

The development of (non-)symbolic comparison skills throughout kindergarten and their relations with basic mathematical skills



Sylke W.M. Toll^a, Sietske Van Viersen^{a,b}, Evelyn H. Kroesbergen^a, Johannes E.H. Van Luit^{a,*}

^a Department of Special Education, Utrecht University, P.O. Box 80.140, 3508 TC Utrecht, The Netherlands

^b Department of Education and Learning Problems, University of Amsterdam, Nieuwe Prinsengracht 130, 1018 VZ Amsterdam, The Netherlands

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ABSTRACT

Although numerical skills have proven to be important precursors for mathematical proficiency, longitudinal studies on numerical development are rather scarce. The overall goal of the present study is to gain insight in numerical skills, that is non-symbolic and symbolic comparison skills, as precursors of mapping skills and basic math achievement of children within a longitudinal design. Over two and a half years, 671 kindergartners (mean age 4.6 years at the start of the study) were assessed on non-symbolic and symbolic comparison skills at six time points, and on their basic math achievement (divided into math fluency and math reasoning), and mapping skills at the end of first grade. Multivariate latent growth curve models show an interrelation between (the development of) non-symbolic and symbolic comparison skills. Results furthermore reveal symbolic comparison skills as the most important predictor of mapping skills and basic math achievement. Growth in non-symbolic comparison skills predicted math fluency in first grade.

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By the age of five, general numerical awareness has started to develop; this leads to individual differences in children's numerical development (e.g., Aunio & Niemivirta, 2010; Stock, Desoete, & Roeyers, 2010; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011). An important aspect of early numerical development is learning the connection between number symbols and their corresponding quantities ('4' matches a set of four objects). These skills are referred to as mapping skills (e.g., Kolkman, Kroesbergen, & Leseman, 2013), and are, like in the present study, frequently measured with a number line task in which children need to estimate the position of a given number on a (horizontal) number line. During a number line task children are challenged to map number words and symbols to their corresponding magnitude (e.g., Booth & Siegler, 2006). Another manner to measure mapping skills is a two-alternative forced-choice task. In this task, children are shown a target representation of one quantity (symbolic or nonsymbolic) and are required to choose which of the two alternative representations (nonsymbolic or symbolic) matches the target (Mundy & Gilmore, 2009). Mapping skills become more accurate (i.e., linear) during development (Booth & Siegler, 2006; Siegler & Booth, 2003) and were found to be important predictors for basic math performance (Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013; Kolkman, Kroesbergen, &

Leseman, 2013). Basic math performance of children aged five to eight years old can roughly be divided into two different skills: fluently memorizing basic math facts (e.g., $4 + 3 = 7$), often measured with a time-pressured task, and more mathematical problem solving in which the context is described and basic reasoning skills are evoked (e.g., seven people are in the bus, two people get out at the next stop; how many people are in the bus after the bus stop?). In the present study those two skills are called math fluency and math reasoning. Mapping skills have found to be important for both skills (for speeded arithmetic, see for example Brankaer, Ghesquière, & De Smedt, 2014; for mathematical problem solving, see for example Kolkman, Kroesbergen, & Leseman, 2013). Little is known, however, about numerical precursors of mapping skills (Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2014) and the basic math performance of children. Hence, the overall goal of the present study is to gain more insight in this underlying developmental path.

A common idea in the existing literature is that the ability to discriminate between magnitudes, both non-numerical, e.g. dots, and numerical, is an important prerequisite for mapping and other basic mathematical skills (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013;). It is assumed that both *non-symbolic* comparison skills (i.e., discriminating between dots, cubes or sticks) and *symbolic* number comparison skills (i.e., discriminating between Arabic symbols; below referred to as symbolic comparison skills) are important for the mapping of number symbols to non-symbolic quantities (e.g. Gilmore, McCarthy, & Spelke, 2010; Jordan, Glutting, & Ramineni, 2010). This distinction between

* Corresponding author. Tel.: +31 30 253 4614; fax: +31 30 253 2352.

