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Benchmark Two-Good Utility Functions

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Benchmark Two-Good Utility Functions

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Abstract
Benchmark two-good utility functions involving a good with zero income elasticity and unit income elasticity are well known. This paper derives utility functions for the additional benchmark cases where one good has zero cross-price elasticity, unit own-price elasticity, and zero own price elasticity. It is shown how each of these utility functions arises from a simple graphical construction based on a single given indifference curve. Also, it is shown that possessors of such utility functions may be seen as thinking in a particular sense of their utility, and may be seen as using simple rules of thumb to determine their demand.

Keywords: Benchmark Utility Functions, Rules of Thumb

JEL classification: D11

1. Introduction

Economists classify goods as normal or inferior goods, luxuries or necessities, substitutes for other goods or complements to other goods, elastic or inelastic goods, and finally ordinary goods or Giffen goods (Varian, 2003 ; Frank, 2003). Two-good utility functions corresponding to the benchmark cases where a good is neither a normal nor an inferior good (quasilinear preferences) and where a good is neither a luxury nor a necessity (homothetic preferences) are well-known and widely used. However, as illustrated in Table 1, benchmark utility functions where a good is neither a substitute for nor a complement to another good, is neither elastic nor inelastic, and is neither an ordinary nor a Giffen good have not been provided. Moreover, homothetic preferences are only treated in the specific cases of the Cobb-Douglas utility function and the constant-elasticity of substitution (CES) utility function. This paper fills these gaps by providing two-good utility functions for each of the missing cases.[1]

good 1 :	neither normal nor inferior	Neither substitute nor complement	neither price elastic nor price inelastic	neither luxury nor necessity	neither ordinary nor Giffen
benchmark utility function	quasilinear in good 2	?	?	homothetic preferences (general algebraic form ?)	?

Table 1 : Known and unknown benchmark utility functions

A *first* reason why these additional utility functions are of importance is a didactical one. While in the theoretical part of their microeconomic courses, by means of graphical examples students are exposed to a wide range of possible cases, as noted by Spiegel (1994), microeconomic exercises provide students with a very limited number of algebraic utility functions, the main ones being Cobb-Douglas preferences (based on Cobb and Douglas (1928) production function) and quasilinear preferences. Students could increase their skills by using a wider range of utility functions. While there is a literature containing examples of utility functions where one good is an inferior good[2] and/or a Giffen good[3], these utility functions have not found their way into microeconomic textbooks. One reason for this is that some of the utility functions presented are relatively complex.[4] Additionally, while homothetic and quasilinear

[1] The subject of this paper bears a relationship to the integrability conditions of Hurwicz and Uzawa (1971), who show under what conditions utility functions can be derived from given demand functions. However, our interest is in concrete algebraic utility functions for specific cases.

[2] See Epstein and Spiegel (2001), Weber (2001), and the references in the latter paper. Some of these references in fact are concerned with production functions involving an inferior input (e.g. Liebhafsky, 1969). However, these examples of production functions can equally well be used as utility functions.

[3] See Spiegel (1994) and Moffatt (2002), and the references in these papers.

[4] For instance, Spiegel's (1994) utility function leading to Giffen behavior consists of two parts, for different levels of the goods.

preferences have a clear graphical interpretation, the utility functions presented in this literature do not. The additional utility functions that we present do have a simple graphical interpretation, and can therefore be treated easily in a unified framework with quasilinear and homothetic utility functions. Needless to say, these utility functions can also be used to provide additional examples of production functions.

A *second* reason why our additional utility functions are useful is for applied theoretical research. Theoretical economists who want to give concrete algebraic specifications to the utility functions they use, also rely predominantly on well-known utility functions such as the Cobb-Douglas utility function or the quasilinear utility function, in part because of their analytic tractability. However, the assumptions underlying these utility functions may not always fit the situation that the theorist tries to model. With a Cobb-Douglas utility function, goods lack reservation prices for nonzero consumption (Yin, 2001), and with quasilinear preferences there is obviously no income effect on one good. Homothetic preferences furthermore do not allow for the case of asymmetric substitutability (the first good is substitute for the second good, while the second good is a complement to the first good[5]), and neither homothetic nor quasilinear preferences allow for the case where one good is an inferior good. As shown in this paper, such cases can be considered while keeping the underlying utility functions analytically tractable.

We argue that, when looking for analytically tractable utility functions, obvious candidates are precisely the missing cases in Table 1. What makes quasilinear and homothetic utility functions analytically tractable is that their underlying preference mappings can be constructed in a simple manner from a single given indifference curve. For quasilinear preferences, indifference curves are vertical shifts of a single indifference curve. For homothetic preferences, indifference curves are 'blown up' versions of a single indifference curve. Each of the benchmark utility functions that we derive can be based on the same type of graphical construction.

A *third* reason why our additional utility functions are of interest is that consumers who behave according to our benchmark utility functions may be seen as using simple rules of thumb to determine their demand (e.g. 'spend any increase in income entirely on good *x*' ; 'spend any decrease in the price of good *x* on other goods', etc). Boundedly rational consumers may find it easier to approach their true preferences by following such rules of thumb, rather than to calculate their optimal utility (Güth and Neuefeind, 2001). Consumers may learn by experience which of several possible simple rules of thumb best fits them. In this manner, by behaving according to one of our benchmark utility functions, they may still approach their true and ideal utility functions.

At the same time, consumers following our benchmark utility functions may be seen as taking as a reference point the welfare corresponding a given indifference curve, and as thinking of their gains or losses with respect to this reference point (cf. Kahneman and Tversky, 1979) in a particular way. For instance, a consumer who spends all extra income on good *x*, and compensates any income loss by saving on good *x*, may be seen as thinking of his or her gains with respect to the reference indifference curve as units of good *x* gained, and of his or her losses with respect to the reference indifference curve as units of good *x* lost.

[5] Consider a poor consumer who almost only consumes rice and meat. When the price of rice goes down, the consumer buys more meat, because the consumer likes eating rice along with meat. Meat is a complement to rice. When the price of meat goes up, the consumer buys more rice, because from the perspective of subsistence, rice is a substitute for meat. Rice is a substitute for meat.

Having thus justified our approach, let us now turn to the benchmark utility functions themselves. Denoting the quantity consumed of good 1 (good 2) by x_1 (x_2), we show how our benchmark utility functions can be constructed from a single given well-behaved indifference curve u_0 with generic form $x_2 = f(x_1)$, where $f'(.) < 0$ and $f''(.) > 0$. Put otherwise, for a single given indifference curve $x_2 = f(x_1)$, we find the utility function $u(x_1, x_2) = U[f(x_1), x_2]$ that meets certain properties. As our utility functions are based on a single given indifference curve, manipulating the shape of this given indifference curve allows the derivation of a myriad of utility functions. Section 2 repeats the well-known derivation of quasilinear preferences. This derivation is repeated to show the analogy with the new utility functions that we derive. Section 3 gives a general algebraic form of the utility function corresponding to homothetic preferences. Sections 4 to 6 respectively give the derivation of the general form of a utility function where one good is neither a substitute for nor a complement to the other good, where one good is neither price elastic nor price inelastic, and finally where one good is neither an ordinary good nor a Giffen good. Section 7 gives utility functions corresponding to more than one of the benchmark cases at the same time.

For each of these cases, we consecutively derive not only the general form of the direct utility function, but also of the indirect utility function. The reason for deriving the indirect utility function is to show that the consumer with a benchmark utility function may be considered as thinking of his or her utility not only in terms of changes in consumption (as follows from the direct utility functions), but also in terms of changes in prices or income. This observation again relates to the fact that consumers with benchmark utility functions can be interpreted as using simple rules of thumb to determine their demand. Such a rule of thumb is stated for each benchmark utility function. We end with a conclusion in Section 8.

2. Zero income effect for the first good

2.1. Zero income effect: direct utility function

Proposition 1: there is no income effect on the first good if and only if the direct utility function takes the form $u(x_1, x_2) = x_2 - f(x_1)$.

Proof:

Step 1. Consider a single given indifference curve $x_2 = f(x_1, u_0)$ of an unknown preference mapping $u(x_1, x_2) = U[f(x_1, u_0), x_2]$, and yielding a utility level of u_0. Consider a utility maximising bundle (x_1^*, x_2^*) on this indifference curve. For this bundle, it must be the case that $p_1 / p_2 = -f'(x_1^*, u_0)$.

