comments and information's that you consider relevant but that is not addressed in the questions	France was the place for a baccalaureat debate in 2007 (see http://educmath.ens-lyon.fr/Educmath/en-debat/epreuve-pratique/, with a contribution of John Monaghan himself! The goal was precisely to integrate ICT in the baccalaureat mathematics, with the idea that "a condition for changing teachers practices is to change the final examination". During five years, this new way of assessing mathematics has been experimented, leading to new forms of assessment, and new practices of the teachers involved, see the official report http://media.education.gouv.fr/file/98/3/4983.pdf , see in particular the examples, evidencing a real process of conjecturing, experimenting, briefly speaking: a real way of integrating ICT as efficient tools of mathematics practice. After this experiment… the project was left behind… The reason: such changes would lead to change the whole French baccalaureat system.	

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	Debate about the baccalaureat, see above!	
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?	Providing quality proved educational resources, a national institution, the CNDP, is in charge of this level, see http://www2.cndp.fr/secondaire/mathematiques/	A national conference, in 2013, evidenced what was still to be done to reach the official objectives http://emiconf-2013.ens-lyon.fr/tables-rondes/table-ronde-5
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	Developing new blended device for teacher training, named Parform@nce http://national.pairformance.education.fr/ On the basis of an appraisal of this program, a new program, more flexible, is developed from this year for the primary teachers, will be developed next year for the secondary teachers: M@gister. French government (more exactly the Ministry of research and higher education) has created a new structure (FUN: France Universités Numérique) dedicated to the design of MOOCs, among them a MOOC dedicated to "Teaching and Training with ICT". This MOOC as four fields of application, one of them is dedicated to math teaching (lead by Ghislaine Gueudet, described in the journal of the French Commission for Mathematics Teaching (on line, p. 5)	Potential and constraints of the Pairform@nce program analysed in Gueudet & Trouche (2011b) Several features of this program are underlined: the links between teacher education and classroom practices, teachers collaborative work as a necessary condition for ICT integration, interrelations between teachers resource system, collective resource systems, and institutional resource systems.
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	See above, the discussions about the baccalaureat	
Please add other comments and information's that you consider relevant but that is not addressed in the questions	A promising way for integrating ICT seems to be the development of interdisciplinary devices: - computer sciences and mathematics (see above, Part 2); - since 2010, there is, for grade 10, a new interdisciplinary program MPS (scientific methods and practices), for developing common work of mathematics, physics and biology teachers, resulting in a common reflection on IBST, and new practices involving the use of ICT in every disciplines; - since 2005, there is, for grades 11 and 12, an interdisciplinary program TPE (personal and monitored works), consisting in projects realised by students (alone or by pairs), involving two disciplines (e.g. math and biology). This work is assessed during a special examination taken into account for the baccalaureat.	

References

Gueudet, G., & Trouche, L. (2011a). ICT in France: Development of Usages, Institutional Hesitations and Research Questions, in C. Knights & A. Oldknow (Eds.), *Mathematics Education with Digital Technologies* 23-29, Continuum Press, London.

Gueudet, G., & Trouche, L. (2011b). Mathematics teacher education advanced methods: an example in dynamic geometry, *ZDM, The International Journal on Mathematics Education,* **43**(3), 399-411 http://www.springerlink.com/content/13733h7321658734/, DOI 10.1007/s11858-011-0313-x

IGEN (Inspection générale de l'éducation nationale) (2004). *Les technologies de l'information et de la communication dans l'enseignement des mathématiques au collège et au lycée.* Prescription for ICT usages (students from grade 6 to 12), and institutional assessement.

IGEN (Inspection générale de l'éducation nationale) (2010). *Mise en oeuvre du programme de mathématiques en classe de seconde*. xxxx
Ce rapport a été rédigé sur la base d'une étude menée par le groupe mathématiques de l'inspection générale menée au cours de la première année de mise en œuvre du programme (année scolaire 2009-2010). Sur la base d'un protocole national, les IA-IPR de mathématiques ont procédé à des enquêtes auprès des établissements, organisé des réunions d'équipes sur ce thème et assuré des observations lors d'inspections individuelles

MEN (French Ministry of education) (2009). *Programme de mathématiques de la classe de seconde* [first year of the French highschool] http://www.education.gouv.fr/cid28928/mene0913405a.html
Program implementation assessed in 2013 (see MEN 2014)

MEN (French Ministry of education) (2010). *Programme de mathématiques de la classe de première scientifique* [second year of the French highschool] http://www.education.gouv.fr/cid53326/mene1019634a.html

MEN (French Ministry of education) (2011). *Programme de mathématiques de la classe de terminale scientifique* [third – and last - year of the French highschool] http://www.education.gouv.fr/pid25535/bulletin_officiel.html?cid_bo=57529

MEN (French Ministry of education) (2012). *Activités mathématiques et TICE*, http://eduscol.education.fr/maths/usages
Specific goals for teaching mathematics with TICE: experimenting, conjecturing, testing… A repository of exercises, designed by collectives of teachers, and checked by inspectors.

MEN (French Ministry of education) (2013). *Resources for mathematics teaching in hisghschool*, http://eduscol.education.fr/cid45766/mathematiques-pour-le-college-et-le-lycee.html#lien0
For mathematics teaching in highschool, the Eduscol website (French ministry of education) proposes reflections on mathematics teaching and pedagogical sheets with exercises aiming to support teachers work, directly related to the current curriculum.

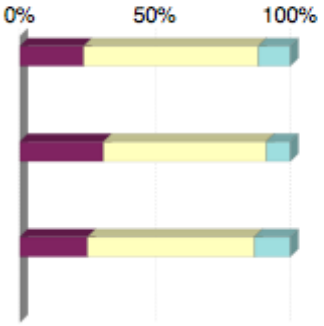
MEN (French Ministry of education) (2014). *Rapport sur la mise en oeuvre du programme de mathématiques en classe de seconde* [Report on the implementation of the mathematics curriculum for grade 10 – first year of French highschool], http://cache.media.eduscol.education.fr/file/Mathematiques/01/6/CSM-projet-rapport2013_293016.pdf
A very recent national survey (January 2014), from a commission involving inspectors, researchers, teacher trainers, and teachers, teacher association (APMEP) and the IREM network.

Sesamath (2014). A national teacher association aiming to design and share resources for mathematics teaching, including texbooks, freely available online, and giving a llarge place to ICT. A textbook for grade 10 is in progress. http://manuel.sesamath.net/index.php?page=manuel_et_cahiers_2nde_2014

Examples

Q47 à Q49. Intégration de l'utilisation des TIC :

	au moins une séance sur quatre	de temps en temps	jamais
Intégrez-vous l'utilisation des TIC (autres qu'exerciseurs et calculatrices) : - en Géométrie :	**23,8%**	**64,5%**	**11,7%**
Intégrez-vous l'utilisation des TIC (autres qu'exerciseurs et calculatrices) : - dans l'étude des fonctions :	**30,9%**	**61,0%**	**8,1%**
Intégrez-vous l'utilisation des TIC (autres qu'exerciseurs et calculatrices) : - en Statistiques et probabilités :	**25,5%**	**61,8%**	**12,7%**



TNI stands for “tableau numérique interactif”, classe mobile is a set of computers that can move from a classroom to another one

****

Q45. À quel type d'équipement pouvez-vous avoir régulièrement accès avec votre classe (*plusieurs réponses possibles*)

vidéoprojecteur	**208**	77,0%
TNI	**77**	28,5%
postes informatiques en accès libre dans la classe	**27**	10,0%
classe mobile	**9**	3,3%
salle informatique	**203**	75,2%
autre	**11**	4,1%

TNI stands for “tableau numérique interactif”, classe mobile is a set of computers that can move from a classroom to another one

****

Q83. Les élèves disposent-ils d'un manuel scolaire ?

oui	253	98,4%
non	4	1,6%
Total	**257**	

Non-réponses : 2 (0,8 %)

Q84. (si Q83 = « oui ») Le manuel scolaire dont disposent vos élèves vous semble-t-il bien adapté au programme ?