E-mail addresses: S.W.M.Toll@uu.nl (S.W.M. Toll), S.VanViersen@uva.nl (S. Van Viersen), E.H.Kroesbergen@uu.nl (E.H. Kroesbergen), J.E.H.VanLuit@uu.nl (J.E.H. Van Luit).

non-symbolic and symbolic comparison skills is based on the triple code model of Dehaene (1992; 2001). According to this model, there are three different types of representations for numbers (so-called codes): the analogue magnitude code (semantic knowledge about the proximity and relative size of quantities), the auditory verbal code (the ability to enumerate the counting row), and the visual code (Arabic representations). Dehaene (2001) hypothesized that all children are born with a system for non-symbolic quantity representation and that this can be measured as early as the first months of life. This suggests that children have an innate understanding of non-symbolic magnitudes (Dehaene, 1992; Feigenson, Dehaene, & Spelke, 2004) and is confirmed by studies showing that preschoolers are capable of comparing and adding large sets of elements without counting (e.g., Barth, La Mont, Lipton, & Spelke, 2005). During early childhood, symbolic knowledge develops based on increasing experience with number words (verbal code) and number symbols (visual code). The ability to translate between those Arabic numerals and verbal number words places critical constraints on young children's math development (Göbel, Watson, Lervåg, & Hulme, 2014). The connection between the three codes and their relation in math development has been studied extensively during the last decade (e.g., Sarama & Clements, 2009).

Besides Dehaene's model (1992; 2001), other models exist that make a distinction between non-symbolic and symbolic comparison skills. Von Aster and Shalev (2007), for example, propose a four-step developmental model with a core system of magnitude (i.e., cardinality) at the first step, the verbal number system at the second step, the Arabic number system at the third step, and a mental number line (i.e., ordinality) as a final step. Within this model, the non-symbolic comparison skills can be found in the first step, whereas the symbolic comparison skills are represented in the second and third steps. Integrating those non-symbolic and symbolic comparison skills directly relates to the ability to approximate calculations or create a spatial image of ordinate numbers (e.g., Siegler & Opfer, 2003). Recently, Geary (2013) distinguished between mechanisms that facilitate children's early numeracy learning. These mechanisms may include an inherent sense of magnitude (i.e., non-symbolic comparison skills), fluent mapping of basic mathematical symbols onto this intuitive number sense, and the ability to explicitly operate on these symbols and understand the logical relations among them (i.e., symbolic comparison skills).

In addition to theoretical frameworks, empirical evidence is available on the existence of non-symbolic and symbolic comparison skills as key aspects of numerical development. However, based on this evidence, it can be concluded that the relation between non-symbolic and symbolic comparison skills remains unclear. Yet, there exists a presumption that non-symbolic comparison skills and symbolic comparison skills are separable but dissociable components; they do not share the same underlying ability (Kolkman, Kroesbergen, & Leseman, 2013). The results of Friso-van den Bos, Kroesbergen, and Van Luit (2014) also support the idea that non-symbolic and symbolic number processing are distinguishable processes at kindergarten age. From the growing body of research on this topic it is furthermore known that both functions follow different developmental trajectories during childhood (Kolkman, Kroesbergen, & Leseman, 2013). Clear evidence about how these two skills are related is nevertheless lacking. Whereas some studies found significant correlations across performance on non-symbolic and symbolic comparison tasks (e.g., Gilmore, Attridge, De Smedt, & Inglis, 2014) or reveal a reciprocal nature of the relation between the non-symbolic and symbolic comparison skills (Gilmore et al., 2010), other studies illustrate the mastering of non-symbolic (comparison) skills as precondition for developing symbolic (comparison) skills (Kolkman, Kroesbergen, & Leseman, 2013). Yet, there are results that reveal no predictive association between non-symbolic number comparison and symbolic comparison six months later (Sasanguie, Defever, Maertens, & Reynvoet, 2014).

Whether non-symbolic or symbolic comparison skills are more important for early numeracy and math development is still a subject of

discussion as well. Recently, De Smedt et al. (2013) reviewed neurocognitive and behavioral studies that tested the association between numerical processing and mathematics achievement and conclude that results are consistent across studies for the symbolic comparison skills, but rather contradictory for the non-symbolic comparison skills. Whereas some studies hypothesize non-symbolic understandings of magnitude as a necessary precondition for learning to associate a perceived number of objects with symbolic number words or number symbols (Von Aster, Schweiter, & Zulauf, 2007), other authors believe that the symbolic numerical skills of children might be more important than the previously developed quantitative abilities (e.g., Bartelet, Vaessen, Blomert, & Ansari, 2014). Predictive relationships were found between non-symbolic comparison skills and basic math performance in five to eight-year-old children (Desoete, Ceulemans, De Weerd, & Pieters, 2012; Gilmore et al., 2010; Inglis, Attridge, Batchelor, & Gilmore, 2011) as well as between symbolic comparison skills and basic math performance in six to eight-year-old children (e.g., Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012). This last group of studies states that non-symbolic comparison skills play a subordinate role in learning math in contrast to the important role of symbolic comparison skills (LeFevre et al., 2010), since an effect of non-symbolic comparison skills on math performance was mediated by symbolic comparison skills (Holloway & Ansari, 2009; Xenidou-Dervou, De Smedt, Van der Schoot, & Van Lieshout, 2013). Results of a longitudinal study in which different measures of math achievement were used (both a timed arithmetic test and a general curriculum-based math test; as will be the case in the present study), also emphasize the dominant role of learning experiences with symbols for later math abilities (Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Moreover, De Smedt et al. (2013) put forth the explanation that the kinds of representations and processes measured by the non-symbolic comparison tasks may be unimportant for school-relevant math skills. But even though no consensus has been reached about the influence of non-symbolic and symbolic comparison skills, most researchers agree that, eventually, the symbolic comparison skills are gradually integrated with existing nonverbal knowledge, resulting in more complex cognitive representations in which number symbols and words are connected to quantity representations (i.e., non-symbolic comparison skills) and that these specific skills can be referred to as mapping skills (Dehaene, 2001; Krajewski & Schneider, 2009; Mundy & Gilmore, 2009; Mussolin, Mejias, & Noël, 2010).