Step 2. We now look for a preference mapping with $x_2 = f(x_1, u_0)$ as one of its indifference curves, and with the property that the consumption of good 1 does not depend on income. For this to be the case, at $x_1 = x_1^*$ and for different levels of x_2, the slope of any other indifference curve $x_2 = g(x_1)$ of the preference mapping must be equal to $-f'(x_1^*, u_0)$. By integration, at $x_1 = x_1^*$ and for different levels of x_2, indifference curves of the preference mapping must be locally described by

$x_2 = g(x_1^*) = k(U) + f(x_1^*, u_0)$, where *k* is a higher constant for indifference curves that lie higher, and where $k(u_0) = 0$.

Step 3. Note now that, as *k* ranks the indifference curves, it can itself be used as a measure of the consumer's utility ($k(U) = U$).

Step 4. The exercise in Steps 1 and 2 can be repeated for any bundle (x_1^{**}, x_2^{**}), to show that at $x_1 = x_1^{**}$, indifference curves must be locally described by $x_2 = g(x_1^{**}) = k + f(x_1^{**}, u_0)$. But as *k* can itself be interpreted as the consumer's utility (Step 3), it follows that for fixed *k*, with $k \neq 0$, the expression $x_2 = k + f(x_1, u_0)$ fully describes a new indifference curve. Also, for different levels of *k*, the expression $x_2 = k + f(x_1, u_0)$ describes the entire preference mapping.

Step 5. Finally, as *k* can itself be interpreted as the consumer's utility (Step 3), and using Step 4, the consumer's utility function takes the form $u(x_1, x_2) = x_2 - f(x_1, u_0)$. QED

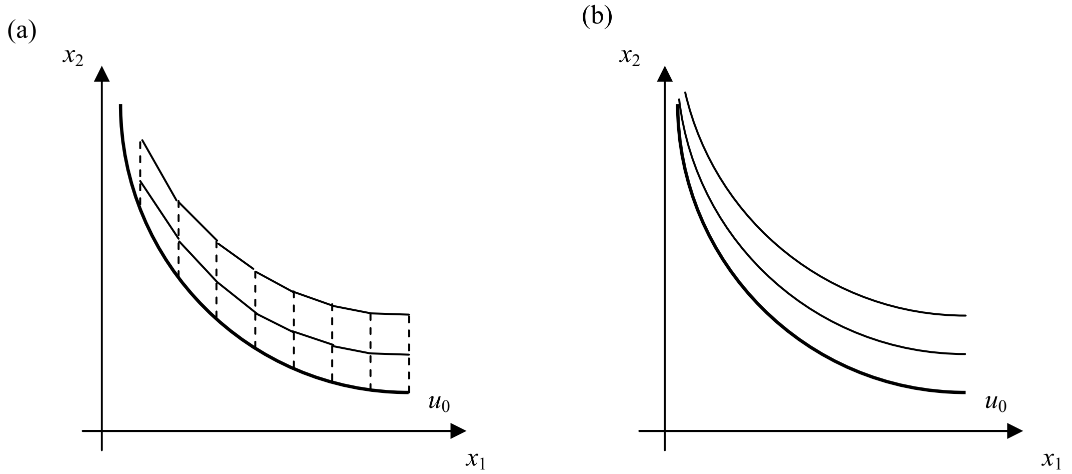

Figure 1 : construction of quasilinear preferences from a given indifference curve u_0 by adding a fixed amount of good 2 to some bundles on u_0. Panel (a) sketches new indifference curves by connecting the newly obtained bundles with straight lines. Panel (b) shows the smooth indifference curves obtained when the procedure is done for an infinite number of consumption bundles.

For a numerical example, let us start from a single given indifference curve $f(x_1) = \left(1 - x_1^a\right)^{1/a}$, with $a \leq 1$.[6] The corresponding quasilinear utility function for which this is one of the indifference curves is described by $u(x_1, x_2) = x_2 - \left(1 - x_1^a\right)^{1/a}$. The corresponding demand functions are $x_1 = \left[1 + \left(p_1 / p_2\right)^{\frac{a}{1-a}}\right]^{-\frac{1}{a}}$ and

[6] The reason for choosing this particular indifference curve is that, in Section 3, the well-known CES utility function is constructed from it. $1/(1-a)$ is therefore the elasticity of substitution along the given indifference curve u_0. By manipulating *a*, one can thus manipulate the curvature of the given indifference curve, with $a = 1$ (linear indifference curve) and $a = -\infty$ (*L*-shaped indifference curves) as extremes.

$x_2 = (m/p_2) - (p_1/p_2)\left[1+(p_1/p_2)^{\frac{a}{1-a}}\right]^{-\frac{1}{a}}$. It follows from Proposition 1 that graphically, when there are zero income effects, new indifference curves can be derived from a given indifference curve u_0 by adding a fixed level k to each level of x_2 on the given indifference curve. As shown in *Figure 1a*, by connecting the points thus obtained, one obtains a sketch of new indifference curves. As one adds k to the consumption level of good 2 for each bundle on the indifference curve u_0, one obtains the smooth indifference curves in *Figure 1b*. From the way in which quasilinear preferences are constructed, it is clear that such a preference mapping is well-behaved. Since indifference curves are vertical shifts of one another, each indifference curve is well-behaved as long as u_0 is well-behaved. Moreover, indifference curves that are vertical shifts of one another cannot cross.

2.2. Zero income effect : indirect utility function

Proposition 2 : there is no income effect on the first good if and only if the indirect utility function takes the form $v(p_1,p_2,m) = (m/p_2) + g(p_1/p_2)$, with $g'(.) < 0$.[7]
Proof :
Given the direct utility function derived in Proposition 1, one obtains the first-order condition $p_1/p_2 = -f'(x_1,U^*)$. Let us denote by $|f'|^{-1}$ the inverse function of the absolute value of the derivative of the given indifference curve. The direct demand function is then $x_1 = |f'|^{-1}(p_1/p_2)$. It follows that $x_2 = (m/p_2) - (p_1/p_2)|f'|^{-1}(p_1/p_2)$. Plugging these demand functions into the utility function derived in Proposition 1, we obtain $v(p_1,p_2,m) = (m/p_2) - (p_1/p_2)|f'|^{-1}(p_1/p_2) - f\left[|f'|^{-1}(p_1/p_2)\right]$, which can be restated as $v(p_1,p_2,m) = (m/p_2) + g(p_1/p_2)$. QED

It follows from Proposition 2 that for a zero income effect on the first good, new indifference curves can graphically be constructed from u_0 in the following alternative way. Take any bundle on the indifference curve u_0, and assume income and price levels such that this bundle is expenditure-minimising. In other words, draw a budget line tangent to u_0 at the chosen consumption bundle. The intercept of this budget line with the Y-axis shows us how much of good 2 the consumer can afford. Now, increase the maximum affordable consumption level of good 2 by a fixed amount, i.e. shift the budget line vertically. This new budget line is tangent to some new indifference curve. We can repeat this procedure for several bundles on the indifference curve u_0, each time using the *same* absolute increase in affordable level of good 2. Each of the new budget lines thus constructed is then tangent to one and the same new indifference curve. The new indifference curve therefore cannot lie below any of the constructed new budget lines. We can thereby sketch new indifference curves as shown in *Figure 2a*. By repeating the procedure for all consumption bundles on u_0, we finally obtain the smooth indifference curve in *Figure 2b*.

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[7] Using Roy's identity (Roy, 1947), which states that $x_1(p_1,p_2,m) = -\partial v(p_1,p_2,m)/\partial p_1 \left[\partial v(p_1,p_2,m)/\partial m\right]^{-1}$, it can be checked that the derived indirect utility functions indeed yield a demand for good 1 with the required property. This same test of the validity of the results can be applied to all indirect utility functions in this paper.