oui	205	82,0%
non	45	18,0%
Total	**250**	

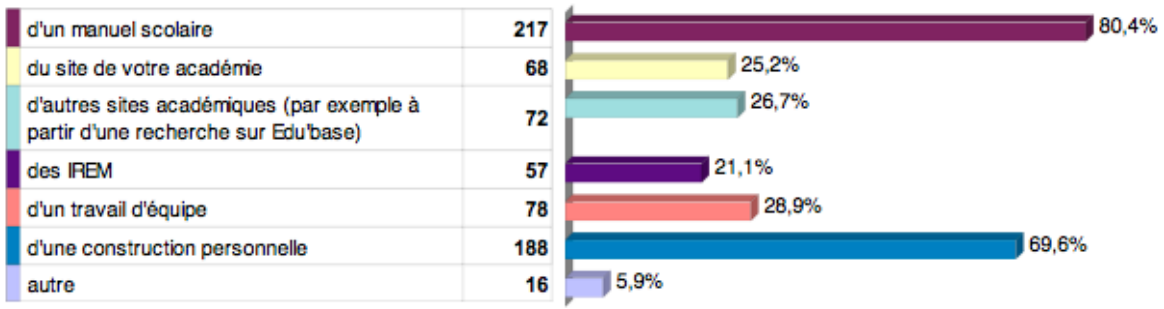
Non-réponses : 3 (1,2 %)

Q60. Les activités TICE que vous proposez aux élèves proviennent : (*plusieurs réponses possibles*)

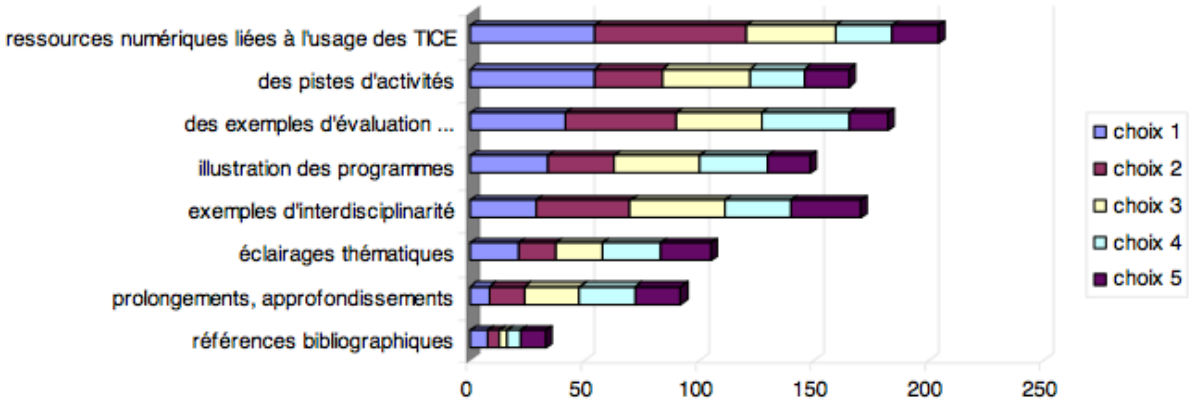
d'un manuel scolaire	217	80,4%
du site de votre académie	68	25,2%
d'autres sites académiques (par exemple à partir d'une recherche sur Edu'base)	72	26,7%
des IREM	57	21,1%
d'un travail d'équipe	78	28,9%
d'une construction personnelle	188	69,6%
autre	16	5,9%

A commercial of the Nathan company

Q60. Les activités TICE que vous proposez aux élèves proviennent : (*plusieurs réponses possibles*)

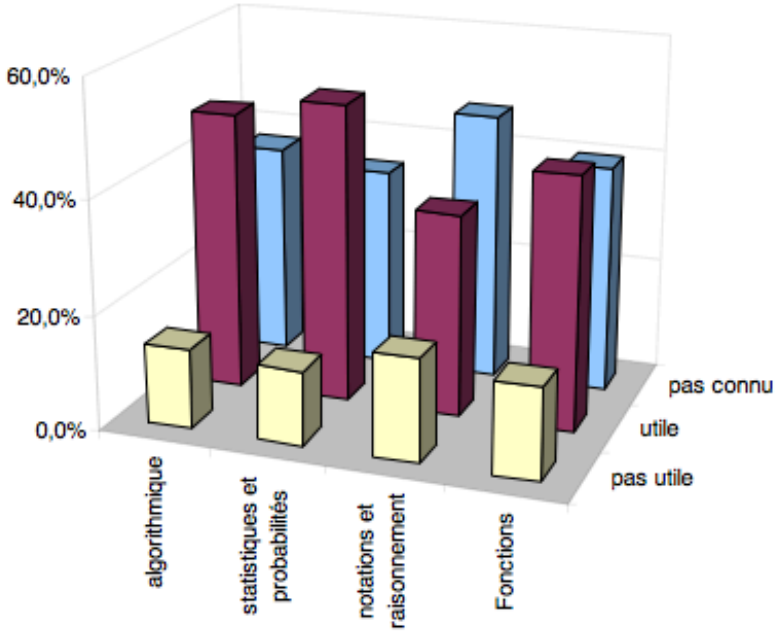


Q88. Que pourraient être ou contenir les ressources d'accompagnement proposées par le ministère ? (*classez vos réponses, 5 choix maximum*)



Q90 à 97. Les documents ressources éduscol :

- utilité



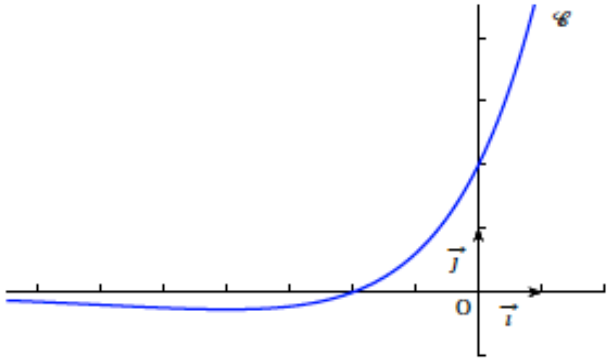
EXERCICE 1 **6 points**

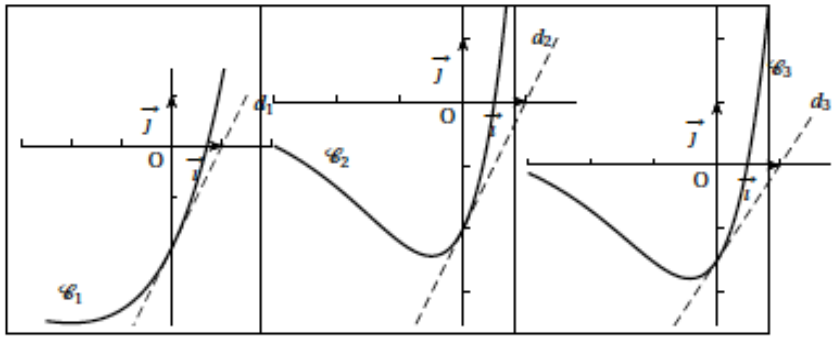
Commun à tous les candidats

Soit f une fonction définie et dérivable sur $\mathbb{R}$. On note $\mathscr{C}$ sa courbe représentative dans le plan muni d'un repère $\left(O, \vec{\imath}, \vec{\jmath}\right)$.

Partie A

Sur les graphiques ci-dessous, on a représenté la courbe $\mathscr{C}$ et trois autres courbes $\mathscr{C}_1$, $\mathscr{C}_2$, $\mathscr{C}_3$ avec la tangente en leur point d'abscisse 0.





1. Donner par lecture graphique, le signe de $f(x)$ selon les valeurs de x.
2. On désigne par F une primitive de la fonction f sur $\mathbb{R}$.
   a. À l'aide de la courbe $\mathscr{C}$, déterminer $F'(0)$ et $F'(-2)$.
   b. L'une des courbes $\mathscr{C}_1$, $\mathscr{C}_2$, $\mathscr{C}_3$ est la courbe représentative de la fonction F.
   Déterminer laquelle en justifiant l'élimination des deux autres.

Exercise for applying an algorithm:

$$u_{n+1} = \sqrt{2u_n}.$$

1. On considère l'algorithme suivant :

Variables :	n est un entier naturel
	u est un réel positif
Initialisation :	Demander la valeur de n
	Affecter à u la valeur 1
Traitement :	Pour i variant de 1 à n :
	| Affecter à u la valeur $\sqrt{2u}$
	Fin de Pour
Sortie :	Afficher u

a. Donner une valeur approchée à 10^{-4} près du résultat qu'affiche cet algorithme lorsque l'on choisit $n = 3$.

