Remembering Johannes J. Duistermaat (1942–2010)

Victor Guillemin, Álvaro Pelayo, San Vũ Ngọc, and Alan Weinstein, Coordinating Editors

We are honored to pay tribute to Johannes (Hans) J. Duistermaat (December 20, 1942–March 19, 2010), a world leading figure in geometric analysis and one of the foremost Dutch mathematicians of the twentieth century, by presenting a collection of contributions by some of Hans's colleagues, collaborators, and students. Duistermaat's first striking contribution was his article "Fourier integral operators II" with Hörmander (published in *Acta Mathematica*), a work that he did after his doctoral dissertation. Several influential results in analysis and geometry have the name Duistermaat attached to them, for instance, the Duistermaat-Guillemin trace formula (1975), Duistermaat's global action-angle theorem (1980), the Duistermaat-Heckman theorem (1982) and the Duistermaat-Grunbaum bispectral theorem (1986). Duistermaat's papers offer an unusual display of originality and technical mastery.

Hans Duistermaat passed away on March 19, 2010, in The Netherlands. Hans had been affiliated with the Mathematics Institute at Utrecht University as Professor of Pure and Applied Mathematics since 1974. He officially retired in 2007 but continued to be active in research until his unexpected death. During the last five years of his career, he held a KNAW Professorship from the Royal Academy of Sciences of the Netherlands, which freed him from teaching and administrative duties. He was also "Ridder in de Orde van de Nederlandse Leeuw" (Knight of the Order of the Dutch Lion) and the recipient of numerous other awards. A conference in honor of the sixty-fifth birthday of Hans Duistermaat took place in Utrecht in 2007, and the proceedings are to be published by Birkhäuser, Boston. The Dutch Research School in Mathematics (WONDER) has decided to establish the Hans Duistermaat Chair for Visiting Professors. With the passing of Hans Duistermaat, mathematics has lost an exceptional and original figure. Hans had a powerful, sharp personality, and he loved a good argument, but, at the same time, he was unusually kind and generous toward others.

Photograph by J. W. H. Kolk.

Hans Duistermaat (December 20, 1942–March 19, 2010) at a gathering following J. A. C. Kolk's thesis defense in 1977.

Victor Guillemin is professor of mathematics at the Massachusetts Institute of Technology. His email address is vwg@math.mit.edu.

Álvaro Pelayo is a member of the School of Mathematics at the Institute for Advanced Study and an assistant professor at Washington University in St. Louis. His email addresses are apelayo@math.wustl.edu *and* apelayo@math.ias.edu.

San Vũ Ngọc is professor of mathematics at the University of Rennes 1 and the Institut Universitaire de France. His email address is san.vu-ngoc@univ-rennes1.fr.

Alan Weinstein is professor of mathematics at the University of California Berkeley. His email address is alanw@math.berkeley.edu.

Hans Duistermaat considered himself an analyst, but his influence on mathematics extends well beyond the field of analysis. He liked geometry very much because he liked to understand the intrinsic nature of the problems that he tackled. The mathematics of Hans is a rare combination of elegance and practical relevance. He did not want to be a theoretician and loved to revisit the most classical examples and demonstrate their universal qualities. And when a theoretical problem appeared before him, amply motivated, his capacity for abstraction was extraordinary.

Hans was genuinely modest, both at a human and at a professional level. He did not care about honors, but when he received them he was at first surprised and then gratified. He refused to have a conference celebrating his sixtieth birthday, but at the insistence of the mathematical community he agreed to have a sixty-fifth birthday celebration provided that the conference lectures consisted of purely scientific communications.

His modesty was matched only by his scientific honesty. Hans thought that most of the so-called new ideas could be traced back to the works of other famous figures in mathematics. His passion for history, evident to all those who knew him, drove him back to the original works of Lie, Cartan, Poincaré, and even Huygens and Newton. Consequently, and contrary to current trends, he published sparingly. Richard Cushman, one of his colleagues in Utrecht, confessed to us that Hans had accumulated a massive amount of unpublished notes that could provide the material for dozens of articles. Fortunately, Hans's taste for history did not take over his enthusiasm for advancing mathematics. He was known to be a strong supporter of young people and always encouraged them to push the boundaries of knowledge without fear.