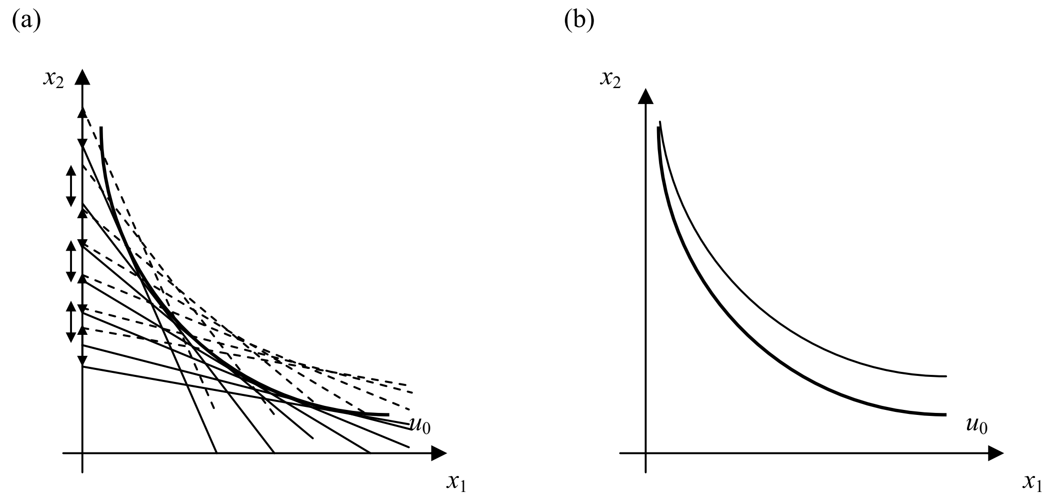

Figure 2 : construction of quasilinear preferences from a given indifference curve u_0 by adding a fixed amount to the intercept with the Y-axis of several budget lines tangent to u_0. Panel (a) sketches a new indifference curve tangent to the constructed dashed budget lines. Panel (b) shows the smooth indifference curves obtained when the procedure is repeated for an infinite number of budget lines tangent to u_0.

2.3. Zero income effect: measures of utility and rules of thumb

We first look at which rules of thumb correspond to quasilinear preferences. The consumer may be seen as using a rule of thumb telling him to spend every extra dollar of income on good 2. An equivalent rule of thumb says that, when the consumer needs to save on consumption after a reduction in income, he should do all his or saving by reducing his or her consumption of good 2.

Intuitively then, the consumer should think of his or her gains and losses in terms of units of good 1 gained, and units of good 1 lost. And indeed, this is the message of Propositions 1 and 2. Taking the welfare corresponding to a given indifference curve as a reference point, the increase in the maximum affordable level of good 2 (see indirect utility function), or the increase in the actual consumption of good 2 (see direct utility function), can be used as a measure of the consumer's welfare. Thus, as long as we know that the consumer is indifferent between two bundles A and B, and if we now add a fixed amount to each consumption level of good 2 in these two bundles, or instead increase the consumer's income such that he can afford a fixed amount extra of good 2, then we again obtain two bundles on the same indifference curve.

Concretely, attributing utility zero to the reference indifference curve, both numbers $(...,-\Delta^2 x_2, -\Delta^1 x_2, 0, \Delta^1 x_2, \Delta^2 x_2, ...)$ and numbers $[..., -(\Delta^2 m)/p_2,\ -(\Delta^1 m)/p_2,\ 0,\ (\Delta^1 m)/p_2,\ (\Delta^2 m)/p_2, ...]$ can be used to rank the consumer's indifference curves. $\Delta^i x_2$ refers to an increase in consumption of good 2 with respect to any bundle on the reference indifference curve, $(\Delta^i m)/p_2$ refers to an increase in the amount of good 2 that can be afforded, where changes are larger for larger *i*.

3. Unit income elasticity

3.1. Unit income elasticity : direct utility function

Proposition 3: if the first good is unit income elastic, the direct utility function is implicitly described by the equation $x_2 - \{u(x_1, x_2) f[x_1 / u(x_1, x_2)]\} = 0$.

Proof :

Step 1 is the same as in the proof of Proposition 1.

Step 2. We look for a preference mapping with $x_2 = f(x_1, u_0)$ as one of its indifference curves, and with the property that, when income changes by a certain percentage, consumption of good 1 changes by the same percentage. Take the bundle (x_1^*, x_2^*) on indifference curve u_0, and let the consumer's income increase by a proportion k. Then, in order to have unit income elasticity, for the bundle (x_1^{**}, x_2^{**}) optimal given this income, it must be the case that $x_1^{**} = kx_1^*$. As only income has changed, the slope of the indifference curve $x_2 = g(x_1)$ through the new optimal bundle must be equal to $-f'[(x_1^{**}/k), u_0]$ at $x_1 = x_1^{**}$. By integration, it follows that, at (x_1^{**}, x_2^{**}), the indifference curve must locally take the form $x_2 = g(x_1^{**}) = kf[(x_1^{**}/k), u_0]$. This applies for any $k(U)$, where k is a higher constant for indifference curves that lie higher, and where $k(u_0) = 1$.

Steps 3 and 4 are analogous to the proof of Proposition 1.

Step 5. As k can itself be interpreted as the consumer's utility function, the utility function is implicitly described by $x_2 - \{u(x_1, x_2) f[[x_1 / u(x_1, x_2)], u_0]\} = 0$. QED

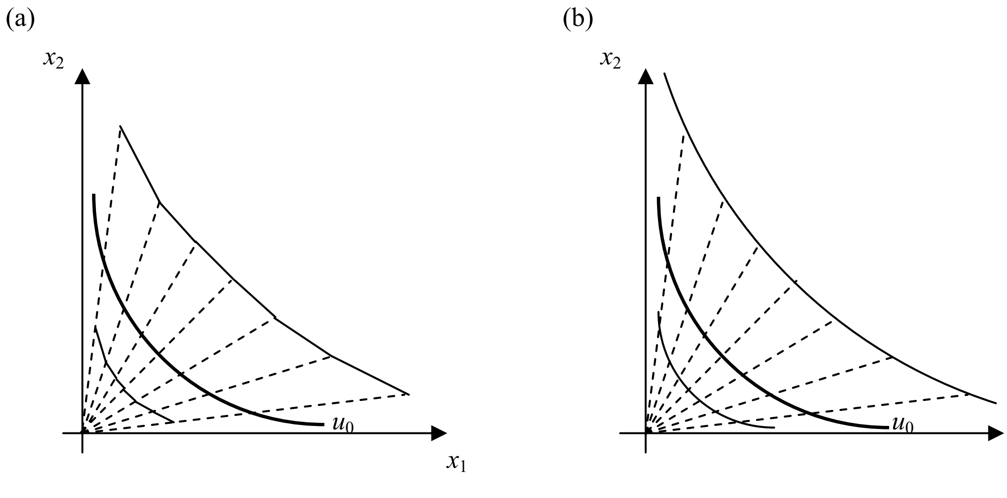

Figure 3 : homothetic preferences constructed from a given indifference curve u_0. Panel (a) takes some bundles on u_0, and changes them by the same proportion (half as much and double as much) ; sketches of the new indifference curves are obtained by connecting the new bundles obtained. Panel (b) shows the smooth indifference curves obtained when this procedure is repeated for every bundle on u_0.

When the given indifference curve takes the form $f(x_1) = (1 - x_1^a)^{1/a}$, the corresponding utility function with unit income elasticity on the first good (of which the given indifference curve is one of the indifference curves) can be checked to be

$u(x_1, x_2) = \left(x_1^a + x_2^a\right)^{1/a}$, which is nothing but the well-known CES utility function. The demand functions are $x_1 = m\, p_1^{-1/(1-a)} \left[p_1^{-a/(1-a)} + p_2^{-a/(1-a)}\right]^{-1}$ and $x_2 = m\, p_2^{-1/(1-a)} \left[p_1^{-a/(1-a)} + p_2^{-a/(1-a)}\right]^{-1}$. It follows from Proposition 3 that one can graphically construct new indifference curves from u_0 by increasing each consumption level in each bundle by the same fixed proportion. Going through this procedure for a few consumption bundles, one obtains a sketch of the indifference curves as in *Figure 3a*. When going through this procedure for every single bundle on u_0, one obtains the smooth indifference curves of *Figure 3b*. As higher indifference curves are just blown-up versions of u_0, the preference mapping is well-behaved : each indifference curve is well-behaved as long as u_0 is well-behaved, and indifference curves do not cross.

3.2. Unit income elasticity : indirect utility function

Proposition 4: if there is a unit income effect on the first good, the indirect utility function takes the form $v(p_1, p_2, m) = (m / p_2)\, g(p_1 / p_2)$ with $g'(.) < 0$, or the equivalent form $v(p_1, p_2, m) = (m / p_1) h(p_2 / p_1)$ with $h'(.) < 0$.

