Quantum Mechanics and No-Hidden Variable Theories

Comparing the Bell, Kochen-Specker and Free Will Theorems

June 18, 2014

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Bachelor Thesis

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Abstract

This thesis explores the exact differences between the two most well-known no-hidden variable theories: the Bell theorem and the Kochen-Specker theorem. Furthermore, it tries to place a newer result called 'the free will theorem', as developed by John Conway and Simon Kochen, in this context. To compare the Bell and Kochen-Specker theorems a reformulation of the two theorems by David Mermin will be discussed, which turns out to lead to an even stronger no-hidden variable theory. From the discussion of the free will theorem and the critical reactions it has elicited, it will become clear that the free will theorem is actually very similar to this stronger no-hidden variable theory.

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Chapter 1

Introduction

Quantum mechanics has puzzled physicists ever since it was first formulated in the twenties and thirties of the twentieth century. On the one hand it is the most successful theory in the sense that it makes the most exact experimentally confirmed predictions. On the other hand the interpretation of the theory gives rise to many conceptual problems and counterintuitive phenomena, such as the probabilistic nature of the theory, quantisation, entanglement, the wave-like behaviour of matter and the curious nature of measurement. Because of this, there are many questions about the interpretation of the quantum mechanical theory. Does it in any way correspond to reality? And if it does, in what way? Is the theory a complete description of nature or is some deeper understanding missing? For many physicists, the choice has been not to give too much attention to these questions and instead focus on using the theory. If it is the most accurate theory ever devised, why worry about its interpretation? Others *have* been bothered by the conceptual issues raised by quantum mechanics and developed theorems and experiments trying to find out the precise relation between quantum theory and reality.

One such person was Albert Einstein. Together with Boris Podolsky and Nathan Rosen he developed an argument (known as the EPR-argument) showing that considering quantum mechanics to be a complete theory leads to a contradiction [11]. It was not until almost thirty years later, that their argument was disproved by John Stewart Bell [3]. Ironically, Bell showed that it was actually the assumptions made by Einstein, Podolsky and Rosen that were wrong, leaving the issue of the completeness of quantum mechanics undecided. In the decades following Bell's paper several other theorems have been developed which put further constraints on the possibility of theories which are more complete than quantum mechanics. Such supposedly more complete theories are called hidden variable theories, with 'hidden variable' referring to the information and parameters absent in quantum theory and representing a more fundamental level of reality. The most important theorem, besides Bell's theorem, which puts constraints on such hidden variable theories is the Kochen-Specker theorem [14], which was also independently developed by Bell himself [4].

The goal of this thesis is twofold. Firstly, the aim is to research the nature of the Bell and Kochen-Specker theorems and the exact relation between the two theorems. To do so − besides discussing both theorems − a reformulation of the two theorems developed in 1990 by David Mermin [15], which is based on an expansion of the EPR-argument by Daniel M. Greenberger, Michael A. Horne and Anton Zeilinger (together GHZ, see [12] and [13]), will be treated. This reformulation of the two so-called no-hidden variable theories enables an exact comparison. Secondly, the aim is to evaluate the 'free will theorem', as developed by John Conway and Simon Kochen (see [8] and [10]). This theorem uses aspects of both the Bell and Kochen-Specker theorems in making rather provocative statements about particles having free will. At the same time, it has been accused of simply coming down to Bell's theorem. To find out what the theorem amounts to, some of the criticisms the theorem has elicited and a reformulation of the theorem by Eric Cator and Klaas Landsman [5] will be discussed. The hope is that this discussion will also help in achieving the first aim of a better understanding of the relation between the Bell and Kochen-Specker theorems. Before discussing the topics above, however, the issues of realism and completeness will first be discussed in some detail, to be able to place the different theorems in context.

Chapter 2

Preliminaries

2.1 Realism and instrumentalism

One of the most important philosophical issues in physics concerns the interpretation of theories. The issue is whether physical theories describe a physical reality existing independently of measurement or that physical theories are merely instruments used for predictions. In the first case, corresponding to a realist interpretation, physical theories are judged on the extent to which they express reality. In the second case, corresponding to an instrumentalist interpretation, theories are judged on the extent to which they are useful in giving the right predictions. The debate over realism and instrumentalism is very interesting in the case of quantum mechanics, due to some of the theory's unique features. It is also important for understanding the idea behind hidden variable theories and therefore the relevant issues of the debate will be discussed in this chapter, which is mainly based on [6].

Whereas in classical mechanics there is a direct link between the mathematical description of some physical system and the physical properties of that system, in quantum mechanics the mathematical description of a system (its wavefunction or more generally a vector in Hilbert space) does not correspond to unique values for its properties. Quantum mechanics only gives a probability distribution for the possible outcomes of a measurement of a physical property of the system. Furthermore, in quantum mechanics there are cases of perfect correlation where a combined system (e.g. two entangled particles) *does* have a well-defined value for some physical property (e.g. the sum of spin), but the constituent systems (the two separate particles) are only described by a probability distribution for the value of this property (their spin). Experiments have confirmed these predicted probability distributions and thereby quantum theory as well.

Therefore the most fundamental description quantum mechanics gives of particles is a probabilistic one. This, however, raises some conceptual difficulties. First of all, in classical physics probabilities indicate a theory is not complete. Stating that throwing a 6 with a dice has a 1 in 6 chance is only an indication of our ignorance about the exact position, mass, etc. of the dice, with which one theoretically should be able to predict the outcome of a throw. Does this mean that quantum mechanics is incomplete in a similar way? A second problem is that quantum mechanics gives a probability distribution for the physical properties of a system, but when a measurement is made of some physical property of this system, one will find some specific value. Furthermore, subsequent measurements − for as long as the system is not disturbed and did not have time to evolve dynamically − will return this same value. So did the system actually have a value for the measured property before measurement? Again, this would mean quantum mechanics is an incomplete theory. Another issue here is how the measurement affected the system. If the system did not have a well-defined value before measurement but does just afterwards, a measurement affects the system in a very curious way. There is clearly an inconsistency between the dynamics of the system, as described by the Schrödinger equation, and the sudden 'collapse' of the wavefunction upon measurement (see [1]).

These conceptual problems raises the question to what extent quantum mechanics describes a physical reality independent of measurement. Does the formalism describe some fundamental reality which exists outside of the theory? Or is there possibly a more fundamental reality than what is described by quantum mechanics, like classical mechanics is more fundamental than classical statistical mechanics? Or, from an instrumentalist perspective, should quantum mechanics merely be considered as a functional tool for predicting outcomes of measurements?