Next, we give a brief glimpse of (some of the) mathematics that Hans Duistermaat encountered in his distinguished trajectory as a mathematician. He was awarded a doctorate in mathematics at the University of Utrecht in 1968. Although H. Freudenthal is listed as his advisor, Duistermaat's doctoral thesis was directed by G. K. Braun, who died one year prior to the thesis defense. Following his thesis, he had the great insight of studying the work of Hörmander regarding a technique to study linear partial differential equations: Fourier integral operators.

After reading certain parts of Hörmander's work, Hans decided to take a postdoctoral position at Lund where Hörmander was teaching, so he could better understand Hörmander's work. For a year, Hörmander lectured about his new theory to a small group of mathematicians, including a young and energetic Duistermaat. Duistermaat's extensive knowledge of Lie's work allowed him to contribute in an essential way to the global theory of Fourier integral operators. It was undoubtedly his first stroke of brilliance: when he returned to the Netherlands in 1970, Hans dedicated all his energy to exploring and understanding the applications of the theory of Fourier integral operators, and after several months of work he started mailing manuscripts with the results of his investigations to Hörmander. After many months of unanswered letters, Hans received a letter from Hörmander saying: "now I understand what is going on—manuscript will follow."

Hans eventually received a manuscript from Hörmander, and with his usual modesty he said that he "recognized" his contributions here and there in the text. Duistermaat had revealed himself to the mathematical community as a pioneer in the field of microlocal analysis. This work [10], published by *Acta Mathematica* in 1972, has become an essential reference in the field. He also wrote about this topic in his notes from Nijmegen and the Courant Institute [4].

Hans witnessed, for the first time during his stay in Lund, a difference in the views about Maslov's theory between Hörmander and Leray, who belonged to different schools of thought. The latter understood the meaning of the WKB ansatz in terms of Lagrangian distributions, but his intuitive approach to this theory, motivated by concrete problems in physics (e.g., the spectral theory of the helium atom), was, for mathematicians at least, difficult to follow. On the other hand, Hörmander, in the Bourbaki vein, wanted a clean and mathematically precise theory that would make no reference to physics. In France, Leray was very interested in Maslov's approach.

Hans stayed open to both views; even better, his phenomenal quickness allowed him to benefit from, and make contributions to, both approaches. Only two years after his seminal article with Hörmander, "Fourier integral operators II", he published in *Communications in Pure and Applied Mathematics* another article on oscillatory integrals [2] that has become a standard reference in the field, in which Maslov's formulation (including the small parameter $\hbar$) is mathematically justified. Moreover, Hans became quite interested in the Maslov index that appears in stationary phase. He was the first to establish a link between the index defined by Hörmander and the Morse index of the variational problem.

Meanwhile, Hans Duistermaat and Victor Guillemin had started collaborating on a paper [7] regarding the link between the spectrum of positive elliptic operators and periodic bicharacteristics, using Fourier integral operators, a paper that was published in *Inventiones Mathematicae* in 1975.

Hans made important contributions concerning harmonic analysis on Lie groups and locally symmetric spaces. It is impossible not to mention

Photograph by Á. Pelayo.

Duistermaat playing chess against himself. Picture taken in the living room of the Duistermaats' residence in the Netherlands in 2006.

his work [9] with Gert Heckman, "On the variation in the cohomology of the symplectic form of the reduced phase space", another classic also published in *Inventiones Mathematicae* in 1982. This work had a tremendous influence on the development of a number of topics in symplectic geometry and analysis. Atiyah stated that this article was one of his main motivations for his work with R. Bott on equivariant cohomology [1].

Formula of Duistermaat-Heckman

$$\int_M e^{J_X} e^{\sigma} = \sum_j \int_{N_j} \frac{e^{i_j{}^* J_X} e^{i_j{}^* \sigma}}{\det \frac{LX+\Omega}{2\imath i}}.$$

Formula of Duistermaat-Heckman

The Duistermaat-Heckman formula is one of the most famous results in the history of symplectic geometry. Some years after he and Heckman proved this theorem, Hans, in collaboration with Guillemin, Meinrenken, and Wu, looked at the issue of commutation of quantization and symplectic reduction in [8], an important paper in the subject. The beautiful book *The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator* [5], published in 1996, consists of a collection of notes on Dirac operators, the index theorem (proved by the heat equation), and equivariant cohomology, which he had written for himself to understand the important developments that were taking place at the time. Hans wrote these notes with the goal of being able to work effectively on the subject himself rather than with the goal of publishing them, but many people encouraged him. Had it not been for his colleagues' enthusiasm, it is very likely that these notes would never have been made publicly available.