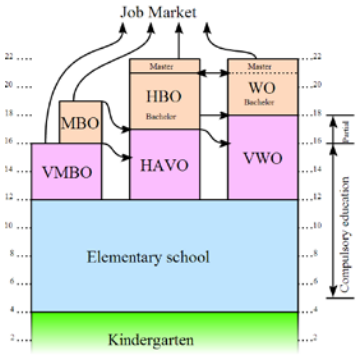
b. Que permet de calculer cet algorithme?

Questionnaire 3

Respondent name: Paul Drijvers **Country described in the response: the Netherlands**

This response is based on the official curriculum (see online resources at the end of this document.

Part 1: Overall description

Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	The two 16-19 programmes of secondary education that grant admission to higher education are HAVO, which lasts five years, and VWO, which lasts six years. Pupils are enrolled according to their ability, and VWO is considered more rigorous. The VWO curriculum prepares pupils for university (also known as WO), while the HAVO diploma prepares students for going to a university of professional education, also known as HBO (http://www.iamexpat.nl/read-and-discuss/education/articles/the-dutch-education-system-primary-secondary-education#sthash.UkqLOEbU.dpuf). See the figure below.  For academic mathematics, we focus on HAVO and VWO mathematics 16-19 curricula. In 2013, the number of students who did the final national examination was 57.600 for HAVO and 40.116 for VWO (http://www.volkskrant.nl/vk/nl/2686/Binnenland/article/detail/3439999/2013/05/12/207-000-scholieren-doen-eindexamen.dhtml). Schooling in the upper years (the last two years of HAVO, or the last three years of VWO) is divided into a common component, a specialised component and an optional component (http://www.government.nl/issues/education/vwo-and-havo). The four specialised subject combinations that pupils can choose from are: - science and technology (NT); - science and health (NG); - economics and society (EM);

	- culture and society (CM). The EM and NG streams are the most popular. All four streams have different mathematics curricula: Math C for CM (only VWO, focus on reasoning, statistics), Math A for EM and NG (focus on applied calculus and statistics), Math B for NT (focus on calculus and geometry), and Math D (optional for NT, focus on pure math and statistics). Students may upgrade C to A, and A to B. Mathematics education in the Netherlands is to an important extent influenced by the theory of Realistic Mathematics Education (see Van den Heuvel-Panhuizen & Drijvers, 2013, for an overview); in the meanwhile, this approach has been criticized and is the topic of a national debate. This also impacts on views on the role of digital technology, as it has been related to opportunities for mathematics education that focuses on higher-order thinking skills such as problem solving and modelling real life problems (Trouche & Drijvers, 2010).

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	Yes. The official curricula - or rather targeted learning objectives and goals, as there are no time schedules or other prescriptions - are described in so-called syllabi available at www.examenblad.nl. The syllabi address different domains. The first domain, called Skills, mentions the use of ICT in general terms, e.g., "The candidate can, also through the use of ICT, gather, select, process, judge and present information" and, under the heading of Algebraic skills, "The candidate can […] perform operations with, but also without ICT means such as a graphing calculator." The domain specific descriptions also in some places refer to the use of ICT, for example in the domain Differential and integral calculus: "In appropriate situations, the candidate can set up an integral, calculate its exact value and approximates it using ICT."	In the reality of the mathematics classroom, ICT seems to be used more and more, see comments below. Pisa 2012 findings and national studies by Kennisnet show that ICT infrastructures are relatively good in Dutch schools. However, exact data for mathematics teaching seem to be lacking.
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones, …)?	This is not made explicit in official curriculum documentation. Meanwhile, stress on, for example, integration techniques has been reduced.	In practice, text books tasks and assessment tasks may involve mathematical problems (e.g., complex equations or integrals) that students are not expected to solve by hand, but that can be approximated by numerical techniques available on graphing display calculators.
Is ICT used in mathematics classes on a	This will depend on the teacher and the school, but the overall answer is yes. Data specific for mathematics	

regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?	do not seem to be available. School facilities vary as well: most schools will have computer labs, many schools have IWB in many classrooms, and/or one computer for the teacher in each classroom with a data projector. Students have graphing calculators, as they are expected during the national final examination. Some schools have laptop classes, mobile laptop sets, tablet classes. Some mathematics classrooms have some student computers available. About who uses it: for IWB and projecting devices, the initiative is mainly for the teacher. For individual laptop / tablet / GDC, the initiative is for the student. In the light of the 'flip-the-classroom' idea, video clips with mathematical explanations that students watch at home are getting more and more popular. Think of Khan Academy (https://www.khanacademy.org/) but also of clips that teachers make and upload to YouTube, in answering to students' needs.	
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?	There are different types of fundings. Kennisnet (www.kennisnet.org) is a national organisation that supports the integration of ICT in education (all levels and subjects). Other research and implementation fundings are available, not specifically focusing on ICT, but ICT-oriented proposals can be granted.	Research fundings are decreasing. A new research organisation, NRO (www.nro.nl) is established and seems to value practice-oriented research, with schools and teachers involved, with a concrete output, a high valorisation focus, and a short project period.
Do textbooks anticipate the availability of ICT?	Yes, they do. Most text book series come with additional software, either for IWB use or for individual student practice. Text books have icons that refer to interactive activities using digital tools.	
Are internet resources used in mathematics courses?	Yes, they are, but not in an institutionalized way.	Teachers and students use applets and online courses. The Freudenthal Institute's Digital Mathematics Environment (www.fisme.uu.nl/dwo/en) is popular, not only because of its high quality content, but also because of the authoring system, which allows teachers to adapt or design content, and because of its student monitoring system.
Are there any plans to extend the use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?	Not on an official level. In the light of the 'back-to-the-basics' and 'get-rid-of-realistic-mathematics-education' movement, the official policy is not so much oriented towards a real extended use of ICT.	In school reality, more and more schools have laptop or tablet classes. This 'sells' in the battle for students. However, many mathematics teachers do not know what to do with these devices, are looking for good content and for efficient ways to use them in their teaching.
Please add other comments and informations that you consider relevant but that is not addressed in the questions		

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	Yes, there are national final examinations for the different mathematics courses (www.examenblad.nl). The 3-hour sessions determine the student's final grade for 50%; the other 50% is the average grade of the school-organized local examinations.	School inspectors check if school examination grades are not much higher than the national examination grades. The teachers grade the national examination of their own students, based on precise guidelines and marking schemes. A second corrector, a teacher from a different school, checks the first corrector's grading.
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	Yes, the use of a GCD is allowed. Specific types are permitted. The list of these types is updated yearly. TI, Casio, HP and Sharp are brands that are on this list. Main criteria are: no communication options, no printing options, and no CAS options.	In reality, Casio and TI share the market. Initially, TI was the market leader, but Casio is coming closer, is my impression. Hard data on this are not available to me. From the beginning of this regulation (1998), HP calculators were allowed, even if they offer symbolic differentiation. The ministry's argument was that they did not want to exclude one brand…
If GDC are allowed, do they need to be reset before the start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?	GCDs do not need to be reset, so additional apps or text files can be used during the examination. The ministry's argument for this is that resetting in school practice is hard to carry out, also because students program reset simulation programs and expert math teachers are not always around during the examination.	If a student would bring a CAS calculator, this probably would not be noticed by the examination officers. So control is weak. The text functionality was the reason to ban the GDC from national examinations in biology, physics, chemistry and business; initially, it was also allowed for these subjects, but this has been changed recently. The increasing capacities of GCD apps now question the current policy (e.g., ZoomMath, see http://www.zoommath.com/). The assessment authorities have installed a committee (chair: Paul Drijvers ☺) to assess the situation and to advise on future strategies.
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?	Yes, there is an official list of 'magic words'. For example, "calculate the exact value" or "prove" means that no GDC facilities may be used, whereas "calculate" means that GDC facilities (including procedures such as calc intersect or zeros or nderiv) may be used.	This list of magic words always raises discussion, as, in spite of efforts to communicate this clearly, teachers seem to miss this, do not explain these conventions to their students and, as a result, want to grade the examinations against the guidelines. Debate…

Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?	Yes, it is. In application tasks, the focus is on modelling, or on mathematization, and the resulting equations or other mathematical problem can be solved using the GDC. If by-hand techniques are to be assessed, the above mentioned magic words are used, and there usually is less context or application in such tasks.	Now that GDCs are so common, there is a tendency to less reward their use than was the case shortly after their introduction. Also, the requirement for a student to describe the techniques used is not as tight as it used to be. All together, the role for the GCD in the examination is decreasing.
Please add other comments and informations that you consider relevant but that is not addressed in the questions		