Since longitudinal evidence for the contribution of non-symbolic or symbolic comparison skills is scarce (De Smedt et al., 2013), the present study aims to provide additional grounds for this topic. In order to achieve this overall goal, there are two research goals. The first goal is to examine the nature of the developmental relationship between non-symbolic and symbolic comparison skills throughout kindergarten over a period of two and a half years. Based on previous longitudinal studies in this age group (e.g., Gilmore et al., 2010, 2014), it is hypothesized that a mutual relationship exists between non-symbolic comparison skills and symbolic comparison skills. The second goal is to examine the contribution of both non-symbolic and symbolic comparison skills in predicting mapping and other basic mathematical skills. In other words, it was tested whether it is possible to predict math performance at the end of first grade, after approximately one school year (i.e., ten months of education) of formal math instruction, based on children's numerical (non-symbolic and symbolic) comparison skills and their growth in those comparison skills throughout kindergarten. Because of the enormous variation in which mathematical performance in the early school years is measured, three ways to measure basic mathematical skills, including the mapping of number symbols to non-symbolic quantities (i.e., mapping skills), fluently memorizing basic math facts (i.e., math fluency) and mathematical problem solving (i.e., math reasoning), are distinguished in the present study. By doing so, we aim to provide more insight in the specific roles of (non-)symbolic comparison skills for learning mathematics.

1. Method

1.1. Participants

This study involves a longitudinal research project about numerical development in kindergartners. The sample consisted of 671 Dutch children (50.2% boys) with a mean age of 4.6 years at the start of the study ($SD = 3.72$ months). The average raw score on Raven's Colored Progressive Matrices, a non-verbal test for reasoning ability (Raven, 1962), was 26.34 ($SD = 4.34$, $min = 14$, $max = 36$). All children attended one of the 23 primary schools that participated in the study. In the Netherlands, children begin attending kindergarten when they reach the age of four. They attend, on average, two years of kindergarten before attending first grade in September of the year in which they turn six years old. During kindergarten, the children received preschool math education for approximately 1 h a week. The socio-economic-status (SES) of the children, according to the classification system developed by the Dutch government that funds primary schools, was predominantly medium or high (93.6%). The SES of 3.9% of the children was low and of 1.5% of the children was very low. Parental consent for participating in this study was obtained for all children.

1.2. Procedure

During this two-and-a-half-year study, all children were screened on non-symbolic and symbolic comparison skills at four time points in kindergarten and two time points in first grade. The measurement moments were approximately six months apart with a margin of seven to eight weeks (i.e., January–March and May–July of each school year). Besides their (non-)symbolic comparison skills, children were also assessed on their intelligence score levels halfway first grade and on their math fluency, math reasoning, and mapping levels at the end of first grade. The assessment of mathematical reasoning skills was administered by the teacher classically. In primary schools in the Netherlands, such assessment happens twice a year to monitor the development of the children. The individual test results were retrieved from the school. The other tasks were administered by trained graduate students with a degree in education or psychology and were conducted in a fixed order. The training of the students comprised 3 h of instruction about the instruments and a trial session with a child, on which they received feedback. The children were tested individually in a quiet room in their own school for one 20-to-30-minute session.

