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Discussion Paper Series nr: 11-17

A Taylor Rule for Fiscal Policy

David A. Kendrick
Hans M. Amman

Universiteit Utrecht

Utrecht School of Economics

Tjalling C. Koopmans Research Institute
Utrecht School of Economics
Utrecht University

Kriekenpitplein 21-22
3584 TC Utrecht
The Netherlands
telephone +31 30 253 9800
fax +31 30 253 7373
website www.koopmansinstitute.uu.nl

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How to reach the authors

Please direct all correspondence to the first author.

David A. Kendrick
Department of Economics
University of Texas
Austin, Texas 78712
USA
e-mail: kendrick@austin.utexas.edu

Hans M. Amman
Utrecht School of Economics
Utrecht University
Kriekenpitplein 21-22
3584 TC Utrecht
Netherlands
E-mail: h.m.amman@uu.nl

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A Taylor Rule for Fiscal Policy

David A. Kendrick[a]
Hans M. Amman[b]

[a]Department of Economics
University of Texas

[b]Utrecht School of Economics
Utrecht University

October 2011

Abstract

In times of rapid macroeconomic change it would seem useful for both fiscal and monetary policy to be modified frequently. This is true for monetary policy with monthly meetings of the Open Market Committee. It is not true for fiscal policy which mostly varies with the annual Congressional budget cycle. This paper proposes a feedback framework for analyzing the question of whether or not movement from annual to quarterly fiscal policy changes would improve the performance of stabilization policy. More broadly the paper considers a complementary rather than competitive framework in which monetary policy in the form of the Taylor rule is joined by a similar fiscal policy rule. This framework is then used to consider methodological improvements in the Taylor and the fiscal policy rule to include lags, uncertainty in parameters and measurement errors.

Keywords: design of fiscal policy, optimal experimentation, stochastic optimization, time-varying parameters, numerical experiments.

JEL classification: C63, E61, E62

1 Introduction

In February of 2008 Congress passed a tax rebate of about 150 billion that had been proposed by the Bush Administration to slow the downturn in the economy. One year later, in February of 2009 Congress passed the Obama's Administration stimulus package of about $800 billion dollars. Between these two dates the unemployment rate rose from about 5 percent to about 8 percent across a twelve month span in which no additional fiscal policy measures were enacted. In contrast, across this same period monetary policy was reviewed monthly - or even more frequently - and corrective actions were taken repeatedly in attempts to mitigate the downturn.

Why across this period, and more broadly across the period 2007-2009, was monetary policy adjusted so frequently and fiscal policy so infrequently? The basic answer is simple - the Open Market Committee meets about once a month to consider modifications in monetary policy while fiscal policy dances to the slow rhythm of the annual budget cycle of the Congress. However, this leaves open the question of whether fiscal policy *should* be modified more frequently. Would smaller and more frequently changes in fiscal policy in the period from the fall of 2007 thru the fall of 2009 have decreased the downward inertia of the economy and thus mitigated substantially the rise in unemployment. And would this have decreased the decline in government revenues and thus the amount of the rise in the federal deficit?

This paper addresses these questions through the framework of feedback rules.

2 Feedback Rules in Macroeconomics

The idea of using control theory methods and feedback rules in macroeconomic stabilization was given it first prominence in the works of A.W. H. Phillips in the 1950's. [1] It was then that he developed the famous water models of the economy while he was living in Great Britain. The idea was simple, namely that the condition of the economy should be feedback to the policy controller so that policies could be adjusted to bring the economy back onto desired paths.

Phillips' idea did not gain traction with economists at that time. However, the idea was revived twenty years later by a group of economists and engineers brought together under the leadership of Michael Athens, Gregory Chow, Ed Kuh and M. Ishaq Nadiri for an NBER conference at Princeton

[1] See Phillips (1954).

University in 1972. This time the idea found strong support and resulted in the formation of the Society of Economic Dynamics and Control which sponsored a series of annual conferences. The group even created their own journal, the *Journal of Economic Dynamics and Control*, which quickly rose in the rankings among economics journals.

