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"Need to Know" Versus "Spread the Word": Collective Action in the Multi-Player Electronic Mail Game

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Abstract
As shown by Rubinstein (1989, AER), in the two-player electronic mail game, players are better off if the extent to which they can check each other's information, check each other's information about each other's information, etc., is limited. This paper investigates to what extent this result extends to the multi-player electronic mail game. It is shown that, contrary to the two-player game, the multi-player game has a plethora of equilibria. If players play inefficient equilibria where they require a specific communication network to be established in order to achieve collective action, then Rubinstein's results extend. However, contrary to the two-player game, the multi-player game also has equilibria where players find many alternative communication networks sufficient to undertake collective action. If players play such equilibria, then contrary to what is the case in the two-player electronic mail game they can become better off with more information.

Keywords: Multi-Player Electronic Mail Game, Collective Action, Communication Networks.

JEL classification: D82, D85, D71

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1. Introduction

While the prisoner's dilemma has been the most popular representation of collective action problems, an equally valuable representation of collective-action problems is the so-called stag hunt (Skyrms, 2004). In a stag hunt, contrary to what is the case for the prisoner's dilemma, cooperation exists as an equilibrium along with non-cooperation. However, undertaking cooperative action when an insufficient number of other players act is assumed to be costly. Thus, in order to do their part of the collective action, players may require a large degree of reassurance, i.e. knowledge that there is an opportunity for collective action, knowledge that everyone knows this, knowledge that everyone knows that everyone knows this, etc. Clearly then, as pointed out by Chant and Ernst (2008), players' interactive knowledge plays a crucial role in the stag-hunt type of collective-action problem.

How can players achieve a sufficient degree of common belief that there is an opportunity to benefit from collective action? The mechanisms that can be found in the literature can be roughly divided along two lines. A *first* mechanism is unregulated communication between the players. The message here from the stag hunt game[1] with asymmetric information known as the *electronic mail game* (henceforth EMG) (Rubinstein, 1989), is that decentralized attempts by players to achieve common belief can have a crippling effect on collective action. Given that acting with too few people is costly, the last player whose information gets checked always has an interest to in turn check for confirmations that his information got through. This again induces further checking for confirmations by the other players. As long as communication is noisy, every additional confirmation required makes it less likely that collective action ever takes place. From this perspective, a strong argument exists for a *second* mechanism, namely what can be broadly described as institutionalized communication. In one type of institutionalized communication, common knowledge gets generated by a common-knowledge generating event such as a public meeting (Chwe, 2001). Another type of institutionalized communication is guided by strict rules of who can talk to whom, and of who needs to know what (Chwe, 1995). Thus, collective action is successful because a hierarchy exists among the agents involved in the collection-action problem. In the simplest form of such a hierarchy, a leader makes a public invitation for collective action, without any other communication taking place (Koessler, 2000). Seeing firms as collective-action problems, this argument would seem to give credit to the so-called classical management's (e.g. Fayol, 1949) arguments in favor of hierarchically-organized firms.

Yet, it is well-documented that instances of collective action such as riots, revolutions and strikes often take place spontaneously, without any public announcement and without any clear leadership. The same is true for collectively produced information goods on the Internet, such as Linux (Flanagin, Stohl & Bimber, 2006). Finally, it has been argued that the network form of organization, without strong hierarchical rules of who reports to whom, coexists along with the hierarchical form of organization for firms (Powell, 1990). Indeed, some even argue that the network form of organization is becoming the dominant form of organization (Powell, 2001).

How can these observations be reconciled with the theoretical prediction of crippling reassurance seeking that would take place if players were freely allowed to check each other's knowledge, each other's knowledge about each other's knowledge, etc.? With a few exceptions, the EMG has been treated as a *two*-player game. We show that, in the *multi*-player EMG (which seems more apt for analyzing collective action problems), a fundamentally different mechanism can be at work, working in the opposite direction of the mechanism of

[1] A multi-player stag hunt game is treated by Van Damme and Carlsson (1993).

crippling reassurance seeking. The ever increasing number of chains of confirmations and re-confirmations need not be used for mutual reassurance, but instead may serve as multiple independent channels through which players can get informed about the opportunity for collective action, thus forming an antidote for the unreliability of communication.[2]

Intuitively, consider a collective action problem with three players (Alice, Bob, Carl), of which one is informed (Alice). Let the threshold be equal to 3 players. Then, if players are allowed to communicate for more than one round, collective action can both be achieved if Carl finds out Alice's information through Bob, and if Bob finds outs Alice's information through Carl. If these two communication networks are considered as equally valuable alternatives, players are less vulnerable to noise. Extra messages that at first sight could only lead to crippling extra rounds of assurance and reassurance instead may be used for generating multiple paths along which information can travel, thus reducing the effect of noise. From this perspective, the longer the social planner lets communication continue and the more players are involved, the more additional paths are created through which the players can find out the same information. Put otherwise, from this perspective, the social planner should design a communication protocol that allows the players to "spread the word". In an extreme version of such an equilibrium, it suffices that each player receives a single message over the two last stages of the communication process, where it does not matter from whom the message is received, as long as the message contains information that each informed player received a positive signal. It is then as if players are acting based on "hearsay". Thus, when it comes to the desirability of putting interactive knowledge available to players, one can come to radically different conclusions in the multi-player EMG and in the two-player EMG.

Yet, it is important to stress that these efficient equilibria coexist with inefficient equilibria where players do engage in in crippling rounds of assurance and reassurance. In fact, in the latter type of equilibria, effects occur in the multi-player EMG that do not exist in the two-player EMG; because of this, *if* crippling reassurance seeking still occurs in the multi-player EMG, it can have severe effects. We show that in the multi-player game, players' mutual expectations can create what we call *endogenous thresholds* and *pseudo-experts*. In case of an endogenous threshold, even though only T players need to act for benefits of collective action to arise, each player only acts when receiving direct information that S players, with $S > T$, find out about the opportunity for collective action. Thus, players' mutual expectations lock them into playing *as if* the threshold were larger than it is in reality. Put otherwise, if all other players require reassurance from S other players, then so will you. Moreover, if everyone believes that collective action is only possible when receiving a message from an uninformed player i, then players act in the same manner as they would if player i were an informed player with a crucial signal determining whether or not the opportunity for collective action exists. Thus, players' mutual expectations can lock them into considering player i as a pseudo-expert. If all other players believe that they need assurance from player i, then so do you. Given the possibility of endogenous thresholds and pseudo-experts, it may on the contrary be a good idea for a social planner who tries to maximize the probability of collective action to restrict how long any given number of players talks, and to restrict the number of players that can talk to one another. More specifically, the social planner from this perspective should design a communication protocol where players are informed on a "need to know" basis. In such a communication protocol, communication from the informed to the

[2] These two mechanisms reflect the two mechanisms by which players attempt to reduce the risk of acting alone, as listed by Chwe (1995), namely *reconfirmation* and *redundancy*. In Chwe's work, however, redundancy means that an identical message is sent several times, whereas in the current paper, it means that players are on multiple chains on which players are ordered in a different way, where these chains form multiple alternative channels along which information can travel.

uninformed players only takes place once the informed players have checked that all their signals are positive, players who are not crucial for collective action are not involved, and the ability of players to check each other's knowledge is limited.

The paper is structured as follows. Section 2 looks at some related literature. The multi-player EMG is described in Section 3. Section 4 characterizes the Nash equilibria. Section 5 focuses on the Nash equilibria where players require specific information from one another, and shows that this can lead players to form endogenous thresholds and to create pseudo-experts. It is shown that, if such equilibria are likely to be played, it makes sense to reduce the information available to the players. Section 6 focuses on Nash equilibria where the players consider several pieces of information as equally valuable alternatives. If such equilibria are likely to be played, then it on the contrary makes sense to make as much information as possible available to the players. In Section 7, we show that the co-existence of efficient and inefficient equilibria remains even if players are unsophisticated, in that they are not able to tell the difference between several long message strings. We end with a conclusion in Section 8.

2. Literature on the multi-player EMG

We here treat the work of three authors who have in various ways extended the EMG to a multi-player setting. Chwe (1995) treats a three-player EMG with one informed player, and two states of nature. Only the uninformed players act, and have the choice between not acting, and two different actions. For each state, coordinated play of one specific action yields a positive payoff to all players. The informed player is only interested in coordinated action in the right state; all other outcomes yield him payoff zero. The uninformed players incur one loss when acting alone, and another loss when taking the wrong action. Communication is *voluntary* (cf. Binmore and Samuelson, 2001; De Jaegher, 2008), and can make use of three different signals. Signals both can get lost, and can get confused with one another. It is possible to send a message to notify that one did not receive a message, or received a garbled message. Chwe (1995) both studies an equilibrium where the informed player informs each uninformed player separately (*star*), and one where the informed player first informs one uninformed player, who then informs the other uninformed player (*line*). Depending on the parameters, one or the other equilibrium exists, or both exist; when both exist, the equilibrium preferred by each player again depends on the parameters.

We next review work of two authors who study a multi-player EMG with *involuntary* communication. Morris (2002a, 2002b) investigates a threshold game where benefits from coordinated action arise if a certain threshold number T among the total number of players N act together. At each stage, a different random sample of M players, with $M < T$, receives messages (this is a generalization of the two-player EMG in the sense that $T = 2$ in this game, and that a subset of the players, namely one player, receives a message at each stage). Either all players in the random sample receive messages, or none of them does, where the latter occurs with small probability. At the first stage, the players receive information about the state of the world; at further stages, they receive information on whether previous messages were received (positive acknowledgements). The interpretation is that there is a sequence of public meetings of parts of the population. As in Rubinstein's (1989) main model, the communication process is only stopped when a message gets lost. Morris shows that for a large enough loss of acting with less than T players, only a single equilibrium exists, where no player acts, and where the results of Rubinstein therefore generalize.