Some of the aspects of quantum mechanics would suggest an instrumentalist interpretation. Quantum mechanics makes very exact and well-confirmed predictions, thereby making it a very good theory according to an instrumentalist's criteria. Furthermore, an instrumentalist interpretation of quantum mechanics would to some extent avoid the conceptual problems as described above. For example, in this perspective, the issue whether quantum mechanics is complete is only to some extent of interest. Understanding quantum theory on a more fundamental level than probabilistic predictions would only be of any use if it would lead to any new useful predictions. In a realist interpretation, however, the issue of completeness is of great interest. Both possibilities − considering quantum mechanics to be complete or incomplete − will be discussed in the next paragraph.

2.2 Completeness

One option is considering quantum mechanics to be incomplete. As discussed, this implies the probabilities predicted by quantum mechanics follow from some more fundamental level of reality. This more fundamental level is associated with a so-called 'hidden variable', an as yet unspecified variable. A major problem with theories based on these hidden variables is that they make the same predictions as quantum mechanics but need more parameters (which the hidden variable represents) to do so. Furthermore, for reproducing the same predictions as quantum mechanics some ad hoc assumptions are made about the nature of the hidden variable. So in this way realism can be upheld, but no empirical evidence can be given for it.

The second option is to consider quantum mechanics a complete theory. For a realist this raises the problem of measurement as described in the previous paragraph. Furthermore, it goes against our intuition that for example a particle simply does not have well-defined values for its properties. Still there have been several versions of this type of realist interpretation, of which some seem quite exotic. For example, in 1961, Eugene Wigner proposed a theory in which consciousness was crucial in defining what a measurement constitutes. And in 1957, Hugh Everett proposed the many-worlds interpretation in which at every measurement the world divides itself in as many worlds as there are possible outcomes of the measurement. Both proposals, however, suffer from methodological problems. In Wigner's case consciousness is assumed not be physically understandable, thereby not improving physical understanding of the theory. In Everett's case extra parameters are introduced without empirical justification, since the other worlds are by their nature unobservable for us. Another possibility is a kind of pure quantumrealism. In this case, considering quantum mechanics to be a complete theory implies that the most fundamental level of nature corresponds to a system's state Ψ as found in the quantum mechanical formalism. So then on this level nature is probabilistic and reality is considered to be directly described by the mathematical formalism of quantum mechanics.

Historically, the counterintuitiveness of the interpretation just described has been a reason for people to oppose the completeness of quantum mechanics. The most well-known example of this is the argument developed by Einstein, Podolsky and Rosen. As stated in the introduction, in 1964 Bell developed a theorem on the basis of a modified EPR-argument that demonstrated that it was actually EPR's assumption, locality, that had to be given up and not the completeness of quantum mechanics. Bell's theorem is in this sense an example of a so-called no-hidden variables theorem (also called a no-go theorem): a theorem that tries to show that certain hidden variable theories lead to predictions which do not conform to quantum mechanical predictions. When these predictions can be experimentally verified, these no-go theorems make it an experimental issue to decide if a specific type of hidden variable theory is possible. As becomes clear in the next chapter, such experiments have been performed in the case of Bell's theorem, clearly in agreement with the theorem.

Chapter 3

Bell's theorem

3.1 EPR-argument

Bell's theorem [3] is a reaction to an argument developed by Einstein, Podolsky and Rosen [11] which clearly points out the conceptual problems of quantum mechanics. Einstein, Podolsky and Rosen start off by arguing that in the case of two noncommuting observables there are two possibilities: either the description of reality by quantum mechanics is not complete or the two noncommuting observables cannot have simultaneous reality. The authors then propose a situation in which two systems, of which the states are known, interact until some time t, after which the two systems are supposed not to interact anymore. From the time t the state of the combined system can be calculated using the Schrödinger equation, whereas the states of the separate systems are unknown.

Now one can measure some physical attribute on one of the systems and then, using the knowledge of the combined system, one immediately knows the value for this same physical attribute of the other system. Depending on what quantity one chooses to measure this other system can be left in different states. However, since there is no interaction between the two systems after the time t (the locality assumption), both states should be elements of the same reality, according to EPR. Now imagine choosing between measuring any of two noncommuting physical quantities, say position and momentum. In both cases one could predict some property (either position or momentum) of the other system with certainty, thereby making both an element of reality (according to EPR's definition of physical reality). This means that they have simultaneous reality, which contradicts the second of the possibilities given by EPR. Therefore only the first option is left: quantum mechanics is incomplete.

The modified EPR-argument on which Bell builds his theorem is the one formulated by Bohm and Aharonov and is one which concretises the original version. It is as follows (Bohm and Aharonov mainly developed their argument with polarisation properties of correlated photons, but this works just the same): two spin 1/2 particles are in the singlet state and move away from a common origin in opposing directions (this could be the decay of a neutral pi meson into an electron and a positron). Now one measures the spin-component in direction $\mathbf{a}$ for both particle 1 and 2. According to quantum mechanics, if the spin-component (in units of $\hbar/2$) of either particle is $+1$, it is -1 for the other particle and vice versa, regardless of the distance between the particles. So if one measures the a-component of spin for particle 1, the a-component of spin of particle 2 is immediately known. Assuming the measuring device and particle on one side do not influence the measuring device and particle on the other side (the locality assumption), this leads to the conclusion that the spin of both particles must be predetermined and therefore that quantum theory is not complete.

3.2 Bell's inequalities

In reaction to the EPR-argument and the modified version by Bohm and Aharonov, Bell followed the strategy for a no-go theorem: make some assumption for a possible hidden variable theory, find out what predictions it leads to and then compare this to the (possibly experimentally confirmed) quantum mechanical predictions. Bell started off with a slight change in setup compared to the argument described before: the direction in which the spin is measured is now different for the two particles, with $\mathbf{a}$ for particle 1 and $\mathbf{b}$ for particle 2. The value in both instances can be either $+1$ or -1, so $A(\mathbf{a}, \lambda) = \pm 1$ and $B(\mathbf{b}, \lambda) = \pm 1$ with λ being the hidden variable. Now Bell proposed to calculate the expectation value $P(\mathbf{a}, \mathbf{b})$ of the product of A and B, using the probability distribution of the hidden variable $\rho(\lambda)$. Thereby he made it possible to compare a hidden variable theory to quantum mechanical statistical predictions. Working out the assumptions about the hidden variable theory lead to the Bell inequalities, which turned out to be in disagreement with what follows from quantum mechanics.