In the most recent years, Hans Duistermaat worked mainly on three projects. The first project, with Á. Pelayo, concerned the structure theory of symplectic manifolds equipped with symplectic torus actions and its relations to Kodaira's work on complex analytic surfaces. This project gave rise to four papers, starting with a paper in which a classification of torus actions with coisotropic orbits is given [13]. The second project concerned QRT (Quispel, Roberts, and Thompson) maps and elliptic surfaces. Hans devoted an enormous amount of energy to this project, which resulted in a six-hundred-page book [6] (his work on this topic was prompted by an interaction with J. M. Tuwankotta); the book presents a self-contained and complete treatment of the subject, analyzing QRT maps using Kodaira's theory of elliptic surfaces. His final project, with N. Joshi, had the goal of gaining a full understanding of Painlevé differential equation $\mathrm{d}^2 y/\mathrm{d}x^2 = 6y^2 + x$.

By the time of Duistermaat's passing, he and Joshi had strengthened considerably the previous asymptotic results about the Painlevé equation (which is the first in Painlevé's classification of those algebraic second-order ordinary differential equations such that every isolated singularity of moderate growth of every solution is a pole) and corrected older, incomplete results. The paper "Okamoto's space for the first Painlevé equation in Boutroux coordinates", with N. Joshi, available in the *Mathematics arXiv* since October 27, 2010, regards this project.

These brief highlights of selected results due to Hans Duistermaat should not hide the fact that Hans made other remarkable contributions on which we have not elaborated. He always had deep motivations for working on every topic he studied. Many of his motivations came from classical mechanics, a topic which he was passionate about. He had a predilection for Hamiltonian systems. His 1980 article on the globalization of action-angle coordinates [3] is still regarded as the best reference on the subject and prompted much subsequent research. His works on symplectic reduction are rooted in his very detailed study of the bifurcations of Hamiltonian periodic orbits, and his works on elliptic curves are motivated by the theory of completely integrable Hamiltonian systems and Lagrangian fibrations and their applications.

Hans had a rare combination of talents: he was often motivated by "applied" problems and loved examples, but at the same time he was a technical and theoretical master. As his contributions show, he was able to continuously use his knowledge of classical examples to make long-lasting theoretical breakthroughs. We should also not forget his pedagogical contributions, for example, in his beautiful volumes on real analysis, in collaboration with his colleague J. A. C. Kolk [12]. His course on Fourier integral operators [4] quickly became a standard reference in the subject, and his book on "Spin-c" is a terrific treatment for those who want to enter the subject. Finally, the book on Lie groups [11], again in collaboration with Kolk, was very quickly recognized by the community as a clear and important reference.

Duistermaat's body of work has been and continues to be an influential driving force in geometric analysis. If you have not done so yet, we encourage you to explore one of Duistermaat's books or papers; you will not regret it. We hope that these pages provide a glimpse of Duistermaat's profound influence on mathematics. The contributions by Atiyah, Cushman, Heckman, van den Ban and Kolk, Nirenberg, Sjamaar, and Sjöstrand that follow provide further testimony of Hans Duistermaat's influence on mathematics.

P.S. An article in memory of Hans Duistermaat, written by Erik van den Ban and Johan Kolk, has also appeared in *Nieuw Archief voor Wiskunde* and is available at:

`http://www.math.leidenuniv.nl/~naw/serie5/dee1011/dec2010/nawdec235.pdf`

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The remainder of this text consists of contributions by a few of Hans's students and close colleagues, discussing aspects of both the mathematician and the person.