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	Yes! Main issues: how to ensure equal opportunities for all students during the national examination. And how to maintain paper-and-pen skills.	The new curricula make a case for 'mathematical thinking activity'. The question is if ICT can be used for this, or if it is detrimental. Of course, the answer depends on the type of ICT, and above all on the type of task and the type of use.
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?	To a limited extent. As the GCD is already around for many years, there is no PD focusing on that. For other ICT tools, it is very limited and only small scale. For using the IWB, there has been some training, rather button oriented.	
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	There are no national PD courses for using ICT in mathematics education. In research projects or pilot PD courses, a blended approach is sometimes used, with Moodle like environments combined with face-to-face meetings. Pre-service teacher education does make use of online content.	
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	Not really. The new curricula that will be implemented in 2015 are a compromise between the back-to-the-basic movement, which is not in favour of ICT, and more 21st century like ideas that match better with ICT integration.	
Please add other comments and informations that you consider relevant but that is not addressed in the questions		

References

Online resources:
http://en.wikipedia.org/wiki/Education_in_the_Netherlands
http://www.government.nl/issues/education/vwo-and-havo
http://www.examenblad.nl/
https://www.cve.nl/ (Board of Examinations)
http://www.cito.nl/Onderwijs/Voortgezet%20onderwijs/centrale_examens/schriftelijke_examens_havovwo/examens_havovwo_2013/vwo_ce_tv1.aspx
www.ctwo.nl (the Dutch mathematics curriculum reform committee)
http://www.onderwijsinspectie.nl/ (The Dutch Inspectorate of Education)
http://duo.nl/organisatie/open_onderwijsdata/databestanden/vo/leerlingen/default.asp
http://www.kennisnet.nl/fileadmin/contentelementen/kennisnet/Over.kennisnet/Vier_in_balans/Vier_in_balans_monitor_2013.pdf

Text resources:
Trouche, L., & Drijvers, P. (2010). Handheld technology: Flashback into the future. *ZDM, the International Journal on Mathematics Education, 42*(7), 667-681.
Van den Heuvel-Panhuizen, M., & Drijvers, P. (2013). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Dordrecht, Heidelberg, New York, London: Springer.

Questionnaire 4

Respondent name: Mike Thomas **Country described in the response: New Zealand**

Part 1: Overall description

Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	New Zealand (NZ) has a national curriculum that is internally and externally assessed through the National Certificate of Educational Achievement (NCEA) at ages 16 (Level 1), 17 (Level 2) and 18 (Level 3). It is Level 3 that determines university entrance. In addition, some schools opt to take other examinations, including Cambridge International Examinations (CIE) and IB. Mathematics is taken by virtually all students at Level 1. There are two mathematical subjects at Level 3, Statistics and Modelling and Mathematics with Calculus. At each level the subjects are divided into separate standards. In 2012, the latest year published, the number of students taking each subject were: Level 1: Mathematics 63956 Level 2: Mathematics 46770 Level 3: Mathematics with Calculus 7448 Statistics and Modelling 16060 Scholarship: Mathematics with Calculus 1450 Statistics and Modelling 1754 (Scholarship is the highest level examination, taken concurrently with Level 3)

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	The written curriculum[9] is divided into 8 levels (with 8 the highest) for the subject Mathematics and Statistics, with Achievement Objectives for each level. It is very short, comprising four pages in total[10], and makes general statements such as: Calculus Level 7 - Sketch the graphs of functions and their gradient functions and describe the relationship between these graphs. - Apply differentiation and anti-differentiation techniques to polynomials. Level 8 - Identify discontinuities and limits of functions.	This depends on the individual school and teacher.

[9] See http://nzcurriculum.tki.org.nz/National-Standards/Mathematics-standards

[10] See attached copy

	- Choose and apply a variety of differentiation, integration, and antidifferentiation techniques to functions and relations, using both analytical and numerical methods. - Form differential equations and interpret the solutions. The only mention of technology is in Level 7 Statistics: "…calculating probabilities, using such tools as two-way tables, tree diagrams, simulations, and technology."	
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones, …)?	Not really as far as I know.	
Is ICT used in mathematics classes on a regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?		This depends on the individual school and teacher. Some schools have a lot of technology integrated into their classroom lessons and others have none. There are two papers that outline this in some detail, see [1] and [2]. Most schools have one, two or three computer labs but they are difficult to access due to popularity with all subjects. There is often only one computer in a mathematics classroom.
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?	There was a programme to provide laptops for Principals and teachers in schools.	
Do textbooks anticipate the availability of ICT?	Yes there are some examples of textbooks with calculator use.	
Are internet resources used in mathematics courses?		This depends on the individual teacher and school.
Are there any plans to extend the use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?		No plans officially. A number of individual teachers and schools are looking to expand technology use into tablets, etc.
Please add other comments and informations that you consider relevant but that is not addressed in the questions		

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	Yes there are national examinations at Levels 1, 2 and 3. There is one examination for each standard but not all standards are externally assessed by examination, some are internally assessed. None of the standards speak about technology directly, the comments are in the additional notes. Four of the internally assessed standards in Level 3 Statistics and Modelling, namely 91580 Investigate time series data 91581 Investigate bivariate measurement data 91582 Use statistical methods to make a formal inference 91583 Conduct an experiment to investigate a situation using experimental design principles all state: - Use of a statistical graphing package is expected.	
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	Yes, in the NCEA assessment, as below Level 1 Equipment to bring …All approved scientific or graphing calculators may be used by candidates entering level 1 Mathematics standards 91028, 91031, and 91037. A graphing calculator is an advantage in 91028. (Note that in the Common Assessment Task for Mathematics 91027, no calculators may be used.) The Common Assessment Task is "Apply algebraic procedures in solving problems" Level 2 Equipment to bring Candidates must bring an approved calculator (preferably a graphing calculator). Candidates who do not have access to graphing calculators will be disadvantaged. Level 3 Equipment to bring Candidates must bring an approved calculator (preferably a graphing calculator).	

Candidates who do not have access to graphing calculators will be disadvantaged.

Special notes
Candidates will be required to answer questions that demonstrate an understanding of the mathematical concepts rather than directly transferring results from their graphing calculator. This may involve use of unknown constants.
…
When graphing calculators are used to solve a problem, candidates must provide evidence of their differentiation and integration skills.

Further clarification
Candidates using graphing calculators will not receive credit for correct solutions to problems assessed against this standard where they have not provided the correct integrated function. That is, candidates must show the results of any integration needed to solve a problem.

Approved Calculators List for 2014:
The Approved Calculators list presently comprises the following calculators:

Scientific and Graphing Calculators approved for ALL subjects

Manufacturer	**Model**	**Type**
Canon	F-717SGA	Scientific
Casio	fx-82, fx-83, fx-100, fx-991ES (all	Scientific
	fx-9750 (all variants), fx-9860 (all	Graphing
Deskmaster	Scientific	Scientific
Home & Office	E6610	Scientific
Jastek	JasCS1	Scientific
Mahobe	Mahobe Scientific, DS-742CQ, DS-	Scientific
	DS-742DQ	Graphing
Sharp	EL531 (all variants)	Scientific
Texas	TI-30XB	Scientific
Instruments	Ti-Nspire (non-CAS), TI-82, TI-83, TI-84+	Graphing

In addition to calculators from the above list, candidates entering Level 3 Statistics and Modelling, Scholarship Statistics and Modelling, and Scholarship Mathematics with Calculus examinations may use approved CAS-capable calculators. CAS calculators that are

	approved for these subjects are: **CAS calculators approved for level 3 Statistics and Modelling, Scholarship Statistics and Modelling, and Scholarship Mathematics with Calculus ONLY** **Manufacturer** — **Model** **Casio** — CFX9970G, ClassPad (all variants), FX Algebra 1.0, FX Algebra 2.0 **Hewlett Packard** — HP40g, HP40gs **Texas Instruments** — TI89, TI89 Titanium, TI92 (all variants), Voyage 200, TI-Nspire CAS	
If GDC are allowed, do they need to be reset before the start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?	They have to be reset. The website[11] states: The NZQA *Assessment and Examination Rules and Procedures* allow the legitimate use of most types of calculator, including graphical and programmable calculators with reset buttons. Any calculator used in NZQA examinations must be silent, hand-held, non-printing and must contain its own power source. It MUST NOT be able to: - wirelessly transmit or receive information to or from another source - be used to bring in stored information - be used as a dictionary.	
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?	Some kind of intermediate working is expected rather than simply a final answer from the Calculator. For example: 1. Find the area enclosed between the graph of $y = \sin(2x)$, the x-axis, and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. *Give the result of any integration needed to solve this problem.* 2. Use integration to find the area enclosed between the graphs of the functions $3y = x^2$ and $y = 2x$. *You must use calculus and give the result of any integration needed to solve this problem.* 3. A curve has the equation $y = (x^3 - 2x)^3$. Find the equation of the tangent to the curve at the point where $x = 1$. *Show any derivatives that you need to find when solving this*	

[11] See http://www.nzqa.govt.nz/ncea/assessment/search.do?query=mathematics&view=exams&level=03

	problem. 4. Find the value of *x* that gives the maximum value of the function $f\ (x)\ =\ 50x\ -\ 30x\ \ln 2x$ *You do not need to prove that your value of x gives a maximum. You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.*	
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?	The central aim seems to be to set calculator neutral examinations where there is no advantage in the calculator. In practice this is not really accomplished.	