Coles (2007) recently treats three versions of a multi-player EMG where there is a single informed player, and where there are a number of uninformed players. Each time, the informed player talks individually with each uninformed player, but uninformed players do *not* talk to each other. In a *first* variant, each individual message gets lost with small probability, but the informed player can only send a positive acknowledgement to an uninformed player when having received the maximum possible messages from *all* uninformed players at the immediately preceding stage. Put otherwise, the informed player's confirmations are confirmations of all immediately preceding messages. The message exchange only stops when a message gets lost. Benefits arise only if all players act when there is an opportunity to benefit from collective action. Coles shows that if communication continues indefinitely, collective action never takes place.

In a *second* variant, everything is the same as in the first variant, with the exception of the payoffs. The informed player still observes a single state of nature, and in the state where benefits from collective action are possible, after the communication process, still decides whether or not to act. Yet, the informed player obtains a positive payoff for each individual uninformed player he can coordinate with, and a negative payoff for each individual uninformed player with whom he or she cannot coordinate, where these payoffs are then added up. The informed player thus plays a two-player EMG with each uninformed player, but must still decide whether to act with respect to all uninformed players, or whether not to act. Coles shows that in this case, contrary to what is the case in Rubinstein (1989), players are satisfied with a finite number of messages. Intuitively, as long as receiving information that a sufficient number of uninformed player act, the informed player will be satisfied.

The *third* variant treated by Coles is again similar to the first variant, but it now additionally is the case that either all the informed player's messages arrive, or none arrive. In this case, when only receiving a few confirmations from uninformed players, the informed player still knows that all his messages arrived. Again, Coles shows for this case that players are satisfied with a finite number of messages.[3]

The relation of the current research to these papers is the following. *First*, though we assume communication to be involuntary, we assume it stops at some given finite stage. This is because our focus is not on approximate common knowledge (which requires the potential of infinite communication), but in a practical EMG with a realistic finite number of communication rounds. There is a deadline at which players need to decide whether or not to act, and at this deadline communication stops. Involuntary communication can simply be interpreted as the ability of players to check each other's information, each other's information about each other's information, etc. *Second*, unlike Coles (2007), we only treat cases where a certain number of players need to act before benefits from joint action arise. This is because we are interested in the multi-player EMG as a collective-action problem. Similar to Morris (2002a, 2002b), we allow this threshold to be smaller than the total number of players. *Third*, unlike any of the papers reviewed, we allow for the case where there are several informed players, who each receive a signal that is essential to determining that there is an opportunity for collective action. This allows us to illustrate the possibility of uninformed players taking up the role of pseudo-experts in equilibrium. *Fourth*, within the

[3] A small literature investigates the robustness of Rubinstein's results to modifications other than introducing multiple players. Dulleck (2007) shows that boundedly rational players with imperfect recall can still coordinate on requiring only a few messages. Dimitri (2004) shows that when the probabilities that a message from Alice and Bob gets lost are not the same, coordinate action can still occur, as the player whose messages arrive with high probability can then be quite sure that his or her message arrived, and that it is the confirmation of the other player that got lost; Coles (2007) provides a similar result for the two-player EMG. Binmore and Samuelson (2001) investigate the effect of communication being voluntary instead of automatic. They show that, while efficient equilibria now exist, players may still coordinate on inefficient equilibria where a large number of messages are sent back and forth. De Jaegher (2008) not only assumes that communication is voluntary, but also that it is possible to send a message even though no previous message was received. In this case, only efficient equilibria survive the intuitive criterion.

finite number of communication stages, we allow for the generation of all possible messages. Thus, in the first stage, an informed player sends a message to each other player, after which an uninformed player who receives a message sends a confirmation back to each other player, including other uninformed players (unlike in Coles, 2007), etc. This is because we want to allow players to check in the widest sense possible each other's information. Also, this is in the spirit of Rubinstein's (1989) original game, in that our communication protocol is able to generate, within a finite number of stages, all possible interactive knowledge. *Fifth*, along the same lines, we leave out all correlation between the probabilities that messages arrive: each message gets lost with a small probability that is independent of the event of other messages arriving or getting lost. If e.g. player 2 can separately check whether player 1 knows that player 2 knows what player 1 knows, and whether player 1 knows that player 3 knows what player 1 knows (and not just once all player 1's knowledge), then it makes sense to assume that each of these information checking processes can independently go wrong. The latter two modeling decisions drive our results, as they allow for the possibility that players allow for multiple channels through which they can become informed. As we will see, equilibria indeed exist where players make use of this possibility – though they coexist with equilibria where they do not.[4]

3. The model

The multi-player EMG studied in this paper takes the following form. A set of players $\mathcal{N}$ with cardinality *N*, $N \geq 3$, play the game. The typical uninformed player is denoted as player *i*. There is a subset of informed players $\mathcal{I}$, with $\mathcal{I} \subseteq \mathcal{N}$, and with cardinality *I*, $I \geq 1$. The typical informed player ι in $\mathcal{I}$ observes a signal α with probability $(1-q)$, and a signal β with probability *q*; the signals are assumed to be uncorrelated. We say that state *b* occurs if *all* informed players observe a signal β; in any other case, we say that state *a* occurs. State *b* thus occurs with probability $p = q^I$, state *a* occurs with probability $(1-p) = (1-q^I)$. We assume that $p < 1/2$. Informed players only observe their own signal, uninformed players do not observe any signals.

The *N* players can choose between two actions, namely actions *A* and *B*. In state *a* (= less than *I* players obtain signal β), a player who does *A* always gets payoff zero, and a player who does *B* incurs loss *L*. In state *b* (= all *I* players obtain signal β), a player who does *A* when less than *T* other players play *B*, obtains payoff zero (where $T \geq 3$, $T \leq N$).[5] A player who does *A* when *T* or more other players play *B*, or who does *B* when *T* or more other players play *B* along with him or her, obtains payoff *M*. It is assumed that $L > M > 0$[6], where we typically consider *L* to be very large.

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[4] A related paper is Chwe (2000), which equally treats a stag-hunt like collective-action problem. In this paper, each player has a different threshold, i.e. holds a different minimal number of players who need to act before benefits arise. Players do not know each other's threshold, but can find these out through exogenously given links. Information transmission is not subject to noise, but having a link with a neighbor only gives you information on this neighbor's threshold, and not on the thresholds of the neigbor's of your neighbor. Chwe shows that any network leading to collective action takes the form of a hierarchy of cliques.

[5] This is a multi-player version of a version of the two-player electronic mail game used by Chwe (1995) and Morris and Shin (1997), and Morris (2002a, 2002b). In Rubinstein's (1989) original game, a payoff *M* is also obtained when players coordinate on playing *A* in state *a*. Our results also apply in Rubinstein's original setting, but the proofs are more complicated.

[6] Together with the assumption that $(1-p) > 1/2$, this implies that, if all other players would play *A* in state *a* and *B* in state *b*, an individual player plays *A*.

Just as in the two-player EMG, before players simultaneously decide on their actions, they observe messages generated by an automatic communication protocol $c(z_S, \mathcal{N}_S)$. In our multi-player EMG, the automatic communication protocol lets a set $\mathcal{N}_S \subseteq \mathcal{N}$ of players communicate with one another, where $\mathcal{J} \subseteq \mathcal{N}_S$. Denote the cardinality of this set as N_S. In the spirit of Rubinstein, this automatic communication protocol allows players to achieve t^{th}-order knowledge at stage t.[7] Concretely, at stage 1, when observing state β, one informed player, denoted as player 1, automatically sends an e-mail to all other players in the set $\mathcal{N}_S$. At all further stages up to the final stage z_S, each player automatically forwards each message received to each other player in the set $\mathcal{N}_S$ (where informed players only forward a message if they have also observed state β).[8] Each e-mail gets lost with small probability ε. We typically assume ε to be small. At any particular stage t, up to $(N_S - 1)^t$ e-mails are sent. The idea of assuming that a single informed player initiates communication is that there is a single communication process between the players, and not several parallel ones.

We assume that, by scrolling down an e-mail, a player in $\mathcal{N}_S$ can observe the sequence of players through which a message was forwarded. Thus, when player i receives a particular message from player j at stage t, player i knows that j knows that k knows that l knows that … player 1 observed signal β, or $K_i^t K_j^{t-1} K_k^{t-2} K_l^{t-3} ... K_1^0(b)$. Superscripts refer to stages, where player 1 learns the state of nature at stage 0. This same message also implies $K_j^{t-1} K_k^{t-2} K_l^{t-3} ... K_1^0(b)$, $K_k^{t-2} K_l^{t-3} ... K_1^0(b)$, etc. Note that, if player l is also an informed player, player i additionally knows that j knows that k knows that l also observed signal β. We call $K_i^t K_j^{t-1} K_k^{t-2} K_l^{t-3} ... K_1^0(b)$ a *message string*, denoted as $m_{i,x}^t$ (where i denotes the last player to receive a message in the message string, t the stage at which i receives this message, and where x labels the message string), and $K_j^{t-1} K_k^{t-2} K_l^{t-3} ... K_1^0(b)$, $K_k^{t-2} K_l^{t-3} ... K_1^0(b)$ and $K_l^{t-3} ... K_1^0(b)$ *sub message strings* of this message string, denoted respectively as $m_{j,w}^{t-1}$, $m_{k,v}^{t-2}$ and $m_{l,u}^{t-3}$, where we say that $m_{j,w}^{t-1} \subset m_{i,x}^t$, $m_{k,v}^{t-2} \subset m_{i,x}^t$, $m_{l,w}^{t-3} \subset m_{i,x}^t$.