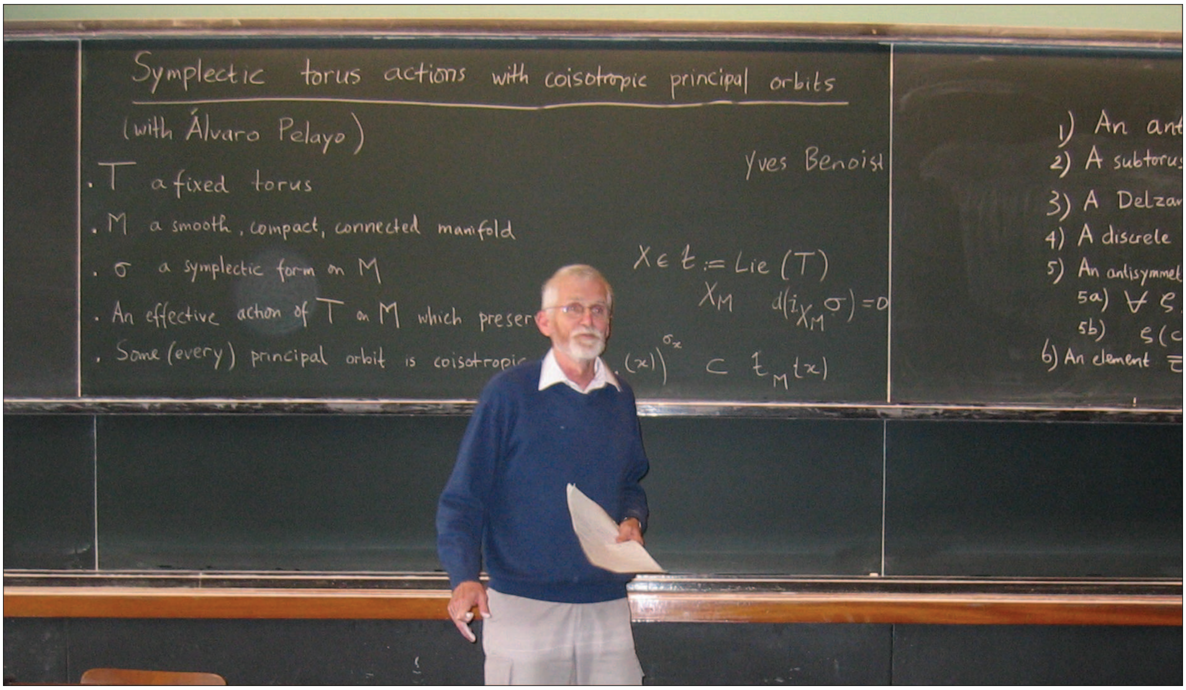
Michael Atiyah

My main memory of Hans is of a colleague with a dry sense of humor and a passionate attachment to mathematics. His enduring love was analysis, centering around PDE and Lie groups. He was a scholar who knew and appreciated the detailed work of our predecessors in past centuries, but he was also modern in outlook, using microlocal analysis and pushing forward the frontiers. He is perhaps most famous for a small spin-off, the Duistermaat-Heckman theorem, giving conditions under which the stationary-phase approximation is exact.

The Duistermaat-Heckman theorem asserts that the stationary-phase approximation is exact for a Hamiltonian arising from the action of a circle on a compact symplectic manifold. The result is striking and the proof is simple and elegant. It stimulated Raoul Bott and me to examine it carefully and to show how it could be interpreted in terms of equivariant cohomology and fixed point formulae. It also connects up with work of Witten on Hodge-Morse theory. This leads on formally to a number of interesting cases in infinite dimensions that occur in quantum physics. Altogether it was very influential.

Very many years ago, when we were both young and hearty, we spent a week together in the Canadian Rockies, after a conference in Vancouver. For someone from the Netherlands where height above (or below) sea level is measured in single digits, the Rockies, where height is measured in thousands, were an irresistible and inevitable attraction. We were not professional mountaineers with ice-axe and rope but enthusiastic hill-walkers with boots and energy. The scenery was fabulous, with the beautifully named Emerald Lake imprinted on my memory. Hans was the perfect companion, slowing me down with his measured words which conveyed his deep appreciation of the landscape. I still visualise him in that role.

Michael Atiyah is professor of mathematics at the University of Edinburgh. His email address is `atiyah@maths.ed.ac.uk`.



Photograph by Michael Pignon.

Duistermaat lecturing on symplectic differential geometry at the Colin de Verdière Fest in 2006.

Richard Cushman

Hans was a mathematical polymath. His scientific papers deal with many areas of geometric analysis. I will focus on one of his contributions to the geometric study of Hamiltonian systems, namely, monodromy.