1.3. Instruments

1.3.1. (Non-)symbolic comparison

Two comparison tasks described by Kolkman, Kroesbergen, and Leseman (2013) were used to measure non-symbolic and symbolic comparison skills. The non-symbolic comparison task was used to measure children's quantity discrimination skills. In the non-symbolic comparison task, the children were presented with two arrays of dots (ranging between zero and 100) and had to indicate the array with the highest number of dots. The symbolic comparison task was used to measure children's number discrimination skills. The children were asked to compare numbers (ranging between zero and 100) and had to indicate which number had the highest numerical value. Both tasks consisted of three practice trials and 30 test items and one point was given for every correct item. In addition, the items in the non-symbolic comparison task were equally divided over three conditions (i.e., congruent (the dot or symbol size of the larger amount is also physically larger), incongruent (the dot or symbol size of the larger amount is physically smaller), and similar (the size of the dots or symbols is similar)) to control for dot size, density, and covered surface. Raw accuracy scores were used in the analyses, because this has proven to be the best manner to index individual differences in non-symbolic acuity (Ingilis & Gilmore, 2014). The internal consistency of the non-symbolic and

symbolic comparison tasks for the total sample was considered satisfactory ($\alpha = .87$ and $\alpha = .67$).

1.3.2. Mapping

Children's mapping skills were measured using a number line task. The children were asked to estimate the position of a given number on a horizontal number line (ranging from zero to 100). After a demonstration of the positions of numbers 1 to 100, the children were presented with ten trials in which they had to estimate number's positions on the number line. Linear fit scores, in which a child's answers are fitted onto a linear curve, were used as a measure of performance (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Reliability of the task is considered good for the total sample ($\alpha = .78$) and in previous studies ($\alpha = .83$; e.g., Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013).

1.3.3. Mathematical reasoning (non-timed)

The Cito standardized math test (SMT; Janssen, Scheltens, & Kraemer, 2005) was used to assess mathematical reasoning ability. The test consisted of 50 picture illustrated math problems concerning numbers, number relations, and simple addition and subtraction that were also read aloud by the teacher. Children received one point per correct answer. Raw test scores were transformed into competence scores (ranging from 0 to 100), which were used in the analyses. Reliability of this test was good in a previous study ($\alpha = .92$; Janssen, Verhelst, Engelen, & Scheltens, 2010).

1.3.4. Mathematical fluency (timed)

The Dutch speeded Arithmetic Test (Tempo Test Rekenen (TTR); De Vos, 1992) was used to assess children's automation of arithmetic facts. The test consisted of two subtests (i.e., addition and subtraction). Children were instructed to solve as many sums of increasing complexity (up to 100) per subtest as they could in 1 min. The children received one point per correct answer, with a maximum of 40 points per subtest. Raw scores of both subtests were combined into a sum score for the analyses. Reliability of both subtests is considered good ($\alpha = .86$).

1.4. Analyses

1.4.1. Data screening

A missing data analysis on all relevant variables showed a random pattern of missing data (MAR). 2.7% of the data points in the relevant variables were missing. The application of full information maximum likelihood (FIML) within structural equation modeling allowed all available scores of the missing children to remain included in the analyses (Enders & Bandalos, 2001). An outlier analysis showed 18 negative outliers on the mapping variable. These values were adjusted to a value that corresponded with a z-score above -3.0 .

1.4.2. Main analyses

The data were analyzed with latent growth curve modeling, using the Mplus statistical package for structural equation modeling (Muthén & Muthén, 1998–2012). In structural equation modeling, models are evaluated based on overall model fit, which is indicated by a specific set of fit indices that address different aspects of the model under investigation. In this study, model fit was evaluated based on three indicators as recommended by Blunch (2008): 1) Chi-square (χ^2) including p -value, 2) comparative fit index (CFI), and 3) root mean square error of approximation (RMSEA) including Pclose-value. Chi square is a discrepancy measure; between the current model and the saturated model. CFI compares the fit of the model to the independence model. RMSEA is a parsimony measure, favoring simpler models. For model fit to be evaluated as good, Chi-square should be as low as possible, with a p -value as high as possible, CFI should be $>.95$ ($>.90$ is acceptable), and RMSEA should be <0.05 (<0.08 is acceptable), with a Pclose-value $>.05$. Here, a p -value below .05 for the Chi-square test can be considered acceptable due to the large sample size (Byrne,

[illegible]

Table 2
Correlations between the predictive and outcome measures.