In part of our analysis, we will assume that prior to the start of the game, a social planner chooses z_S, with $z_S \leq z$ (where z is determined by the deadline at which action needs to occur), and chooses the set $\mathcal{N}_S$. It is assumed that the social planner's purpose is to maximize the probability of collection action.

4. Characterization of the set of Nash equilibria allowing for collective action

We first describe a minimal sufficient set of messages as the basic unit for describing a strategy of a player in the multi-player EMG.

[7] Everybody knows that state b occurs by state 1, everybody knows that everybody knows that state b occurs by stage 2...

[8] Players are only allowed to send confirmations of receipt to each other, thus to say: 'I've received your message.' But one can envisage that players would also send notices of non-receipt, where they would say: 'I didn't hear from you. Are you sure you did not contact me?'. As long as noise is small, a justification for only considering confirmations of receipt is obtained when adding costs of paying attention to the model (see Binmore and Samuelson, 2001). When noise is small, notices of non-receipt will send so infrequently that it is not worth to pay attention to them.

Definition 1. Define as a *minimal sufficient set* $s_{i,x}$ a set of message strings $m^t_{i,x}$, $m^t_{i,y},\ldots,m^s_{i,v}$, $m^s_{i,w},\ldots$ with $\forall m^t_{i,x}, m^s_{i,v} \in s_{i,x} : m^t_{i,x} \not\subset m^s_{i,v}, m^s_{i,v} \not\subset m^t_{i,x}$, that describe part of player *i*'s strategy in the sense that

(i) player *i* plays *B* when observing all messages in $s_{i,x}$ (which makes the set $s_{i,x}$ *sufficient*);

(ii) player *i* plays *A* when observing no messages *outside* of the set $s_{i,x}$, and not all messages *in* the set $s_{i,x}$ (which makes the set $s_{i,x}$ *minimal* sufficient).[9]

A strategy S_i by a player *i* can be described as a set of one ore more alternative minimal sufficient sets, $\{s_{i,x}, s_{i,y},\ldots\}$, where *x*, *y*,… label the sets. Effectively, the assumed automatic communication protocol reflects that, within bounds, players are able to check any type of higher-order knowledge with one another, where a player's strategy then says which higher-order knowledge this player requires in order to play *B*. A player's strategy can be seen as a requirement that a particular network, or one out of a particular set of networks, should be established before the player is willing to play *B*. As we show below, any such network consist of one or more so-called *brooms*. We now define for each one-but-final message string as the corresponding *broom* the set consisting of, first, all submessage strings of this one-but-final message string, and second, all the final confirmation of this one-but-final message string.

Definition 2. Consider an automatic communication protocol $c(z_S, \mathcal{N}_S)$. For any message $m^{z_S-1}_{i,x} \subset c(z_S, \mathcal{N}_S)$, define as the *broom* corresponding to $m^{z_S-1}_{i,x}$, denoted by b_x, the set of *all* messages $m^t_{h,w}$, $m^{z_S}_{j,y} \subset c(z_S, \mathcal{N}_S)$ with the following characteristics:

(i) $m^t_{h,w} \subset m^{z_S-1}_{i,x}$;

(ii) $m^{z_S}_{j,y} \supset m^{z_S-1}_{i,x}$.

Call a set of such brooms, $\mathcal{B}_{\mathrm{I}} = \{b_v, b_w, b_x,\ldots\}$ generated from messages $m^{z_S-1}_{j,v}$, $m^{z_S-1}_{k,w}$, $m^{z_S-1}_{i,x},\ldots \subset c(z_S, \mathcal{N}_S)$ a *broom set*. Call $\mathcal{B}[c(z_S, \mathcal{N}_S)] = \{\mathcal{B}_{\mathrm{I}}, \mathcal{B}_{\mathrm{II}}, \mathcal{B}_{\mathrm{III}},\ldots\}$ the set of all possible broom sets generated from all possible sets of one-but-final messages in $c(z_S, \mathcal{N}_S)$.

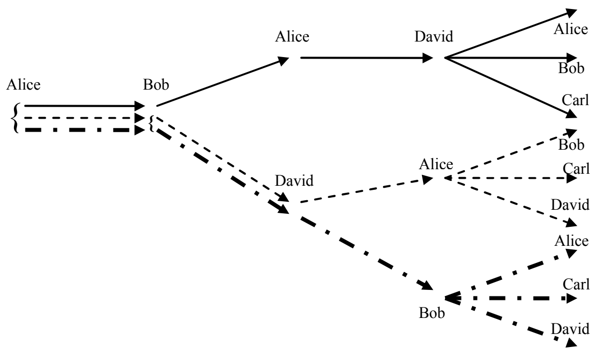

Figure 1 Broom set for $N_S = 4$, $z_S = 4$. *The broom set consists of three brooms, indicated by solid arrows, dashed arrows, and dash-dot arrows.*

[9] For a similar definition of minimal sufficient sets, see Chwe (2000).

An example of a broom set is given in Figure 1. Note that brooms in a broom set may partially overlap, as indicated by the brackets in Figure 1. Consider now the class of broom sets where each informed player gets to send a message. Proposition 1 shows that, for sufficiently large *L*, to each possible set of such broom sets corresponds a unique Nash equilibrium. The advantage of assuming a large *L* is that all equilibria where collective action takes place with positive probability can be described by means of one and the same basic structure, namely the broom set, and that different such basic structures need not be described for different parameter levels. More importantly, assuming a large *L* drives home the point that we make in Section 6 below. For large *L*, it would seem that the risk is high that giving the players the liberty to check each other's information will only lead to a crippling exercise of assurance and reassurance: for large enough *L*, everyone wants to know with certainty that all informed players received positive signals, and, as long as confirmations are available, everyone wants to receive direct confirmations that everyone else achieved a minimal sufficient set. Indeed, this is entirely in the spirit of the two-player EMG, and makes our game comparable to the two-player EMG. Yet, our analysis shows that, even if *L* is very large, equilibria exist where players consider several channels as substitutes for getting informed about the profitability of collective action. While players require confirmations up to the final stage for each of these channels, the benefit of having more channels available when increasing the number of players participating in communication and/or increasing the length of communication exceeds the cost of more messages being required in each individual channel. Before showing this, however, we characterize the Nash equilibria in Proposition 1.

Proposition 1 (Characterization). For $c(z_S, \mathcal{N}_S)$, consider *any* set of broom sets $\mathcal{B}^N$ with $\mathcal{B}^N \subseteq \mathcal{B}$ such that for every $\mathcal{B}_X, \mathcal{B}_Y \in \mathcal{B}^N$: $\mathcal{B}_X \not\subset \mathcal{B}_Y$, and such that $\forall\, \mathcal{B}_X \subset \mathcal{B}^N$: $\forall \imath \in \mathcal{J}$: $\exists\, \mathscr{b}_x \subset \mathcal{B}_X$: $m^s_{\imath,x} \subset \mathscr{b}_x$. Then a level of *L* exists such for any ε in $]0, \varepsilon(L)[$, this set $\mathcal{B}^N$ describes a unique Nash equilibrium, where for each player *i* and for each broom set $\mathcal{B}_X$ in $\mathcal{B}^N$, the set of all messages $m^t_{i,x}$ such that $m^t_{i,x} \subset \mathcal{B}_X$ describes a minimal sufficient set $s_{i,x}$.[10]

Proof:

Step 1 shows that, for sufficiently large *L*, each equilibrium minimal sufficient set must contain at least one message string received by an informed player. Step 2 shows, for the case where *L* is large, the form of player *j*'s best response minimal sufficient set $s_{j,x}$ when player *i* has a minimal sufficient set $s_{i,x}$. Step 3 shows that, when *L* is large, any equilibrium minimal sufficient set must only contain messages received at the two final stages. Step 4 shows that, for large *L*, all equilibria can be described by means of sets of broom sets, where as shown in Step 5 no one broom set can be a subset of another broom set.

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[10] Consider for instance the case $\mathcal{J} = \{1\}$, $N_S = 3$, $T = 3$, $z_S = 2$. The automatic communication protocol can then generated two brooms $\mathscr{b}_1$ and $\mathscr{b}_2$. The set of Nash equilibria is decribed as ({$\mathscr{b}_1$,$\mathscr{b}_2$}, {$\mathscr{b}_1$}, {$\mathscr{b}_2$}, [{$\mathscr{b}_1$}, {$\mathscr{b}_2$}]), where {.} denotes an individual minimal sufficient set, and [.] denotes a set of alternative minimal sufficient sets. Consider the restriction on ε and *L* for such an equilibrium to exist. The player facing the maximal uncertainty from acting is the uninformed player in equilibrium {$\mathscr{b}_1$,$\mathscr{b}_2$}, in that 3 messages need to arrive for payoff *M* to be obtained. Thus, it must be the case that $(1-\varepsilon)^3 M - \left[1-(1-\varepsilon)^3\right] L > 0$. The player facing the minimal uncertainty from acting when receiving too few messages is the informed player who observes state *b* and does not get any messages in equilibrium [{$\mathscr{b}_1$}, {$\mathscr{b}_2$}]. The probability that no uninformed player received a message at stage 1 then equals $1/(2-\varepsilon)^2$. As $\varepsilon \rightarrow 0$, this probability approaches ¼, and is the probability that informed player incurs a loss of *L*. It follows that the informed player acts according to equilibrium even for vanishing ε iff $(3/4)M - (1/4)L < 0$ or $L > 3M$. Thus, the described equilibria exist for e.g. $L = 4$, $M = 1$, $\varepsilon = 0.10$.