To describe what monodromy is we look at a completely integrable, two-degree-of-freedom Hamiltonian system on four-dimensional Euclidean space. In other words, we assume that there is a function, called an integral, which is constant on the motions of the original Hamiltonian system in addition to the Hamiltonian. The integral map of such a system is given by assigning to each point in Euclidean 4-space the value of the Hamiltonian and the extra integral. We suppose that this integral map is proper and the preimage of each point is connected. Then the theorem of action-angle coordinates states that the preimage of a suitably small open 2-disk in the set of regular values of the integral map is symplectically diffeomorphic to a product of a 2-torus and 2-disk. In [1] Hans showed that this local theorem need not hold globally on the set of regular values. In particular, if we have a smooth closed curve in the set of regular values of the integral map, which is diffeomorphic to a circle that is not contractible to a point, then its preimage under the integral map is the total space of a 2-torus bundle over a circle. In general this bundle is twisted, that is, it need not be diffeomorphic to a product of a circle and a 2-torus. A 2-torus bundle over a circle may be constructed from a product bundle over a closed interval with a typical fiber a 2-torus, which is Euclidean 2-space modulo the lattice of points with integer coordinates, by identifying each of its two end 2-tori by an integer 2×2 matrix with determinant 1.

When Hans was starting to write [1], he asked me to find an example of a Hamiltonian system with monodromy. The next day I told him that the spherical pendulum has monodromy and gave him a proof. When writing up the paper Hans found a much simpler geometric argument. In the early days, showing that a particular integrable system had monodromy was not easy. In [2] Hans did this for the normal form of the Hamiltonian Hopf bifurcation.

Eight years after the discovery of monodromy in classical mechanics came its discovery in quantum mechanics [3]. In particular, Hans and I showed that the quantum analogue of monodromy appears in the joint spectrum of the energy and angular momentum operators of the quantized spherical pendulum as the failure of the local lattice structure of this joint spectrum to be a global lattice. This discovery has now been recognized as fundamental by chemists who study the spectra

Richard Cushman is professor of mathematics at the University of Calgary. His email address is r.h.cushman@gmail.com.

of molecules and has led to a very active area of scientific research.

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Gert Heckman

In my recent contribution to a memoriam for Hans Duistermaat in the *Notices of the (Dutch) Royal Mathematical Society*, I described the important role that Hans played in the early days of my career. The interested reader can also find this on my website (`http://www.math.ru.nl/~heckman/`). Here I would like to talk about our contact in the last year of his life.

It started with a master class I have been teaching over the past few years for high school students in their final grade. In collaboration with Maris van Haandel, a high school teacher in Wageningen (with a Ph.D. in functional analysis), we taught, over the course of six Wednesday afternoon sessions, the basics of calculus, finishing with a discussion of the proofs of the three Kepler laws of planetary motion. It was a wonderful experience to work with these talented high school students.

In the process of going over the various proofs of the ellipse law in the literature, Maris and I found yet another proof that we liked best for use in our class. When I mentioned this proof to Hans he reacted positively, but, when I told him that the original proof by Newton was incomplete (as I had read somewhere in the literature), Hans was critical and suggested that I read Newton's original text. This was a great advice, since we were to discover—after some translation into modern language—how beautiful and complete the original proof of Newton really was. We explained this in an article in the *Mathematical Intelligencer*.

In September 2009 Maris and I organized a training weekend for high school teachers on the possibilities of teaching the Kepler laws in high school. For the Friday evening we invited Hans to give an informal lecture on Poincaré and his influence on classical mechanics, and he happily accepted. However, a few weeks before the training weekend, I got an email from Hans telling me that

Photograph by Michael Pignon.

(Left to right) Yves Colin de Verdière, Hans Duistermaat, and Victor Guillemin. Picture taken at the Colin de Verdière Fest in Grenoble in 2006.

there was a problem. That same Friday evening there was a ping-pong competition, and his club simply could not do without him. We arranged the following deal: Hans would write up his lecture and explain some of the details to me, then I would give his lecture.

When I visited Hans in Utrecht to discuss his lecture it occurred to me that he had grown older. He mentioned very briefly that he had not been too well of late: that spring his spleen had been removed, but things were now under control. Hans gave me a wonderful two hours of private tutoring on Poincaré's work on celestial mechanics: the restricted three-body problem, the Poincaré slice, the return map, the twist map, the stable and unstable manifold and their intersection, leading to the homoclinic disorder. Looking back at his notes, it struck me how his interest in Poincaré is related to his early (and in my opinion best) work with Victor Guillemin on spectral asymptotics of positive elliptic operators.

On top of this Hans gave me yet another hour of his time, answering and commenting on some of my questions on Lagrange points and related spectral asymptotics. It was a truly delightful morning with Hans at his best. The next and last time I spoke with Hans was in December 2009. With enthusiasm he told about his ping-pong evening, how his club had won and had been promoted to a higher division. I did not know what to say when Erik van den Ban called me up that Friday in March and told me that Hans had passed away early that morning. All I can do now is to repeat what I wrote in my previous eulogy, namely that I am very lucky that Hans shared his godsent talent with me so generously.