Task	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Non-symbolic M1	–														
2. Non-symbolic E1	.34**	–													
3. Non-symbolic M2	.23**	.29**	–												
4. Non-symbolic E2	.26**	.21**	.28**	–											
5. Non-symbolic M3	.14**	.11**	.23**	.26**	–										
6. Non-symbolic E3	.14**	.14**	.10*	.18**	.18**	–									
7. Symbolic M1	.45**	.22**	.18**	.20**	.19**	.13**	–								
8. Symbolic E1	.24**	.43**	.27**	.23**	.21**	.11**	.47**	–							
9. Symbolic M2	.25**	.35**	.52**	.28**	.27**	.17**	.31**	.52**	–						
10. Symbolic E2	.21**	.26**	.24**	.48**	.28**	.23**	.19**	.46**	.49**	–					
11. Symbolic M3	.17**	.21**	.21**	.24**	.47**	.19**	.20**	.28**	.33**	.43**	–				
12. Symbolic E3	.15**	.19**	.16**	.16**	.21**	.38**	.19**	.22**	.35**	.27**	.42**	–			
13. Math reasoning	.23**	.22**	.15**	.28**	.21**	.20**	.32**	.38**	.36**	.40**	.31**	.33**	–		
14. Math fluency	.19**	.14**	.14**	.22**	.21**	.19**	.34**	.38**	.36**	.34**	.28**	.30**	.53**	–	
15. Mapping	.12**	.20**	.17**	.10*	.15**	.22**	.23**	.31**	.30**	.26**	.31**	.26**	.36**	.37**	–

Note. M = mid-year, E = end year

* $p < .05$.

** $p < .01$.

each other mutually or whether both skills develop hierarchically (e.g., Gilmore et al., 2010, 2014; Kolkman, Kroesbergen, & Leseman, 2013; Sasanguie et al., 2014). Correlations between the non-symbolic and symbolic comparison skills were allowed at each combined time point (i.e., T2, T3, T4, and T5) to improve the model fit. The relation between the two intercepts was found to be strong (.63) and the relation between the two slopes was moderate (.39). The intercept–slope and slope–intercept correlations were both strong, but turned out to be negative as an artifact of the model (–.92 and –.98). The fit indices of the model are presented in Table 3 under multivariate growth. The model fit was evaluated as good. The results suggest that both the overall level of the child and growth during the 2.5 years of the study in non-symbolic as well as symbolic comparison skills were significantly related to each other.

2.3. Final model

The second aim of the study was to examine the contribution of both non-symbolic and symbolic comparison skills in predicting mapping and other basic mathematical skills. Therefore, mapping, math fluency and math reasoning were added to the multivariate model and regressed on both intercepts and slopes. All non-significant paths were stepwise omitted from the model (i.e., non-symbolic intercept on TTR, SMT and mapping; non-symbolic slope on SMT and mapping), which resulted in a good model fit (see Table 3). The final model is displayed in Fig. 2.

The results show that mapping skills as well as math fluency and math reasoning ability are mainly predicted by children's average level and development in symbolic comparison skills. Children's growth in non-symbolic comparison skills was found to *only* predict math fluency levels at the end of first grade. Note that the standardized coefficients that are reported here are not numerically bounded by ± 1 and that the interpretation of standardized coefficients greater than one is the same as all other rates of change (Deegan, 1978). 19.8% of the

variance in mapping skills and 32.3% of the variance in math reasoning were accounted for by the average level and growth in symbolic comparison skills. These effects are interpreted as medium and large, respectively. 29.4% of the variance in math fluency was accounted for by the average level and growth in symbolic comparison skills and the growth in non-symbolic comparison skills. This can be considered a large effect.

3. Discussion

Despite the growing body of research on numerical development, a lot remains uncertain in this field of study; many presumptions are a topic of debate. In the present study, two of those topics were subject of a longitudinal investigation. Since the relation between non-symbolic and symbolic number comparison skills remains unclear, the first goal was to examine the developmental relationship between non-symbolic and symbolic comparison skills throughout kindergarten over a period of two and a half years. The second goal related to the question whether non-symbolic or symbolic comparison skills are more important for math development. Thus, the second aim of the study was to examine the contribution of both non-symbolic and symbolic comparison skills in predicting mapping and other basic mathematical skills.

Regarding the first goal, focusing on the existence of a mutual relationship between non-symbolic and symbolic comparison skills, it can be concluded that the initial status of both skills as well as the growth rate are interrelated and influence each other reciprocally. These results reflect a strong relationship between non-symbolic and symbolic comparison skills. Especially the slope–slope effect suggests that the two comparison skills are factors that influence each other mutually. The ability to process non-symbolic quantities seems to be a vital component in processing number symbols and vice versa; without knowledge or competence of one, it seems impossible to complete the task of the other. Children with lower non-symbolic processing capacities are likely to perform worse (i.e., make more errors) on the ability to discriminate between symbolic magnitudes. This conclusion is congruent to previous longitudinal studies in this age group (e.g., Gilmore et al., 2010, 2014). However, it should be taken into account that later points in symbolic comparison skills than in non-symbolic comparison skills were used. This could suggest that development of symbolic processing commences later, as was the case in studies by Kolkman, Kroesbergen, and Leseman (2013) and Sasanguie et al. (2014). It could be argued that the found reciprocal relationship in the present study in fact has been influenced by the developmental time frame. Notwithstanding that is quite unlikely, this calls for a cautious interpretation of the current

Table 3
Fit indices for the univariate and multivariate latent growth curve models.