Early in this period Gregory Chow and his undergraduate assistant, Andrew Abel, developed at Princeton a quarterly macroeconomic model and applied stochastic control theory methods to it.[2] That model is small and simple enough to serve as a good starting point for the discussion in this paper. It had two state variables, consumption (C) and investment (I) and two control variables, government expenditures (G) and the money supply (M). These variables were embedded in the system equations for the econometric model which were written as

$$x_{k+1} = A_k x_k + B_k u_k + \xi_k \tag{1}$$

where

$$\begin{aligned}
k &= 0, ..., N-1, \text{being the time subscript} \\
x_k &= \text{the state vector in period } k \text{ of dimension } n \times 1 \\
u_k &= \text{the control vector in period } k \text{ of dimension } m \times 1 \\
A_k &= \text{state vector coefficient matrix in period } k \text{ of dimension } n \times n \\
B_k &= \text{control vector coefficient matrix in period } k \text{ of dimension } n \times m \\
\xi_k &= \text{vector of additive noise terms in period } k \text{ of dimension } n \times 1
\end{aligned}$$

and where the state and control vectors were

$$x_k = \begin{bmatrix} I_k \\ C_k \end{bmatrix} \qquad u_k = \begin{bmatrix} G_k \\ M_k \end{bmatrix} \tag{2}$$

There were desired paths for both the state and control variables which were specified with a quadratic tracking criterion function. Minimization of the criterion function with respect to the systems equation (1) yielded the feedback rule

$$u_k = \hat{G}_k x_k + \hat{g}_k \tag{3}$$

where

[2]See Chow (1967) and Abel (1975).

$$\hat{G}_k = \text{the feedback gain matrix in period } k$$
$$\hat{g}_k = \text{the vector of feedback parameters in period } k$$

Thus deviations of the consumption or investment state variables from their desired paths worked through the feedback rule to increase or decrease the government expenditure and/or money supply variables as necessary to bring the economy back onto track.

One could develop a variant of the Abel model with output and inflation as the state variables and government expenditures and the interest rate as the control variables. The feedback rule in this case would be of the form

$$\begin{bmatrix} G_k \\ r_k \end{bmatrix} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} \\ \hat{G}_{21} & \hat{G}_{22} \end{bmatrix} \begin{bmatrix} Y_k \\ \pi_k \end{bmatrix} + \begin{bmatrix} \hat{g}_1 \\ \hat{g}_2 \end{bmatrix} \tag{4}$$

dropping the k time subscript for the feedback gain matrix $\hat{G}$ and feedback gain vector $\hat{g}$. The second of the two equations in equation (4) is the familiar Taylor rule in which the interest rate is determined by a feedback of output and the inflation rate.[3]

$$r_k = \hat{G}_{21} Y_k + \hat{G}_{22} \pi_k + \hat{g}_2 \tag{5}$$

Feedback rules of this kind can be derived from quadratic-linear control models of the type described above or handcrafted and varied over runs in simulation models until satisfactory values of the feedback gain coefficients ($\hat{G}_{21}$, $\hat{G}_{22}$, $\hat{g}_2$) are determined.[4] Also historical values of the interest rate, output and inflation can be used to estimate the feedback gain coefficients.

Our focus here is not on the second equation in equation (5), i.e. the Taylor rule, but rather on the first equation which provides a feedback rule for fiscal policy, i.e.

$$G_k = \hat{G}_{11} Y_k + \hat{G}_{21} \pi_k + \hat{g}_1 \tag{6}$$

in which the level of government expenditures is determined by feedback of the income and inflation state variables.

[3]See Taylor (1993, 1999).

[4]See Kendrick (1981, 2002).

3 Quarterly Fiscal Policy

The feedback rule in equation (6) assumes that fiscal policy would be modified each quarter while, as was discussed above, the practice is that government expenditure levels are modified once a year in the Congressional budget cycle.

It seems apparent that small quarterly changes in fiscal policy would provide a less volatile path for the economy than large annual changes. However, so far as we know, model experiments have not yet been done to compare results with quarterly fiscal policy changes to those with annual fiscal policy changes. Therefore one of the high priorities for research in this field would be to focus on this question.

Such an experiment could be done by using time varying weights in the quadratic tracking criterion function. This function has the form

$$\begin{aligned} J &= \frac{1}{2}(x_N - \tilde{x}_N)' W_N (x_N - \tilde{x}_N) \\ &+ \frac{1}{2}\sum_{t=1}^{T}\left[(x_k - \tilde{x}_k)' W_k (x_k - \tilde{x}_k) + (u_k - \tilde{u}_k)' \Lambda_k (u_k - \tilde{u}_k)\right] \qquad (7) \end{aligned}$$

where additionally

$\tilde{x}_k$ = the desired state vector in period k of dimension $n \times 1$

$\tilde{u}_k$ = the desired control vector in period k of dimension $m \times 1$

W_N = symmetric state variable penalty matrix in terminal period N of dimension $n \times n$

W_k = symmetric state variable penalty matrix in period k of dimension $n \times n$

Λ_k = symmetric control variable penalty matrix for period k of dimension $m \times m$

Thus high weights (penalties) could be used in the diagonal element of the matrices corresponding to government expenditures in three of the four quarters in each year and a low weight could be used in the quarter when Congress usually enacts the budget. The weights in the matrices corresponding to the money supply would be low in all quarters. In contrast, for experiments in which there are quarterly changes in fiscal policy the weights in the matrices corresponding to both government expenditure and the money supply would be low in all quarters.