Gert Heckman is professor of mathematics at Nijmegen University. His email address is `G.Heckman@math.ru.nl`.

Erik van den Ban and Johan Kolk

On March 19, 2010, mathematics lost one of its leading geometric analysts, Johannes Jisse Duistermaat. At age sixty-seven he passed away, after a short illness following a renewed bout of lymphoma the doctors thought they had controlled. "Hans", as Duistermaat was universally known among friends and colleagues, was not only a brilliant research mathematician and an inspiring teacher, but also an accomplished chess player and very fond of several physical sports.

Hans dropped the subject of thermodynamics because the thesis had led to dissent between mathematicians and physicists at Utrecht University. Nevertheless, this topic exerted a decisive influence on his further development: in its study, Hans had encountered contact transformations. These he studied thoroughly by reading S. Lie, who had initiated their theory. In 1969–70 he spent one year in Lund, where L. Hörmander was developing the theory of Fourier integral operators; this class of operators contains partial differential operators as well as classical integral operators as special cases. Hans's knowledge of the work of Lie turned out to be an important factor in the formulation of this theory. His mathematical reputation was then firmly established by a long joint article with Hörmander concerning applications of the theory to linear partial differential equations. In 1972 Duistermaat was appointed full professor at the Catholic University of Nijmegen, and in 1974 at Utrecht University, as the successor to Freudenthal.

It is characteristic of Hans's work, that on the basis of a complete clarification of the underlying geometry, deep and powerful results are obtained in the area of geometric analysis: in his case, in ordinary or partial differential equations, discrete integrable systems, classical mechanics, analysis on Lie groups, and symplectic differential geometry. Furthermore, after a period of intense concentration on a particular topic, he would move to a different area of mathematics, thereby bringing acquired insights often to new fruition. Nevertheless, on closer inspection, most of these transitions are not as abrupt as they look at first sight.

For example, Hans's interest in semisimple Lie groups was stimulated by efforts to improve, in the more specific setting of such groups, error estimates for the asymptotics of the eigenvalues of elliptic partial differential operators, obtained in previous joint work with V. W. Guillemin. In turn, some results derived along the way acted as a catalyst for the Duistermaat-Heckman formula in symplectic differential geometry. The latter field forms the structural basis for large parts of classical mechanics.

In the later part of his life, Hans had an intense interest in application of mathematics elsewhere in society. For instance, he worked with geophysicists on the inversion of seismic data and on modeling the polarity reversals of the Earth's magnetic field; with mathematical economists on barrier functions and on options; and with biomedical technologists on computer vision.

In 2004 Hans was honored with a special professorship at Utrecht University endowed by the Royal Netherlands Academy of Arts and Sciences. This position allowed him to focus exclusively on research. He frequently described his work at that time as "an obsession". It concerned QRT mappings; these arise in mathematical physics, and Hans studied them with methods from dynamical systems and algebraic geometry. His final work was concerned with Painlevé functions.

While Hans clearly exerted a substantial influence on mathematics through his own research, often with co-authors, and through that of his twenty-three Ph.D. students, the eleven books that he was involved with cover a wide spectrum of mathematical exposition, both in topic and level of sophistication. But in this case, again, there is a common characteristic: every result, however commonplace it might be, had to be fully understood and explained in its proper context. In addition to this, when writing, he insisted that the original works of the masters be studied. He often expressed his admiration for the depth of their treatment, but he could also be quite upset about incomplete proofs that had survived decennia of careless inspection.

On many occasions Hans worked together with colleagues from the United States. Even though several U.S. departments offered him positions and he enjoyed coming to the United States for sabbatical periods, he preferred Utrecht as his permanent base. There he had easy access to mathematicians with a wide range of expertise, while he could still enjoy his favorite physical activities, such as biking and skating.

Hans took teaching seriously and considered it an indispensable counterbalance for his tremendous concentration when involved in research. As a lecturer, he was quite aware that not every student was as gifted as he. Despite the fact that he could ignore all restrictions of time and demanded

Erik van den Ban is professor of mathematics at Utrecht University. His email address is `E.P.vandenBan@uu.nl`.