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	There was a big debate about the use of CAS calculators in the curriculum and assessment. As a result during 2006 the Ministry of Education developed two Level 1 CAS Mathematics achievement standards that permitted the use of this new technology. This was followed by the development of three Level 2 CAS Mathematics achievement standards in 2007. The development of the three Level 3 achievement standards provided an ongoing pathway for students who used the technology. They were available for voluntary, opt-	The CAS standards were reviewed with teachers invited to comment on them. Following consultation this announcement was made: There will be no NCEA examinations offered for any of the CAS Mathematics external achievement standards in 2012. This applies to the following standards: - Level 1 - 90799 and 90800 (CAS Mathematics 1.1, 1.2), which are now expired standards - Level 2 - 90806, 90807, 90808 (CAS Mathematics 2.1, 2.2, 2.3), which are now expired standards - Level 3 - 90833, 90834, 90835 (CAS Calculus 3.1, 3.2,

	in use from 2009 onwards and support material and examinations[12] were produced.	3.3). These standards were not assessed in 2011. There will not be separate CAS Mathematics external achievement standards post-Standards Alignment and NZQA will not be setting examinations for any of the current CAS standards in 2012 or beyond. Note: CAS Mathematics refers to mathematics involving the use of Computer Algebraic Systems calculators. These advanced calculators are capable of manipulation of mathematical expressions in symbolic form.
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?		Only on an ad hoc basis, often organised locally by the teacher organisations. No central government or Ministry of Education assistance.
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	There are a lot of curriculum resources online to assist teachers, but little on the use of ICT.	
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	Not as far as I know.	
Please add other comments and information that you consider relevant but that is not addressed in the questions		

References

Thomas, M. O. J. (2006). Teachers using computers in the mathematics classroom: A longitudinal study. *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education,* Prague, Czech Republic, *5*, 265–272. Available from http://www.emis.de/proceedings/PME30/5/265.pdf

Thomas, M. O. J., Hong, Y. Y., Bosley, J., & delos Santos. A. (2007). Calculator use in the mathematics classroom: A longitudinal study, In Wei-Chi Yang, T. de Alwis & Jen-chung Chuan (Eds.), *Proceedings of the 12th Asian Technology Conference in Mathematics* (C, pp. 37-47). Taiwan: ATCM. ISSN 1940-4204. Available from http://atcm.mathandtech.org/EP2007/EP2007.htm.

[12] See the example examination paper questions at the end

Examples

Some questions that were set in the 2010 CAS examinations.

1. Find, in polar form, expressions in terms of *n* for all the solutions of the equation *z*3 – 64*n* = 0, where *n* is a positive real number.
2. For what value(s) of *p* does the equation $x^2 + px + p = 0$ have no real roots?
3. Find the value of *k* if the roots of $2x^2 - 12x + k + 2 = 0$ are of the form α, $\alpha + 2$, ie, there is a difference of 2 between the two roots.
4. Prove the identity $\frac{1+\sin\theta}{1-\sin\theta} = \tan^2\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$.
5. Write an expression for the *n*th derivative of $\frac{1}{x}$.
6. and 7.

Questionnaire 5

Respondent name: Mike Thomas **Country described in the response: Singapore**

Part 1: Overall description

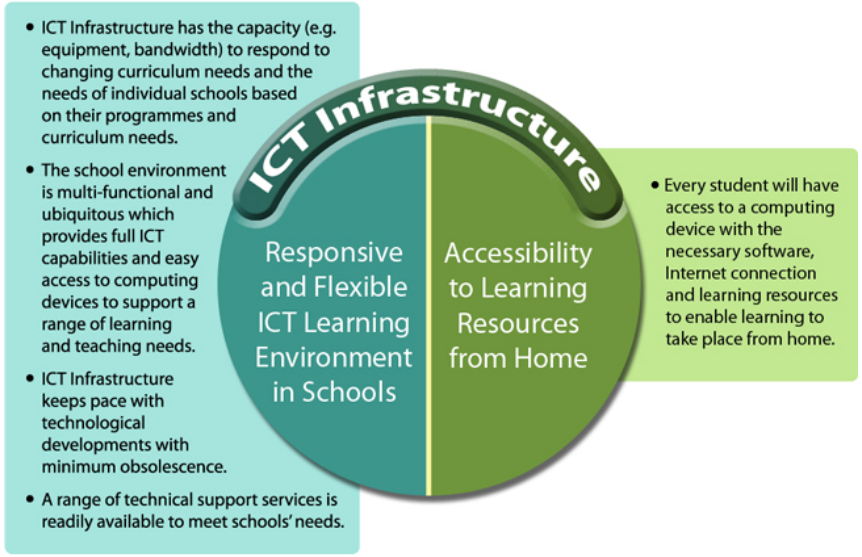
Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	There is a national curriculum in Singapore. Syllabus documents[13] provide the main overview descriptors of this, giving the design, structure, aims, content, outcomes and approaches to learning. One such document states that: "It is the goal of the national mathematics curriculum to ensure that all students will achieve a level of mastery of mathematics that will serve them well in their lives, and for those who have the interest and ability, to pursue mathematics at the highest possible level. Mathematics is an important subject in our national curriculum. Students begin to learn mathematics from the day they start formal schooling, and minimally up to the end of secondary schooling. This gives every child at least 10 years of meaningful mathematics education." (p. 2) At 16 students take UK-type O and AO level examinations. At 17 and 18 they study at levels H1, H2 and H3 for UK A level examinations.

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	Yes, the stated aim is to have a 'technology-enabled learning environment'. Some explicit ideas are expressed in terms of: Use…technology to present and communicate mathematical ideas; and perform calculations with calculator. It also says about skills: In today's classroom these skills also include the abilities to use spreadsheets and other software to learn and do mathematics …students should have opportunities to use and practise the skills. and: Learning experiences should provide opportunities for students to enhance conceptual understanding through use of various mathematical tools, including ICT tools. The Ministry of Education has a 'Masterplan3' (mp3) for ICT[14] use to transform the learning environments of students and equip them to succeed in the knowledge economy.	
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed, approximated solutions rather than exact ones,	Graphing software use is totally integrated into the syllabus. Examples are: Use graphing software to investigate the characteristics of various functions, to see how the sign of the discriminant influences the position of a quadratic graph and find the relationship between the solutions of simultaneous equations and the intersections of two polynomial graphs.	