The shift from thinking of fiscal policy as annual to quarterly also heightens the interest in the time distribution of government expenditures.

4 Institutional Considerations for Quarterly Fiscal Policy[5]

If the studies of quarter fiscal policy should show that there is a substantial advantage to quarterly rather than annual changes in fiscal policy, then the question will arise of the institutional changes to implement this alteration in policy procedures.

It seems unlikely that the Congress would want to pass quarterly changes in government expenditure; however the Congress might create in each annual budget cycle a pipeline of projects and programs. These could be packaged in tranches and given priorities. Then the speed with which these projects and programs are released from the pipeline would be determined by a Fiscal Policy Agency governed by a board of Senators and Representatives as well as Administration officials.

This agency would also need to have a small but highly qualified technical staff to do fiscal policy research just as is now done on the monetary side by the Federal Reserve Board staff. One of the issues that such a staff would need to consider is lags in the actual expenditure of funds on projects and programs of different types.

5 Lags in Fiscal and Monetary Policy

At the time the Obama stimulus package was passed by the Congress in February of 2009 there was much discussion about *shovel ready* projects amidst a debate about when the effects of the legislation would be felt in the economy. This suggests that econometric models like those discussed above should distinguish between actual expenditure of government funds on projects and obligations passed by the Congress. This could be modeled with equations like

$$G_k = \alpha_0 O_k + \alpha_1 O_{k-1} + \alpha_2 O_{k-2} \tag{8}$$

[5] We are indebted to Douglas Dacy for some ideas that underlie institutional considerations for quarterly fiscal policy including the notion of a separate agency with its own technical staff.

where

$$G_k = \text{Government expenditures in period } k$$
$$O_k = \text{Government obligations voted by Congress in period } k$$
$$\alpha_j = \text{percentage of obligations in period } k \text{ spent in period } k+j$$

The effect of a specification like that in equation (8) on the system equation (1) is to add a distributed lag in the control variable, thus the system equation becomes

$$x_{k+1} = A_k x_k + B_k u_k + B_{k-1} u_{k-1} + B_{k-2} u_{k-2} + \xi_k \tag{9}$$

Models with distributed lags in the system equations are converted to models with the usual single lag like equation (1) by augmenting the state vector to include the lagged controls.[6] Thus the augmented state would be

$$z_k = \begin{bmatrix} x_k \\ u_{k-1} \\ u_{k-2} \end{bmatrix} \tag{10}$$

More generally in most macro-econometric models there are also distributed lags beyond one period in the state variables. Thus in a model with two quarter lags in the states and three quarter lags in the controls the augmented state vector would be

$$z_k = \begin{bmatrix} x_k \\ x_{k-1} \\ u_{k-1} \\ u_{k-2} \end{bmatrix} \tag{11}$$

and the feedback rule would be

$$u_k = G_k z_k + g_k \tag{12}$$

Therefore the control variable would be determined by the feedback rule as a function of the current state vector as well as by that vector lagged once and by the control vector lagged once and twice. Thus the current value of government expenditure would be a feedback function of current and past output and inflation as well as of past government expenditure.

[6] See Kendrick (1981, Section 2-1, page 9).

Thus in contrast with the presently discussed Taylor rules for monetary policy, it is likely that a similar feedback rule for fiscal policy should include lagged values of the state and control variables.

With a fiscal policy feedback rule in hand it is useful to think about combining it with the Taylor rule to have a set of feedback rules for both fiscal and monetary policy. When this is done it is useful to consider the role of uncertainty in feedback rules.

6 Uncertainty in Feedback Rules

There has long been discussion of the comparative advantage - and even complementarity - of monetary and fiscal policy. This discussion has traditional been about the size of the effects of each policy on output, inflation, balance of payments, etc. This discussion can focus on the size of the coefficients of these policies in the econometric models of the economy.

Also, in the context of the discussion of lags in the previous section it is worthwhile to consider the timing of monetary and fiscal policy. Does fiscal policy have a shorter mean lag than monetary policy or vice versa?

However, it is also useful to consider a third kind of comparative advantage - namely which policy is more reliable. Are the effects on the economy of monetary policy more or less uncertain than the effects of fiscal policy? One way to address this question is by considering the standard errors of the estimates of the coefficients multiplied by policy variables in econometric models, i.e. in the B_k matrix in the system equation

$$x_{k+1} = A_k x_k + B_k u_k + \xi_k \tag{13}$$

If the t-test for the coefficient on government expenditures is larger than the t-test for the coefficient on the Fed funds rate then - at least as a first approximation - fiscal policy is more reliable than monetary policy.