The derivation of the inequalities is as follows: define the hidden variable as λ (which could be any type of variable) with

$$\int \rho(\lambda) d\lambda = 1. \tag{3.1}$$

The locality assumption implies that both A and B are independent of the configuration $\mathbf{b}$ and $\mathbf{a}$ of the other detector, that is:

$$A(\mathbf{a}, \mathbf{b}, \lambda) = A(\mathbf{a}, \lambda), \tag{3.2}$$

$$B(\mathbf{a}, \mathbf{b}, \lambda) = B(\mathbf{b}, \lambda). \tag{3.3}$$

Now the expectation value of the product of A and B is

$$P(\mathbf{a}, \mathbf{b}) = \int \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) d\lambda. \tag{3.4}$$

Furthermore we know that if the spin of both particles is measured in the same direction, they will be each others opposites, that is

$$A(\mathbf{a}, \lambda) = -B(\mathbf{a}, \lambda). \tag{3.5}$$

So equation 3.4 becomes

$$P(\mathbf{a}, \mathbf{b}) = -\int \rho(\lambda) A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) d\lambda. \tag{3.6}$$

Now if $\mathbf{c}$ is another vector

$$P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) = -\int \rho(\lambda)[A(\mathbf{a}, \lambda)A(\mathbf{b}, \lambda) - A(\mathbf{a}, \lambda)A(\mathbf{c}, \lambda)]d\lambda \tag{3.7}$$

$$= -\int \rho(\lambda)[1 - A(\mathbf{b}, \lambda)A(\mathbf{c}, \lambda)]A(\mathbf{a}, \lambda)A(\mathbf{b}, \lambda)d\lambda, \tag{3.8}$$

with the last equation following from $A(\mathbf{b}, \lambda)^2 = 1$. Now considering that $A(\mathbf{a}, \lambda)A(\mathbf{b}, \lambda) = \pm 1$ and $\rho(\lambda)[1 - A(\mathbf{b}, \lambda)A(\mathbf{c}, \lambda)] \geq 0$ it follows that

$$\mid P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) \mid \leq \int \rho(\lambda)[1 - A(\mathbf{b}, \lambda)A(\mathbf{c}, \lambda)]d\lambda. \tag{3.9}$$

Using equation 3.5 this can be more simply expressed as

$$\mid P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) \mid \leq 1 + P(\mathbf{b}, \mathbf{c}), \tag{3.10}$$

which is the original Bell inequality [3, 7].

3.3 Consequences

Quantum mechanics, however, predicts [3, p. 404]

$$P(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}. \tag{3.11}$$

It can easily be seen that this does not agree with the Bell inequality. Imagine $\mathbf{a}$ and $\mathbf{b}$ make a 90 degree angle with each other and $\mathbf{c}$ is exactly in between them in the same plane. Then, according to quantum mechanics, $P(\mathbf{a}, \mathbf{b}) = 0$ and $P(\mathbf{a}, \mathbf{c}) = P(\mathbf{a}, \mathbf{c}) = -\frac{1}{2}\sqrt{2}$, which does not fulfill the Bell inequality. Since the quantum mechanical result has been experimentally confirmed [13], this leads to the conclusion that the hidden variable theory assumed in Bell's theorem is incorrect. More specifically, local hidden variable theories are incorrect.

At first, one might think nonlocality would contradict relativity theory, since in the case of entanglement there is evidently some influence which travels faster than light (otherwise the perfect correlation cannot be explained). The reason that influences with superluminal speeds are prohibited in relativity theory is that in some inertial frames those influences travel back in time. If the influence is a causal one, this would mean the effect precedes the cause which leads to a logical contradiction. One option to avoid this problem would be to state that the results of the spatially separated measurements were predetermined. In this case this does not refer to the particles having well-defined values before measurement, but it means that everything, including the choice of the experimenter in which direction to measure, was determined in such a way as to preserve the correlation. This so-called superdeterminism, however, has few proponents (with the notable exception of Gerard 't Hooft), since it excludes any form of free will.

So what is the way out of this problem? Well, it is not clear whether there actually is a problem. That is to say, it is not clear in what way the nonlocality found in entanglement actually contradicts relativity theory. This has to do with the probabilistic nature of the outcome of measurements, due to which someone performing a measurement has no influence on the outcome of the measurement. From this it follows that if an experimenter A decides to do a measurement on particle 1, another experimenter B measuring particle 2 cannot in any way know whether experimenter A has performed a measurement. Furthermore, if both these two measurements are repeated, both experimenters will get the same statistics for the measurement results. Only when the experimenters compare their results, the correlation between the particles becomes apparent. Without any knowledge of the measurement results of the other experimenter, there is no information as to whether the other experimenter has performed any measurements. The conclusion is that no information can be send using entanglement and that the influence therefore is not causal, at least not in the sense that, for superluminal speeds, it can lead to a logical contradiction. So at least there seems to be no obvious contradiction with relativity theory.

Chapter 4

The Kochen-Specker theorem

4.1 Introduction

Following Bell's theorem many similar no-hidden variable theorems have been developed. Differences between these theorems are that they attack hidden variable theories based on different assumptions and therefore also lead to different restrictions on possible hidden variable theories. As stated in the introduction, the most important of these is the Kochen-Specker theorem [14], which has been developed independently by Bell as well [4]. While Bell's original theorem focuses on statistical correlations (the expectation value of the product of the spin in two different directions), the Kochen-Specker theorem focuses on perfect correlations. The Kochen-Specker theorem is in this sense closer to the original EPR-argument where the direction of both detectors is the same, a situation about which Bell's theorem makes no statement.

For comparing the two theorems it is important to note the strategy of no-go theorems as outlined in section 3.2: compare some functional relation which follows from quantum mechanics with that what follows from the assumptions made in the hidden variable theory. As became clear in the previous chapter, in the case of Bell's theorem the functional relation is given by $P(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}$ (equation 3.11) and this is compared to the Bell inequalities that follow from the locality assumption made for the hidden variable theory. In the case of the Kochen-Specker theorem, which concerns perfect correlations, this works in a slightly different way. In this case we are not talking about statistics, but about specific values for observables. It turns out that this makes it possible to directly find out if a physical system can have well-defined values for its physical properties.

The method for doing so is as follows. First note that any mutually commuting set of observables is simultaneously measurable and that the outcome of such a measurement will be a set of simultaneous eigenvalues. From this it follows, according to quantum mechanics, that any relation between operators belonging to these mutually commuting observables should also be valid for the simultaneous eigenvalues. Furthermore, if you try to assign values to observables, these should be their eigenvalues, since these are the ones that are revealed upon measurement. Now these remarks are true for any commuting subset of the total set of observables. Therefore the question is whether values can be assigned to the observables in such a way that the functional relations hold in any commuting subset.