Proof :

The general form of the first-order condition is $p_1/ p_2 = -f'(x_1 / k)$, where k is utility (see Proposition 3). Denoting by $|f'|^{-1}$ the inverse function of the absolute slope of the given indifference curve, one finds the conditional demands $x_1 = k|f'|^{-1}(p_1/ p_2)$ and $x_2 = (m / p_2) - (p_1 / p_2) k |f'|^{-1}(p_1 / p_2)$. Plugging these conditional demands into the implicit specification of the direct utility function obtained in Proposition 3, and recalculating, one obtains the indirect utility function $v(p_1, p_2, m) = (m / p_2) \left\{(p_1 / p_2) |f'|^{-1}(p_1 / p_2) + f\left[|f'|^{-1}(p_1 / p_2)\right]\right\}^{-1}$. This expression again is equal to $(m / p_1)\ \left\{(p_1^2 / p_2^2) |f'|^{-1}(p_1 / p_2) + (p_1 / p_2) f\left[|f'|^{-1}(p_1 / p_2)\right]\right\}^{-1}$. Both of these expressions can be rewritten in the form given in the proposition. QED

It follows from Proposition 4 that for unit income effects, new indifference curves can graphically be constructed from u_0 in the following way. Take any bundle on the indifference curve u_0, and assume income and price levels such that this bundle is expenditure-minimising. In other words, draw a budget line tangent to u_0 at the chosen consumption bundle. The intercept of this budget line with the Y-axis shows us how much of good 2 the consumer can afford. Next, assume that the maximum affordable amount of good 2 increases by some fixed percentage, that is shift the budget line upwards. This new budget line is tangent to some new indifference curve. We can repeat this procedure for several bundles on the indifference curve u_0, each time using the same percentage change in affordable level of good 2. Each of the new budget lines thus constructed is then tangent to one and the same new indifference curve. The new indifference curve thus cannot lie below any of the constructed new budget lines. We can thereby sketch new indifference curves as shown in *Figure 4a*. By repeating the procedure for all consumption bundles on u_0, we finally obtain the smooth indifference curves in *Figure 4b*. The same indifference curves can also be constructed by changing the maximum affordable amount of good 1 by a fixed percentage.

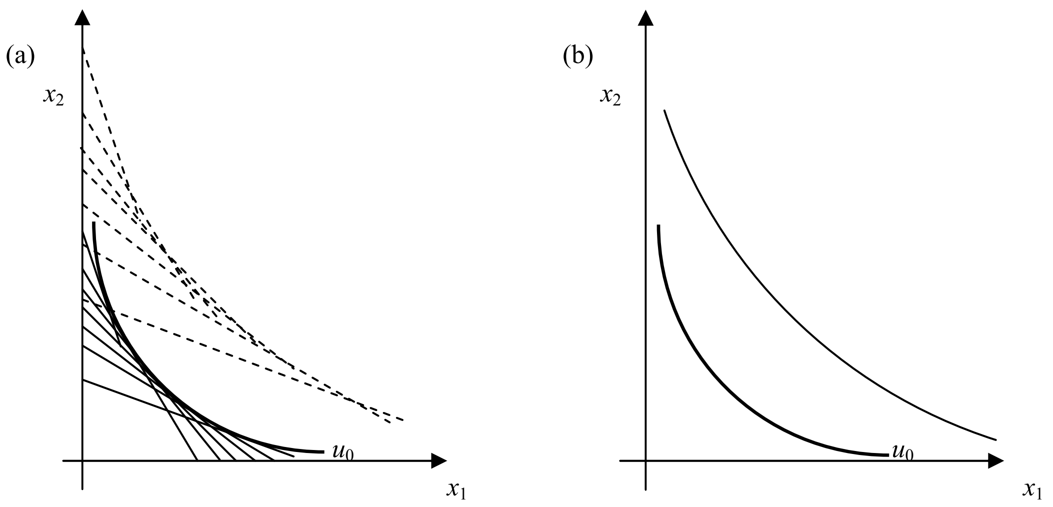

Figure 4 : homothetic preferences constructed from a given indifference curve u_0. Panel (a) takes some bundles on u_0, and doubles the intercept with the Y-axis of some budget lines tangent to u_0 ; the new indifference curve is tangent to the new budget lines thus obtained. Panel (b) shows the smooth indifference curve obtained when this procedures is repeated for every budget line tangent to u_0.

3.3. Unit income elasticity : measures of utility and rules of thumb

The rule of thumb used by the consumer for his consumption decision in this case is to let his consumption level of each good increase by the same percentage as the one by which income increases. Intuitively, the consumer then thinks of gains as percentage extra obtained of each good, and of losses as percentage lost of each good. Indeed, Propositions 3 and 4 can be interpreted as follows. With respect to a given reference indifference curve, the *percentage* by which a consumer's consumption of each good increases, or the *percentage* by which the maximum affordable consumption level for each good increases, can both be used as measures of the consumer's utility. Put otherwise, as long as we know that the consumer is indifferent between two bundles A and B, and if we now scale up the consumption levels in each bundle by a fixed percentage, or instead increase the consumer's income such that he can afford a fixed percentage more of each good, then we again obtain two consumption bundles lying on one and the same indifference curve.

Concretely, attributing utility zero to the reference indifference curve, both numbers $\left[\ldots,-(\Delta^2 x/x),-(\Delta^1 x/x),0,(\Delta^1 x/x),(\Delta^2 x/x),\ldots\right]$ and numbers $[\ldots,-(\Delta^2 m/p)/(m/p),$ $-(\Delta^1 m/p)/(m/p),0,\ (\Delta^1 m/p)/(m/p),\ (\Delta^2 m/p)/(m/p),\ldots]$ can be used to rank the consumer's indifference curves. $(\Delta^i x/x)$ refers to a percentage change in consumption of both goods with respect to any point on the reference indifference curve, and $(\Delta^i m/p)/(m/p)$ refers to a percentage change in the amount of each good that can be afforded with respect to any point on the reference indifference curve, where larger i refers to a larger change.

4. Zero cross-price effect

4.1. Zero cross-price effect : direct utility function

Proposition 5: there is no cross-price effect on the first good if and only if the direct utility function takes the form $u(x_1,x_2)=x_2 f(x_1)^{-1}$.

Proof :

Step 1 is the same as in the proof of Proposition 1.

Step 2. Starting from the bundle (x_1^*, x_2^*), let p_2 decrease by a proportion k (= be divided by a proportion k). Then, in order to have a zero cross-price effect on good 1, at $x_1 = x_1^*$, and for different levels of x_2, the slope of any other indifference curve $x_2 = g(x_1)$ must be equal to $-kf'(x_1^*,u_0)$, as this slope must also equal $(k/p_2)p_1$. By integration, at $x_1 = x_1^*$ and for different levels of x_2, indifference curves must be locally described by $x_2 = g(x_1^*) = k(U)f(x_1^*,u_0)$, where $k(U)$ is a higher constant for indifference curves that lie higher, and where $k(u_0)=1$.

Steps 3 and 4 are analogous to the proof of Proposition 1.

Step 5. As k can itself be interpreted as the consumer's utility function, the utility function is implicitly described by $u(x_1,x_2)=x_2 f(x_1,u_0)^{-1}$. QED

As an example, when the given indifference curve u_0 takes the form $x_2 = \left(1-x_1^a\right)^{1/a}$, the corresponding utility function with a unit price-elastic good 1 takes the form $u(x_1,x_2)=x_2\left(1-x_1^a\right)^{-1/a}$. The demand functions are $x_1 = (m/p_1)^{1/(1-a)}$ and $x_2 = (m/p_2)-(p_1/p_2)(m/p_1)^{1/(1-a)}$.

Evidently, any monotonically increasing function of the utility function derived in Proposition 5 is perfectly equivalent. Therefore, we can describe the same preference mapping by taking the natural logarithm of the derived utility function, to obtain $w(x_1,x_2) = -\ln f(x_1) + \ln x_2$. This function may be termed as *quasiloglinear* in the second good. It follows that, if the consumption of good 2 is measured on the logarithmic scale and the consumption of good 1 on the linear scale, that indifference curves are again vertical translates of u_0. When both goods are measured in the linear scale, any new indifference curve can be constructed from a given indifference curve u_0 by increasing each consumption level of good 2 on this indifference curve by a fixed percentage. By connecting the new bundles thus constructed from a few bundles on the original indifference curve, one thus obtains a sketch of the new indifference curves as in *Figure 5a*. As one performs this procedure for every single bundle on u_0, one obtains the smooth indifference curves of *Figure 5b*.