Step 1. $\forall s_{i,x} \in \{s_{i,x}, s_{i,y}, ...\} : \forall \imath \in \mathcal{J} : \exists m^t_{\imath,x} \subset s_{i,x}$. To show this, note that at best, a player who does not find all informed players' signals finds out positive signals for $(I-1)$ informed players, and at best estimates the probability that state b occurs to be q (normally speaking, non-receipt of messages will increase his or her belief that state b occurs above q). For large enough L, $qM-(1-q)L$ is such that the player prefers to do A. By the same reasoning, an uninformed player not in set $\mathcal{N}_S$ always plays A.

Step 2. Consider an equilibrium minimal sufficient set $s_{i,x}$, with typical element $m^s_{i,x}$ such that no $m^t_{i,x} \in s_{i,x}$ exists with $m^s_{i,x} \subset m^t_{i,x}$ ("final" messages to be received by i in $s_{i,x}$). Consider the set $s_{j,x}$ consisting of 1) for every $m^s_{i,x} \in s_{i,x}$, all messages $m^r_{j,x}$ with $m^r_{j,x} \subset m^s_{i,x}$; 2) for every $m^s_{i,x} \in s_{i,x}$, all messages $m^{s+1}_{j,x} : m^s_{i,x} \subset m^{s+1}_{j,x}$; 3) if $s_{i,x}$ contains any message $m^{z_s}_{i,x}$, for all $m^{z_s-1}_{h,x} \subset m^{z_s}_{i,x}$, all messages $m^{z_s}_{j,x} : m^{z_s-1}_{h,x} \subset m^{z_s}_{j,x}$. We show that, for sufficiently large L, it is a best response for player j to hold the set $s_{j,x}$ as a minimal sufficient set.

We first show that it is a best response for player j to hold $s_{j,x}$ as a *sufficient* set. If player j observes the constructed message set, then player j believes with high probability that player i achieved this particular message set $s_{i,x}$. So player j believes that player i believes that collective action is sufficiently likely to take place. But beliefs should be confirmed in equilibrium, so that player j should then also believe that collective action is sufficiently likely to take place. Note that if player j observes every message in $s_{j,x}$, then for any message string $m^t_{h,x}$ with $m^t_{h,x} \subset m^s_{i,x}$ (where $m^s_{i,x} \in s_{i,x}$), player j does not need any additional confirmation $m^{t+1}_{j,x}$ such that $m^t_{h,x} \subset m^{t+1}_{j,x}$, as the information that player h observed message $m^t_{h,x}$ is already contained in $s_{j,x}$.

Second, we show that $s_{j,x}$ is a *minimal* sufficient set to player j. Suppose that player j does not observe any messages outside of the set $s_{j,x}$, and not all messages in $s_{j,x}$; we show that for large L it is then a best response for player j to play A. At best, only a single message $m^{z_s}_{j,x}$ in $s_{j,x}$ did not arrive to j,[11] and for a message $m^{z_s-2}_{g,v}$ with $m^{z_s-2}_{g,v} \subset m^{z_s}_{j,x}$, player j still observes a message $m^{z_s-1}_{j,y}$ such that $m^{z_s-2}_{g,v} \subset m^{z_s-1}_{j,y}$. Player j then believes that with probability $\varepsilon/[\varepsilon+(1-\varepsilon)\varepsilon] = 1/(2-\varepsilon)$, player i did not observe all messages in $s_{i,x}$. Of any other minimal sufficient set $s_{i,x}'$ that player i may hold, player j also at best believes with probability $1/(2-\varepsilon)$ that it has not been established. Player j thus at best believes with probability $[1/(2-\varepsilon)]^{\sigma_i-1}$ that player i has not achieved any other minimal sufficient set, where σ_i is the number of minimal sufficient sets held by player i. For a finite N_s and z_s, σ_i is finite, and this probability is positive. It follows that player j believes that with a positive probability, player i believes it to be too unlikely that $(T-1)$ or more other players play B for player i to play B. In equilibrium, player i's beliefs should be confirmed. Finally, for sufficiently large L, player j thus plays A.

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[11] Note that for any message $m^t_{j,x}$ in $s_{j,x}$ with $t < z_S$, by virtue of not having received any messages after t, player j will consider it even more likely that player i did not observe all messages in $s_{i,x}$.

Step 3. We show here that any minimal sufficient set $s_{i,x}$ consists of messages that player *i* requires at stages $(z_s - 1)$ and z_s.

Assume player *i* observes only the messages in set $s_{i,x}$. Note that player *i* then does not receive messages confirming that player *j* achieved every message in any other minimal sufficient set than set $s_{j,x}$, where such other sets are denoted as $s_{j,x}'$. At best, player *i* believes with probability $[1/(2-\varepsilon)]^{\sigma_j - 1}$ that player *j* has not achieved any other minimal sufficient set, where σ_j is the number of minimal sufficient sets of player *j*. For a finite N_s and z_s, σ_i is finite, and this probability is positive. For large enough *L*, the risk of playing *B* based on the belief that player *j* has achieved some alternative minimal sufficient set $s_{j,x}'$ is too high. It follows that player *i* will only play *B* when believing with sufficiently high probability that at least $(T-1)$ players of the type *j* observe the minimal sufficient set $s_{j,x}$ as defined in Step 2. Such beliefs cannot be achieved by player *i* getting confirmations of messages in $s_{j,x}$ – otherwise such messages would already be included in the $s_{i,x}$ itself. The only alternative is that $c(z_S, \mathcal{N}_S)$ does not allow player *i* to receive confirmations of messages in $s_{i,x}$, which is only possible if $s_{i,x}$ consists of messages sent at stages $(z_s - 1)$ and/or z_s. Note that a by-product of this result is that player *i*, when observing set $s_{i,x}$, believes that with high probability that $(N_S - 1)$ other players (rather than $(T-1)$ other players) play *B*.

Step 4. If $s_{i,x}$ is a minimal sufficient set in equilibrium, then by Step 2, it is a best response for players $j_1, j_2, ..., j_m, ... j_{N_S - 1}$ to consider respectively $s_{j_1,x}, s_{j_2,x}, ..., s_{j_m,x}, ..., s_{j_{N_S-1},x}$ as minimal sufficient sets, where each such set is as defined in Step 2. Given that by Step 3, $s_{i,x}$ must consist of messages sent at stages $(z_s - 1)$ and/or z_s, it follows that any such minimal sufficient set $s_{j_m,x}$ itself consists of messages sent at stages $(z_s - 1)$ and/or z_s. By Step 2, when a player j_m holds any such minimal sufficient set $s_{j_m,x}$, it is a best response for players $j_1, j_2, ..., j_{m-1}, j_{m+1}, ... j_{N_S - 1}$ to consider respectively sets $s_{i,x}, s_{j_1,x}, s_{j_2,x}, ..., s_{j_{m-1},x}, s_{j_{m+1},x}, ..., s_{j_{N_S-1},x}$ as minimal sufficient sets. We thus have mutual best responses. Note finally that the set $s_{i,x}, s_{j_1,x}, s_{j_2,x}, ..., s_{j_m,x}, ..., s_{j_{N_S},x}$ of minimal sufficient sets that are mutual best responses taken together constitute a *broom set*.

Step 5. If one equilibrium broom set is a subset of another equilibrium broom set, then this is not compatible with the corresponding sets of messages that players observe being minimal sufficient sets. QED

The intuition for the broom, consisting of a single chain of messages up to the final stage and an explosion of messages at the final stage, as the unit for describing all equilibria is the following. Because there is a possibility that information does not get through, and because there is a large loss *L*, if other players require information from you or from others, and if you can receive confirmations on whether other players received this information, you will only do your part of the action if you receive confirmations that all the required information got through to the other players. If information is required from you at the last stage, confirmations on whether your information got through is not available; in this case, as long as noise is small enough, you will trust that your information got through.

By the same reasoning, for all information that is required by other players at the one-but-last stage, you will want to receive a message at the last stage. Given that each received message contains information on a string of players through which a message was forwarded, you do not need earlier confirmations of messages that are already contained in the forwarded messages which you receive. For this reason, the number of required messages explodes, which explains why the broom is the basic unit by means of which the Nash equilibria can be described.

We note that, broadly speaking, there are two types of Nash equilibria allowing for collective action described by Proposition 1. In a *first* type, each player holds only a *single* minimal sufficient set. To each single broom set letting each informed player send a message that can be constructed from $c(z_S, \mathcal{N}_S)$ corresponds one such equilibrium. Note that this both includes an equilibrium described by a single broom (at least if all informed players are included so that everyone finds out the signal of each informed player), and an equilibrium described by the broom set consisting of *all* brooms, such that each player wants to receive the maximum possible number of messages. This we call the *total-welfare minimizing equilibrium*, as the probability of collective action is minimized in such an equilibrium. In a second type of equilibrium, each players considers several alternative minimal sufficient sets as equally valuable. This includes the equilibrium described by the maximal number of broom sets allowing each player to receive a signal from each informed player, where each player then has the largest possible number of alternative minimal sufficient sets. As this equilibrium allows for the maximum number of different ways in which collective action can be achieved, we call this the *total-welfare maximizing equilibrium*. For instance, for the case where there is a single informed player, this is the equilibrium described by set of all singleton broom sets, or simply put the set of all brooms.