[13] See http://www.moe.gov.sg/education/syllabuses/sciences/

[14] See http://ictconnection.moe.edu.sg/cos/o.x?c=/ictconnection/pagetree&func=view&rid=665

…)?		
Is ICT used in mathematics classes on a regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?	The Masterplan, mp3, states that: The key elements in providing an ICT infrastructure that supports learning anytime, anywhere are: 1. A responsive and flexible ICT learning environment in schools 2. Accessibility to learning resources from home.  *Source: The ICT Connection [http://ictconnection.edumall.sg]* In mathematics: Guided by generative design principles, the participatory learning in Mathematics approach provides opportunities for students to perform Mathematical practices in an online collaborative learning environment. Anonymity and versatility in identities allow students to adopt different roles in a collaborative space which encourages peer evaluation and self-review. As students exchange knowledge and ideas about the Mathematical problems, meaningful discussions and a critical exploration of Mathematical concepts can happen. [2] page 302 states In the mathematics classroom, the goals of the MPI translate into a vision of "integration of ICT to enhance the mathematical experience" (University of Cambridge Local Examination / Ministry of Education, 2000, emphasis added).	The goals of ICT use are student-directed learning (SDL) and collaborative learning (CoL). Teachers develop and share lessons that use ICT with these as the aim. - Teachers should select the appropriate SDL and CoL elements to integrate into their lessons. Otherwise the teacher might be overwhelmed. - Teachers can role model SDL by reflecting on their own teaching practices and making incremental adjustments on a daily basis. - *The ICT Connection* provides a platform for teachers to contribute lesson packages to share with the teaching fraternity their own ideas about how SDL and CoL attributes can be developed in the students through the use of ICT. Goals: Enabler Goal 1 highlights the important role that school leadership, particularly the Principal, plays in casting a vision for the use of ICT in learning and teaching of ICT use within the curriculum. Enabler Goal 2 focuses on the need for teachers to have the capacity to plan, as well as deliver ICT-enabled learning experiences that will foster self-directed and collaborative learning among learners as well as guiding them in using ICT safely and responsibly. Enabler Goal 3 highlights the need to create the infrastructure to achieve the proposed mp3 vision, both at the individual school and MOE levels. While the main focus

	This statement of intent points to ICT policy objectives in at least the follwing ways: - As “integrate” would imply, ICT should not merely be a bit-part player, but rather, it should feature prominentlyin mathematics classrooms. - “Integrate” also implies that ICT should shouldnot be viewed or acted upon as an isloates part of instruction, but it should be weaved tightly into other components of teaching practice to form a well-coordinated whole. - Students are expected to use technological tools directly in order to “enhance the mathematical experience”.	of this goal refers to the infrastructure within the school, it is important that a combination of factors be considered to make possible the extension of ICT use for learning to the home environment for students.
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?	There is a website portal to resources: http://ictconnection.moe.edu.sg/masterplan-3/resources-for-schools Also [2] page 302 states: In Singapore the eduaction authorities have, over the last decade, taken concrete steps to encourage the use of computers to enhance teaching and learning. Much resources, in the region of S$2 billion, were channeled into infrastructure, computer hardware and software, and teacher training in the first phase of the information technology (IT) master plan (MPI) from 1997 to 2002 with the target that every student would have access to technology learning (Ministry of Education, 1997). In the second phase of the master plan (MP2), which is ongoing at the time of writing, the aim is to further harness the power of ICT in bringing together key areas of education such as curriculum, assessment, instruction, and professional development to build school environments that are conducive for engaged and holistic learning (Ministry of Education, 2002).	
Do textbooks anticipate the availability of ICT?	The following textbooks are endorsed by Cambridge for use with the syllabuses – Cambridge University Press will supply further information. **Author Title Publisher ISBN** Neill & Quadling *Pure Mathematics 1* Cambridge University Press 0 521 53011 3 Neill & Quadling *Pure Mathematics 2 & 3* Cambridge University Press 0 521 53012 1 Quadling *Mechanics 1* Cambridge University Press 0 521 53015 6 Quadling *Mechanics 2* Cambridge University Press 0 521 53016 4 Dobbs & Miller *Statistics 1* Cambridge University Press 0 521 53013 X Dobbs & Miller *Statistics 2* Cambridge University Press 0 521 53014 8	
Are internet resources used in mathematics courses?	Yes; see above	
Are there any plans to extend the	Unclear	

use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?		
Please add other comments and information that you consider relevant but that is not addressed in the questions		

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	Yes. The GCE 'N' Level examinations, otherwise known as the 'N' Levels, are conducted annually in Singapore[15]. It is taken after four years in the normal academic or normal technical stream. The GCE N(T) Level-Normal Technical Level, is taken by Normal Technical students after four years of secondary school education. This will eventually lead them to the ITE or Institute of Technical Education. The GCE 'O' Level examinations, or more commonly known as 'O' Levels, are conducted annually in Singapore. Like the 'N' Levels, they are done after four years of express or five years of normal academic secondary education and under the same examining authority. The Singapore-Cambridge GCE Advanced Level examination, like the other examinations, is conducted annually. It is taken before the completion of 2 years of Junior College or 3 years at Millenia Institute (tertiary education) at the post-secondary level. The Singapore-Cambridge GCE 'A' Level examinations require students to read a compulsory H1 General Paper subject or alternative-H2 Knowledge and Inquiry (KI) alongside with 3 Higher-2 and 1 Higher-1 subjects (minimum of 10 Academic Units (A.U)).	
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	Yes calculators are allowed. There is a list of approved scientific and graphing calculators (see attached file).	
If GDC are allowed, do they need to be reset before the start of the	The regulations (also in the file) are: 2. The following guidelines are to be adhered to in the use of scientific	

[15] See http://www.seab.gov.sg/

examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?	calculators in national examinations. (a) The calculator must be silent, with a visual display only. (b) The working condition of the calculator is the responsibility of the pupil and a fault in a calculator cannot be used as a reason for seeking special consideration for the user. (c) Calculators must not be borrowed from other pupils in the course of the examination for any reason. (d) External storage media, e.g. card, tape, and plug-in modules, must not be in the possession of pupils during the examination. (e) No unauthorised materials, e.g. instructions leaflets, formulae printed on the lid or cover of a calculator or similar materials, must be in the possession of pupils during the examination (where the instructions cannot be removed they should be securely covered). (f) No programmable calculators are allowed. (g) No calculators with permanent features of a programmed kind are allowed, e.g. calculators capable of numerical integration, numerical differentiation, and/or expressing in irrational number form. (h) No calculators with special communication features are allowed, e.g. calculators with the capability of remote communication via infra-red or blue-tooth with other machines. (i) No calculators with capabilities for storing and displaying visual and verbal information are allowed. The CIE A level syllabus says: It is expected that candidates will have a calculator with standard 'scientific' functions available for use for all papers in the examination. Computers, and calculators capable of algebraic manipulation, are not permitted.	
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic	Exam rubric states: Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an electronic calculator is expected, where appropriate.	

words' to indicate this?	Eg (iii) Use the iterative formula $\theta_{n+1} = \frac{1 - sin\ \theta_n}{cos\ \theta_n}$, with initial value θ_1 = 0.5, to determine the value of θ[16] correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3] versus (iii) Find the exact value of $\int_0^{\frac{\pi}{4}} (1 + \tan^2\theta - 3\sec\theta\ \tan\theta)\ d\theta$. [5]	
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are tasks designed in such way that technology just supports the solution process, or that is of no value at all?	Just a support in the solution process.	
Please add other comments and information that you consider relevant but that is not addressed in the questions		

[16] Where $\theta = \frac{1 - sin\ \theta}{cos\ \theta}$

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	No, not really.	
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?	Website support, as above, for both technical and pedagogical use of ICT.	
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	Strong online teacher support.	
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	The mp3 plan is underway until 2015 and then there may be a change.	
Please add other comments and information that you consider relevant but that is not addressed in the questions		

References

http://ictconnection.moe.edu.sg/masterplan-3/implementation-strategies/ict-in-curriculum-pedagogy-n-assessment
Khoon Yoong Wong (Ed.) (2009). Mathematics education: The Singapore journey. World Scientific Publishing Co. Ltd: Singapore.

Examples

See files

Questionnaire 6

Respondent name: Mike Thomas **Country described in the response: Australia, Victoria**

Part 1: Overall description

Question	Answer
Please describe the place of 16-19 academic mathematics within the national system	Prior to 2011 Australia had a system where each state had its own curriculum. On 20-12-2012 a national curriculum for senior secondary students was first published (see http://www.australiancurriculum.edu.au) in 14 subjects, including mathematics. The curriculum is run by the Australian Curriculum, Assessment and Reporting Authority (ACARA – http://www.acara.edu.au/default.asp). Version 5.2 of the mathematics curriculum is dated 24-12-2013 and is used here. It says (p. 6) "The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years." "The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training. General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series. Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation. Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that has been designed to not be taken as a stand-alone subject."