Johan Kolk is professor of mathematics at Utrecht University. His email address is `j.a.c.kol@uu.nl`.

serious work from the students, he was very popular among them. On several occasions he gave nonscheduled courses on their request.

In mathematics, Hans's life was a search for exhaustive solutions to important problems. This quest he pursued with impressive single-mindedness, persistence, power, and success. In our minds he remains very vivid, one of the most striking among the mathematicians we have met. We, as his students, co-authors, and colleagues at Utrecht University, deeply mourn his loss; yet we can take comfort in memories of many years of true and inspiring friendship.

Louis Nirenberg

I first met Hans Duistermaat when he visited Courant Institute in New York as a postdoc around the year 1971. It was September, and New York was hot and muggy, like a steam bath. Almost the first thing Hans said was: "It smells like Jakarta"—he was raised there.

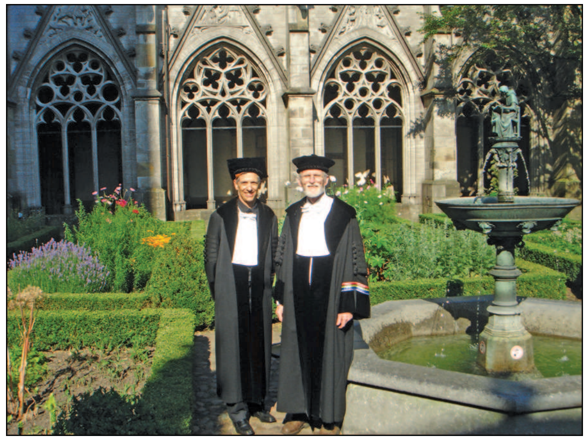
Our families became good friends immediately. He was full of warmth and good humor and radiated intelligence. It soon became clear that he was very talented. Lars Hörmander was visiting about the same time and they wrote their famous paper, "Fourier integral operators II". Hans's research then spread to a wide variety of subjects—in which he made fundamental discoveries.

Over the years we would meet occasionally at conferences. Once, my wife and I had rented an apartment in London, and Hans, who was visiting Oxford, or perhaps Cambridge, came to stay with us for a weekend. At one point I quoted a line from a London guidebook: "It is inadvisable to leave London without having had tea at the Ritz." We decided to do so, but I pointed out that he would have to wear a suit. He promptly went out and bought one.

It was always an enormous pleasure to be together with him, always full of fun. All his friends miss him as well as his mathematics.

Reyer Sjamaar

I got to know Hans Duistermaat as an undergraduate when I took his freshman analysis course at Utrecht. I quickly became hooked and realized that I wanted one day to become his graduate student. This came to pass and I finished in 1990 my thesis work on a combination of two of Hans's favorite topics, Lie groups and symplectic



Photograph by Marius Crainic.

Duistermaat and Alan Weinstein. Picture taken at a thesis defense in Utrecht, 2009.

geometry. He was an older friend as much as an advisor, and I greatly enjoyed my ride with him. Although I was born too late to join him in the kite fighting he engaged in during his boyhood in Indonesia, we did go speedskating together for a few winters at the local rink. Portions of the Utrecht analysis sequence are now available in the form of Duistermaat and Kolk's beautiful textbook *Multidimensional Real Analysis*.

A good way to honor Duistermaat's memory is to display one of his mathematical gems, which the reader can find in Chapter 1 of his *Lie Groups*, another book he cowrote with Joop Kolk. This work is much closer to Lie's original point of view pertaining to differential equations than modern treatments such as Bourbaki, which are more algebraic in spirit. Nevertheless the book is notable for several innovations, particularly its proof of Lie's third fundamental theorem in global form, which in outline runs as follows.

Given a finite-dimensional real Lie algebra $\mathfrak{g}$ with a norm, the space $P(\mathfrak{g})$ of continuous paths $[0,1] \to \mathfrak{g}$ equipped with the supremum norm is a Banach space. For each γ in $P(\mathfrak{g})$ let A_γ be the continuously differentiable path of linear endomorphisms of $\mathfrak{g}$ determined by the linear initial-value problem

$$A'_\gamma(t) = \mathrm{ad}(\gamma(t)) \circ A_\gamma(t), \qquad A_\gamma(0) = \mathrm{id}_{\mathfrak{g}}.$$

Duistermaat and Kolk prove that the multiplication law

$$(\gamma \cdot \delta)(t) = \gamma(t) + A_\gamma(t)\delta(t)$$

turns $P(\mathfrak{g})$ into a Banach Lie group. Next they introduce a subset $P(\mathfrak{g})_0$, which consists of all paths γ that can be connected to the constant path 0 by a family of paths γ_s which is continuously differentiable with respect to s and has the property that

$$\int_0^1 A_{\gamma_s}(t)^{-1} \frac{\partial \gamma_s}{\partial s}(t)\, dt = 0$$

Louis Nirenberg is professor of mathematics at the Courant Institute of Mathematical Sciences. His email address is nirenberg@cims.nyu.edu.