It should be noted that a larger z_S and a larger N_S makes the results in Proposition 1 tight from two sides. *First*, a larger z_S and a larger N_S mean that the total-welfare minimizing equilibrium, where players require all messages from each other, only exists for a smaller and smaller ε. When z_S and/or N_S is increased, the number of required messages increases exponentially. A player who has received all messages is then to a higher extent uncertain about other players also having received all their messages, and the risk for such a player of playing B is then larger. For given L, such equilibria therefore only exist for ever smaller ε as z_S and/or N_S is increased. *Second*, a larger z_S and a larger N_S means that the described total-welfare maximizing equilibrium, where players consider each smallest possible minimal sufficient set as an equally valuable alternative, only exists for ever larger L. Given that many alternative sets of messages can lead to collective action, when not receiving any confirmation from uninformed players, an informed player may still consider it quite likely that each uninformed player received at least one message. This probability will be higher the larger z_S and a larger N_S, such that an ever larger L is needed to support the described equilibrium where players require strong evidence that at least one minimal sufficient set has been established. The point of still assuming large L and small ε, in spite of these tight results, is the following. Small ε means that equilibria exist where players require all available messages from one another. Large L means that even in the equilibria where players consider several alternative minimal sufficient sets as substitutes, players still require quite a lot of messages from one another: each individual minimal sufficient set runs all the way to the last stage, and in each minimal sufficient set, the number of required messages explodes at the last stage. If, as we show, players can even under these unfavorable circumstances become better when having more information, then we have convincingly shown that this is generally possible (thus also for larger ε and smaller L).[12]

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[12] Consider again the case $\mathcal{J} = \{1\}$, $N_S = 3$, $T = 3$, $z_S = 2$ (see Footnote 10), but assume that $L = 2$, $M = 1$, $\varepsilon = 0.15$. Then $(1-\varepsilon)^3 M - \left[1-(1-\varepsilon)^3\right]L < 0$, such that $\{\mathscr{b}_1,\mathscr{b}_2\}$ no longer describes an equilibrium. As $(1-\varepsilon)^2 M - \left[1-(1-\varepsilon)^2\right]L > 0$, $\{\mathscr{b}_1\}$ and $\{\mathscr{b}_2\}$ continue to describe equilibria. To show that $[\{\mathscr{b}_1\}, \{\mathscr{b}_2\}]$ no

For an application of Proposition 1, consider the game with $N = T = N_s = 3$, $z_s = 3$ $\mathcal{J}=\{1\}$, and consider the broom set in the bottom part of Figure 2, represented by solid arrows. An equilibrium exists where each player holds a single minimal sufficient set consisting of the messages in this broom set, where $S_1 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1, 1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{3}{\rightarrow} 1\}\}$, $S_2 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1 \overset{3}{\rightarrow} 2$, $1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{3}{\rightarrow} 2\}\}$, $S_3 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3$, $1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1 \overset{3}{\rightarrow} 3\}\}$. In this equilibrium, player 1 and player 3 obtain payoff $p(1-\varepsilon)^4\{(1-\varepsilon)^3 M - [1-(1-\varepsilon)^3]L\}$, and player 2 obtains payoff $p(1-\varepsilon)^5\{(1-\varepsilon)^2 M - [1-(1-\varepsilon)^2]L\}$.

Compare this to a similar strategy profile where, however, players ignore messages at stage 3: $S_1 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1\}\}$, $S_2 = \{\{1 \overset{1}{\rightarrow} 2\}\}$, $S_3 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3\}\}$. In this strategy profile, player 1 and player 3 obtain payoff $p(1-\varepsilon)^2\{(1-\varepsilon)M - \varepsilon L\}$, and player 2 obtains payoff $p(1-\varepsilon)\{(1-\varepsilon)^2 M - [1-(1-\varepsilon)^2]L\}$. Thus, all players are better off. If so, why is this not an equilibrium? Let player 2 observe $\{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{3}{\rightarrow} 2\}$, but not $\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1 \overset{2}{\rightarrow} 2\}$. Then player 2 believes with probability $1/(2-\varepsilon)$ that player 1 did not observe $\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1\}$. For $L > M$, player 2 plays *A*; it follows that player 2 also plays A when not getting any messages at stage 3. Thus, given $S_1 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1\}\}$, $S_3 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3\}\}$, it is a best response for player 2 to put $S_2 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 1 \overset{2}{\rightarrow} 2$, $1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{2}{\rightarrow} 2\}\}$. But players 1 and 3 will then in turn play the best responses as specified in the above Nash equilibrium. In general, in equilibria where each player holds a single broom set, players require messages up to the final stage of communication. Simply, as long as extra information is available, players will inevitably require it, even if this triggers further information being required by other players. Moreover, the number of messages required explodes at the final stage. The intuition for this is that there is no further opportunity to find out information after the final stage.

Note that there is a Pareto superior Nash equilibrium with $S_1 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{2}{\rightarrow} 1\}\}$, $S_2 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3 \overset{2}{\rightarrow} 2\}\}$, $S_3 = \{\{1 \overset{1}{\rightarrow} 2 \overset{2}{\rightarrow} 3\}\}$. Why does the Pareto inferior Nash equilibrium specified above then exist as well? If player 2 believes that players 1 and 3 each only play *B* when receiving a message at stage 2, then it is a best response for player 2 to require both messages at stage 3. But this in turn makes it a best response for players 1 and 3 to require the messages at stage 2. Thus, the players' mutual best responses can lock them into requiring a large number of messages from one another. A difference with the two-player EMG is that players do not necessarily require every single message generated by the automatic communication protocol.

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longer describes an equilibrium, consider what happens if players 2 and 3 play the corresponding strategy profile, and player 1 does not receive any messages. Denoting $\alpha = (1-\varepsilon)/(2-\varepsilon)$, player 1's expected payoff from playing *B* equals $\{(1-\alpha)^2(1-\varepsilon^2) + 2\alpha(1-\alpha)(1-\varepsilon)\}M - \alpha^2 L > 0$, so that player 1 plays *B* even when not receiving any signals. Nevertheless, it is easy to see that an equilibrium continues to exist where player 1 does not require any messages, player 2 (3) either wants to hear directly from 1 or wants to hear from 1 through 3 (2). Thus, increasing ε eventually eliminates the equilibrium where players require all available messages. Decreasing *L* eventually means that total-welfare maximizing equilibrium is described by alternative lines rather that alternative brooms. Note that in examples with more than three players, equilibria described by a single large communication network will continue to exist (even though this will not include the maximum number of messages). Thus, our results extend to such cases, but the description of the equilibria is only made more complex.

Let us next use the same example to get the intuition for the existence of additional equilibria, where players consider several minimal sufficient sets as equally valuable alternatives. We first show that, even if other players have several alternative minimal sufficient sets, for sufficiently large *L*, each individual player will require strong evidence that the other players have observed at least one minimal sufficient set. Consider the strategies $S_1 = \{\{1 \xrightarrow{1} 2 \xrightarrow{2} 1, 1 \xrightarrow{1} 2 \xrightarrow{2} 3 \xrightarrow{3} 1\}, \{1 \xrightarrow{1} 3 \xrightarrow{2} 2 \xrightarrow{3} 1\}\}$ and $S_3 = \{\{1 \xrightarrow{1} 2 \xrightarrow{2} 3, 1 \xrightarrow{1} 2 \xrightarrow{2} 1 \xrightarrow{3} 3\}, \{1 \xrightarrow{1} 3 \xrightarrow{2} 2 \xrightarrow{3} 1\}\}$, such that it is efficient for players 1 and 3 to either receive all message in the top (singleton) broom set (dashed arrows), or in the bottom broom set (solid arrows). Let player 2 only observe message $\{1 \xrightarrow{1} 2 \xrightarrow{2} 1 \xrightarrow{3} 2\}$. Then by the same reasoning as above, player 2 believes that with probability $1/(2-\varepsilon)$, player 3 has not observed all messages in the bottom broom set (note that the same then certainly applies when player 2 only observes $\{1 \xrightarrow{1} 2\}$). Additionally for this case, player 2 believes that with the same probability $1/(2-\varepsilon)$, player 3 has not observed all messages in the top broom set. The probability that player 3 plays A therefore equals $1/(2-\varepsilon)^2 > 1/4$. For sufficiently large *L*, player 2 plays A. Thus, even if other players hold several alternative minimal sufficient sets, the individual player continues to demand strong evidence that at least one minimal sufficient set has been established. For this reason, each individual minimal sufficient set runs to the last stages of the communication protocol.

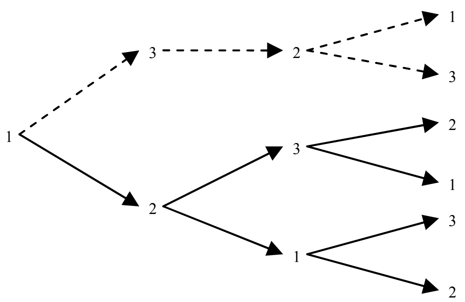


Figure 2 Game $N = T = N_s = 3$, $z_s = 3$ $\mathcal{J} = \{1\}$. Equilibria described by broom set consisting of dashed lines, and broom set consisting of solid lines.