It can easily be seen that as long as the indifference curve u_0 is well behaved, then so will any indifference curve constructed from it in this way. As $x_2 = kf(x_1)$, it is true for any k that $\partial x_2/\partial x_1 < 0$, and that $\partial x_2^2/\partial x_1^2 > 0$. However, it is not necessarily the case that quasiloglinear preferences are transitive. As illustrated in *Figure 6*, as soon as u_0 has an intercept with the X-axis, all indifference curves will intersect on the X-axis. Consumers should now prefer bundle A to bundle C, but at the same time be indifferent between bundles A and B, and between bundles B and C, an inconsistency. We can either ignore this fact, e.g. by assuming that a minimum level of good 2 is always consumed, or we can impose the additional condition that u_0 should not have an intercept with the X-axis.

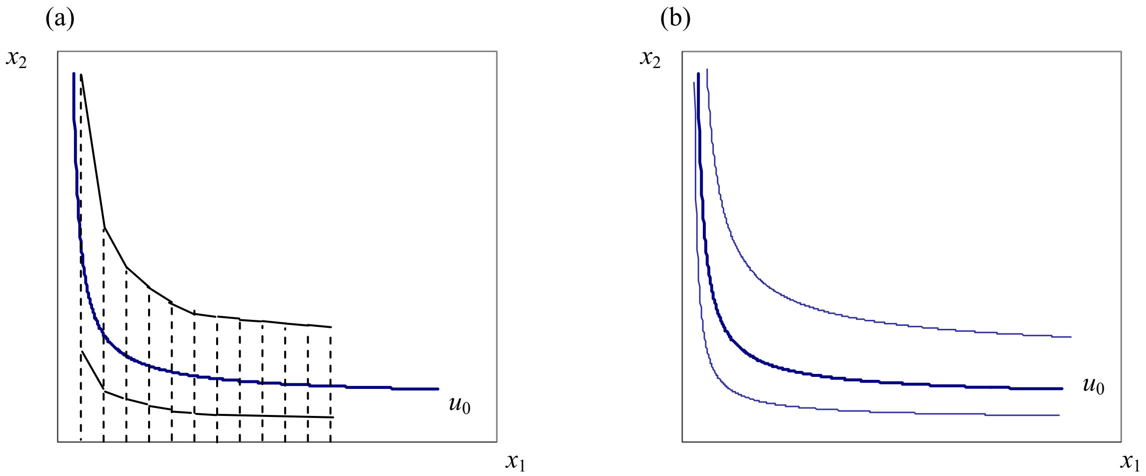

Figure 5 : Construction of quasiloglinear indifference curves from a given indifference curve u_0 by increasing the consumption level of good 2 by the same proportion (twice as much, half as much) for some consumption bundles on u_0.

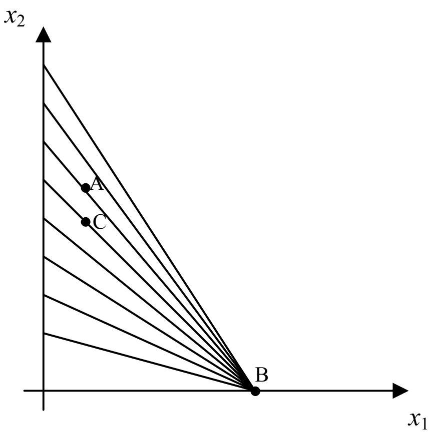

Figure 6 : quasiloglinear preferences constructed from a linear indifference curve.

4.2. Zero cross-price effect : indirect utility function

Proposition 6: there is no cross-price effect on the first good if and only if the indirect utility function takes the form $v(p_1, p_2, m) = (p_1 / p_2)\, g(m / p_1)$ with $g'(.) > 0$ or equivalently $v(p_1, p_2, m) = (m / p_2) h(p_1 / m)$ with $h'(.) < 0$.

Proof:

Given the direct utility function obtained in Proposition 5, it is easily calculated that the first-order conditions imply the equality $(p_1 / m) = \left[f(x_1) + x_1 |f'|(x_1)\right] / |f'|(x_1)$. The right-hand side is a function of x_1, and let us denote this function by $i(x_1)$. The direct demand functions are $x_1 = i^{-1}(p_1 / m)$, $x_2 = (m / p_2) - (p_1 / p_2) i^{-1}(p_1 / m)$. Plugging these into the direct utility function obtained in Proposition 5, one obtains

$v(p_1, p_2, m) = \left[(m/p_2) - (p_1/p_2) i^{-1}(p_1/m)\right] \left\{f\left[i^{-1}(p_1/m)\right]\right\}^{-1}$. This expression again is equal to $(p_1/p_2) \left[(m/p_1) - i^{-1}(p_1/m)\right] \left\{f\left[i^{-1}(p_1/m)\right]\right\}^{-1}$, and to (m/p_2) $\left[1-(p_1/m) i^{-1}(p_1/m)\right] \left\{f\left[i^{-1}(p_1/m)\right]^{-1}\right\}$, two expressions that can each time be re-expressed as the given indirect utility functions. QED

It follows from Proposition 6 that for zero cross-price effects on the first good, new indifference curves can be graphically constructed from u_0 in the following alternative way. Again take any bundle on the indifference curve u_0, and assume income and prices such that this bundle is the expenditure-minimising one. Now construct a new budget line by increasing the affordable amount of good 2 by a fixed percentage, keeping the affordable amount of good 1 fixed. Equivalently, construct a new budget line by increasing the opportunity cost of good 1 in terms of units of good 2 foregone by a fixed percentage, again keeping the affordable amount of good 1 fixed. The new budget line thus obtained is tangent to a new indifference curve. We can now repeat the same procedure for several bundles on u_0. As long as each time, we used the same proportion for every bundle along u_0, each new budget line obtained will be tangent to one and the same new indifference curve. Given that none of the new budget lines will lie above the new indifference curve, we can thus sketch the new indifference curves as shown in *Figure 7a*. Going through this procedure for every single bundle on u_0, we obtain the smooth indifference curves of *Figure 7b*.

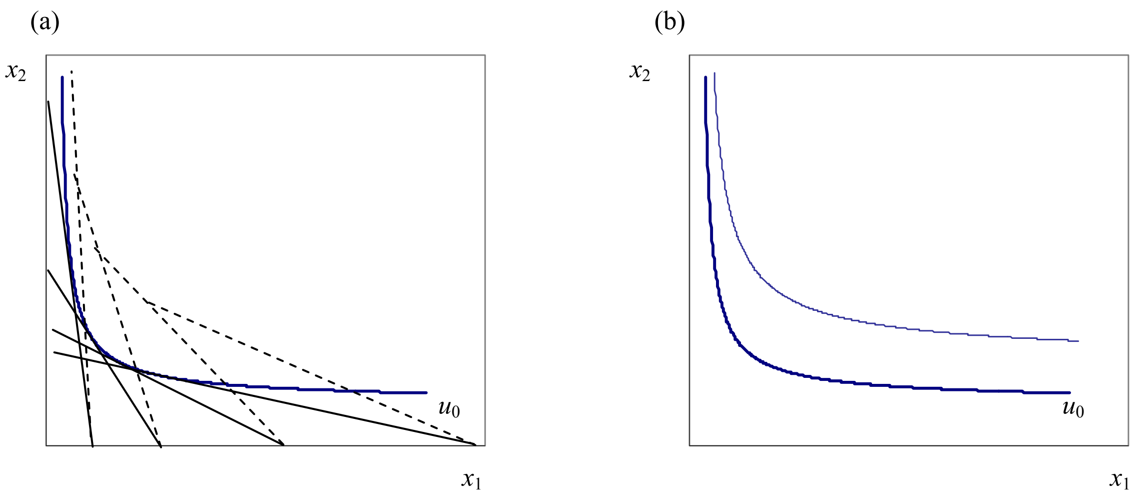

Figure 7 : Construction of quasiloglinear indifference curves from a given indifference curve u_0 by increasing the slope of the budget line tangent to u_0 by a fixed proportion (twice as large).

4.3. Zero cross-price effect : measures of utility and rules of thumb

A consumer with quasiloglinear preferences may be seen as using a rule of thumb telling him or her to spend any extra budget available because of a decrease in the price of good 2 on consumption of good 2, and to keep the consumption of or expenditure on good 1 fixed in response to changes in the price of the good 2.

