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Kris de Jaegher

Universiteit Utrecht

Utrecht School of Economics

How to reach the authors

Please direct all correspondence to the first author.

Kris de Jaegher
Utrecht University
Utrecht School of Economics
Janskerkhof 12
3512 BL Utrecht
The Netherlands
E-mail: k.dejaegher@econ.uu.nl

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The Evolution of Horn's Rule

Kris de Jaegher[a]

[a]Utrecht School of Economics
Utrecht University

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Abstract
Horn's rule says that messages can be kept ambiguous if only a single interpretation is plausible. Speakers only perform costly disambiguation to convey surprising information. This paper shows that, while noncooperative game theory cannot justify Horn's rule, evolutionary game theory can. In order to model the evolution of signalling, the pooling equilibrium needs to be one's starting point. But in such an equilibrium, the plausible interpretation is made, and the receiver is therefore already predisposed to interpret absence of a signal as referring to a plausible event. From there on, a marked signal referring to an implausible event can evolve. At the same time, the paper identifies an exception to Horn's rule. If giving a plausible interpretation for an implausible event is very costly, then in the pooling equilibrium the implausible interpretation is always made. In this exceptional case, only an inefficient separating equilibrium disobeying Horn's rule can evolve.

Keywords: Horn's Rule, Signalling, Evolutionary Game Theory, Evolutionary Drift, Pragmatics.

JEL classification: C72, D82

1. Introduction

Horn's rule (1984) is the general rule in pragmatics saying that (un)marked expressions get an (un)marked interpretation. For instance, if I argue that every ten minutes, a man gets mugged in New York, then normally you will interpret this as meaning that this concerns a different man every ten minutes. Yet, strictly speaking, I could make the same statement to imply that every ten minutes, the same very unlucky New Yorker gets mugged. Thus, strictly speaking, my statement is ambiguous. The reason that you give my statement the first interpretation rather than the second is because you estimate it to be far more likely that the first interpretation is the correct one. But why do I not make my statement less ambiguous? Presumably, disambiguation is costly. If I have to eliminate all ambiguity, this will cost me more time. And as you are unlikely to make the second interpretation, I can leave my statement ambiguous without too much danger.

Some efforts have been made to account for Horn's rule by means of noncooperative game theory (Parikh 1991, 2000, 2001), using signalling games. Yet, from the perspective of noncooperative game theory, there is no reason why Horn's rule would apply. It may very well be that you and I both know that costs will be saved if I do not need to disambiguate. But if I believe that you will interpret the sentence 'Every ten minutes, a man gets mugged in New York' as meaning one and the same man, then each time I want to make the more plausible statement, I will have to point out that I do not mean one and the same man. And once I talk in this way, if I do not specify that I do *not* mean one and the same man, you will interpret it as meaning one and the same man. Simply, there are multiple Nash equilibria to the signalling game, and our mutual beliefs can lock us into an inefficient equilibrium.

Parikh argues that the fact that the signalling equilibrium that follows Horn's rule is Pareto efficient will make it a focal point (Schelling 1960), and will cause it to be played. Yet, as pointed out by Van Rooij (2004), equilibrium selection based on Pareto efficiency is not a standard game-theoretic argument.

Additionally, there have been some recent efforts to check whether evolutionary game theory selects signalling equilibria that obey Horn's rule. As pointed out by Van Rooij (2004), all signalling equilibria, including equilibria that do not obey Horn's rule, are evolutionary stable equilibria (ESS), and therefore the ESS concept does not bring one any closer to a solution. Under replicator dynamics, Benz, Jäger and Van Rooij (2005) show for a specific game that the basin of attraction of an equilibrium that obeys Horn's rule is larger than the one of an equilibrium that does not, and that starting from a strategy profile where all strategies are played with equal probability, under replicator dynamics an equilibrium obeying Horn's rule evolves. Yet, this paper does not give much intuition for why this result is obtained.