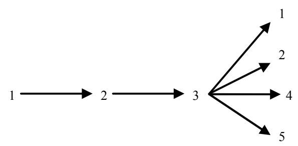
Proposition 1 reveals a plethora of Nash equilibria for the game: to every possible set of non-fully overlapping broom sets corresponds an equilibrium. Equilibrium refinements have no cutting ground in selecting among this plethora of equilibria. All these equilibria are sequentially rational; moreover, since communication is automatic, forward induction arguments do not have cutting ground. As we would still like to provide arguments on how much information a social planner should allow the players to have with a view of maximizing the probability of collective action, we investigate two different types of equilibria in the following two sections. In Section 5, each player requires a specific set of messages for playing *B*, and does not allow for any alternatives. As such equilibria involve the most scope for inefficiency, from this perspective the available information should be limited. The intuition here is that the more information you make available, the more danger that the players get locked into requiring a large number of messages from one another. In Section 6, the focus is on equilibria where there are several alternative information states that induce the players to undertake collective action. These type of equilibria have the most scope for efficiency, and it is shown that from this perspective the available information should on the contrary be maximized. Intuitively, the more players are involved, and the longer they talk, the larger the different manners in which the players can find out about the opportunity for collective action.

5. "Need to know": in favor of limited information

In order to illustrate the danger of making a lot of information available, we start by showing that equilibria exist in which players act in exactly the same manner as if there would be more informed players than there are in reality. We call such uninformed players performing the same role as informed players "pseudo-experts". Simply, as we have shown, every set of brooms sets describes an equilibrium as long as each broom set allows each informed player to send a message. But a set of brooms sets where, along with each informed player, a certain subset of uninformed players also send a message in each broom set, then also describes an equilibrium. Players' mutual expectations may then be seen to lock them in considering certain uninformed players as pseudo-experts.

Corollary 1 (Pseudo-experts). Consider the set of equilibria as described in Proposition 1 for a multi-player EMG with a set $\mathcal{J}_1$ of informed players. Consider an identical EMG, except that there is smaller set $\mathcal{J}_2$ of informed players, with $\mathcal{J}_2 \subset \mathcal{J}_1$. Then to the set of equilibria of the game with a set $\mathcal{J}_1$ of informed players corresponds an identical set for the game with set $\mathcal{J}_2$, where this set is a subset of the complete set of equilibria in the game with set $\mathcal{J}_2$.
Proof: It is clear that to all equilibria of the game with set $\mathcal{J}_1$ correspond equilibria of the game with set $\mathcal{J}_2$, as the minimal sufficient sets of the equilibria for the game with set $\mathcal{J}_1$ contain all informed players in $\mathcal{J}_2$.

Given the possibility of pseudo-experts, the social planner may decide to put z_S low. To see why, consider the case where $N_S = 5$, $\mathcal{J} = \{1, 3\}$. For $z_S = 3$, an equilibrium described by a single broom set consisting of a single broom exists where player 1 sends a message to player 2, after which player 2 forwards this message to player 3, who finally forwards this message to all other players. This is illustrated in the left part of Figure 3. In this case, player 2 plays the role of a pseudo expert: nobody acts unless a message is received from player 2, as well as from player 1 and 3. This is in spite of the fact that player 2 does not observe any signal that determines the state of nature. The effect is generated by players' mutual expectations that they will need such a signal from player 2. The social planner can avoid this possibility by putting $z_S = 2$, as illustrated in the right part of Figure 2. In a similar way, more players can assume the role of pseudo-experts the longer is z_S. If the social planner attaches a high probability to the event where players play an equilibrium involving pseudo-experts, then the social planner should put z_S low.



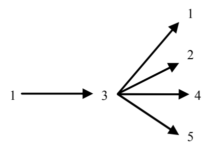


Figure 3. Player 2 as a pseudo-expert

A second way in which we illustrate the potential inefficiency caused by allowing for a lot of information is by showing that for any automatic communication protocol with given z_s and $\mathcal{N}_S$, the equilibrium where players require the maximum number of available messages from one another exists for any game, independently of the level of the threshold *T*. Thus, players' mutual expectations can lock them into acting as if the threshold is larger than it really is.

Corollary 2 (Endogenous thresholds). Consider the automatic communication protocol for given $\mathcal{N}_S$ and z_S. Then for *any* multi-player EMG with *T* such that $3 \le T \le N$, an equilibrium exists where each player in $\mathcal{N}_S$ only plays *B* when receiving each message in the automatic communication protocol .
Proof:
This follows directly from Proposition 1. As, for a given automatic communication protocol, an equilibrium is described by every set of broom sets that let every player receive a message from every informed player, this includes the case of a set consisting of a single broom set spanning all messages that can be generated by the automatic communication protocol. QED

For an example of an endogenous threshold, consider the case with N = 5, $\mathcal{J} = \{1\}$, and suppose that the social planner has put z_S = 2. Assume that $T = 4$. Let the social planner adopt a minimax criterion, and for each given N_S take into account only the worst possible outcome, where the probability of collective action is minimal. Consider first the case where N_S = 4. By Proposition 1, the probability of collective action is lowest in the equilibrium where all players require all messages from one another, as indicated in the right part of Figure 4. Consider next the case where N_S = 5. Then the probability of collective action is lowest in the equilibrium represented by the broom set in the left part of Figure 4. This involves five players even though $T = 4$. If everyone believes that everyone else needs to be reassured that four uninformed players know the state of nature, then it is a best response for the individual player to require such assurances, which again justifies the other players' strategy of requiring such assurances. Thus, even though the real threshold is lower, players' mutual expectations may induce them to adopt a virtual threshold. In the example in Figure 4, player 5 is a "fifth wheel on the cart". The social planner can increase the probability of collective action by only involving four people in the communication process (N_S = 4), so that the threshold T = 4 is exactly achieved.

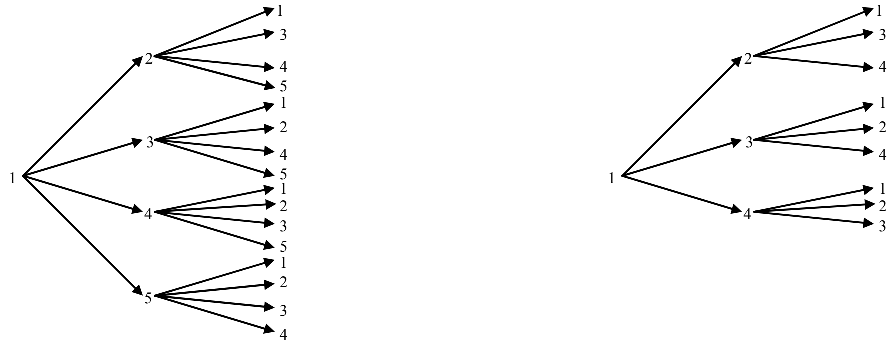


Figure 4. Player 5 as a" fifth wheel on the cart".

Pseudo-experts and endogenous thresholds illustrate a wider principle, saying that the more information players are given, the more they can potentially lock each other into requiring a large number of confirmations and confirmations of confirmations from one another. This follows directly from Corollary 2, as equilibria where the players require the maximal number of messages from each other in the communication protocol exist for any multi-player EMG.

Having looked at a social planner who can only change z_S and/or N_S, let us now go one step further and look at a social planner who can also remove messages in any given $c(z_S, \mathcal{N}_S)$.

Proposition 2 ("Hierarchy"). Consider any EMG as described in Section 3 with $T < N$.[13] Assume that players only play the subset of the set of Nash equilibria described in Proposition 1 where players hold only a single broom set to be a minimal sufficient set. Let a total-welfare maximizing social planner be able to design the communication network by, first, choosing $c(z_S, \mathcal{N}_S)$, and second, by removing messages at will from the chosen $c(z_S, \mathcal{N}_S)$. Then

(i) The social planner chooses as a communication network a single broom ℓ with length $z_S = I$ (= the cardinality of the set of informed players);

(ii) In the first $(z_S - 1)$ stages of this broom, only the informed players are involved, and are ordered along a line. By stage z_S, one informed player has aggregated all other informed players' information, and informs all other players about this information.[14]

Proof:

Note first that it does not make sense that messages are sent to uninformed players before it has been established that state *b* occurs. In order to establish this with the least possible links, this should be done by first letting only the informed players send messages between each other. The shortest way to establish whether state *b* has occurred is to order all informed players along a line, where the last informed player in the line can then find out that state *b* occurs.

This informed player can then inform other players that state *b* occurs. With $T < N$, the probability of collective action is highest when at a final stage a message is sent from the last informed player in line to each other player. QED

Proposition 2 has a natural interpretation. First, a committee of informed player needs to establish whether state *b* has occurred, without any involvement of the uninformed players. Once one informed player has gathered the signals of all other informed players, he or she directly informs everyone, in order to maximize the probability that *T* players are reached. Note that as long as $z_S = I$, and $N_S = N$, for the complete $c(z_S, \mathcal{N}_S)$, an equilibrium as described in Proposition 1 exists that replicates the total-welfare maximizing outcome, in that players can require messages from a single broom and ignore all other messages. But the removal of messages in $c(z_S, \mathcal{N}_S)$ prevents that, *first*, pseudo-experts would be included in any given expert committee, and that, *second*, players would require evidence from multiple committees.

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[13] The case $T = N$ is special in that there are multiple total-welfare maximizing communication networks. In each of these networks, it continues to be the case that in the first $(I - 1)$ stages all the informed players need to be ordered along a line. However, once one informed player has found out that the state is *b*, any structure involving $(N - 1)$ further messages may follow. Still, if this structure also involves $(N - 1)$ players ordered along a line, so that the whole communication network consists of a single line of length $(I + N - 2)$, then the risk of players is reduced to the maximal extent, in that players at a later stage increasingly get more certainty that collective action takes place. There is no conflict here between maximizing the probability of collective action and the players individual incentives to take little risk.