4.2 Proof

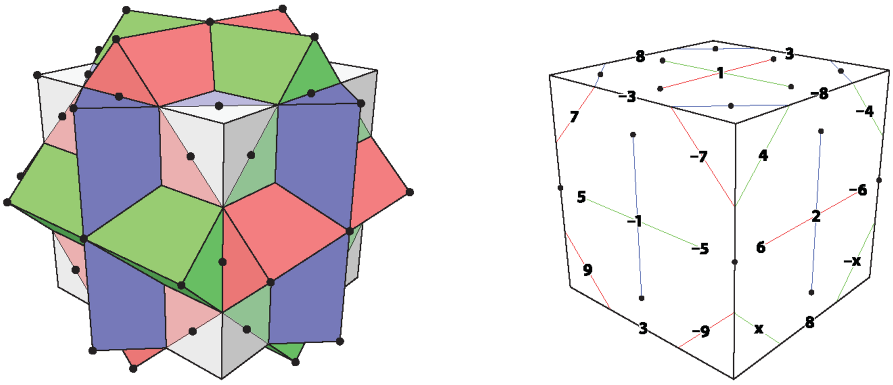
In the case of the Kochen-Specker theorem the observable to be discussed can be the square of orthogonal spin-components of a particle with spin 1. These can be either 1 or 0, since the non-squared spin-component can be either 1, 0 or -1. It follows from quantum mechanics that in this case the squared orthogonal spin-components commute and are therefore simultaneously measurable. Furthermore, for the sum of the squared spin-components quantum mechanics gives the functional relation

$$S_u^2 + S_v^2 + S_w^2 = s(s+1) = 2, \tag{4.1}$$

with u, v and w specifying orthogonal directions and $s = 1$ since the particle has spin 1. Now the question in this case is whether a particle can have well-defined values for it squared spin in all directions, while still satisfying the functional relation as just specified. It turns out that this is not possible.

From equation 4.1 it follows that of each three orthogonal components two components should have value 1 and one component value 0. This means no two orthogonal components can have value 0 and pairs of opposite directions have the same value. Using a geometrical argument it can then be shown that no configuration can be made where *both* the particle's squared spin-component is defined in all directions *and* these functional relations apply. Kochen and Specker used a set of 117 directions to prove this, but the same argument has been made using 33 and 31 (respectively by Peres, and Conway and Kochen) directions. The configuration by Peres can be represented graphically most clearly, using the three superimposed colored cubes of figure 4.1a. One obtains the three cubes by rotating the uncolored cube by 45 degrees about its coordinate axes. The 33 directions consist of the symmetry axes of the three superimposed colored cubes, all going through the common center of the cubes.

In their 2009 article about the free will theorem Conway and Kochen present the proof of Kochen-Specker in the conceptually easiest way, using the 33 directions of Peres [10]. The task is to find a set of directions for which it is impossible to assign each direction either the value 1 or the value 0 with



(a) (b)

Figure 4.1: (a) The three superimposed cubes of Peres' configuration and (b) the representation of the proof by Conway and Kochen [10].

the property that each subset of three mutually orthogonal directions contain twice the value 1 and once the value 0. Conway and Kochen associate each direction which has value 1 with an odd number and each direction which has value 0 with an even number, thereby making it easier to refer to the specific direction while at the same time knowing its value. The result can be seen in figure 4.1b. Here the 33 directions are represented by points on the surface of the uncolored cube. In the proof the first time a number of a direction is used it is used only as a label for referring to that specific direction. Only afterwards it is determined whether this direction should be even or odd, though of course the directions are immediately given the right parity.

Conway and Kochen start with the coordinate axes, giving two axes an odd number (axes 1 and −1, corresponding to twice the value 1 for squared spin) and one an even number (axis 2, corresponding to the value 0 for squared spin). Rotating about axis 2 gives the triple (2,3,−3), making axes 3 and −3 odd. Furthermore, both (3,4,$-x$) and (−3,−4,x) form a triple, since they each correspond to the axes through the centers of two edges and the center of a face of one of the superimposed cubes (e.g. directions 3, 4 and $-x$ correspond to the green cube). Due to symmetry considerations, directions 4 and −4 can, without loss of generality, be chosen to be even (choosing other directions comes down to rotating about direction 2). From a 90 degrees rotation about direction 1 that moves 7, 5 and 9 to 4, 6 and x it can be seen that 5 is orthogonal to 4, 6 and x, while 6 is orthogonal to 7, 5 and 9. Since we have taken direction 4 to be even, 5 must be odd, making 6 even, making 7 and 9 odd. In the same way it can be seen that directions −7, −5 and −9 are odd and −6 is even. Now both (8,−7,9) and

(-8,7,-9) form a triple (just like the (3,4,$-x$) triple), making both 8 and -8 even. These directions, however, are orthogonal and both even, leading to a contradiction.

4.3 Consequences

The conclusion of the Kochen-Specker theorem is then that attributes (such as spin) cannot be assigned to all particles in such a way that the relations between these attributes still hold (in this case equation 4.1). Therefore the Kochen-Specker theorem shows that the result of any individual measurement (of spin in this case) is not predetermined independently of the choice of measurements. In other words, it only allows contextual hidden variable theories. Essential for proving this is that the Kochen-Specker theorem makes use of a three-dimensional space (squared spin-component in three directions). Two dimensions do not give enough degrees of freedom to prove the above point. Actually in the case of the EPR-argument a classical, deterministic, local model can be made that reproduces the results of quantum mechanics [4].

Furthermore, it is important to note that the way the Kochen-Specker theorem is presented here is much less mathematical than how it was stated originally. In its original version, the Kochen-Specker theorem $-$ a special case of a more general result by A.M. Gleason from 1957 $-$ actually does not make any reference to a particular state. Using a particular state, as was done in this chapter, does make the proof conceptually insightful. However, unlike in the case of Bell's theorem, it is not necessary for proving the Kochen-Specker theorem.

Chapter 5

Mermin's reformulation

5.1 Greenberger-Horne-Zeilinger

As has become clear in the two previous chapters, the Bell and Kochen-Specker theorems are both no-hidden variable theories, but apply to slightly different situations. Like the EPR-argument, on which Bell's theorem is a reaction, the Kochen-Specker theorem discusses perfect correlations. Furthermore, while Bell's theorem needs a particular state (entanglement) to arrive at a contradiction, the Kochen-Specker theorem does not. The Kochen-Specker theorem has, however, historically received significantly less attention from physicists, due to its mathematical and − at least seemingly − complex nature. Therefore a logical question is whether it it possible to formulate a conceptually easier theorem, in the mind of Bell's theorem, which applies to perfect correlations. This is exactly what Greenberger, Horne and Zeilinger showed in their 1989 paper [12] and subsequently worked out in another paper (together with Abner Shimony, [13]).

They did this by changing the setup into one with the spin of three or four entangled spin $1/2$ particles being measured instead of just two. For certain configurations of the direction of the detectors perfect correlation occurs. For example in the case of the four entangled particles the product of the spins in four given directions equals ± 1 if the angles the four detectors make with a specified direction add up to 0 or 180 degrees. This is the equivalent of the functional relation of Bell's theorem (equation 3.11). The situation thus created is very similar to the EPR-argument as formulated by Bohm and Aharonov, but then with three or four particles.