	χ^2	df	p	CFI	RMSEA	Pclose
Univariate growth						
Non-symbolic	10.50	7	<.16	.99	.03	.86
Symbolic	24.66	6	<.01	.97	.07	.12
Multivariate growth						
Non-symbolic and symbolic	54.23	30	<.01	.99	.04	.96
Final model with outcome measures	95.83	53	<.01	.98	.04	.99

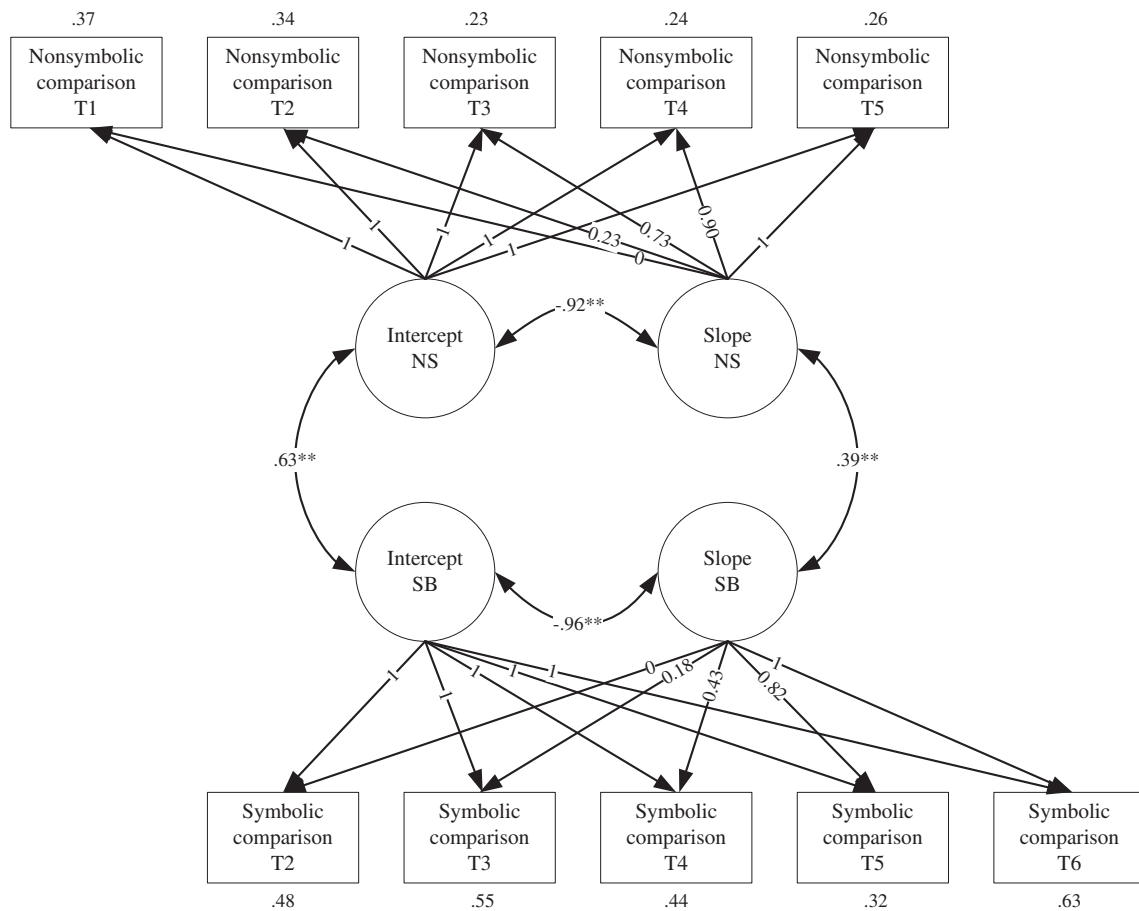


Fig. 1. Multivariate latent growth curve model of nonsymbolic comparison skills (top), symbolic comparison skills (bottom), and the correlation between the two (middle). * $p < .05$, ** $p < .01$, NS = nonsymbolic comparison, SB = symbolic comparison. Note. Factor loadings (above arrows) are un-standardized; explained variance (next to each rectangle) and correlations between latent factors are standardized. To aid visibility, error terms and correlations between T2–T5 are not displayed.

results and conclusions. Another point that needs to be critically reflected are the similarities, other than the comparison of values, between the two comparison tasks; perhaps those similarities, such as the (computer-based) mode of delivery and the colors and size of the items, could explain part of the correlation.