Intuitively then, as all extra budget available from decreases in the price of good 2 is spent on good 2, and as the consumer thereby increases consumption of good 2 by the same percentage as the percentage decrease in price, the consumer may be seen as thinking of his or her gains in terms of the percentage change in the consumption of good 2. This indeed is the message of Propositions 5 and 6. For quasiloglinear preferences, from the viewpoint of a reference indifference curve, the *percentage* increase in consumption of good 2 or alternatively the *percentage* increase in consumption of good 2 affordable can be interpreted as a measure of utility. Additionally, one can use the *percentage* decrease in the opportunity cost of good 2 in terms of units of good 1 as a measure of utility. This is because to a percentage increase in consumption of good 2 corresponds an identical percentage decrease in opportunity cost.

Concretely, attributing utility zero to the reference indifference curve, both numbers $[\ldots,-(\Delta^2 x_2/x_2),-(\Delta^1 x_2/x_2),0,(\Delta^1 x_2/x_2),(\Delta^2 x_2/x_2),\ldots]$ and numbers $[\ldots,-(\Delta^2 m/p_2)/(m/p_2), \quad -(\Delta^1 m/p_2)/(m/p_2),0, \quad (\Delta^1 m/p_2)/(m/p_2), (\Delta^2 m/p_2)/(m/p_2),\ldots]$ can be used to rank the consumer's indifference curves. $(\Delta^i x/x)$ refers to a percentage increase in consumption of good 2 with respect to any point on the reference indifference curve, and $(\Delta^i m/p_2)/(m/p_2)$ refers to a percentage increase in the amount of good 2 that can be afforded with respect to any point on the reference indifference curve. Alternatively, numbers $[\ldots,\Delta^2(p_2/p_1)/(p_2/p_1), \quad \Delta^1(p_2/p_1)/(p_2/p_1),0, \quad -\Delta^1(p_2/p_1)/(p_2/p_1), -\Delta^2(p_2/p_1)/(p_2/p_1),\ldots]$ can be used to rank the indifference curves. $\Delta^i(p_2/p_1)/(p_2/p_1)$ refers to a percentage decrease in the opportunity cost of good 2 with respect to any point on the reference indifference curve. Each time, larger *i* refers to larger changes.

5. Unit own-price elasticity

5.1. Unit own-price elastic : direct utility function

Given that unit own-price elasticity of the first good implies zero cross-price elasticity of the second good[8], we can simply refer to the analysis in Section 4. Restating the indifference curve as $x_1 = f^{-1}(x_2)$, where f^{-1} is the inverse of *f*, the general form of a utility function where the demand for the first good is neither elastic nor inelastic is $u(x_1,x_2) = x_1\left[f^{-1}(x_2)\right]^{-1}$. We can describe the same preference mapping by taking the natural logarithm of the derived utility function, to obtain

[8] Suppose that we change p_1. Then the x_1 and x_2 need to change in such a manner that the budget constraint is still met. This implies that $x_1 + p_1\frac{\partial x_1}{\partial p_1} + p_2\frac{\partial x_2}{\partial p_1} = 0$, or $x_1\left(1+\frac{p_1}{x_1}\frac{\partial x_1}{\partial p_1}\right)+\frac{1}{x_2}\left(\frac{p_2}{x_2}\frac{\partial x_2}{\partial p_1}\right)=0$, or $x_1\left(1+\varepsilon_{x_1p_1}\right)+\frac{1}{x_2}\varepsilon_{x_2p_1}=0$. It follows that if $\varepsilon_{x_1p_1}=-1$ iff $\varepsilon_{x_2p_1}=0$. Intuitively, with unit own price elasticity, your expenditure on good 1 remains fixed. But if your income does not change, and your expenditure on good 1 does not change, then your consumption of good 2 cannot change.

$w(x_1,x_2) = \ln x_1 - \ln f^{-1}(x_2)$. This function may be termed as quasiloglinear in the first good.[9]

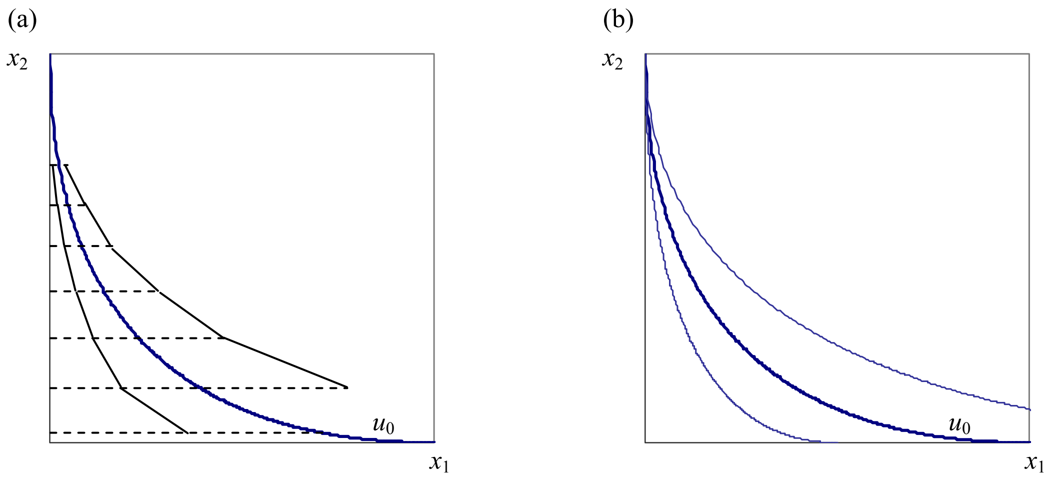

Figure 8 : Construction of preferences that are quasiloglinear in good 1 from a given indifference curve u_0. New indifference curve are sketched in panel (a) by connecting the new bundles obtained after changing the consumption level of good 1 for some bundles on u_0 by a fixed proportion (half as much and doubles as much in this case). Panel (b) shows the smooth indifference curve obtained when this procedure is repeated for many bundles along u_0.

Graphically, from a given indifference curve u_0, one can construct a new indifference curve corresponding to such preferences by increasing each consumption level of good 1 along u_0 by a fixed proportion. Connecting these points gives us a sketch of the shape of a new indifference curve, as shown in *Figure 8a*. Going through this procedure for each consumption level of good 1 yields us the smooth indifference curves of *Figure 8b*. It is again easy to check that indifference curves constructed in this way will be well-behaved as long as u_0 is well-behaved. However, preferences are only intransitive when u_0 has no intercept with the Y-axis.

When the given indifference curve u_0 takes the form $x_2 = \left(1 - x_1^a\right)^{1/a}$, the corresponding utility function with a unit price-elastic good 1 takes the form $u(x_1,x_2) = x_1\left(1 - x_2^a\right)^{-1/a}$. The demand functions are $x_1 = (m/p_1) - (p_2/p_1)(m/p_2)^{1/(1-a)}$ and $x_2 = (m/p_2)^{1/(1-a)}$.

5.2. Unit own-price elastic : indirect utility function

It follows from Section 4.2 that the indirect utility function takes the form $v(p_1,p_2,m) = (p_2/p_1)g(m/p_2)$ with $g'(.) > 0$, or equivalently $v(p_1,p_2,m) = (m/p_1)h(p_2/m)$ with $h'(.) < 0$. This also implies an analogous graphical construction as in Section 4.2, which we do not repeat here.

[9] An example of a utility function that is quasiloglinear in good 1 can be found in Liebhafsky (1969, p. 933), and takes the form $u = A\ln x_1 + 0.5x_2^2$. Liebhafsky's purpose is to provide a utility function where good 1 is an inferior good, which is indeed the case for this numerical example.

5.3. Unit own-price elastic : utility measures and rules of thumb

It is clear from Section 4.3 that, with respect to a reference indifference curve, the consumer may in this case be seen as thinking of his gains in terms of the percentage increase in actual or potential consumption of good 1, or in terms of the percentage decrease in the opportunity cost of good 1 (and similarly for his or her losses). This is because the consumer may be seen as using a rule of thumb telling him to respond to a decrease in the price of good 1 by keeping his expenditure on good 1 fixed. An equivalent rule of thumb says that, when the consumer needs to save on consumption after an increase in the price of good 1, he should do this by keeping his expenditure on good 1 fixed.