[14] If the social planner is only able to control N_S in an automatic communication protocol $c(z_S, \mathcal{N}_S)$, then by Corollary 2, it may be a good idea to limit N_S. If, however, the social planner is able to design all aspects of the communication protocol, then all players should be included. Having each other player receive a message at the final stage then does not create a risk of further confirmations being asked.

Yet, such an analysis is based on the premise that players play equilibria where a single broom set is considered to be a minimal sufficient set. They may, however, play equilibria where they consider many such minimal sufficient sets as equally valuable substitutes. In this case, as we now go on to show, a total-welfare maximizing social planner should not remove any messages from a given $c(z_S, \mathcal{N}_S)$; moreover, the social planner should make z_S and N_S as large as possible.

6. "Spread the word": in favor of unlimited communication

We start by showing that, if players play total-welfare maximizing equilibria where they consider many alternative information sets as sufficient, the probability of collective action is increased the more players are included in the automatic communication protocol. Thus, if players choose "good" equilibria, including many players does not create the risk of inefficiencies arising because of endogenous thresholds or pseudo-experts, but on the contrary creates more ways in which the players can achieve collective action. Intuitively, the chances of collective action are increased if the word is spread over many players. To formalize this argument, for simplicity, we concentrate on the case where player 1 is the only informed player. We start by showing that, for any given automatic communication protocol, the unique total welfare maximizing Nash equilibrium is described by a set of singleton broom sets, where each broom set consists of a single broom, and where all brooms that can be constructed from the automatic communication protocol are included in the set.

Corollary 3 ("Total-welfare maximizing equilibria"). For any automatic communication protocol with given z_S and $\mathcal{N}_S$, the unique total welfare maximizing Nash equilibrium of an EMG with $\mathcal{J} = \{1\}$ and any T is described by the set $\{\{\mathscr{b}_1\}, \{\mathscr{b}_2\}, \{\mathscr{b}_3\},\ldots\}$ consisting of all brooms contained in the automatic communication protocol.
Proof:
It was already shown in Proposition 1 that each broom set in an equilibrium set of broom sets must let each informed player send at least one message. In the case of one informed player, this is the case for any singleton broom set contained in the automatic communication protocol.
Consider any equilibrium where at least one broom set contains more than one broom. Then the probability of collective action is increased if it suffices for each individual player to receive all the messages in one of these brooms. QED.[15]

Next, we show that, whatever the level of T, for any automatic communication protocol with fixed number of stages, total welfare can potentially be increased by involving more players in the game. This is true even if the number of players in the automatic communication protocol surpasses the threshold.

Corollary 4 ("Spread the word"). For any EMG with $\mathcal{J} = \{1\}$ and general T, across all automatic communication protocol with fixed z_S, the equilibrium with the highest total welfare exists for the automatic communication protocol with $\mathcal{N}_S = \mathcal{N}$.

[15] Consider again the case $\mathcal{J} = \{1\}$, $N_S = 3$, $T = 3$, $z_S = 2$ (see Footnote 10), and consider the parameters $L = 4$, $M = 1$, $\varepsilon = 0.10$. In the equilibrium $\{\mathscr{b}_1,\mathscr{b}_2\}$, collective action takes place in state b with a probability equal to $(1-\varepsilon)^6 =$ 53%; in equilibria $\{\mathscr{b}_1\}$ and $\{\mathscr{b}_2\}$ with a probability equal to $(1-\varepsilon)^3 =$ 72%, and in equilibrium $[\{\mathscr{b}_1\}, \{\mathscr{b}_2\}]$ with a probability of $2\varepsilon(1-\varepsilon)^3 + (1-\varepsilon)^2\left\{(1-\varepsilon)^2 + \left[1-(1-\varepsilon)^2\right](1-\varepsilon)^2\right\}$ = 93%.

Proof:

Start from any set $\mathcal{N}_S$' included in the automatic communication protocol, and enlarge this set to any set $\mathcal{N}_S$''. Consider the new total welfare maximizing equilibrium for the case with $\mathcal{N}_S$''. All the brooms that where sufficient for the case $\mathcal{N}_S$' are now also sufficient for the case $\mathcal{N}_S$'', with the only modification that extra players receive messages in these brooms at stage z_S. This increases the probability that each individual broom leads to collective action. Second, note that the fact that $\mathcal{N}_S$'' is larger means that, in the new total-welfare maximizing equilibrium, extra brooms will be included, again increasing the probability of collective action. QED[16]

To complete the argument that having more information available potentially makes players better off, we show that for a fixed number of players in the automatic communication protocol, total welfare can be increased by adding stages to the automatic communication protocol (taking as a premise that players continue to play the total-welfare maximizing equilibrium after the increase in the number of stages).

Proposition 3. ("Talk as long as possible.") For any EMG with $\mathcal{J}$ = {1} and general T, across all automatic communication protocol with fixed $\mathcal{N}_S$, for sufficiently small ε, the equilibrium with the highest total welfare exists for the automatic communication protocol with $z_S = z$.

Proof:

For any EMG with $\mathcal{J}$ = {1} and general T, consider the total welfare maximizing Nash equilibrium described by a set of singleton broom sets $\{\{\mathscr{b}_I\}, \{\mathscr{b}_{II}\}, \{\mathscr{b}_{III}\},\ldots\}$. Take any singleton broom set $\{\mathscr{b}_X\}$ from this set, and extend the automatic communication protocol with one more stage, by making it possible for each player to receive a confirmation at stage (z_S+1) of each message in $\{\mathscr{b}_X\}$ received at stage z_S. This leads to (N_S-1) new brooms $\{\mathscr{b}_{X,1}\},\{\mathscr{b}_{X,2}\}, \ldots, \{\mathscr{b}_{X,(NS-1)}\}$ being formed. Consider the strategy profile described by the set $\{\{\mathscr{b}_I\}, \{\mathscr{b}_{II}\}, \{\mathscr{b}_{III}\},\ldots,\{\mathscr{b}_{X,1}\},\{\mathscr{b}_{X,2}\}, \ldots, \{\mathscr{b}_{X,(NS-1)}\},\ldots\}$ where each player plays B as soon as receiving at least all messages of one broom in this set. We show that in this new strategy profile, the probability of collective action is increased. If this is true for any single $\{\mathscr{b}_X\}$, it is also true when one stage is added to *all* brooms in the set $\{\{\mathscr{b}_I\}, \{\mathscr{b}_{II}\}, \{\mathscr{b}_{III}\},\ldots\}$.

Consider the probability that broom set $\{\mathscr{b}_X\}$ leads to collective action in the original communication protocol. This probability equals $(1-\varepsilon)^{z_S-1}\pi$, where $\pi = \sum_{k=T-1}^{N_S-1}\binom{N_S-1}{k}(1-\varepsilon)^k\varepsilon^{N_S-1-k}$, or the probability that $(T-1)$ or more players receive a message from $\{\mathscr{b}_X\}$ at stage z_S (such that the threshold is achieved). For reasons that will become clear below, we rewrite the expression $(1-\varepsilon)^{z_S-1}\pi$ as

$$(1-\varepsilon)^{z_S-1}\left[(1-\varepsilon)+\varepsilon\right]^{N_S-1}\pi\,, \tag{1}$$

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16 Consider the case $\mathcal{J}$ = {1}, N = 4, T = 3, z_S = 2, and consider the parameters L = 8, M = 1, ε = 0.10. The automatic communication protocol can then generate three brooms $\mathscr{b}_1$, $\mathscr{b}_2$ and $\mathscr{b}_3$. The reason that a larger L is needed compared to Footnote 10 is that in the case where N_S = 4, the informed player who does not receive any messages in equilibrium $[\{\mathscr{b}_1\}, \{\mathscr{b}_2\}, \{\mathscr{b}_3\}]$ for vanishing ε estimates the probability that two other players act to be 7/8. In this equilibrium, the probability of collective action in state b is equal to $3\varepsilon^2(1-\varepsilon)\alpha + 3\varepsilon(1-\varepsilon)^2\left[\alpha+(1-\alpha)\alpha\right] + (1-\varepsilon)^3\{\alpha+(1-\alpha)\left[\alpha+(1-\alpha)\alpha\right]\}$=99.8%, where $\alpha = \left[(1-\varepsilon)^3+3\varepsilon(1-\varepsilon)^2\right]$. This is larger than the probability 93% in the case where N_S = 3, z_S = 2 (see Footnote 15).

where the term $\left[(1-\varepsilon)+\varepsilon\right]^{N_S-1}=1$ can be interpreted as the probability that any number of messages arrive at a fictitious stage between (z_S-1) and z_S, where receipt of messages at this fictitious stage is independent on the event of receipt of previous or future messages. Note that the term $\left[(1-\varepsilon)+\varepsilon\right]^{N_S-1}$ can be rewritten as $\sum_{k=1}^{N_S-1}\binom{N_S-1}{k}(1-\varepsilon)^k\varepsilon^{N_S-1-k}$.