Regarding the second goal, the current results support the idea that symbolic comparison skills are substantial predictors of basic math achievement. Except for basic arithmetic facts (math fluency), no interference was found for the initial status or the growth rate of non-symbolic comparison skills. These results are congruent with the majority of studies in this field (e.g., Desoete et al., 2012; or see De Smedt et al., 2013 for a review) and are enhanced by the decision to use various mathematical tests as output measure for mathematical proficiency, as was only investigated in one former study (Sasanguie et al., 2013). For mapping skills as well as more complex mathematical reasoning skills, the comparison of symbols turns out to be a unique predictor. Thus, being able to compare Arabic symbols based on their numerical value is a necessity for estimating the position of a given number on a horizontal number-line correctly as well as for the reasoning of a correct answer to more complicated, often linguistically involved, math assignments. For the speeded basic math fact tests, however, the growth in non-symbolic comparison skills predicted a portion of children's performance. Although it must be mentioned that the effect size was rather small, this indicates that the degree of success when completing simple addition and subtraction problems depends partly on a child's former development in non-symbolic comparison skills, in addition to their initial status and development on symbolic comparison skills. This result can possibly be attributed to the complexity of the offered challenges.

Whereas in mapping skills and the word problem of the math reasoning task several other (meta)cognitive skills such as (visual) orientation and planning are appealed to (Jacobse & Harskamp, 2012), the number of skills elicited in number facts is more or the less limited to attention and the processing of numerical information. This could explain why, in the execution of this specific math task, non-symbolic comparison skills turn out to be prominent as well. Furthermore, it must be noted that the explained variance was substantial (varying between 19.8% and 32.3%), but leaves room for other explaining variables that were not included in the present study, such as counting skills (e.g., Gelman, 2008), working memory (e.g., Noël, 2009) or executive functions (Espy et al., 2004; Mazzocco & Kover, 2007), and subitizing (e.g., Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009).

Research on numerical cognition raises several points of discussion that were accounted for in the present study. Both points relate to the way numerical representations are measured with comparison tasks, in which participants are shown two arrays of dots or symbols and are asked to select the more numerous. A first problem with comparison tasks is the task impurity problem. The task impurity problem is often referred to in tasks that elicit more complex cognitive functions such as focus control and attention switching (Miyake et al., 2000), and concerns tasks that measure other (cognitive) processes in addition to the targeted abilities. For example, Gilmore et al. (2013) show that, rather being driven by the nature of numerical representation, the relationship between numerical magnitudes and mathematical learning may be an artifact of the inhibitory control demands of some trials of a non-symbolic comparison task. One method to overcome this problem, which is used in the current study, is alternating between congruent,