Part 2: Curriculum

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is the use of ICT explicitly part of the mathematics curricula? If yes, how is this addressed and described?	The curriculum, page 8 says "**Role of technology** It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects." and one of 7 general capabilities is: **"ICT in Mathematics** In the senior years students use ICT both to develop theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations." (p. 9) Access to technology to support the computational aspects of these topics is assumed. It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills. A stated aim is: [Develop a] capacity to choose and use technology appropriately Examples of curriculum statements: 1. using technology to translate two-dimensional house plans into three-dimensional buildings 2. use a calculator for multi-step calculations (ACMEM005) 3. use technology to find the line of best fit (ACMEM142)	In a critique of the new curriculum and its ability to influence classroom practice, Goos in [1] states "The pedagogical opportunities afforded by the curriculum are still restricted to the level of *tasks* in Pierce and Stacey's (2010) taxonomy, in that teachers are encouraged to use technology to improve speed and accuracy, link mathematical representations, or work with real data. To be fair, it is unrealistic to expect a curriculum document to transform *classroom interactions* (the second level of Pierce and Stacey's framework), since this remains in the realm of pedagogy. Nevertheless, a truly future-oriented mathematics curriculum might make a more serious attempt at transforming the *subject* itself, by (1) supporting curriculum goals that increase emphasis on concepts, applications, and mathematical thinking, or (2) changing the way that mathematical topics are approached and sequenced." (p. 150) "The second part [of the curriculum] offers some snapshots of practice to illustrate what effective classroom practice can look like when technologies are used in creative ways to enrich students' mathematics learning." (*ibid*., p. 136) "The intention was that use of ICT was to be referred to in content descriptions and achievement standards. Yet this was done superficially throughout the first published version of the curriculum, with technology often being treated as an add-on that replicates by-hand methods.) (*ibid*., p. 146)

	4. perform simulations of experiments using technology (ACMEM150) 5. find distances between two places on Earth using appropriate technology. (ACMEM161) 6. with the aid of a calculator or computer-based financial software, solve problems involving… 7. use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights. (ACMGM012) 8. perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate (ACMGM015) 9. construct straight-line graphs both with and without the aid of technology (ACMGM040) 10. solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained. (ACMMM019) 11. solve equations involving exponential functions using technology, and algebraically in simple cases. (ACMMM067) 12. estimate the limit of $\frac{a^h-1}{h}$ as $h \to 1$ using technology, for various values of $a > 0$ (ACMMM098) 13. calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve problems (ACMMM170) 14. use numerical integration using technology (ACMSM126) Note: these appear to be primarily pragmatic in nature rather than epistemic.	
Do the opportunities that ICT offers impact on curriculum choices (e.g., integration by parts no longer needed,	"Irrational numbers can be approximated as closely as desired by rational numbers, and most electronic calculators use a rational approximation when performing calculations involving an irrational number." (Curriculum, p. 175)	

approximated solutions rather than exact ones, …)?		
Is ICT used in mathematics classes on a regular basis? If yes, what type of technology (IWB, GDC, laptop, desktop, …)? Who uses it, the teacher or the student? Are there specific computer labs in schools, or do regular classes have ICT facilities?		"Although the technology messages contained in the *Australian Curriculum: Mathematics* do not do justice to what research tells us about effective teaching and learning of mathematics, it is almost inevitable that there are gaps between an intended curriculum and the curriculum enacted by teachers and students in the classroom…Many teachers are already using technology to enhance students' understanding and enjoyment of mathematics." ([2], p. 150) In [3, p. 1] we read "**CAS use in Australia** Some Queensland schools, such as the Brisbane School of Distance Education have been using CAS calculators extensively in Mathematics B since the late 1990s. [4] Some other schools have been using them to a greater or lesser extent, both as hand-held and as computer based systems for similar or shorter periods. Victorian schools have been experimenting with a CAS enabled syllabus for several years and as a result of their trials (which have been supported by extensive research and professional development) the Victorian studies authority has mandated the use of CAS in their Mathematics B equivalent courses for 2009. Western Australia has recently decided to also mandate the use of CAS calculators from 2010. In other states, graphing calculators only are allowed. In New South Wales, they can only be used in middle school mathematics." A doctoral study [5] conducted in 26 public secondary schools in NSW 2005 to 2006 concluded that "These results indicate that past and current computer policies and professional development programs integrating the use of ICT in mathematics seem to be not effective in making teachers adopt the use of computer technology in their teaching practices. There should be a thorough monitoring technique or strategy that include[s] implementation, feedback and evaluation of technology plans by the stakeholders of the secondary education sector. Possibly more structured and ongoing professional development programs for mathematics teachers

		should be aligned to their needs and beliefs. Therefore, it is highly recommended that a leadership role of school executives should be a preference when implementing and encouraging teachers to use ICT in the mathematics classrooms". The study in [6] involved 929 teachers from 38 Queensland state schools. 10% never used ICT in teaching upper secondary mathematics, 66% sometimes, 21% often and 3% very often.
Is there any funding, e.g., by governmental institutions, for ICT integration? Or other kinds of resources?	Not that I am aware of.	
Do textbooks anticipate the availability of ICT?	Yes there are textbooks specifically addressing use of ICT.	
Are internet resources used in mathematics courses?		This will depend on the teacher but I would expect that they are quite widely used.
Are there any plans to extend the use of digital technology in mathematics classes in the nearby future? If yes, what kind of plans? What kind of technology? Are GDCs being replaced by other hardware such as tablets or smartphones?	With the new curriculum this is not likely, although note that the curriculum is under review by the new government.	
Please add other comments and		

informations that you consider relevant but that is not addressed in the questions	

Part 3: Assessment

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Are there national examinations for mathematics? If yes, how are they set up (duration, one or more parts, …)	Since the national curriculum is very new and the first assessment will likely be in 2014 these comments are based on what happens in the state of Victoria.	
Is the use of ICT allowed during the national examination? If yes, which types of technology? What are criteria? Are specific types or brands allowed?	Victoria developed a course with CAS integrated[17]: "Mathematical Methods (CAS) Units 3 and 4 consists of the following areas of study: 'Functions and graphs', 'Calculus', 'Algebra' and 'Probability', which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods (CAS) Units 3 and 4 are contained in Mathematical Methods Units (CAS) Units 1 and 2, and will be drawn on, as applicable in the development of related content from the areas of study, and key knowledge and skills for the outcomes of Mathematical Methods (CAS) Units 3 and 4. In Unit 3, a study of Mathematical Methods (CAS) would typically include a selection of content from the areas of study 'Functions and graphs', 'Algebra' and applications of derivatives and differentiation, and identifying and analysing key features of the functions and their graphs from the 'Calculus' area of study. This unit is studies in Year 12." According to [2, p. 45] "Victoria is the only jurisdiction to have moved from an established study, Mathematical Methods (1992–2009) to concurrent piloting of a related equivalent and	

[17] See http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/cas/casexams.aspx and the Table below.

	alternative study, Mathematical Methods CAS (2001–2005); then concurrent implementation of both fully accredited studies as equivalent but alternative (2006–2009) with a transition to the CAS version replacing the 'parent' version of the study from 2009 (Units 1 and 2 – Year 11 level) and 2010 (Units 3 and 4 – Year 12 level)." MM CAS has two exam papers: the first is 1 hour and no calculators are allowed. The second is two hours and all CAS calculators are allowed. The second paper has two parts; the first has around 22 multiple choice questions and the second has 4 extended response questions with multiple parts. Paper 1 states: "Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type." Paper 2 states that: "…one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used." The non-CAS examinations have a similar format of two papers. In the first: "**Examination 1** The examination will consist of short answer questions which are to be answered without the use of technology." while "**Examination 2** The examination will consist of two sections. Section 1 will consist of 22 multiple-choice questions worth 1 mark each and Section 2 will consist of extended answer questions, involving multi-stage solutions of increasing complexity…" For the second paper: "The following materials are permitted in this examination. …- A CAS calculator **or** CAS software, and, if desired, one scientific calculator. The memories of calculators need not be cleared for this examination."	
If GDC are allowed, do they need to be reset before the	For MM CAS paper 2 the explicit statement is made for the examination that: "…one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific	

start of the examination? Are additional applications and text files allowed? Is press-to-test mode used? How are all these regulations controlled in schools?	calculator. For approved computer-based CAS, their full functionality may be used." For Specialist Mathematics paper 2 it says "The memories of calculators need not be cleared for this examination." and "The VCAA publishes details of approved technology for use in mathematics examinations annually. Details of approved calculators are published in the October *VCAA Bulletin*. The current list may be found at the VCE Specialist Mathematics Study page on the VCAA website." See below for the approved calculator list[18].	
Are tasks phrased in such a way that the student knows if algebraic / exact answers are required, or if approximations found with the GCD will do? Are there 'magic words' to indicate this?	The rubric states "A decimal approximation will not be accepted if an **exact** answer is required to a question."	
Is the use of ICT during examinations rewarded, in the sense that the student gets credits for appropriate use, or for answers that are found by just using ICT? Or are	The aim in the MM CAS curriculum was to make the curriculum and assessment CAS positive and CAS –produced answers would be credited. In the 'standard' courses it is more of an 'add-on'.	