The purpose of the current paper is to show that the evolution of an equilibrium that selects Horn's rule follows straightforwardly from the fact that a signalling equilibrium must at some point have evolved from a pooling equilibrium (otherwise, it does not make sense to study the evolution of signalling). In a pooling equilibrium, the optimal action taken by the receiver will correspond to the most likely event. The absence of a signal is therefore already interpreted as referring to a frequent event. From there on, the interpretation of a signal as referring to an infrequent event can evolve. For a signalling equilibrium where a signal refers to a frequent event to evolve from the pooling equilibrium, the receiver would somehow have to completely change his initial interpretation of the absence of the fact that no signal is received. It is this necessary reversal of meaning that makes it difficult for a signalling equilibrium that does not obey Horn's rule to evolve from a pooling equilibrium. The paper formalises this point, applying replicator dynamics to a simple signalling game.

Van Rooij (2006) recently gives an equilibrium selection argument in favour of signalling equilibria meeting Horn's rule that is somewhat similar to the one in this paper. Using the so-called *intuitive criterion* (Cho and Kreps 1987), he argues that, when the players are playing the pooling equilibrium, and when the signaller deviates from the equilibrium by sending a message, the receiver will then always interpret this as referring to the infrequent event. This is because only the signaller who has observed the infrequent state can become better off by deviating from the pooling equilibrium. A rational receiver will therefore infer that any out-of-equilibrium signal must have been sent in the infrequent state. In turn, expecting the receiver to make such an inference, the signaller will send a message in the infrequent state. However, in its standard form, the intuitive criterion only eliminates the pooling equilibrium, and not the inefficient signalling equilibrium. Once the players play the inefficient signalling equilibrium, the signaller who observes the infrequent state has no reason to start sending a message, because he is already doing the best he possibly can. Van Rooij (2006) solves this problem by arguing that one should take a pooling equilibrium as one's starting point. In this sense, his argument is similar to the one in this paper.

On top of the evolutionary argument in favour of Horn's rule, this paper also suggests an exception to the evolution of this rule, based on the same evolutionary argument. If it is extremely costly to take the wrong action when the infrequent state occurs, then the pooling equilibrium can be such that the action that is optimal in the infrequent state is always taken. In this case, by the argument in this paper, only the inefficient signalling equilibrium can evolve, because the pooling equilibrium has already predisposed the receiver who does not receive a signal to taking the action optimal in the infrequent state.

The paper is structured as follows. Section 2 sets out a basic signalling game, and provides the evolutionary argument backing up Horn's rule. Section 3 shows the exception to Horn's rule. The paper ends with a conclusion in Section 4.

2. The model

The model we use is the following basic discrete sender-receiver game. The sender is denoted by symbol *S*, and the receiver by symbol *R*. Let there be two states of the world, namely a frequent state *F*, and an infrequent state *I*. The probability that the frequent state occurs is denoted p_F, so that the probability that the infrequent state occurs is $(1-p_F)$. By assumption, $p_F > 0.5$. The receiver can choose one of two actions, namely action *F*, or action *I*. Player *i*'s utility when the receiver takes action *j* in state *k* is denoted by $U_i(j|k)$, where $i = S, R$ $j,k = I, F$. We assume that $U_i(j|j) > U_i(k|j)$, meaning that both the sender and the receiver prefer the receiver to take action *F* (respectively *I*) in state *F* (respectively *I*). The sender and the receiver thus have common interests. Also, we assume that

$$p_F U_i(F|F) + (1-p_F)U_i(F|I) > p_F U_i(I|F) + (1-p_F)U_i(I|I) \qquad (1)$$

meaning that a player *i* (where $i = S, R$) who does not know what state of the world occurs, prefers to action *F* to be taken. The sender knows what state of the world occurs, but the receiver does not. Finally at a cost $d > 0$, the sender can take an action (= sends a signal).[1]