It turns out, just as was the case for the Kochen-Specker theorem, that these extra dimensions offers enough freedom to demonstrate a contradiction for perfect correlations. In other words, there is no possibility to assign values to the spin-components of the four particles in such a way that the functional relation is always satisfied. So while in the case of perfect correlations in the EPR-argument a classical, local, deterministic model can be made that reproduces the results of quantum mechanics, this is not possible in the cases with three or four particles. Therefore Greenberger, Horne and Zeilinger conclude that the impossibility of a classical, local, deterministic theory cannot only be proven in the general case (Bell's theorem), but also in the EPR case of perfect correlations [12, 13].

5.2 Mermin

Following these publications, David Mermin reformulated the no-go theorem of GHZ [15, 16], thereby making it conceptually easier to understand and at the same more suitable for a comparison of the Bell and Kochen-Specker theorems. Mermin considers the spin-component of particles in the two orthogonal directions x and y. The operators for these spin-components can be written as σ_x^1 for the x-component of particle 1 and so on for the y-component and other particles. Consider, as an example, a generalization Mermin made of an argument similar to Kochen-Specker by Peres. In this case we take the four spin operators σ_x^1, σ_x^2, σ_y^1 and σ_y^2, combinations of those (but not the combinations $\sigma_x^1\sigma_y^1$ and $\sigma_x^2\sigma_y^2$ since those are combinations of two noncommuting operators) and the operator $\sigma_z^1\sigma_z^2$. These can be seen in figure 5.1a.

Now the assumption made is that all the physical attributes corresponding to these operators have well-defined values. As stated before, the strategy is to find out what this leads to and compare it to some functional relation following from quantum mechanics. First, note that the observables on every row and on every column are mutually commuting. In the case of the bottom row and the column to the right this is true because for every possible pair there are two anticommutations. Furthermore, it follows from quantum mechanics that $\sigma_i^2 = 1$ and $\sigma_i\sigma_j = -\sigma_j\sigma_i = i\sigma_k$ (and so on for cyclic permutations) with i, j and k orthogonal directions. From this it follows that the product of the three observables in the column to the right is -1 and the product of the three observables on the other columns and on all rows is $+1$. So these are some functional relations that follow from quamtum mechanics.

Now we have to find out what the assumption of all observables having well-defined values leads to. Since the observables on every row and column are mutually commuting, any relation between these observables must also be satisfied by the values assigned to these observables. So the product of the values assigned to the observables on every row and column should be $+1$, except for the column on the right where it should be -1. This, however, means that the product of all nine observables is $+1$ when using the products of the rows and -1 when using the products of the columns. Because of this contradiction the assumption has to be false. This proof is a version of the Kochen-Specker theorem and arrives at the same conclusion:

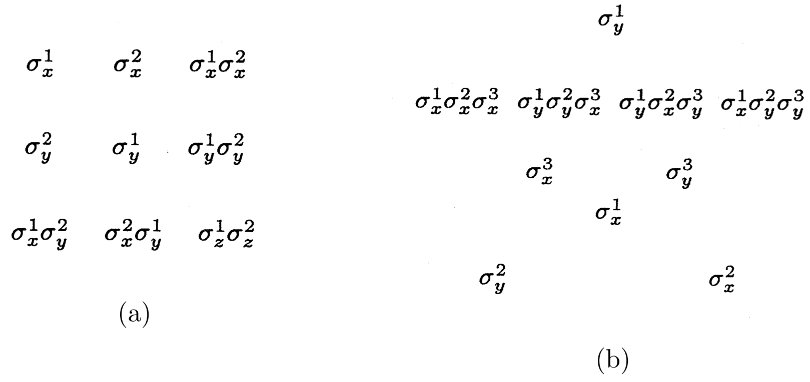


Figure 5.1: Mermin's (a) square and (b) pentagram [16].

only contextual hidden variable theories are possible. The outcome of a measurement apparently depends on which set of observables it is measured in.

In order to reformulate the argument by GHZ, Mermin followed a similar method for three particles. Now the relevant spin operators are the 6 operators for the x- and y-components of the three particles together with the 4 products of three commuting operators $\sigma_x^1\sigma_x^2\sigma_x^3$, $\sigma_y^1\sigma_y^2\sigma_x^3$, $\sigma_y^1\sigma_x^2\sigma_y^3$ and $\sigma_x^1\sigma_y^2\sigma_y^3$ to form 10 operators in total. Again, the assumption is made that the observables belonging to the 10 operators all have well-defined values. The operators can be put on the vertices of a pentagram in such a way that the operators on each of the straight lines of the pentagram commute, see figure 5.1b. For the horizontal line this is true because, again, for every possible pair there are two anticommutations.

Just as in the case of the previous example, we now look to the product of observables on the different lines and compare this to product over all lines. From the relations between spin operators given in the previous example it follows that the product of the observables on the horizontal line is -1 and it is $+1$ on the other lines (the functional relation). Since the operators on the straight lines commute, they have simultaneous eigenstates with corresponding simultaneous eigenvalues which should also satisfy the these relations. So the product of the values assigned (the assumption) to the operators on the straight lines should be $+1$, except for the horizontal line where it should be -1. From this it follows that the product of all five lines of products must be -1. However, since every operator appears twice in this total product the outcome should be $+1$, again leading to a contradiction. So just like the previous example, this proof is equivalent to the Kochen-Specker theorem. This version, however, has the advantage that it can be made into a theorem in the mind of Bell, while still applying to perfect correlations.

To do so, Mermin takes the three-particle system to be in the simultaneous eigenstate of the operators $\sigma_y^1\sigma_y^2\sigma_x^3$, $\sigma_y^1\sigma_x^2\sigma_y^3$ and $\sigma_x^1\sigma_y^2\sigma_y^3$ for which all three gave eigenvalue 1. Because these commute with $\sigma_x^1\sigma_x^2\sigma_x^3$ this eigenstate is also an eigenstate of that operator. Furthermore, since $\sigma_x^1\sigma_x^2\sigma_x^3 = -(\sigma_x^1\sigma_y^2\sigma_y^3)(\sigma_y^1\sigma_x^2\sigma_y^3)(\sigma_y^1\sigma_y^2\sigma_x^3)$, it has eigenvalue -1. Now the connection with GHZ becomes clear. Take, in line with the EPR-argument, the particles to be spatially separated. Then one can with certainty predict the outcome of a measurement of for example the x-component of the spin of some particle by measuring the y-component of the two particles far away. The same prediction can be made about a particle's y-component, but then by measuring the y-component of one other particle and the x-component of the other particle. In both cases this follows from the eigenvalue equations, which serve as the functional relations. Also, in both cases there is a perfect correlation.