[18] See also http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/approvedcalculators.aspx

tasks designed in such way that technology just supports the solution process, or that is of no value at all?		
Please add other comments and informations that you consider relevant but that is not addressed in the questions	Australia is considering a move to national online assessment. See http://www.acara.edu.au/assessment/assessment.html In MM CAS [2, p. 45] states that "mean score data on the technology free Examination 1 for 2006–2009 consistently indicate that, in general, the Mathematical Methods (CAS) cohort perform at least as well as the Mathematical Methods cohort on related questions."	

Part 4: Implementation Strategy

Question	Answer regarding the official, intended curriculum	Answer regarding the real, implemented curriculum
Is there a debate going on concerning the use of ICT in mathematics classes? If yes, what are the main issues and opinions?	There has been some debate over how much ICT should be in the new curriculum, but this has now been more or less settled.	
Is there support for teachers' professional development with respect to integrating ICT in their teaching? If yes, is this technically oriented, or also pedagogical?	Not known, but I suspect not a great deal.	
Is ICT used for supporting ICT integration, for example blended teacher education (pre- and in-service), online courses for professional development, MOOCs?	I'm not aware of any.	
Are there any future plans to implement new curricula with a different role for ICT than is the case at present? If yes, how would you describe this changing role?	On 10-1-2014 the government announced "To ensure we have the best possible curriculum the government has appointed Professor Ken Wiltshire AO and Dr Kevin Donnelly to conduct a review of the Australian Curriculum. The review will evaluate the robustness, independence and balance of the Australian Curriculum, examining the content and development process."	
Please add other comments and informations that you consider relevant but that is not addressed in the questions		

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Hudson, R. (2010). ICT Use to Improve Mathematics Learning in Secondary Schools In D. Gronn, & G. Romeo (Eds.) *ACEC2010: Digital Diversity.* Conference Proceedings of the Australian Computers in Education Conference 2010, Melbourne 6-9 April. Carlton, Victoria: Australian Council for Computers in Education (ACEC). Retrieved from, http://acec2010.acce.edu.au/proposal/970/ict-use-improve-mathematics-learning-secondary-schools

Jamieson-Proctor, R. M., Burnett, P. C., Finger G., & Watson, G. (2006). ICT integration and teachers' confidence in using ICT for teaching and learning in Queensland state schools. *Australasian Journal of Educational Technology*, 22(4), 511-530.

Examples

Table below from [2]

Table 1: Assumed technology for end of year 12 final examinations in Victoria from 1970

Stage	Assumed technology for end of Year 12 examinations in Victoria
Pre-1978	Four-figure logarithm tables and/or an approved slide rule.
1978–1996	Scientific calculator. Until 1990 there was a single 3-hour examination. From 1991 there were two 1½-hour examinations.
1997	Scientific calculator – approved graphics calculator permitted but not assumed.
1998–1999	Approved graphics calculator assumed for Mathematical Methods and Specialist Mathematics (both examinations). Scientific calculator with bivariate statistical functionality or approved graphics calculator assumed for Further Mathematics (both examinations).
2000–2005	Approved graphics calculator for Further Mathematics, Mathematical Methods and Specialist Mathematics (both examinations). Approved CAS (calculator or software) for Mathematical Methods CAS pilot study, 2002–2005 (both examinations).
2006–2009	Approved graphics calculator or CAS for Further Mathematics (both examinations). Mathematical Methods and Mathematical Methods (CAS) were alternative but like studies with a common technology free Examination 1 (worth 40 marks) and a separate technology assumed Examination 2 (worth 80 marks), with around 70% – 80% common material, approved graphics calculator assumed for Mathematical Methods Examination 2, approved CAS assumed for Mathematical Methods (CAS) Examination 2. Specialist Mathematics – technology free Examination 1. Approved graphics calculator or CAS assumed for Examination 2 (technology active but graphics calculator/CAS neutral).
2010–2013	Approved CAS or graphics calculator assumed for Further Mathematics (both examinations). Mathematical Methods (CAS) and Specialist Mathematics each have a 1-hour technology free examination. Mathematical Methods (CAS) and Specialist Mathematics each have a 2-hour technology active examination. An approved CAS (calculator or software) is the assumed technology.
2014 and beyond	(Draft) Australian curriculum has four senior secondary mathematics studies: Essential mathematics (Course A); General mathematics (Course B); Mathematical methods (Course C) and Specialist mathematics (Course D), currently under consultation. If things proceed well, 2014 could be the first year of implementation in Victoria. Assessment remains the province of states and territory jurisdictions for the interim.

Victoria approved calculators from http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/approvedcalculators.aspx

Approved Calculators for Specified VCE Mathematics Examinations 2013

- CAS Calculators
- CAS Software
- Graphics calculators

Use of CAS and calculators in 2013 VCE Mathematics examinations

Further Mathematics

Either one approved CAS or one approved graphics calculator may be used in Further Mathematics Examinations 1 and 2.
A scientific calculator may also be used, if desired, in Further Mathematics Examinations 1 and 2.

Mathematical Methods (CAS) and Specialist Mathematics

One approved CAS may be used in Mathematical Methods (CAS) Examination 2 and Specialist Mathematics Examination 2 only.
No CAS or calculators of any kind are permitted in Mathematical Methods (CAS) Examination 1 and Specialist Mathematics Examination 1.
The use of a graphics calculator is NOT permitted in either Mathematical Methods (CAS) Examination 2 or Specialist Mathematics Examination 2.
A scientific calculator may also be used, if desired, in Mathematical Methods (CAS) Examination 2 and Specialist Mathematics Examination 2.

CAS Calculators

In 2013, the following CAS calculators are approved by the VCAA for use in Further Mathematics Examination 1 and Further Mathematics Examination 2, Mathematical Methods (CAS) Examination 2 and Specialist Mathematics Examination 2.
The full functions of approved CAS calculators may be used (that is, the memories of these calculators do not require clearing prior to entry to the examination).

Casio

Algebra FX2.0, Algebra FX2.0 PLUS, *ClassPad* 300, *ClassPad* 300 PLUS, *ClassPad* 330, *ClassPad* 330 PLUS

Hewlett Packard

HP 40G, HP 40GS, HP 48G, HP 48G II, HP 49G, HP 49G PLUS, HP 50G

Texas Instruments

TI-89, TI-89 (*Titanium*), TI-92/TI-92 PLUS/Voyage 200, TI-*n*spire CAS, TI-*n*spire CAS with *Touchpad*, TI-*n*spire CX CAS

CAS Software

For approved schools only, students enrolled in Mathematical Methods (CAS) either by itself or in addition to Further Mathematics or Specialist Mathematics, will be permitted to use computer-based CAS software *Derive*, *Maple*, *Mathcad*, *Mathematica*, TI-Interactive, TI-*n*spire CAS and *ClassPad Manager* and stored files on a CD-ROM, DVD or USB for examinations in these studies, where the use of technology is permitted, provided they meet VCAA specifications for the conduct of computer-assisted examinations.
Schools wishing to use computer-based CAS software should apply in writing to the VCAA for approval.
Enquiries can be directed to Examinations Unit at examinations.vcaa@edumail.vic.gov.au

Graphics calculators

In 2013, the following graphics calculators are approved by the VCAA for use in Further Mathematics Examination 1 and Further Mathematics Examination 2 only. The full functions of approved graphics calculators may be used (that is, the memories of these calculators do not require clearing prior to entry to the examination).

Casio

FX-7300G, FX7400G, FX7400G PLUS, FX7700G, FX-8500G, FX-9700G, CFX-9800G, CFX-9850G, CFX-9850G PLUS, CFX-9850GB PLUS, CFX-9850GB PLUS-WE, CFX-9850GC, FX 9860G AU, CFX-9950G, FX 9860G AU Plus

Citizen

SRP-400G

Hewlett-Packard

HP 38G, HP39G, HP39G PLUS, HP39GS, HP39GII

Sharp

EL-9200, EL-9300, EL-9400, EL-9600, EL-9650, EL-9900

Texas Instruments

TI-80, TI-81, TI-82, TI-83, TI-83 PLUS, TI-83 PLUS (*Silver*), TI-84 PLUS, TI-84 PLUS (*Silver*), TI-85, TI-86

NB Examination papers are available.