For sufficiently small *d*, it is clear that this simple signalling game has two Nash signalling equilibria, namely one where the sender sends the signal in state *F*, and one where the sender sends the signal in state *I*. Given that $p_F > 0.5$ and that $d > 0$, it is clear that the Pareto efficient signalling equilibrium is the one where the sender sends the signal in state *I*. This equilibrium obeys Horn's rule. However, from the perspective of noncooperative game theory, this is no reason to discard the inefficient signalling equilibrium where the sender sends a signal in state *F*. Moreover, there are also Nash pooling equilibria where the sender does not send any signal, and where the receiver always takes action *F* when not receiving a message, and with sufficiently high probability also plans to take action *F* when receiving a message. These equilibria equally well cannot be discarded. Parikh's (2001) argument in favour of the efficient signalling equilibrium can be interpreted as follows. Given that it is common knowledge that one equilibrium is efficient, this makes this equilibrium a focal point, and will make the receiver make an interpretation following Horn's rule. However, this argument neglects the fact that players' mutual expectations can still lock them into playing a pooling equilibrium or playing the inefficient signalling equilibrium.

We will now reinterpret the sender-receiver game presented above as being a repeatedly played asymmetric population game[2], and let the frequency with which the sender and the receiver population each play their strategies follow the so-called replicator dynamics. This means that, from one period to another, the percentage change in the probability with which any sender or receiver plays a certain strategy is equal to the difference between the payoff obtained from the strategy played by the player himself, and the average payoff of the population to which the player belongs.

These replicator dynamics can be justified in several ways. The players may be successive generations of senders and receivers, who each time play the specified game one time, and where the utilities given are interpreted as their fitness (= survival value). Alternatively, the players may be interpreted as learning from the experience of previous players by a process of proportional imitation (Schlag 1998), where the probability that a strategy of a previous player is imitated is proportional to this player's payoff.

One way to apply evolutionary game theory is to check whether the two signalling equilibria and the pooling equilibrium described above are evolutionary stable, i.e. are robust against an invasion of small subpopulations playing alternative strategies. Both signalling equilibria are evolutionary stable, as they are strict Nash equilibria (see e.g. Swinkels (1992) for this argument in the context of asymmetric population games). The pooling equilibrium is not evolutionary stable for the following reason. It contains a description of what receivers do when they receive a signal. As receivers actually never receive a signal in the pooling equilibrium, any response to a signal is a weak best response. One could still argue in favour of the existence of an evolutionary stable set (ES set, see Balkenborg and Schlag 1998) of weak pooling equilibria if it does not matter which weak best response the receiver chooses. However, the receiver's response clearly does matter, and there is therefore no ES set of pooling equilibria. Another interpretation of this fact is that *evolutionary drift* may cause some receivers to start planning to take action *I* when receiving a signal. If it happens that a large enough proportion of receivers follow this strategy, it will become a best response for senders to send a signal in state *I*. This suggests that, starting from the pooling equilibrium, the signalling equilibrium which follows Horn's rule can evolve. We now formally look at the evolutionary dynamics through which the signalling equilibrium obeying Horn's rule can evolve from an initial pooling equilibrium, and at the same time we show why it is difficult for the signalling equilibrium that does not obey Horn's rule to evolve.[3]

We *first* separately consider the strategies in the pooling equilibrium and the signalling equilibrium that obeys Horn's rule, and abstract from any of the other possible strategies that players can take. In this case, the senders either do not send a signal at all, or send a signal in state *I*. The receivers either always choose action *F*, or choose action *F* when they do not receive a signal and choose action *I* when they do. These strategies, and the corresponding payoffs to the players, are represented in Table 1.

<*Table 1 about here*>

It should be stressed that Table 1 is not the game in strategic form, as it does not include all possible pure strategies for the players. However, Table 1 allows us to derive the replicator dynamics assuming that all other strategies are played with zero probability. This enables us to represent a face of the phase diagram of the entire game, including all strategies. Such a representation makes sense, as there are no streamlines pointing away from the face of the phase diagram, towards the strategies that are played with probability zero on the face. As soon as there are a few receivers taking action *I* when they receive a signal (and take action *F* otherwise), it is better for the sender to send a signal in state *I* rather than to only send a signal in state *F*, and rather than to send a signal in both states *F* and *I*. Similarly, as soon as there are a few senders who send a signal only in state *I*, it is better for the receiver to take action *I* when receiving a signal and action *F* when not receiving a signal, rather than to do the opposite, and rather than to take action *I* in any case.