Just like in Bell's theorem, the assumption is now that, since the particles are spatially separated, no influence can occur and therefore the particles already have well-defined values for their spin-components before measurement. Like before, these values should satisfy the relations that are true for the operators, in this case the eigenvalue equations. So if, say, σ_x^1, σ_x^2 and σ_x^3 are measured in an eigenstate of $\sigma_x^1\sigma_x^2\sigma_x^3$ with eigenvalue -1, the product of the results of the three measurements should also be -1. This gives four equations:

$$
\begin{aligned}
v(\sigma_y^1)v(\sigma_y^2)v(\sigma_x^3) &= 1,\\
v(\sigma_y^1)v(\sigma_x^2)v(\sigma_y^3) &= 1,\\
v(\sigma_x^1)v(\sigma_y^2)v(\sigma_y^3) &= 1,\\
v(\sigma_x^1)v(\sigma_x^2)v(\sigma_x^3) &= -1,
\end{aligned}
$$

with $v(\sigma_x^1)$ signifying the value assigned to the x-component of the spin of particle 1 and so on for the y-component and the other particles. It turns out that, in a similar way as before, this leads to a contradiction: the product of all the terms on the left gives $+1$ (since every value appears twice), while the product of the terms on the right gives -1. Again, the conclusion is that not all physical attributes can have well-defined values. There is, however, an important difference with the previous examples, as will be discussed below.

5.3 Consequences

The question asked at the beginning of this chapter was whether it is possible to formulate a theorem that applies to perfect correlation but is conceptually not as difficult as the Kochen-Specker theorem. It turns out that this is possible through Mermin's reformulation of both Peres and GHZ. There is, however, another important advantage of the reformulation of GHZ. To see this, first consider the case of the Kochen-Specker theorem and its reformulations by Mermin (see figure 5.1). Here a contradiction is derived with the assumption of noncontextual values for any arbitrary observables. So the Kochen-Specker theorem says assigning to *all* observables a noncontextual value is impossible. The conclusion, as stated before, is that the outcome of a measurement apparently depends on the context, or set of measurements, it is measured in. One could think, however, that this is not so surprising. If you measure some observable in two sets of mutually commuting observables, of which the additional observables of one set do not commute with the additional observables of the other, you need two different measurement setups. So is it not possible that the value of an observable depends on this measurement context? On the other hand the statistics of a measurement *is* independent of the context, a fact for which such a contextual hidden variable theory would have no explanation.

Altogether, it is not totally clear whether noncontextuality is a reasonable assumption for a hidden variable theory. However, the strength of GHZ and Mermin's reformulation of this result is that it makes this discussion irrelevant. Here noncontextual values are only assigned to the observables of which the locality assumption requires them to have noncontextual values. So this theorem says you cannot even just assign noncontextual values in the cases where this is required by locality. In other words, the assumption of noncontextuality is replaced by the weaker assumption of locality. Still, a contradiction is derived and therefore this version can be considered a stronger result since it places the most restrictive constraints on possible hidden variable theories. The weaker assumption of locality was of course also made in Bell's theorem, but Bell only applies to statistical correlations. By expanding the situation to three particles, GHZ introduce extra degrees of freedom with which the stronger result can be proven even in the case of perfect correlations [12, 13].

To understand another difference between the theorems, it is useful to find out why Mermin's reformulation of the argument by Peres (see figure 5.1a) cannot be used to formulate an argument similar to GHZ. The method would be to assume the two particles are spatially separated and in the singlet state. Furthermore, noncontextual values are only assumed in the cases this is required by locality, so only for the local observables σ_x^1, σ_x^2, σ_y^1 and σ_y^2. Since the other five observables involve spin operators of both particles they are nonlocal and will therefore not be assumed to have noncontextual values (an assumption which *is* made in the Kochen-Specker theorem). In Mermin's reformulation of GHZ an eigenstate of the nonlocal observables was now chosen to still be able to assign values to the nonlocal observables (from which the necessity of using a particular state can be seen). To do so, however, the nonlocal observables have to be mutually commuting. This is not the case for the five nonlocal observables in Peres' argument and therefore no GHZ-type argument can be developed. From this it becomes clear why Bell and GHZ need a particular state and Kochen-Specker does not, a difference which is sometimes regarded as an advantage of the Kochen-Specker theorem. Mermin argues, however, that this difference should be seen as merely a technical consequence of the less restrictive assumptions made by Bell and GHZ, leaving them less degrees of freedom [16].

Chapter 6

The free will theorem

6.1 The original free will theorem

As became clear in the previous chapter, Mermin's reformulation of the argument by GHZ combined aspects of both the Bell and Kochen-Specker theorems. This is also true for the so-called 'free will theorem', developed by Conway and Kochen [8]. As its name indicates, the theorem arrives at conclusion about free will instead of locality. In Conway and Kochen's words, "it asserts, roughly, that if indeed we humans have a free will, then elementary particles already have their own small share of this valuable commodity" [10, p. 226]. It is quite a controversial theorem and it has generated several critical responses. Some of these criticisms concern their supposedly inappropriate use of the term 'free will' and others state that the theorem actually comes down to Bell's theorem. Especially this last criticism is of interest for evaluating the different no-go theorems, since the free will theorem also explicitly uses the results of the Kochen-Specker theorem. In reaction to the critical responses, Conway and Kochen formulated the 'strong free will theorem' [10], which will also be discussed in order to evaluate the theorem. Lastly, an article by Cator and Landsman [5], in which they follow a method similar to Mermin's, will be treated briefly. They reformulate the Bell and the free will theorem in a mathematical way, enabling an exact comparison. However, before discussing all these issues, the theorems itself will first be outlined.

Conway and Kochen's argument proceeds from the Kochen-Specker theorem and starts off with three axioms:

1. SPIN. The SPIN axiom states that "measurements of the squared (components of) spin of a spin 1 particle in three orthogonal directions always gives the values 1, 0 and 1 in some order" [10, p. 227].

2. TWIN. The TWIN axiom states that a measurement of the squared spin-components of two twinned spin 1 particles in parallel directions

will yield the same result.

3. FIN. The FIN axiom states that "there is a finite upper bound to the speed with which information can be effectively transmitted" [8, p. 3].

The SPIN axiom is the same quantum mechanical prediction the Kochen-Specker theorem uses to prove attributes of physical systems cannot all have well-defined values. The content of the TWIN axiom also follows directly from quantum mechanics and it is similar to the perfect correlation discussed before in the EPRB-argument with two spin 1/2 particles, the particles being in an entangled state with total spin zero. Conway and Kochen formulate the TWIN axiom more precisely as follows: when experimenter A measures the squared spin-components of particle 1 in the three orthogonal directions x, y and z and experimenter B measures the squared spin-component of particle 2 in the direction w, and w is in the same direction as either x, y or z, the values will correspond. Here the direction w is taken to be one of the 33 directions of Peres' configuration (of the Kochen-Specker proof) and the triple x, y and z is taken to be one of the 40 orthogonal triples that one gets from completing all orthogonal pairs in that configuration. While both of these axioms follow from quantum mechanics, the FIN axiom does not. Maybe not surprisingly, this is the axiom which has been criticised.