The feedback framework lends itself well to consideration of this issue. Indeed when there is parameter uncertainty the feedback rule is called - optimal feedback with parameter uncertainty, see Kendrick (1981, Chapter 6), and is written as [7]

$$u_k = G_k^{\dagger} x_k + g_k^{\dagger} \tag{14}$$

[7] See Amman and Kendrick (1999). The F_k term in equation (15) comes from the cross terms between the state and control in the criterion function omitted in equation (7) - see Kendrick (1981, pages 42-45).

with

$$G_k^\dagger = -\left[E\left\{B_k'K_{k+1}B_k\right\} + \Lambda_k'\right]^{-1}\left[F_k' + E\left\{B_k'K_{k+1}A_k\right\}\right] \quad (15)$$

$$g_k^\dagger = -\left[E\left\{B_k'K_{k+1}B_k\right\} + \Lambda_k'\right]^{-1} E\left\{B_k'p_{k+1}\right\} \quad (16)$$

In equation (15) the E is the expectations operator that is taken over the uncertainty in the parameter estimates in the matrices A_k and B_k. While the parameters of greatest interest in this regard are those in the B_k matrix which are multiplied by the control vector, the methodology is general enough to treat uncertainty in the A_k matrix as well and therefore is able to treat not only direct but indirect uncertainty effects of different policies.

Also the expectations operator plays a similar role in the computation of the Riccati matrices, K_k, and tracking vector p_k. It is significant that the only difference between the optimal feedback rule in the deterministic case in equation (3) and the optimal feedback rule with parameter uncertainty in equation (14) is the expectations operator used in the calculation of the feedback gain matrix, $G_k^\dagger$. The same is also true of the computations for the vector of feedback parameters, $g_k^\dagger$.

Much discussion has occurred among economists in recent years about the effectiveness of tax changes (and even of government expenditures) for stabilization policy. The use of parameter uncertainty in feedback rules puts this debate in a more constructive framework but shifting it from whether or not there is any effect of policy changes to the comparative degrees of uncertainty of different policies.

7 Measurement Errors

Most policy studies ignore measurement error and yet it is obvious from the size of changes when economic data are updated repeatedly in the months and quarters after they are first issued that measurement errors are widespread. However, stochastic control theory methodology includes measurement error relationships on the state variables. Thus if some states are measured with less error than others then they can be relied on more heavily in the feedback rules. For example consumption and investment might both appear in a feedback rule for fiscal policy and yet consumption probably is measured with less error than investment and thus can be relied on somewhat more in the feedback rule.

Measurement error also plays a role in models with distributed lags in the state variables. An innovative recent paper in this area by Coenen, Levin and

Wieland (2001) considers the case in which there are both forward variables and lags in the systems equations. The lagged terms can be modeled in the control framework by augmenting the state vector as was discussed above. For example in a model with three-period lags the augmented state would be

$$z_k = \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \\ u_{k-3} \end{bmatrix} \tag{17}$$

and the feedback rule would be

$$u_k = G_k z_k + g_k \tag{18}$$

Therefore the control variable would be determined by the feedback rule as a function of the current state vector as well as by that vector lagged once, twice and thrice. Of course, due to revision of the data the states with the longest lags most likely have the smallest measurement error. Thus the optimal controller must consider the fact that one would like to feedback most heavily on the current state; however it is the noisiest. Therefore the feedback rule must strike a balance between depending on the most recent state of the economy and the state with the least measurement error.

8 Conclusions

In times of rapid macroeconomic change it would seem useful for both fiscal and monetary policy to be modified frequently. This is true for monetary policy with monthly meetings of the Open Market Committee. It is not true for fiscal policy which mostly varies with the annual Congressional budget cycle. A feedback framework with time-varying weights in a quadratic tracking function is proposed for analyzing the question of whether or not movement from annual to quarterly fiscal policy changes would improve the performance of stabilization policy.

More broadly the paper considers a complementary rather than competitive framework in which monetary policy in the form of the Taylor rule is joined by a similar fiscal policy rule. Recent research has been oriented too much to the question of whether one should use monetary policy *or* fiscal policy. In fact the complementarity between the two policies in magnitude of effects, lag structures and degrees of uncertainty in parameters suggests that it is imperative that the two policies be analyzed fully in a complementary rather than a competitive framework. This framework is provided when the Taylor rule for monetary policy is augmented by a similar fiscal policy rule. This joint framework with both feedback rules can then used to consider methodological improvements in the Taylor and the fiscal policy rule to include lags, uncertainty in parameters and measurement errors.

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