As indicated in Table 1, the proportion of the senders who send a signal in state *I* is denoted by *p*, and the proportion of receivers who take action *I* when receiving a signal is denoted by *r*. To Table 1 then correspond the following replicator dynamics :

$$\frac{dr}{dt} = r(1-r)(1-p_F)\left[U_R(I|I) - U_R(F|I)\right] \tag{2}$$

$$\frac{dp}{dt} = p(1-p)(1-p_F)\left\{r\left[U_S(I|I) - U_S(F|I)\right] - d\right\} \tag{3}$$

The streamlines can now be described by dividing (2) by (3), and by applying partial integration, to obtain

$$(1-r)^{\left[U_S(I|I)-U_S(F|I)-d\right]} r^d (1-p)^{-\left[U_R(I|I)-U_R(F|I)\right]} = K \tag{4}$$

where to different values of the constant *K* correspond different streamlines. For $U_i(j|j) = 1$, $U_i(j|k) = 0$, $p_F = 2/3$, the streamlines are described by

$$p = 1 - K^{-1}(1-r)^{1/2} r^{1/2} \tag{5}$$

This case is represented in the phase diagram in Figure 1. It should be noted that for other values, the phase diagram will be similarly shaped. As represented by the solid line, the pooling equilibrium remains an equilibrium as long as there are relatively little receivers who interpret a signal as meaning that state *I* has occurred. However, as soon as a sufficient number (in this case, more than half) of the receivers interpret a signal as referring to state *I*, the streamlines lead to the signalling equilibrium that follows Horn's rule. Adding evolutionary drift to the replicator dynamics, there is therefore an evolutionary path from the pooling equilibrium to the efficient signalling equilibrium.

<Figure 1 about here>

Second, we consider in isolation the strategies in the pooling equilibrium and the signalling equilibrium that does not obey Horn's rule, and abstract from any of the other possible strategies that players can take. In this case, the senders either do not send a signal at all, or send a signal in state *F*. The receivers either always choose action *F*, or choose action *I* when they do not receive a signal and choose action *F* when they do. These strategies, and the corresponding payoffs to the players, are represented in Table 2.

<Table 2 about here>

Again, Table 2 is not the game in strategic form, as it does not include all possible pure strategies for the players, but still allows us to derive the replicator dynamics assuming that all other strategies are played with zero probability. Table 2 enables us to represent a face of the phase diagram of the entire game, including all strategies. Such a representation again makes sense, as there are no streamlines pointing away from the face of the phase diagram, towards the strategies that are played with probability zero on the face. As long as there are an insufficient number of receivers taking action F when receiving a signal and action I otherwise, it is best for the senders not to send any signals; when there is a sufficient number of receivers, it is a best response to send a signal in state F. As long as there is an insufficient number of senders sending a signal in state F, it is a best response for the receivers to always take action F; when there is a sufficient number of such senders, it is a best response for receivers to take action F when a signal is received, and action I otherwise.

As indicated in Table 2, the proportion of the senders who send a signal in state F is denoted by q, and the proportion of receivers who take action F when receiving a signal is denoted by s. To Table 2 then correspond the following replicator dynamics :

$$\frac{ds}{dt} = s(1-s)\left\{(1-p_F)\left[U_R(I|I) - U_R(F|I)\right] - p_F(1-q)\left[U_R(F|F) - U_R(I|F)\right]\right\} \qquad (6)$$

$$\frac{dq}{dt} = q(1-q)p_F\left\{s\left[U_s(F|F) - U_S(I|F)\right] - d\right\} \qquad (7)$$