From the three axioms Conway and Kochen deduce their free will theorem. It concerns the experiment specified above, in which experimenter A measures the squared spin-component of particle a in three orthogonal directions of the 40 directions and experimenter B measures the squared spin-component of particle b in one of the 33 directions. It states that "if the choice of directions in which to perform spin 1 experiments is not a function of the information accessible to the experimenters, then the responses of the particles are equally not functions of information accessible to them" [8, p. 3]. From this conclusion it can be seen that statements about the supposedly 'free will' of the particle only apply when one makes the assumption an experimenter can freely choose a measurement direction. Here Conway and Kochen define free choice of the experimenters as a choice that is "not a function of the information accessible to them" [8, p. 4].

To prove their theorem Conway and Kochen make the assumption that the responses of the particles *are* a function of the information available to them. By α they signify all the information contained in the backward light-cone of particle a and by β the same for particle b. They split this information into the choice of measurement directions and respectively α' and β' signifying the information available just before making the choice of measurement directions. Then the outcome of a measurement of the squared spin-component of particle a in some direction z (as part of a simultaneous measurement of the squared spin-component in the three orthogonal directions x, y and z) depends on the other two directions x and y and on

α':

$$S^2_{a,z} = S^2_{a,z}(x, y, \alpha'), \tag{6.1}$$

and the same for a measurement in the direction w for particle b:

$$S^2_{b,w} = S^2_{a,w}(u, v, \beta'). \tag{6.2}$$

Now the TWIN axiom states that if experimenters A and B choose the same direction for measurement (that is $z = w$) the two outcomes show a perfect correlation:

$$S^2_{a,z}(x, y, \alpha') = S^2_{b,w}(u, v, \beta'). \tag{6.3}$$

Furthermore, the assumption about free choice implies B can choose to measure w as part of any orthogonal triple u, v and w. Due to the FIN axiom, this can have no effect on the outcome of A's measurement. This means that the outcome of B's measurement is independent of the two orthogonal directions u and v, that is

$$S^2_{b,w}(u, v, \beta) = S^2_{b,w}(\beta), \tag{6.4}$$

and the same is true for A's measurement. This, however, directly contradicts the Kochen-Specker theorem (and thereby the SPIN axiom), since it implies the value of any squared spin-component is independent of the orthogonal triple it is measured in. Conway and Kochen conclude from the contradiction that equations 6.1 and 6.2 do not hold, meaning that the outcomes of the measurements do not depend on the information contained in the backward light-cone of the respective particles. Therefore the response of both particles is free in the sense that it is not a function of the information accessible to them [2, 8].

6.2 Criticism

As stated at the beginning of this chapter, the free will theorem has elicited several critical responses. One main point of criticism concerns Conway and Kochen's use of the term 'free will'. The way they define a choice as being free – "not a function of the information accessible" [8, p. 4] or "not a function of the past" [10, p. 228] – suggests the rather peculiar idea that a choice can be independent of the history of the one making the choice. Furthermore, Conway and Kochen's use of the term differs from what is commonly meant by free will. An example can be found in Conway and Kochen's discussion of the possibility of spontaneous information becoming available to a particle just after the measurement directions are chosen (say at a time $t = t_0$). Such information could influence the outcome of a measurement, but is not included in the derivation of the theorem (equations 6.1

and 6.2). Conway and Kochen state that such information would not change anything about the conclusion of the free will theorem, since it would mean the universe has taken a free decision at the time t_0. However, it seems to make more sense not to talk of a 'free decision' in this case, but instead of an inherent probabilistic nature of the universe.

The same line of thought can be applied to the use of the term 'free will' for the experimenter. Regardless of philosophical problems surrounding the meaning and existence of 'free will', one can reasonably state that the choice of the experimenter for a certain measurement direction is a purely arbitrary choice. That is to say, the choice of the experimenter is independent of other factors relevant to the problem at hand (e.g. the incoming particle) and can therefore be considered random. In this way it can be seen that it matters not so much whether and to what extent the experimenter has 'free will', but the more what the assumption of free choice comes down to: the choice of measurement direction being random. The only other option is the superdeterminism discussed in section 3.3, implying particles (or as such the universe) already know in advance in what direction an experimenter will measure. Therefore, being able to cope with choice of measurement directions as random is a very reasonable requirement for a theory.

Other objections have been raised against the validity of theorem itself, most notably by Angelo Bassi and Giancarlo Ghirardi [2], and Roderich Tumulka [17]. Both reacted to the 2006 article by stating that Conway and Kochen arrive at the wrong conclusion based on their proof. They state that the right conclusion is exactly the one made by Bell in 1964: nature is non-local. Therefore the original FIN axiom should be considered wrong. They indicate that Bell's theorem only required the TWIN analogue for two spin 1/2 particles, Bell's version of locality and the assumption about free choice. Without any functional relation of the sort of equations 6.1 and 6.2 (which relates the outcome of measurements with past information), Bell still arrived at a contradiction. Therefore one of his assumptions had to be wrong. Since the TWIN axiom follows directly from quantum mechanics (and is experimentally verified) and almost no-one is willing to deny free choice, locality had to be given up.

Conway and Kochen, however, do include a fourth assumption on determinism − meaning outcomes of measurements are a function of the past − in their proof, as formulated in equations 6.1 and 6.2. Furthermore, they state that the FIN axiom cannot be given up since it follows from relativity theory and the authors therefore conclude the determinism assumption is false. It is interesting to compare this conclusion to the EPR-argument. Here, the similar assumptions of free choice, quantum formalism and locality lead Einstein, Podolsky and Rosen to conclude determinism (which implies quantum mechanics is incomplete). So if, like Conway and Kochen, one assumes locality, this implies determinism. Then, however, we have made all assumptions of the free will theorem which leads to a contradiction. Therefore the free will theorem itself should also be regarded as a nonlocality proof. The objection then is that even if Conway and Kochen would have formulated the right conclusion, they would not have presented any new result.

Another point of criticism concerns Conway and Kochen's inaccurate use of relativity theory, locality and the term 'information'. Tumulka indicates that Conway and Kochen are unclear in their distinction between locality, no-signaling and what Tumulka calls "effective locality", equating all of these, at different points of their theorem, with the FIN axiom. Furthermore, Bassi and Ghirardi note that for Conway and Kochen information seems to include everything that can determine the outcome of a measurement. The influences involved in perfect correlations, however, would then also fall under this category of 'information'. At the same time, Conway and Kochen state that "there is a finite upper bound to the speed with which information can be effectively transmitted" [8, p. 3]. However, as was discussed in section 3.3, it is not clear why such influences could not travel faster than the speed of light.