6. Zero own-price effect

6.1. Zero own-price effect : direct utility function

Proposition 7: there is no own-price effect on the first good if and only if the direct utility function takes the form $u(x_1, x_2) = [x_2 - f(x_1)]/x_1$.
Proof:
Step 1 is the same as in the proof of Proposition 1.
Step 2. Starting from the bundle (x_1^*, x_2^*), let relative price p_1/p_2 increase by an amount *k*. Then, in order to have a zero own-price effect on good 1, at $x_1 = x_1^*$ and for different levels of x_2, the slope of any other indifference curve $x_2 = g(x_1)$ must be equal to $k - f'(x_1^*, u_0)$, as this slope must also equal $k + (p_1/p_2)$. By integration, at $x_1 = x_1^*$ and for different levels of x_2, indifference curves must be locally described by $x_2 = g(x_1^*) = kx_1^* + f(x_1^*, u_0)$, where $k(U)$ is a higher constant for indifference curves that lie higher, and where $k(u_0) = 0$.
Steps 3 and 4 are analogous to the proof of Proposition 1.
Step 5. As *k* can itself be interpreted as the consumer's utility function, the utility function is implicitly described by $u(x_1, x_2) = [x_2 - f(x_1, u_0)]/x_1$. QED

We again give a numerical example for the case where u_0 takes the form $x_2 = (1 - x_1^a)^{1/a}$. The corresponding utility function with a zero own-price effect on the first good takes the form $u(x_1, x_2) = \left[x_2 - (1 - x_1^a)^{1/a}\right]/x_1$. The demand functions are $x_1 = \left[1 - (m/p_2)^{a/(1-a)}\right]^{1/a}$ and $x_2 = (m/p_2) - (p_1/p_2)\left[1 - (m/p_2)^{a/(1-a)}\right]^{1/a}$.

From Proposition 7, it follows that a preference mapping involving everywhere a zero own-price effect on the first good can graphically be constructed from a given indifference curve u_0 in the following way. First, for each consumption bundle on u_0, take a fixed proportion of the chosen level of consumption of the first good. The straight lines in *Figure 9a* represent fixed proportions of the consumption level of good 1 along the Y-axis. Second, add the amount thus obtained (indicated by the black distances) to the level of good 2 in each bundle (indicated by the grey distances). Therefore, the levels obtained from the straight line along the Y-axis should be added to consumption level of good 2 for each consumption bundle along u_0. As long as the one and the same proportion is used for every bundle, all the new bundles obtained will lie on one and the same new indifference curve. Going through this procedure for a few bundles, we can obtain a sketch of a new indifference curve by connecting the points, as shown in *Figure 9a*. Going through this procedure for every single bundle, we obtain the smooth indifference curves shown in *Figure 9b*. It should be noted that, for small consumption levels of good 1, the indifference curves do not coincide, but rather lie very close to one another.

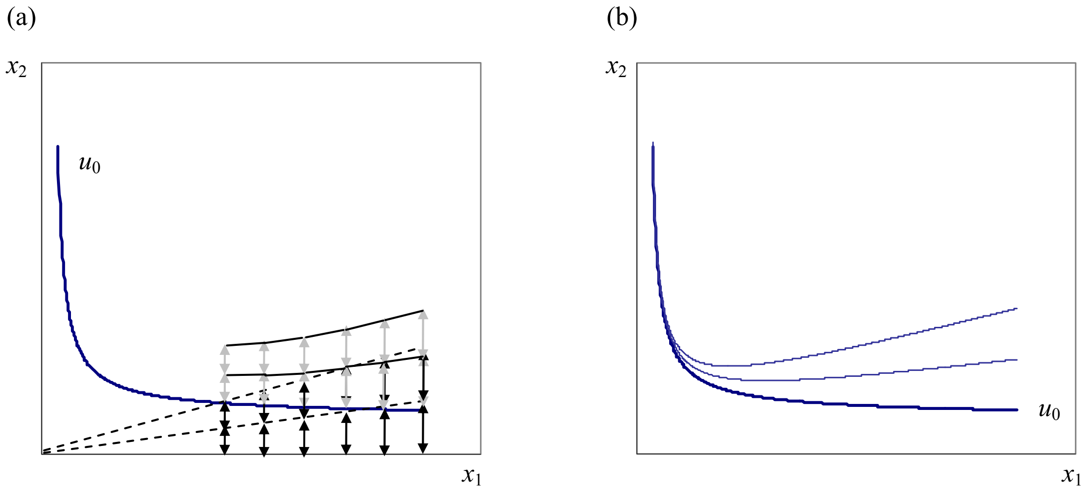

Figure 9: Construction of preferences with a zero own-price effect on the first good from a given indifference curve u_0. In panel (a), a fixed proportion of the consumption level of good 1 is added to the consumption level of good 2 for some bundles; a sketch of two new indifference curves is obtained by connecting the newly obtained bundles. Panel (b) shows smooth indifference curves.

The first derivative of an indifference curve for some level of equals $k + f'(x_1)$, and the second derivative $f''(x_1)$. While any indifference curve obtained is therefore convex as long as u_0 is convex, for $k > 0$ indifference curves will eventually have increasing sections, and good 1 eventually becomes a bad. It follows that these preferences violate the axiom of monotonicity. However, as the second-order conditions are always met, this violation is not problematic.[10] As long as u_0 is convex, commodity 1 becomes a bad for relatively high consumption levels of good 1. Moreover, it is clear that the higher the indifference curve (the higher k), the lower the consumpion level of commodity 1 for which commodity 1 becomes a bad, as illustrated in *Figure 9b*. From the way in which indifference curves are constructed from u_0, it is also clear that if u_0 has an intercept with the Y-axis, then so will all other indifference curves, and the preference mapping is therefore intransitive. We therefore again assume that u_0 does not have an intercept with the Y-axis.

[10] It should be noted that, whenever assuming a linear demand function $p = a - bx$, and calculating a consumer surplus, we are implicitly assuming a utility function $u(x, y) = y + ax - 0.5bx^2$, where for high consumption levels of good x ($x > a/b$), it becomes a bad.

6.2. Zero own-price effect : indirect utility function

Proposition 8: there is no own-price effect on the first good if and only if the indirect utility function takes the form $v(p_1, p_2, m) = -(p_1 / p_2) + g(m / p_2)$, with $g'(.) > 0$.

Proof:

Given this direct utility function, it is easily calculated that the first-order conditions imply the equality $(m / p_2) = f(x_1) + x_1 |f'|(x_1)$. The right-hand side is a function of x_1, and let us denote this function by $i(x_1)$. The direct demand function is $x_1 = i^{-1}(m / p_2)$, which indeed has a zero own-price effect. It is then met that $x_2 = (m / p_2) - (p_1 / p_2) i^{-1}(m / p_2)$. Plugging this into the direct utility function one obtains that $v(p_1, p_2, m) = (m / p_2)\left[i^{-1}(m / p_2)\right]^{-1} - (p_1 / p_2) - f\left[i^{-1}(m / p_2)\right]\left[i^{-1}(m / p_2)\right]^{-1}$. This expression can be rewritten in the form given in the proposition. QED

Proposition 8 implies that for zero own-price effects on the first good, new indifference curves can be constructed from u_0 in the following alternative manner. Take any bundle on the indifference curve u_0, and assume income and price levels such that this bundle is expenditure-minimising. Stated graphically, draw a budget line tangent to u_0 through the chosen consumption bundle. The slope of this budget line is the opportunity cost of the first good in terms of units of the second good foregone. Decrease this opportunity cost by a fixed amount via changes in p_1. This line is tangent to some new indifference curve. Go through the same procedure for some other consumption bundles on u_0, each time decreasing p_1/p_2 by the same fixed absolute amount. Since the new indifference curve cannot lie below any of the constructed budget lines, we can draw a sketch of the new indifference curves as in *Figure 10a*. Repeating this procedure for each consumption bundle on u_0, we obtain the smooth indifference curves in *Figure 10b*.