The streamlines can again be derived by dividing (6) by (7), and by applying partial integration, to obtain

$$(1-s)^{\left[U_S(F|F)-U_S(I|F)-d\right]} s^d q^{(1-p_F)p_F^{-1}\left[U_R(I|I)-U_R(F|I)\right]-\left[U_R(F|F)-U_R(I|F)\right]} (1-q)^{-(1-p_F)p_F^{-1}\left[U_R(I|I)-U_R(F|I)\right]} = L \qquad (8)$$

where to different values of the constant L correspond different streamlines. For $U_i(j|j) = 1$, $U_i(j|k) = 0$, $p_F = 2/3$, the streamlines are described by

$$q = 1/2 \pm \frac{\left[1 - 4L^{-2}(1-s)s\right]^{1/2}}{2} \qquad (9)$$

This case is represented in the phase diagram in Figure 2. This time, there is no evolutionary path from the pooling equilibrium to the signalling equilibrium. Intuitively, this is because in the pooling equilibrium, the receiver is taking action *F* when not receiving any signal. In order for the signalling equilibrium that does not obey Horn's rule to evolve, however, the receiver needs to learn to take action *I* when not receiving any signal. Contrary to what was the case for the face in Figure 1, the receiver is not already predisposed to take the appropriate action when not receiving any signal.

<*Figure 2 about here*>

3. Exception to Horn's rule

Let it now still be the case that $p_F > 0.5$, but let it be, contrary to equation (1), the case that

$$p_F U_i(F|F) + (1 - p_F)U_i(F|I) < p_F U_i(I|F) + (1 - p_F)U_i(I|I). \tag{10}$$

Then it continues to hold that the efficient signalling equilibrium is the one where a signal gets sent in the *I*-state. However, given the analysis above, the equilibrium that will evolve is the signalling equilibrium that does *not* obey Horn's rule, and where a signal gets sent in state *F*. This is because in the pooling equilibrium, the receiver now takes action *I*, i.e. the action that is optimal in the infrequent state. The reason for this in turn is that the marginal cost of taking the wrong action in state *I* is much larger than the marginal cost of taking the wrong action in state *F*. Put otherwise, taking action *F* is more costly, because mistakenly taking action *F* is very costly. As the receiver is predisposed to take action *I* when not receiving a signal, only the signalling equilibrium that does *not* obey Horn's rule can evolve.[4]

As argued by Parikh (2000, 2001), $[U_i(I|I) - U_i(F|I)]$ = $[U_i(F|F) - U_i(I|F)]$ and $p_F > 0.5$ are sufficient conditions for equilibria obeying Horn's rule always to evolve. However, it is possible that taking the action that is best in the frequent state is considered very risky by the receiver. Going back to the example at the start of the paper, suppose that you are considering going to New York, suppose that you are worried about your safety, and that you ask me be about the crime rate in New York. Suppose that I tell you : « Every ten minutes, a man gets mugged in New York ». Then, even though you consider it unlikely that I mean to tell you that on average, you will get mugged every ten minutes, if you do not have any further information, you may prefer not to go to New York. It may be that it is only when I specify to you that I mean that every ten minutes, a different man gets mugged in New York, that you feel safe to go.

In general, if taking the action that is most frequently optimal is in fact risky, then contrary to Horn's rule, it may be the frequent state that is marked. Intuitively, suppose that a receiver either does or does not face a danger, and that in the infrequently occurring case of danger, precautionary action needs to be taken. Also, let it be the case that not taking precaution when there is danger is very costly. Then the type of signal that a signaller can evolve to send will not be an alarm signal sent when there is danger, but an all-clear signal when there is no danger. This is in spite of the fact that this equilibrium is Pareto-inefficient, as the cost of sending a signal is incurred more often than needed. Thus, it is interesting to see that in this case, evolutionary game theory provides a different prediction than focal-point theory.

4. Conclusion

The paper has shown that there is an evolutionary argument for Horn's rule if the fact that one state is more frequent than another also leads the uninformed receiver to take the action that is optimal in the frequent state. Intuitively, the fact that the receiver is initially predisposed to taking the action corresponding to the frequent state when no signal is received, enables the evolution of an equilibrium where a signal is sent in the infrequent state. However, if taking the action corresponding to the frequent state in the infrequent state is very risky, the uninformed receiver will still take the action corresponding to the infrequent state. In this case, the evolutionary path is towards an equilibrium that does *not* correspond to Horn's rule.

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23rd January 2007; Word Count: 3980.