Reyer Sjamaar is professor of mathematics at Cornell University. His email address is jamaar@math.cornell.edu.

for $0 \leq s \leq 1$. The subset $P(\mathfrak{g})_0$ is a closed connected normal Banach Lie subgroup of $P(\mathfrak{g})$ of finite codimension, and the quotient $P(\mathfrak{g})/P(\mathfrak{g})_0$ is a simply connected Lie group with Lie algebra $\mathfrak{g}$!

For the details I refer you to the book; also be sure to read the historical and bibliographical notes at the end of the chapter. One of the many virtues of this proof is that it is manifestly functorial: a Lie algebra homomorphism $\mathfrak{g} \to \mathfrak{h}$ induces a continuous linear map on the path spaces $P(\mathfrak{g}) \to P(\mathfrak{h})$, which is a homomorphism of Banach Lie groups and maps the subgroup $P(\mathfrak{g})_0$ to $P(\mathfrak{h})_0$, and therefore descends to a Lie group homomorphism. A few years after publication, the Duistermaat-Kolk proof became at Alan Weinstein's suggestion a central feature of Marius Crainic and Rui Loja Fernandes's resolution of two long-standing problems in differential geometry; see "Integrability of Lie brackets", *Ann. of Math.* (2) **157** (2003), no. 2, 575–620, and "Integrability of Poisson brackets", *J. Differential Geom.* **66** (2004), no. 1, 71–137. My Cornell colleague Leonard Gross has a paper in preparation that adapts the Duistermaat-Kolk argument to certain infinite-dimensional situations.

Johannes Sjöstrand

Hans spent an academic year in Lund, maybe the one of 1969–70. This was a very active time in the development of (what would somewhat later be called) microlocal analysis, and Hans had precious knowledge in symplectic geometry. I especially remember his short lecture notes on the subject, containing precisely what was needed for a Ph.D. student in one of his first years. Hans's curiosity and his interest in original historical mathematical texts were a great stimulation.

It was a great pleasure to collaborate with him on a subject close to my thesis. Here, his original suggestion was to introduce and study microlocally defined projections onto the kernel and the cokernel of pseudodifferential operators, rather than adding auxiliary operators as in my thesis. This collaboration was not very central to Hans's work and he soon changed his interests, but for me it remained useful through the years.

Later I had the pleasure of meeting Hans in Paris and at other places, though less often during the last decades. All this time he has remained a very important person for me, partly because of his great mathematical results and principally because of his very strong and incisive personality that one cannot forget.

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Johannes Sjöstrand is Directeur de Recherche at the CRNS/Université de Bourgogne. His email address is `johannes.sjostrand@u-bourgogne.fr`.

For more on Hans's scientific work, one may consult *Geometric Aspects of Analysis and Mechanics: A Conference in Honor of the 65th birthday of Hans Duistermaat*, to be published by Birkhäuser, Boston. WONDER (The Dutch Research School in Mathematics) has decided to establish the Hans Duistermaat Chair for Visiting Professors.

Ph.D. Students of Johannes J. Duistermaat

J. A. C. Kolk, 1977
G. J. Heckman, 1980
E. P. van den Ban, 1982
S. J. van Strien, 1982
H. E. Nusse, 1983
J. C. van der Meer, 1985
M. Poel, 1986
P. J. Braam (University of Oxford), 1987
P. H. M. van Mouche, 1988
R. Sjamaar, 1990
H. van der Ven, 1993
J. B. Kalkman, 1993
J. Hermans, 1995
O. Berndt, 1996
E. A. Cator, 1997
M. V. Ruzhansky, 1998
C. C. Stolk, 2000
B. W. Rink, 2003
A. M. M. Manders, 2003
H. Lokvenec-Guleska, 2004
T. Gantumur, 2006
P. T. Eendebak, 2007
A. Q. Vélez, 2009