Consider next the probability of achieving collective action in the set of brooms $\{\mathscr{b}_{X,1}\},\{\mathscr{b}_{X,2}\}, \ldots, \{\mathscr{b}_{X,(NS-1)}\}$. This equals

$$(1-\varepsilon)^{z_S-1}\sum_{S=1}^{N_S-1}\rho_S\pi_S \ , \tag{2}$$

where $\rho_S=\binom{N_S-1}{S}(1-\varepsilon)^S\varepsilon^{N_S-1-S}$ is the probability that S messages are received at stage z_S, and π_S is the probability that $(T-1)$ or more players receive messages at stage (z_S+1) in at least one out of S of the brooms $\{\mathscr{b}_{X,1}\},\{\mathscr{b}_{X,2}\}, \ldots, \{\mathscr{b}_{X,(NS-1)}\}$.

Note that the probabilities ρ_S together consist of all the terms in the sum $\left[(1-\varepsilon)+\varepsilon\right]^{N_S-1}$, with the exception of term ε^{N_S-1}. It follows that expressions (1) and (2) consist of analogous terms – one for each term of $\left[(1-\varepsilon)+\varepsilon\right]^{N_S-1}$ – with the exception of the extra term

$$(1-\varepsilon)^{z_S-1}\varepsilon^{N_S-1}\pi \tag{3}$$

contained in (1). Note that $\pi_1=\pi$, so that for S = 1 in (2), the corresponding term in (1) is identical. For any $S\geq 2$ it is the case that $\pi_S>\pi$, so that for $S\geq 2$, the term in (2) is always larger than the corresponding term in (1). This reflects the gains that are obtained from the fact that, contrary to what is the case in (1), collective action is possible even if less than $(T-1)$ messages arrive at the last stage in one particular broom (= the equivalent of the single broom in the equilibrium corresponding to (1)). Such gains are obtained for all cases where *less* than $(T-1)$ messages arrive at the last stage in one particular broom set. At the last stage, k messages, with $0\leq k\leq T-2$, arrive with probability $\binom{N_S-1}{k}(1-\varepsilon)^k\varepsilon^{N_S-1-k}$; to obtain the individual gain of (2) over (1), this must then be multiplied by the probability π_{S-1} that $(T-1)$ or more players receive messages in at least one of the $(S-1)$ other brooms. Thus, the gains of having the brooms $\{\mathscr{b}_{X,1}\},\{\mathscr{b}_{X,2}\}, \ldots, \{\mathscr{b}_{X,(NS-1)}\}$ instead of a single broom $\{\mathscr{b}_{X,1}\}$ equal

$$(1-\varepsilon)^{z_S-1}\left[\sum_{k=0}^{T-2}\binom{N_S-1}{k}(1-\varepsilon)^k\varepsilon^{N_S-1-k}\right]\sum_{S=2}^{N_S}\rho_S\pi_{S-1} \ . \tag{4}$$

Note now that, as long as $T>2$, (4) contains at least one term that contains an expression ε^{N_S-1-k} with $(N_S-1-k)<(N_S-1)$; this is because $\sum_{S=2}^{N_S}\rho_S\pi_{S-1}$ contains terms where all messages arrive. Finally, note that the loss of (2) over (1), given in (3), contains a probability

ε^{N_S-1}. It follows that, for sufficiently small ε, the loss of (2) over (1) (given in (3)) vanishes compared to the gains of (2) over (1) (given in (4)). QED[17]

Thus, concluding, players can benefit from more information not only by the fact that it may be more likely that a threshold of players act if the word is spread over more players. A fixed number of players can also benefit from talking a longer time. The longer players talk to each other, the more different avenues are created along which information can travel, and the more robust information transmission is to disruption of one of these avenues; the benefit from having extra avenues exceeds the cost of more messages being required in every single avenue. In terms of the analysis in the previous section, it may not matter if players consider the information of several committees as important, as long as they are considered as substitutes, and not as complements. Also, it may not matter if individual committees include pseudo-experts. Indeed, for any committee involving only informed players, one can make $(N-I)$ new committees by putting each uninformed player in a committee once. If these are then all considered as alternatives, the probability of collective action increases.

7. Hearsay: unsophisticated players

A criticism on the analysis so far may be that players are assumed able to process a large degree of interactive knowledge. For instance, in the case of five players with one informed player, it may be that Edward wants to hear from David that David heard from Carl that Carl heard from Bob that Bob heard from Alice that she found out that there is an opportunity to benefit from collective action. It does not suffice for Edward to hear from David that David heard from Bob that Bob heard from Carl that Carl heard from Alice that she found out that there is an opportunity for collective action.

Yet, an unsophisticated player may not be able to distinguish between such knowledge. Let us consider an unsophisticated player who is only able to count how many messages he or she receives at individual stages, but is not able to distinguish between messages. We now show that, even with such unsophisticated players, efficient equilibria continue to exist along with inefficient ones.

Corollary 5 (Unsophisticated players). For the case $\mathcal{J} = \{1\}$, consider a variant of the game described in Section 3 where players are only able to count how many messages they receive at each stage. Then the subset of the Nash equilibria described in Proposition 1 where in each minimal sufficient broom set, players receive at least a sum of X messages over the stages $(z_S - 1)$ and z_S are also Nash equilibria of this modified game. This subset includes both the total-welfare maximizing equilibrium of the original game (where players require the messages in any one broom from one another), and total-welfare minimizing equilibrium of the original game (where players require all messages from one another).

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[17] Consider again the case $\mathcal{J} = \{1\}$, $N_S = 3$, $T = 3$, $z_S = 2$ (see Footnotes 10 and 15), and consider the parameters $L = 4$, $M = 1$, $\varepsilon = 0.10$. In case $z_S = 2$, in equilibrium $[\{\mathfrak{b}_1\}, \{\mathfrak{b}_2\}]$ collective action takes place 93% of the time (see Footnote 15). In case $z_S = 1$, in the unique equilibrium collective action takes place with probability $(1-\varepsilon)^2 = 81\%$ of the time. The latter probability can be rewritten as $\left[(1-\varepsilon)^2 + 2\varepsilon(1-\varepsilon) + \varepsilon^2\right](1-\varepsilon)^2$, the former probability equals $(1-\varepsilon)^2\left\{(1-\varepsilon)^2 + \left[2\varepsilon(1-\varepsilon) + \varepsilon^2\right](1-\varepsilon)^2\right\} + 2\varepsilon(1-\varepsilon)(1-\varepsilon)^2$. In terms of the proof of Proposition 3, the loss of switching from $z_S = 1$ to $z_S = 2$ thus equals $\varepsilon^2(1-\varepsilon)^2$, while the gain equals $(1-\varepsilon)^2\left[2\varepsilon(1-\varepsilon) + \varepsilon^2\right](1-\varepsilon)^2$.

Proof:
Proposition 1 shows for the case $\mathcal{J} = \{1\}$ that a Nash equilibrium corresponds to any set of broom sets where one broom set is not a subset of another broom set. This includes sets of broom sets such that, in each minimal sufficient broom set, players receive a fixed number of messages X over stages $(z_S - 1)$ and z_S, where $1 \le X \le (z_S - 1)(N_S - 1)$. It follows that these are also equilibria of a modified game where players are only able to count how many messages they receive at each stage. Note that for $X = 1$, the total-welfare maximizing equilibrium described in Corollary 3 is replicated. For $X = (z_S - 1)(N_S - 1)$, we have the total-welfare minimizing equilibrium where players require all available messages from each other. QED

While unsophisticated players are equally well able to play inefficient equilibria, in spite of the large assumed loss of acting with less than T players, in the total-welfare maximizing equilibrium, players act only based on "hearsay": each player acts when having recently (i.e., over the two last stages) heard from someone (i.e., *any* other player) that there is an opportunity for collective action.

8. Conclusion

The literature on the two-player EMG shows that two players involved in a collective action problem can lock themselves into requiring a large number of assurances and reassurances from each other that there is an opportunity for collective action, thus reducing the probability of collective action. We have shown that for the multi-player EMG, equilibria exist that generalize this effect. Particular aspects of this effect for the multi-player EMG are that players' mutual expectations can create endogenous thresholds, where players only act when receiving information that more people know about the opportunity for collective action than is strictly necessary, and can create pseudo-experts, where a player who does not have information on whether there is an opportunity for collective action still gets to be considered as having such information. From this perspective, a rationale is obtained for limiting the extent to which players can check each other's knowledge, and players are best ordered in a hierarchy where the informed players first form a committee establishing whether there is an opportunity for collective action, after which all other players are informed about this opportunity. Thus, it seems players' knowledge should be restricted, so that each is informed on a need-to-know basis. From this perspective, we obtain a rationale for setting up hierarchies to solve collective-action problems.

Yet, unlike what is the case for the two-player EMG, in the multi-player EMG equilibria also exist where players use the many messages that can be generated by a process of confirmation and reconfirmation not only as an instrument of mutual reassurance, but as an instrument to generate several alternative channels through which players can be informed about the opportunity for collective action. In the equilibrium of this form that maximizes the probability of collective action, each player acts as soon as receiving at least a single message at the one-but-last or last stage. From this perspective, the players are better off if, first, more players are involved in the collective action problem, and, second, if these players are allowed to talk for a longer time. This is simply because more alternative channels for informing players are then created, thus reducing the effect of noise. Therefore, the probability of collective action may be increased when players are allowed to spread the word as much as possible, and if players are to the maximal extent allowed to check each other's information. From this perspective, players' knowledge should not be restricted, and hierarchies should not be created to solve collective-action problems. Our aim in this paper has been to point out the possibility of such an effect. Yet, we are unable to eliminate inefficient equilibria which justify a completely opposite measure where communication between players is severely restricted. The selection between such equilibria, on further theoretical grounds and/or on experimental grounds, is the subject of future research.

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