Endnotes

[1] As pointed out by a referee, in the standard statement of Horn's rule, the choice is between a cheap message and an expensive message, and not between sending a costly message and sending no message at all, as is the case in the model. However, it suffices to interpret the sender's action of "doing nothing" as the sending of a costless signal to obtain a model that fits the standard statement of Horn's rule. Moreover, the argument extends to the case where the choice is between a cheap signal (which still has a positive cost attached to it) and a costly signal. It should be noted that in these alternative models, in the pooling Nash equilibrium, the cheap signal will always be sent.

[2] The argument extends to a more realistic environment where players sometimes play the role of signallers, and sometimes play the role of receivers. An asymmetric population game is considered here for simplicity.

[3] Why take the Nash pooling equilibrium as a starting point, and not for instance a population state where both sender types send the signal, and where receivers take action *F*? From the latter population state, by the same argument as used in the paper, the *inefficient* separating equilibrium can evolve. However, this neglects the question of how such a population state could ever arise. Surely, if we are to model the evolution of signalling, our starting point should be a situation without signalling. From such a pooling equilibrium, a situation where all sender types send the signal can never evolve.

[4] As pointed out by an anonymous referee, one could still argue that the separating equilibrium that can evolve under condition (10) is Pareto efficient for the following reason. Suppose that the cost of sending a message is negligible, and that the message gets lost with probability ε. Then sending a message in state *F* is Pareto superior to sending a message in state *I* if $p_F[(1-\varepsilon)U_i(F|F)+\varepsilon U_i(I/F)] + (1-p_F)U_i(I/I) > p_F U_i(F|F) + (1-p_F)\ [(1-\varepsilon)U_i(I|I) + \varepsilon U_i(F/I)]$.

It is easy to see that this boils down to equation (10). Simply, if the cost of taking action *F* in state *I* is very high, and if messages sometimes get lost, then it is better to send a message in state *F* than to send a message in state *I*. It should be stressed that this argument is quite different from Horn's rule, which focuses on the cost of sending a message, and does not consider noise.

References

Balkenborg, D. and Schlag, K. (1998) 'Evolutionary Stable Sets in Population Games', working paper, Department of Economics, University of Bonn.

Benz, Anton, Jäger, Gerhard and Rooij, Robert van (2005) 'An introduction to game theory for linguists', in Anton Benz, Gerhard Jäger and Robert van Rooij (eds) *Games and Pragmatics*, London: Palgrave Macmillan.

Cho, I.K. and Kreps, D. (1987) 'Signalling games and stable equilibria', *Quarterly Journal of Economics* 102, 179-222.

Horn, L. (1984) 'Towards a new taxonomy of pragmatic inference : Q-based and R-based implicature', in Deborah Schiffrin (ed.) *Meaning, Form, and Use in Context : Linguistic Applications* GURT84, Washington: Georgetown University Press, pp.11-42.

Parikh, P. (1991) 'Communication and strategic inference', *Linguistics and Philosophy* 14: 473-513.

Parikh, P. (2000) 'Communication, meaning, and interpretation', *Linguistics and Philosophy* 23: 185-212.

Parikh, P. (2001) *The Use of Language*, Stanford, California: CSLI Publications.

Rooij, R. van (2004) 'Signalling games select Horn strategies', *Linguistics and Philosophy* 27: 493-527.

Rooij, R. van (2006) 'Games and Quantity Implicatures', working paper, Department of Philosophy, Universiteit van Amsterdam.

Schelling, T. (1960), *The Strategy of Conflict*, New York: Oxford University Press.

Schlag., K.H. (1998) ‘Why Imitate, and If So, How?’, *Journal of Economic Theory* 78: 130-56.

Swinkels, J. (1992) ‘Evolutionary Stability with Equilibrium Entrants’, *Journal of Economic Theory* 57: 306-32.