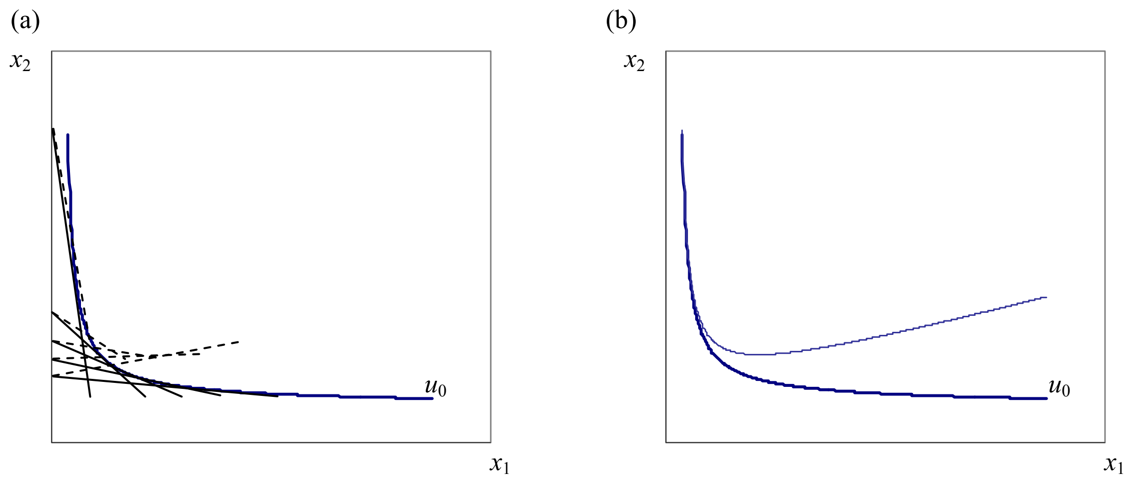

Figure 10 : Construction of preferences with a zero own-price effect on the first good from a given indifference curve u_0. In panel (a), the slope of some budget lines tangent to u_0 are increased by a fixed amount ; the newly constructed dashed budget lines are all tangent to one and the same indifference curve, and one thus obtains a sketch of the shape of a new indifference curve. In panel (b) the smooth indifference curve is shown when repeating this process for many budget lines.

6.3. Zero own-price effect : measures of utility and rules of thumb

The consumer in this case may be seen as using a rule of thumb telling him to spend any extra budget available because of a decrease in the price of the *first* good on consumption of the *second* good, and to keep the consumption of the first good fixed. Intuitively then, starting from any given indifference curve, the consumer should think of his or her gains as the proportion of the consumption level of *good 1* that is added to the consumption of *good 2*. Alternatively, starting from any given indifference curve, the decrease in the opportunity cost of the first good can be used as a measure of utility. Put otherwise, from the viewpoint of a reference indifference curve, the consumer may be seen as thinking of his welfare in function of decreases in the opportunity cost of good 1 in terms of units of good 2 foregone.

7. Hybrid benchmark utility functions

Hybrid cases of the benchmark utility functions, along with the benchmark utility functions themselves, are listed in Table 2. It is easy to show that there are two cases of preferences that are quasiloglinear in good 1 which coincide with other benchmark preferences. In particular, preferences are at the same time quasiloglinear in good 1 and quasilinear in good 2 if they take the form $u(x_1, x_2) = a \ln x_1 + x_2$ with $a > 0$. As the demand for good 2 is $x_2 = (m / p_2) - a$, the parameter *a* can be interpreted as the additional amount of good 2 that the consumer could have afforded, but abstained from buying. In other words, *a* can be interpreted as the abstinence of consumption of good 2. The rule of thumb followed by the consumer is to keep abstinence fixed. To this direct utility function corresponds the indirect utility function $v(p_1, p_2, m) = (m / p_2) + a \ln(p_2 / p_1)$. The consumer may think of his or her gains with respect to a reference indifference curve both in terms of extra units of good 2 that can be afforded or that are actually consumed, and in terms of percentage increases in the consumption of good 1 or percentage decreases in the opportunity cost of good 1.

<table>
<tr><td>good 1 :</td><td>neither normal nor inferior</td><td>Neither substitute nor complement</td><td>neither price elastic nor price inelastic</td><td>neither luxury nor necessity</td><td>neither ordinary nor Giffen</td></tr>
<tr><td>benchmark utility function</td><td>quasilinear in good 2</td><td>Quasi-loglinear in good 1</td><td>quasi-loglinear in good 2</td><td>homothetic preferences</td><td>$u(x_1, x_2) = [x_2 - f(x_1)]/x_1$</td></tr>
<tr><td rowspan="2">hybrid cases</td><td colspan="2">quasilinear in good 2, quasiloglinear in good 1</td><td colspan="3"></td></tr>
<tr><td></td><td colspan="3">Loglinear (Cobb-Douglas)</td><td></td></tr>
</table>

Table 2 : Benchmark utility functions and hybrid cases

Also, preferences are at the same time quasiloglinear in good 1 and quasiloglinear in good 2 if they take the loglinear form $u(x_1, x_2) = a \ln x_1 + (1-a) \ln x_2$. This is nothing but the Cobb-Douglas preference mapping, and this preference mapping is also homothetic. This is no surprise, as in quasiloglinear preferences, the consumer thinks of his or her gains in terms of percentage extra of one good, whereas in homothetic preferences, all goods must increase by the same percentage. The parameter *a* is now interpreted as the budget share that the consumer spends on good 1, and the consumer follows the rule of thumb of spending a fixed share of his or her budget on each good. To this direct utility function corresponds the indirect utility function $v(p_1, p_2, m) = a\ln(m/p_1) + (1-a)\ln(m/p_2) = \ln(m/p_2) + a\ln(p_2/p_1)$.[11]

8. Conclusion

In applied microeconomic theory, very few types of two-good, analytically tractable utility functions are used, and a limited number of possible consumer preferences is therefore only considered. This paper has tried to repair this defect by introducing some additional analytically tractable utility functions, allowing one to cover a wider range of possible preferences. However, there are reasons beyond this for using such utility functions in applications.

A *first* reason is that to such benchmark utility functions correspond simple rules of thumb that a boundedly rational consumer may use to approach his or her true preferences. Indeed, what makes the benchmark utility functions *analytically* tractable is that, from the price and income variables that could affect demand, one variable is assumed not to affect demand, or to affect demand in a very specific way (e.g. price changes do not affect expenditure). This fact is *also* the reason why benchmark utility functions correspond to simple rules of thumb for making one's consumption decisions.

A *second* reason to use two-good analytically tractable utility functions is that consumers using a rule of thumb may apply this rule to a single reference good. Application of the rule of thumb to the reference good then determines the demand for all non-reference goods. E.g., a boundedly rational consumer could use the rule of thumb to spend any income increases on a single reference good, say, holidays. This means that the consumption of all other goods is kept fixed in response to income changes. A consumer who applies such a rule can be seen as thinking with respect to a reference welfare level, of extra holiday time as gains, and of less holiday time as losses. This corresponds to preferences that are quasilinear in holidays. Similarly, a boundedly rational consumer could use the rule of thumb of setting out a certain budget to be spent on a reference good, say CDs, and of keeping his or her expenditure on the reference good fixed in response to changes in the price of the reference good. With respect to a given welfare level, a consumer who applies such a rule of thumb can be seen as thinking of his or her gains as *percentage* extra consumption of CDs, and of his or her losses as *percentage* less CDs consumed. This corresponds to preferences that are quasiloglinear in CDs.

It should be noted that, from any given situation, a *different* benchmark utility function may constitute the best approximation of the consumer's true preferences depending on whether income, own price, or other price changes, and where the

[11] Other hybrid cases do not meet standard assumptions. Utility functions of the form $u(x_1, x_2) = ax_1 + x_2$ (with $a > 0$) do not meet the assumption of convexity of indifference curves. Preferences that are quasiloglinear in good 2, and are constructed from a linear indifference curve, could be considered as a hybrid case between the case where good 1 is neither a Giffen good nor an ordinary good, and where good 1 is neither price elastic nor price inelastic. However, this utility function again violates the assumption of convexity of indifference curves, and additionally violates the assumption of transitivity, as all indifference curve go through a single intersection point with the Y-axis.

reference good chosen may also differ according to which variable changes. Put otherwise, the simple rules of thumb that best approach the consumer's true preferences may correspond to different benchmark utility functions depending on which variable is changed.

In conclusion, whether consumers actually make their consumption decisions according to such simple rules of thumb of course needs to be investigated empirically. Yet, it may very well be that, when modelling boundedly rational consumers, it is sound to attribute simple two-good utility functions to them. Possibly, microeconomics textbooks of the future, taking into account the fact that consumers are boundedly rational, will not contain a completely rewritten consumer theory, but only one where boundedly rational consumers are integrated into standard consumer theory.

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