6.3 The strong free will theorem

Conway and Kochen's 2009 article 'The Strong Free Will Theorem' [10] was in part a response to these criticisms. In this article the authors state that Bassi and Ghirardi miss the point regarding the FIN axiom. They state that the FIN axiom was only applied to the choice made by the experimenter and the response of the spatially separated particle and not to any other information. By changing the axiom FIN into the MIN axiom they hope to have made this clear. In doing so they also included the assumption about free choice implicit in the 2006 article:

3. MIN. The MIN axiom states that, assuming the experiments performed by experimenters A and B are space-like seperated, A and B can freely choose a measurement direction and the response of respectively particle b and a is independent of this choice.

Again, Conway and Kochen restrict the choice of measurement directions to the 40 orthogonal triples for experimenter A and the 33 directions of Peres' configuration for experimenter B. And like before, free choice is "that the choice an experimenter makes is not a function of the past" [10, p. 228]. The conclusion of the strong free will theorem is that the response of a spin 1 particle to the described experiment is free in the sense that it is not a function of the past. They specify this definition of freedom as "not a function of properties of that part of the universe that is earlier than this response with respect to any given inertial frame" [10, p. 228].

In their 2009 article and in an earlier response from 2007 [9], Conway and Kochen repeatedly state that the critical responses of Tumulka and Bassi and

Ghirardi mistook their FIN axiom for Bell's locality condition. They indicate that they are aware that Bell's locality condition was proven to be incorrect by Bell's theorem (under the assumption of free choice), but state that their FIN axiom follows from causality and relativity. They do not anywhere seem to discuss, though, the importance of the distinction between locality and no-signalling. Several times they simply state that the influences involved in twinned entanglement cannot travel faster than the speed of light, since this would contradict causality. But again, as discussed before, it is not in any sense clear that this is the case. The experimenters have no influence whatsoever on the outcome of their measurements and as such no influence on the outcome of the other experimenter's measurement. The only influence is that some specific but random outcome of both measurements becomes reality.

6.4 Cator and Landsman

To conclude the discussion on the free will theorem, a recent article by Eric Cator and Klaas Landsman [5] will be treated. The authors discuss the similarities between the strong free will theorem and Bell's original theorem and state that the two theorems arrive at very similar conclusions on the basis of very similar assumptions. Therefore the authors make mathematical reformulations of both theories, making an exact comparison possible. Cator and Landsman's conclusion is that "the Strong Free Will Theorem uses fewer assumptions than Bell's 1964 theorem, as no appeal to probability theory is made." Roughly, this also became clear in the previous paragraphs. Like Bell's theorem, the strong free will theorem uses a particular state and assumes locality. However, while Bell's theorem is based on statistical correlations, the free strong free will theorem is based on perfect correlations and therefore does not need probability theory. Cator and Landsman also indicate one drawback of Conway and Kochen's theorem: since experiments confirming Bell's theorem only make use of spin 1/2 particles, the strong free will theorem so far lacks experimental confirmation.

Another issue that the authors address is the apparent inconsistency between assuming a deterministic hidden variable theory and assuming free choice for the experimenter. The problem is that the hidden variables should contain all the information relevant to the experiment, but leave the experimenters free to choose a measurement direction. To resolve this issue, Cator and Landsman at first treat the apparatus settings as random variables, in the line of the discussion in section 6.2. Subsequently, independence assumption are made about the choice for apparatus settings being independent of the hidden variable. In this way any vague notions about free will are excluded and and an exact, mathematical comparison can be made of which the result was described above.

Chapter 7

Conclusion

From the preceding chapters some conclusions regarding the two aims of the thesis can be made. The first of these was to evaluate the two most important hidden variable theorems − the Bell and Kochen-Specker theorems − and the relation between them. On the basis of chapters 3 and 4, several differences can be distinguished. Bell's theorem makes use of statistical correlations, needs a particular state for its proof and assumes locality, which is to say it only assumes noncontextual values if required by locality. The Kochen-Specker theorem, however, applies to perfect correlations, does not need a particular state and assumes noncontextuality for all values. So on the one hand Bell's theorem makes weaker assumptions for the hidden variable theory, thereby placing the most restrictive constraints on such a theory. On the other hand the Kochen-Specker theorem applies to the perfect correlations as also found in the original EPR-argument, whereas Bell's theorem does not.

As became clear in chapter 5, it is possible to construct a stronger result combining aspects of both the Bell and Kochen-Specker theorems. The argument by GHZ and Mermin's subsequent reformulation apply to perfect correlations, need a particular state and assume nonlocality, again meaning they only assume noncontextual values if required by locality. So in this case a no-hidden variables theory can be developed on the basis of the weaker assumptions even in the case of perfect correlations. Another advantage of this argument is that it is significantly easier to convey than the two original no-go theorems. The only possible downside would be that, like Bell's theorem, it needs a particular state for its proof to work. It seems, however, that this should in the first place be seen as a technical consequence of the fact that a weaker assumption leaves less degrees of freedom for the no-hidden variable theorem.

In line with this, it is interesting to compare the number of dimensions needed for proving the several theorems in the case of perfect correlations. In the two-dimensional case of the EPR-argument a no-hidden variable theory is impossible (actually a classical, local, deterministic model can actually be developed). In the three-dimensional case of the Kochen-Specker theorem (represented by the three orthogonal squared spin-components) a no-hidden variable theory is possible, but rather difficult. In the four-dimensional case of Mermin's reformulation of Peres (2 orthogonal spin-components of 2 particles giving $2*2 = 4$ dimensions) a no-hidden variable theory is conceptually very easy. And lastly, in the eight-dimensional case of Mermin's reformulation of GHZ (2 orthogonal spin-components of 3 particles giving $2*2*2 = 8$ dimensions) the strongest no-hidden variable theory can be developed.

The second aim of this thesis was to evaluate Conway and Kochen's (strong) free will theorem. Altogether, both the free will theorem and the strong free will theorem at least roughly come down to Bell's theorem. In this sense the theorem has not so much to do with free will as with nonlocality. Actually, as became clear from the discussion of the critical responses to the theorem, Conway and Kochen's choice for the term 'free will' does not seem to improve understanding. Even more importantly, the authors were unclear in their use of the different forms of locality and therefore arrived at the wrong conclusion. Crucial to this is the probabilistic nature of quantum mechanics, due to which the superluminal influences of entanglement cannot be used to convey information. Therefore there seems to be no contradiction between relativity theory and the nonlocality of entanglement.

If on the basis of the strong free will theorem the right conclusion − nonlocality − *is* formulated, it is however a very interesting result. The free will theorem uses perfect correlations, a particular state and only assumes noncontextuality if required by locality. In this way it is similar to the argument by GHZ and Mermin's subsequent reformulation. So possibly a more accurate name for Conway and Kochen's result is 'the strong nonlocality theorem'.

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