		Sender	
		$(1-p)$	p
Receiver		Don't send any signal	Send signal in infrequent state
$(1-r)$	Always interpret frequent	$(p_F\ U_R(F\vert F) + (1-p_F)\ U_R(F\vert I),$ $p_F\ U_S(F\vert F) + (1-p_F)\ U_S(F\vert I))$	$(p_F\ U_R(F\vert F) + (1-p_F)\ U_R(F\vert I),$ $p_F\ U_S(F\vert F) + (1-p_F)\ [U_S(F\vert I) - d])$
r	Interpret frequent when no signal, and infrequent when signal	$(p_F\ U_R(F\vert F) + (1-p_F)\ U_R(F\vert I),$ $p_F\ U_S(F\vert F) + (1-p_F)\ U_S(F\vert I))$	$(p_F\ U_R(F\vert F) + (1-p_F)\ U_R(I\vert I),$ $p_F\ U_S(F\vert F) + (1-p_F)\ [U_S(I\vert I) - d])$

Table 1: Pooling equilibrium versus efficient separating equilibrium

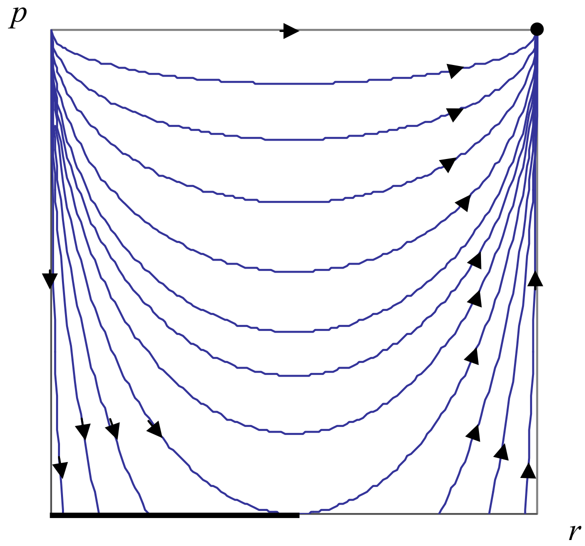

Figure 1 : Face of the phase diagram corresponding to Table 1

Sender

Receiver		Don't send any signal	Send signal in frequent state
$(1-s)$	Interpret frequent	$(p_F\, U_R(F\|F) + (1-p_F)\, U_R(F\|I)$, $p_F\, U_S(F\|F) + (1-p_F)\, U_S(F\|I))$	$(p_F\, U_R(F\|F) + (1-p_F)\, U_R(F\|I)$, $p_F\, [U_S(F\|F) - d] + (1-p_F)\, U_S(F\|I))$
s	Interpret infrequent when no signal, and frequent when signal	$(p_F\, U_R(I\|F) + (1-p_F)\, U_R(I\|I)$, $p_F\, U_S(I\|F) + (1-p_F)\, U_S(I\|I))$	$(p_F\, U_R(F\|F) + (1-p_F)\, U_R(I\|I)$, $p_F\, [U_S(F\|F) - d] + (1-p_F)\, U_S(I\|I))$

Table 2 Pooling equilibrium versus inefficient separating equilibrium

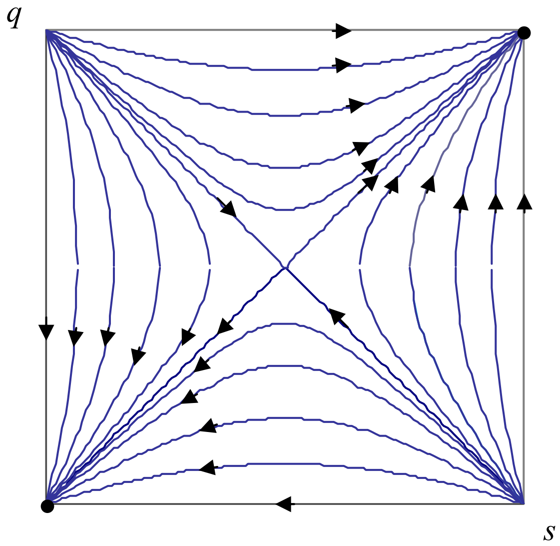

Figure 2 : Face of the phase diagram corresponding to Table 2