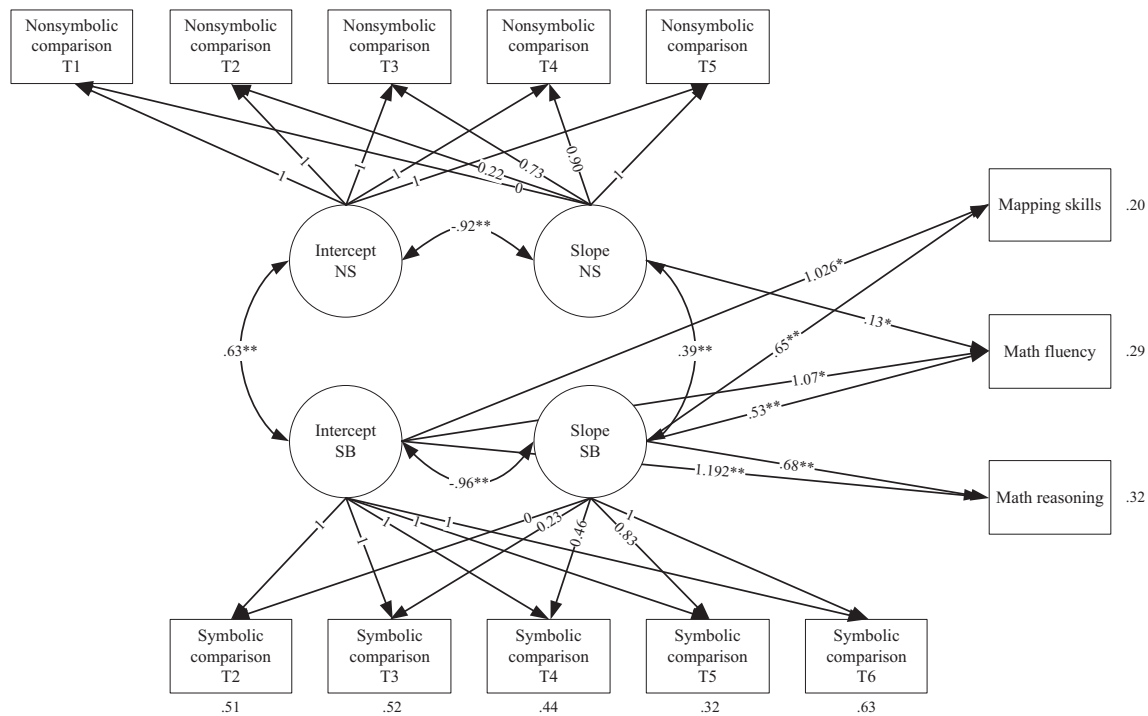


Fig. 2. Multivariate growth model of nonsymbolic comparison skills and symbolic comparison skills with mapping skills, math fluency, and math reasoning as outcome variables. * $p < .05$, ** $p < .01$. Note. Factor loadings (above arrows) are un-standardized; explained variance (next to each rectangle) and (cor)relations between latent factors are standardized. To aid visibility, error terms and correlations between T2–T5 are not displayed.

incongruent and similar items within the task. In this way a correction is made for the visual characteristics of the displays (e.g., dot size, density, total area) in such a manner that those characteristics do not correlate with numerosity across the task. It could be questioned also whether the control for visual characteristics succeeded by including three different conditions, since Gebuis and Reynvoet (2011) indicate that it is impossible to perfectly control for all those non-numerical parameters. Also, despite the effort that was made to include three different conditions in order to correct for visual characteristics, it is debatable whether the tasks did not suffer from the task impurity problem and the results basically are an artifact of the inhibitory control demands of some trials of the two comparison tasks. In that case, the result that growth on non-symbolic comparison contributes solely to the ability to solve as many bare sums within a certain time frame, could indicate that inhibition is involved when recalling the answers from (long term) memory.

Another popular topic of discussion in measuring comparison skills is the type of output that is used to measure performance. Typical measures of performance include overall accuracy, response time, or ratio or distance effects (De Smedt et al., 2013). For the use of all types of these performance measures, valid arguments are available (Ingilis & Gilmore, 2014). In our study, the number of correct items (i.e., accuracy) was chosen as measure of performance, because a recent study on the psychometric properties of different indices of non-symbolic comparison skills shows accuracy as the best manner to index individual differences in non-symbolic acuity (Ingilis & Gilmore, 2014). Nevertheless, including other measures (reaction time, ratio score or the Weber's fraction) as additional support for our findings could have been of value.

The results of the present study should be interpreted in the light of several limitations. First, it should be mentioned that the reliability (internal consistency) of the symbolic comparison task was nearly acceptable. Second, note must also be taken of the ceiling effect on the non-symbolic comparison task at the sixth time point. Only when omitting this time point the model fit could be evaluated as good. Since the results reflect the selected model, this omission can be seen as a third limitation.

Apart from the discussed points of debates and limitations, the present study clearly shows, thanks to a long time frame of more than 2.5 years and a relative large sample size, that first, (the development of) non-symbolic and symbolic comparison skills influence each other mutually, and that second, symbolic comparison skills are a substantial predictor of mapping skills and basic math achievement. Those conclusions are informative and valuable for the current knowledge on numerical development and should therefore be noticed and acknowledged by educational practitioners and curriculum designers for kindergartens and the first years of primary school. Within classroom education on early numerical knowledge, the attentional focus should be on learning the value of Arabic number symbols rather than on non-symbolic abilities. Another practical implication relates to the need of considering symbolic comparison skills as good predictive measures for identifying children at risk of developing learning difficulties at an early stage. Clinicians are encouraged to take those skills into account when assessing children who are expected to be at risk of developing math learning disabilities. It could be helpful to include symbolic comparison skills in screening batteries or other diagnostic instruments in kindergarten. Apart from facilitating the identification of at-risk children, this will also give useful insights into possible gaps in the skills of these children.

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